Cours/Lecture Series

1987-1988 ACADEMIC TRAINING PROGRAMME

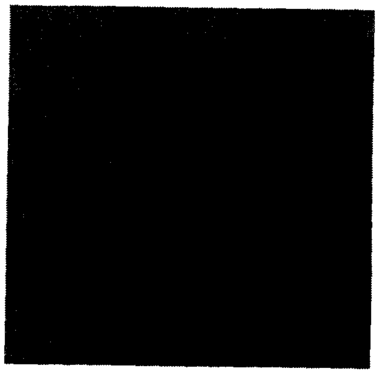
SPEAKERS : Ph. BLOCH / CERN-EP & L. MAIANI / University of Rome

TITLE : Mixing and CP violation in heavy quark states

TIME : 13, 14, 15, 16 & 17 June at 11.00 hrs

PLACE : Auditorium

Lectures 1 to 3 by L. Maiani
Lectures 4 & 5 by Ph. Bloch



ABSTRACT

1. *Mixing of neutral mesons. Description in the Standard Model. Relevant weak parameters*
2. *Mixing in $B\bar{B}$ states, limits to t-quark mass*
3. *CP violation in K-decays. CP violation in B-decays*
4. *Experimental status of mixing and CP violation effects in heavy quarks*
5. *Future possibilities*

Mixing and CP violation in heavy quark states (K-included)

L. Maiani 3 lects
Ph. Bloch 2 lects

- A beautifull subject
  theoretical prejudices &
  brilliant experim. discoveries

- $K_0 - \bar{K}_0$ oscillations predicted by
  Gell-Mann, Pais (1955)
- regeneration : Pais, Piccioni (1955)
- CP violation : ~~[illegible]~~ Christensen, Cronin, Fitch, Turlay (1964)
- $D - \bar{D}$ mixing (correctly ?) believed small
- $B_d - \bar{B}_d$ " (incorrectly) " "
  ARGUS 1987 finds large mix
  $\Rightarrow$ large m_t
- superweak theory : $\epsilon'/\epsilon = 0$

2 (2)

PHYSICAL REVIEW VOLUME 97, NUMBER 5 MARCH 1, 1955

Behavior of Neutral Particles under Charge Conjugation

M. GELL-MANN,* *Department of Physics, Columbia University, New York, New York*

AND

A. PAIS, *Institute for Advanced Study, Princeton, New Jersey*

(Received November 1, 1954)

Some properties are discussed of the θ^0, a heavy boson that is known to decay by the process $\theta^0 \to \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and K particles, the θ^0 possesses an antiparticle $\bar{\theta}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the θ^0 must be considered as a "particle mixture" exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all θ^0's undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.

I

IT is generally accepted that the microscopic laws of physics are invariant to the operation of charge conjugation (CC); we shall take the rigorous validity of this postulate for granted. Under CC, every particle is carried into what we shall call its "antiparticle". The principle of invariance under CC implies, among other things, that a particle and its antiparticle must have exactly the same mass and intrinsic spin and must have equal and opposite electric and magnetic moments. A charged particle is thus carried into one of opposite charge. For example, the electron and positron are each other's antiparticles; the π^+ and π^- and the μ^+ and μ^- mesons are supposed to be pairs of antiparticles; and the proton must possess an antiparticle, the "antiproton".

Neutral particles fall into two classes, according to their behavior under CC:

(a) Particles that transform into themselves, and which are thus their own antiparticles. For instance the photon and the π^0 meson are bosons that behave in this fashion. It is conceivable that fermions, too, may belong to this class. An example is provided by the Majorana theory of the neutrino.

In a field theory, particles of class (a) are represented by "real" fields, i.e., Hermitian field operators. There is an important distinction to be made within this class, according to whether the field takes on a plus or a minus sign under CC. The operation of CC is performed by a unitary operator $\mathfrak{C}$. The photon field operator $A_\mu(x)$ satisfies the relation

$$\mathfrak{C}A_\mu(x)\mathfrak{C}^{-1} = -A_\mu(x), \qquad (1)$$

while for the π^0 field operator $\phi(x)$ we have

$$\mathfrak{C}\phi(x)\mathfrak{C}^{-1} = \phi(x). \qquad (2)$$

Equation (1) expresses the obvious fact that the electromagnetic field changes sign when positive and negative charges are interchanged; that the π^0 field must not change sign can be inferred from the observed two-photon decay of the π^0.

We are effectively dealing here with the "charge conjugation quantum number" C, which is the eigenvalue of the operator $\mathfrak{C}$, and which is rigorously conserved in the absence of external fields. If only an odd (even) number of photons is present, we have $C = -1(+1)$; if only π^0's are present, $C = +1$; etc. As a trivial example of the conservation of C, we may mention that the decay of the π^0 into an odd number of photons is forbidden.[1]

We may recall that a state of a neutral system composed of charged particles may be one with a definite value of C. For example, the $^1S_0$ state of positronium has $C = +1$; a state of a π^+ and a π^- meson with relative orbital angular momentum l has $C = (-1)^l$; etc.

For fermions, as for bosons, a distinction may be made between "odd" and "even" behavior of neutral fields of class (a) under CC. However, the distinction is then necessarily a relative rather than an absolute one.[2] In other words, it makes no sense to say that a single such fermion field is "odd" or "even", but it does make sense to say that two such fermion fields have the same behavior under CC or that they have opposite behavior.

(b) Neutral particles that behave like charged ones in that: (1) they have antiparticles distinct from themselves; (2) there exists a rigorous conservation law that prohibits virtual transitions between particle and antiparticle states.

A well-known member of this class is the neutron N, which can obviously be distinguished from the antineutron $\bar{N}$ by the sign of its magnetic moment. The law that forbids the virtual processes $N \rightleftarrows \bar{N}$ is the law

[1] For other consequences of invariance under charge conjugation see A. Pais and R. Jost, Phys. Rev. 87, 871 (1952); L. Wolfenstein and D. G. Ravenhall, Phys. Rev. 88, 279 (1952); L. Michel, Nuovo cimento 10, 319 (1953).

[2] This is due to the fact that fermion fields can interact only bilinearly. For example, one easily sees that the interactions

- Gilman, Wise (& many others): ϵ'/ϵ not small[3]

$10^{-2} \div 10^{-3}$

- CERN expt. (1987) : evidence for $\epsilon'/\epsilon \neq$

$$\epsilon'/\epsilon = (3.3 \pm 1.1)\, 10^{-3}$$

kills superweak.

- <u>Present status</u> :

CP-viol. ~~and~~ mixing in K, B, B

Fits in Standard Theory nicely

provided $m_{top} \gtrsim 50$ GeV

(90 GeV preferred?

- large theoretical uncertainties may be removed in near future

- limits on new physics (SUSY, 4th gen...

Not speak much about theory too uncertain

- <u>Next issue</u> : find CP violation in B (& determine m_t, V_{ub})

Plan of the lectures

1. – Standard Weak Intes.
   – Mixing & CP viol. in $M_0 - \bar{M}_0$ systems

2. – CP violation in K decays (ε'/ε)
   – status of calculation of weak parameters in lattice QCD (and other schemes)

3. – Mixing in $B_0 - \bar{B}_0$ vs t-quark mass.
   – Prospects for observing CP-viol. in B decays

4. (Ph. Bloch) s, c quarks

5. (" ") b quarks

Summary of first lecture

1. Weak Currents
2. Non leptonic Processes } reminder
3. Vacuum Saturation
4. Mixing & CP-violation in $M_0 \div \bar{M_0}$ systems } general formulae
5. Intensity rules for mixing

1. The Standard Weak Interactions

- 3 quark doublets
- 3 lepton doublets
- charged currents only

$$\mathcal{L}_{int} = g\, W_\mu \left(J^\mu_{quark} + J^\mu_{lepton} \right)$$

J^μ_{lepton} = diagonal in generations =

$$= \bar{\nu}_e \gamma^\mu (1-\gamma_5)\, e + \dots$$

$$J^\mu_{quark} = (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu (1-\gamma_5)\, U \begin{pmatrix} d \\ s \\ d \end{pmatrix}$$

$U^+ U = 1$

U = Kobayashi – Maskawa – Cabibbo matrix
parametrized by 3 real angles
1 (not elim. phase

T-violation $\Longleftarrow$ phase
$\Downarrow$
CP-violation because of $\boxed{CPT}$

Notice: if we had only 2 generations

$$U = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

no phase (K–M)
no T-violation (CP-viol.

Parametrizations of the C-K-M matrix 7

$$\mathcal{U} \equiv \begin{bmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{bmatrix} \equiv$$

$$\begin{bmatrix} c_\beta c_\theta & c_\beta s_\theta & s_\beta e^{i\varphi} \\ -(c_\gamma s_\theta + c_\theta s_\gamma s_\beta e^{-i\varphi}) & (c_\gamma c_\theta - s_\gamma s_\beta s_\theta e^{-i\varphi}) & c_\beta s_\gamma \\ (s_\theta s_\gamma - c_\gamma c_\theta s_\beta e^{-i\varphi}) & -(c_\theta s_\gamma + c_\gamma s_\theta s_\beta e^{-i\varphi}) & c_\gamma c_\beta \end{bmatrix}$$

L. Maiani (19..
Harari, Leurer (19..

$c_\beta \equiv \cos\beta$, $s_\beta \equiv \sin\beta$ etc.

$$\tan\theta = \frac{A(\Delta S=1)}{A(\Delta S=0)} \qquad (\theta = \text{Cabibbo angle})$$

$\sin\beta \sim A(b \to u)$

$\sin\gamma \sim A(b \to c)$

→ CP-violating phase associated only with β

Hierarchy in mixing angles observed: suggests:

$$s_\theta \equiv \lambda$$
$$s_\gamma \simeq A\lambda^2$$
$$s_\beta \simeq A\rho\lambda^3$$

Wolfenstein

neglecting terms of order λ^4 or higher:

$$\begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\rho\lambda^3 e^{i\varphi} \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-e^{-i\varphi}\rho) & -A\lambda^2 & 1 \end{bmatrix}$$

Notes on CP-violation

- it is really a Time-reversal violation. CP violation because TCP is exact.

- No CP-violation in semileptonic dec.

$$A_{SL}(Q \to q) = V_{Qq} [\gamma_\mu (1-\gamma_5)]$$

no relative phase between vector/axia

- CP-violation in Non-leptonic amplitudes vanishes in the limits

  1- Any of $\theta, \gamma, \beta = 0$

  2- Any pair of equal charge quarks degenerate (or massle

- We know that (2) is not realized [but may be useful, see later on $B_0-\bar{B}_0$ mixing]

- Until recently, no direct proof that $\beta \neq 0$!!

ARGUS result (1987) $B(B^{\pm} \to p\bar{p}\pi^{\pm}) = (3.7 \pm 1.3 \pm 1$ $\times 10$

$B(B^{0} \to p\bar{p}\pi^{+}\pi^{-}) = (6.0 \pm 2 \pm 2$ $\times 10^{-4}$

- If confirmed indicates $\beta \neq 0$

- supports K-M model for K-decays.

- Weak trigonometry

Most useful until now : { Inclusive semileptonic decays

Parton model:

$$\Gamma(B \to e + c + \dots) = \frac{G^2 M_b^5}{192\pi^3} \underbrace{g\left(\frac{m_c}{M_b}\right)}_{(\sin\gamma)^2} \left[1 + \text{QCD correcs.}\right.$$

$$\Gamma(B \to e + u + \dots) = \frac{G^2 M_b^5}{192\pi^3} (\sin\beta)^2 g\left(\frac{m_u}{M_b}\right) \left[1 + \text{QCD corrects}\right.$$

Angles can be obtained from these rates
(Cabibbo, L.M.
Ali, Pietarinen,
Suzuki
~~[illegible]~~)

Too much dependent from (unknown) M_b, m_c

- use the electron spectrum to fit
M_b, m_c, p_F = momentum spectrum of spectator quark

(Altarelli et al. Nucl. Phys. (1982))

- good results for $\sin\gamma$ from $\Gamma_{SL}(b \to c)$

- upper bounds to $\sin\beta$ from non-observ of $b \to u$ inclusive transitions

(other methods : ~~[illegible]~~ Grinstein, Wise, Isgur (1986)
. see : Thorndike, Poling, Phys. Reports 157 (1988))

Summary of the experimental informa[tion]
on θ, γ, β

[See Altarelli, Franzini
CERN TH-4914/87

More about that in
Bloch's lectures]

$\sin\theta \equiv \lambda = 0.221 \pm 0.002$ from K-ℓ3
and hyperon dec[ays]

$\sin\gamma = 0.051 \pm 0.009$
$\sin\gamma = A\lambda^2 \Rightarrow A = 1.05 \pm 0.17$ } inclusive semilept. B-decay

$0.3 \times 10^{-2} \leq \sin\beta \leq 1.0 \times 10^{-2}$
$\sin\beta = A\rho\lambda^3 \Rightarrow \underbrace{0.3 \leq \rho \leq 1.0}$

[upper bound from
limits on SL $b \to u$

$$R = \frac{\Gamma(b \to e + u \cdot\cdot)}{\Gamma(b \to c + u \cdot\cdot)} \lesssim 0.10$$

(most recent values $R < 0.04$ (CLEO)
< 0.06 (CUSB))
but I will be
conservative

[lower bound ~~from~~ estimate
from ARGUS results on
$B \to p\bar{p}\pi$, $p\bar{p}\pi\pi$ quoted
before

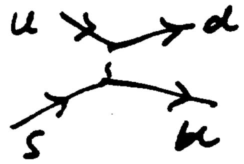
Why are A and ρ so close to 1 ??

2- Non leptonic Interactions ($\Delta S = \pm 1$ case)

Most studied case, see Donoghue, Golowich, Holstein, Phys. Reports

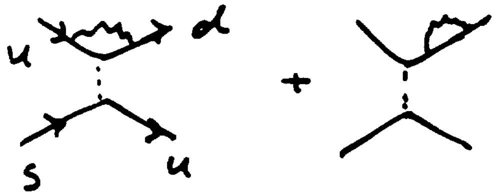
start with: - CP-conserving amplitudes
- drop t-quark altogether

Bare (no-strong interaction):



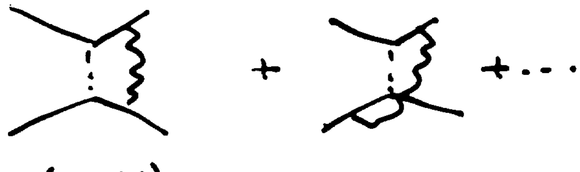
$$H_{eff}^{(0)} = \frac{G}{\sqrt{2}} \overbrace{\;}^{\sin\theta} (\bar{u}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu u_L)$$

QCD-corrections



+ .. (no mixing) $= 0$

current conservation



$\sim \alpha_s \ln\left(\frac{M_W}{\mu}\right)$

$H_{eff}^{(\Delta S = \pm 1)}$ [redacted] $= \frac{G}{\sqrt{2}} (\sin\theta) \times \left[c^{(-)}(\mu)\, \theta^{(-)} + c^{(+)}(\mu)\, \theta^{(+)} \right.$

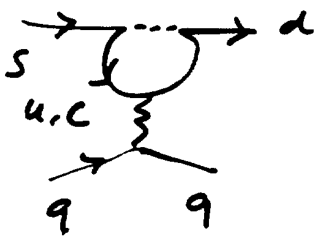
$$\theta^{(\pm)} = (\bar{u}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu u_L) \pm (\bar{u}_L \gamma^\mu u_L)(\bar{d}_L \gamma^\mu s_L)$$

$\mu \simeq$ momentum scale $\gg \Lambda_{QCD}$

$c^{(-)} \sim 1.7$, $c^{(+)} \sim 0.7$ at $\mu \simeq 2\,\text{GeV}$

<u>This is not enough</u>

If we go at lower momentum scale, new operators come in in H_{eff}.
(Shifman, Vainstein, Zakarov)



$\ln\left(\frac{m_c}{\mu}\right)$ <u>PENGUIN DIAGRAMS</u>

$\mu < m_c$

new terms in H_{eff}:

$$c_5 O_5 + c_6 O_6$$

$$\left.\begin{aligned} O_5 &= (\bar d_L \gamma_\mu t^A s_L)(\bar q_R \gamma^\mu t^A q_R) \\ O_6 &= (\bar d_L \gamma_\mu s_L)(\bar q_R \gamma^\mu q_R) \end{aligned}\right\} \text{pure } \Delta I = 1/2$$

$$c_5 \sim \alpha_s \ln \frac{m_c}{\mu}$$

<u>In conclusion</u>:

$$H_{eff}\,(\text{CP-conserving}) = \frac{G}{\sqrt{2}} \sin\theta \cos\theta \times$$

$$\begin{cases} \times\left[c^{(-)}(\mu)\, O^{(-)} + c^{(+)}(\mu)\, O^{(+)}\right] & \mu > m_c \\ \times\left[\sim \text{same} + c_5 O_5 + c_6 O_6\right] & m_K \sim \mu < \end{cases}$$

c_5 (c_6) ~ small (~ 0.2)

but matrix elements of $O_{5,6}$ are enhanced by a factor ~ $\left(\frac{m_K}{m_s}\right)^2 \sim$

$\sim \left(\frac{500 \text{ MeV}}{150 \text{ MeV}}\right)^2 \sim 10$ [see below]

- Penguin may explain (at least) part of the $\Delta I = 1/2$ enhancement:

$\Delta I = 1/2$ only

$$\frac{\langle (\pi\pi)_{I=0} | H_W | K^0 \rangle}{\langle (\pi\pi)_{I=2} | H_W | K^0 \rangle} \simeq 20$$

$\Delta I = 3/2$ only

<u>Criticism</u>:

1) $\langle \pi\pi | O_{(\mu)} | K \rangle$ can be evaluated in the valence quark approx. [vacuum saturation see below]

This is reliable if $\mu \sim m_K$
not " " $\mu \gg m_K$ [gluons may be irradiated from $\mu \to m_K$]

2) coefficients $c^{(\pm)}(\mu)$ are reliable if $\mu \gg \Lambda_{QCD}$
not " " " $\mu \sim m_K$

there is a conflict:

$$H_{eff} \approx C(\mu)\, O(\mu)$$

known at large μ — matrix el. reliable at low μ

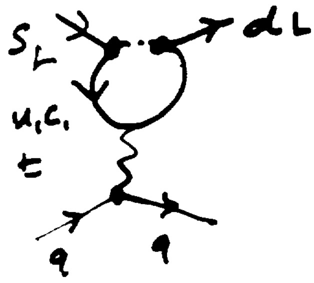
A good program for lattice QCD:

- take H_{eff} at $\mu > m_c \gg \Lambda_{QCD}$ where it is good
- Compute matrix elements of $\mathcal{O}$ on a lattice of lattice spacing: $a^{-1} \approx 2 \div 3$ GeV and let the lattice work out gluon corrections exactly.
- Results still preliminary (to be discussed in 2^{nd} lecture)

What about CP violation?

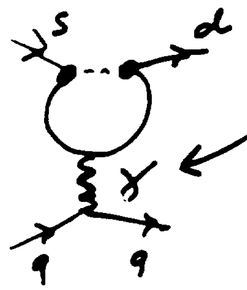
$$(\bar{s}_L \gamma^\mu t^A d_L)(\underbrace{\bar{q}\, t^A \gamma_\mu q}_{\bar{q}_L t^A \gamma_\mu q_L + \bar{q}_R t^A \gamma_\mu q_R})$$

$\Delta S = \pm 1$: 3 different sources

1) 

$\sim V^*_{td} V_{ts}$, $V^*_{cd} V_{cs}$ have complex phases

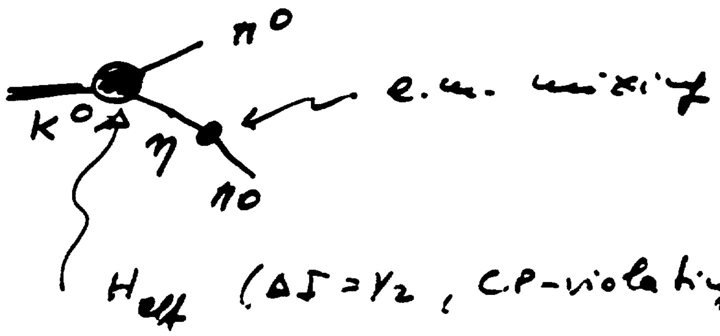
contributes to CP-violation in $\Delta I = 1/2$ amplitudes [Gilman, Wise]

2) 

Electropenguin

feeds CP-violation from $\Delta I = 1/2$ amplitude to $\Delta I = 3/2$

3) η and η' mixing



again feeds CP-viol. from $\Delta I = 1/2 \Rightarrow \Delta I = 3/2$

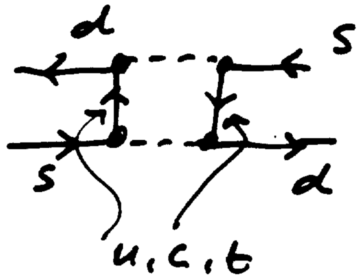
$\longrightarrow$ [Bijnens, Wise]

2-3 may be important: if CP-viol. in $\Delta I = 1/2$ is enhanced like the CP-conserving part, the factor $\frac{A^{1/2}}{A^{3/2}} \simeq 20$ may compensate the e.m. suppression $\alpha_{e.m.} \sim \frac{1}{137}$ -

Effect 2 is further enhanced by the factor $\left(\frac{m_K}{m_s}\right)^2$ in the matrix elements

$\Delta S = 2$ $\quad K_0 - \bar{K}_0$ mixing (and $B_0 - \bar{B}_0$)

CP-violation introduced by the box diagram



via the complex elements of K-M Matrix

MORE ABOUT ALL THAT TOMORROW

3. Vacuum Saturation Approximation

- Important ingredient in the theory of
  - non leptonic decay
  - weak $K_0 - \bar{K}_0$, $B_0 - \bar{B}_0$ mixing

- first introduced by Feynman (~64)

  extensively used by Joffe - Shabalin (~69,

  B.W. Lee, M.K. Gaillard

  ....

- $\langle M | \mathcal{O} | M' \rangle$ M, M' ~ pseudoscalar mesons

  $\mathcal{O}$ ~ four-fermion (quark) operator.

<u>Simplest case</u> :

$$\langle \bar{K}^0 | (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L) | K^0 \rangle$$

$$\bar{d} s \qquad \qquad \bar{s} d$$

$$\sim \langle \bar{K}_0 | \frac{1}{2}(\bar{s}\gamma_\mu \gamma_5 d)|0\rangle \langle 0|\frac{1}{2}\bar{s}\gamma^\mu \gamma_5 d|K^0\rangle * 2$$

$$= \frac{1}{2} f_K P_\mu \quad f_K P^\mu = \frac{1}{2} f_K^2 m_K^2$$

this is only part of the result, there is a second possibility :

$$\bar d s \qquad\qquad \bar s d$$

by the Fierz-identity : $(\bar s_L^a \gamma_\mu d_{L_a})(\bar s_L^b \gamma^\mu d_{L_b}) =$

$$= \bar s_L^b \gamma_\mu d_{L_a} \; \bar s_L^a \gamma^\mu d_{L_b}$$

$$= \langle \bar K^0 | \frac{1}{2} \left(\bar s^b \gamma_\mu \gamma_5 d_a\right) |0\rangle \langle 0 | \frac{1}{2} \bar s^a \gamma_\mu \gamma_5 d_b | K^0 \rangle *$$

$$\left[\frac{1}{3} \delta^b_d (\bar s \gamma_\mu \gamma_5 d) + \ldots\right]$$

$$= \frac{1}{2} \times \frac{1}{3} f_K^2 m_K^2 = \frac{1}{3} * (\text{previous contribution}$$

In total :

$$\langle \bar{K}^0 | (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L) | K^0 \rangle =$$

$$\equiv (B_{LR}) * \frac{2}{3} f_K^2 m_K^2$$

B_{LR} = B-parameter $= 1$ in vac. sat.

→ Try with a L−R operator [recall penguins !!

$$\langle \bar{K}^0 | (\bar{s}_L \gamma_\mu d_L)(\bar{s}_R \gamma^\mu d_R) | K^0 \rangle$$

1st contraction : $-\frac{1}{4} \langle \bar{K}^0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma^\mu \gamma_5 d | K^0 \rangle$

$$= -\frac{1}{2} f_K^2 m_K^2$$

2nd contraction : remember that :

$$\bar{s}_L^a \gamma_\mu d_{La} \; \bar{s}_R^b \gamma^\mu d_{Rb} =$$

minus

$$= -2 \, \bar{s}_R^b d_{La} \; \bar{s}_L^a d_{Rb}$$

so we get :

$$(2^{nd} \text{ contr.}) = -2 \times \frac{1}{3} \times \frac{1}{4} \times 2 \langle \bar{K}^0 | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle$$

Dirac equation for quarks :

$$i \partial^\mu \bar{s} \gamma_\mu \gamma_5 d = (m_s + m_d) \bar{s} \gamma_5 d \sim m_s \bar{s} \gamma_5 d$$

$$(2^{nd} \text{ contr.}) = -2 \times \frac{1}{3} \times \frac{1}{4} \times 2 f_K^2 m_K^2 \left(\frac{m_K^2}{m_s^2} \right)$$

So that

$$\langle \bar{K}^0 | (\bar{s}_L \gamma_\mu d_L)(\bar{s}_R \gamma^\mu d_R) | K^0 \rangle =$$

$$\equiv B_{LR} \left\{ -\frac{1}{3} f_K^2 \, m_K^2 \frac{m_K^2}{m_s^2} - \frac{1}{2} f_K^2 m_K^2 \right\}$$

note for $m_s \to 0$ $\quad m_K^2 = C\, m_s \quad C \neq 0$

so that the 1st term is $\neq 0$ for $m_s \Rightarrow 0$

$$\langle \bar{K}^0 | (\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma^\mu d_L) | K^0 \rangle \xrightarrow[m_s \to 0]{} 0$$

$$\langle \bar{K}^0 | (\bar{s}_L \gamma^\mu d_L)(\bar{s}_R \gamma^\mu d_R) | K^0 \rangle \xrightarrow[m_s \to 0]{} -\frac{1}{3} f_K^2 C^2 \neq 0$$

L-R operators are enhanced by a factor

$$\left(\frac{m_K}{m_s}\right)^2 \approx \left(\frac{500\,\text{MeV}}{150\,\text{MeV}}\right)^2 \approx 10$$

w.r.to L-L operators

[what makes penguins & electropenguins so important]

How good is vacuum saturation?
we will discuss tomorrow

- QCD sum rules
- lattice QCD

- For non leptonic amplitudes, we need : $\langle \pi\pi | \mathcal{O} | K \rangle$ $\qquad \mathcal{O}$ = 4-fermion op

<u>Soft pion theorems</u>

Nambu
Weinberg ...
(See Coleman
Erice lecture)

$\langle \pi\pi | \mathcal{O} | K \rangle$.

can be reduced to : $\langle \pi | \mathcal{O} | K \rangle$ evaluated as before

typical relation :

$$\langle \pi \; \pi(q=0) | \mathcal{O} | K \rangle = \frac{i}{f_\pi} \langle \pi | [\underbrace{Q^5}_{\int d^3x \, A_0(\vec{x},t)}, \mathcal{O}] | K \rangle$$

4. Mixing and CP-violation in Neutral P-S mesons with flavour $\neq 0$

$$M_0 \equiv \begin{cases} K^0 & (\bar{s} d) \\ D^0 & (c \bar{u}) \\ B_d^0 & (\bar{b} d) \\ B_s^0 & (\bar{b} s) \\ \vdots & \end{cases}$$

[T.D. Lee
Particle Physics and Introduction to Field Theory
Harwood Academ
1981]

- 2^{nd} order weak interactions (and other new interactions ?) induce $M_0 \leftrightarrow \bar{M}_0$ transitions
- Because of CPT : $\langle M^0 | H | M^0 \rangle = \langle \bar{M}^0 | H | \bar{M}^0 \rangle$

so that even a very small mixing may give large effects.

In meson rest frame:

$$H = \underbrace{M}_{\text{mass}} - \frac{i}{2} \underbrace{\Gamma}_{\text{lifetime}} \qquad \begin{matrix} M^+ = M \\ \Gamma^+ = \Gamma \end{matrix}$$

$M, \Gamma = 2 \times 2$ matrices in the basis $\begin{pmatrix} M^0 \\ \bar{M}^0 \end{pmatrix}$

– CP and T

$$M = a + b\tau_1 + c\tau_2 + d\tau_3 \qquad (\text{same for } \Gamma)$$

- Naive CP-definition:

$$CP\,|M^0\rangle = |\bar{M}^0\rangle$$

i.e. $$CP \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \tau_1$$

$$CP\ H\ CP \equiv \tau_1\ H\ \tau_1$$

- T is a antiunitary operator:

$$T\ M\ T = M^*$$
$$T\ \Gamma\ T = \Gamma^*$$

	CP	T	CPT	
a	+	+	+	CP, T conserving
$b\tau_1$	+	+	+	
$c\tau_2$	−	−	+	CPT o.k
$d\tau_3$	−	+	−	← violates CPT

$$M \equiv \begin{bmatrix} M & M_{12} \\ M_{21} & M \end{bmatrix} \qquad M_{12}^* \neq M_{12}$$

$$\Gamma \equiv \begin{bmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{21} & \Gamma \end{bmatrix} \qquad \Gamma_{12}^* \neq \Gamma_{12}$$

CP viol.

A matter of phases

those definition too naive. Even if M_{12}, Γ_{12} complex, H could be invariant under another definition of CP.

Most general:

$$(CP)_\theta |M^0\rangle = e^{i2\theta} |\bar{M}_0\rangle$$
$$(CP)_\theta |\bar{M}^0\rangle = e^{-i2\theta} |M^0\rangle$$

in this case:

$$(CP)_\theta = \begin{bmatrix} 0 & e^{-2i\theta} \\ e^{2i\theta} & 0 \end{bmatrix} = \tau_1 e^{2i\theta\tau_3}$$
$$= e^{-i\tau_3\theta} \tau_1 e^{i\theta\tau_3}$$

A $(CP)_\theta$ conserving hamiltonian is such that

$$(CP)_\theta H (CP)_\theta = H \Longrightarrow$$

$$e^{-i\tau_3\theta} \tau_1 e^{i\theta\tau_3} H e^{-i\theta\tau_3} \tau_1 e^{i\theta\tau_3} =$$
$$= H$$

$$\Rightarrow \quad \tau_1 H_\theta \tau_1 = H_\theta \qquad \left(H_\theta = e^{i\theta\tau_3} H e^{-i\theta\tau_3} \right)$$

$$H_\theta = \begin{bmatrix} M & m e^{2i\theta} \\ e^{-2i\theta} & M \end{bmatrix} \quad m \text{ real}$$

$$\Gamma_\theta = \begin{bmatrix} \Gamma & \gamma e^{2i\theta} \\ e^{-2i\theta} & \Gamma \end{bmatrix} \quad \gamma \text{ real}$$

Conclusion

to have CP-violation in H, it must be impossible to find a CP_θ such that $CP_\theta \; H \; CP_\theta = H$

$\Rightarrow$ $\arg M_{12} \neq \arg \Gamma_{12}$

i.e. The intrinsic CP violating parameter is :

$$\arg\left(\frac{M_{12}}{\Gamma_{12}}\right) \begin{cases} \neq 0 & \text{CP violation} \\ = 0 & \text{CP, T conserv.} \end{cases}$$

– Diagonalization (Assume CPT from now on.
CPT violation in K-dec. analyzed by Okun et al.

$$H = \begin{pmatrix} M - i\frac{\Gamma}{2} & 0 \\ 0 & M - i\frac{\Gamma}{2} \end{pmatrix} + \begin{pmatrix} C & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - i\Gamma_{12}^* & 0 \end{pmatrix}$$

define : $M_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad M_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

in the basis (M_1, M_2):

$$H \equiv \begin{bmatrix} M - \frac{i}{2}\Gamma + (\mathrm{Re}\, M_{12} - \frac{i}{2}\mathrm{Re}\,\Gamma_{12}) & (i\,\mathrm{Im}\,M_{12} + \frac{1}{2}\mathrm{Im}\,\Gamma_{12}) \\ +(\quad) & M - \frac{i}{2}\Gamma - (\quad) \end{bmatrix}$$

<u>Assuming all Im's to be small</u> :

•

$$M_S - i\frac{\Gamma_S}{2} \equiv$$ eigenvalue ~~~~ corresponding to $|M_S\rangle \sim |M_1\rangle$

$$= M - \frac{i}{2}\Gamma + \left(Re\, M_{12} - \frac{i}{2}\, Re\, \Gamma_{12} \right)$$

$$M_L - i\frac{\Gamma_L}{2} = M - \frac{i}{2}\Gamma - \left(\qquad\qquad \right)$$

<u>and</u> :

$$\left. \begin{aligned} |M_S\rangle &\cong |M_1\rangle + \varepsilon\, |M_2\rangle \\ |M_L\rangle &\cong \varepsilon\, |M_1\rangle + |M_2\rangle \end{aligned} \right\} + O(\varepsilon^2)$$

$$\varepsilon = \left[\frac{2\, Im\, M_{21} - i\, Im\, \Gamma_{21}}{(\Gamma_S - \Gamma_L) - 2i\,(M_L - M_S)} \right]$$

also

$$= -\frac{i}{2} \left[\frac{Im\, M_{21} - \frac{i}{2}\, Im\, \Gamma_{21}}{Re\, M_{21} - \frac{i}{2}\, Re\, \Gamma_{21}} \right]$$

<u>Note</u> : $\langle M_S | M_L \rangle \sim \varepsilon^* + \varepsilon = 2\, Re\, \varepsilon \neq 0$

Note. 1

if $\theta = \frac{\Im m\, M_{21}}{Re\, M_{21}} \sim \arg(M_{21}) = \frac{\Im m\, \Gamma_{21}}{Re\, \Gamma_{21}} \sim \arg \Gamma_{21}$

we expect no C.P-violation

In this case:

$$\varepsilon = -\frac{i}{2} \frac{(2\, Re\, M_{21} - i\, Re\, \Gamma_{21})\theta}{Re\, M_{21} - \frac{i}{2} Re\, \Gamma_{21}} = \underline{-i\theta}$$

ε pure imaginary

$$|M_S\rangle = \frac{1}{\sqrt{2}}\left(\underline{|M_0\rangle + |\bar{M}_0\rangle}\right) - \frac{i\theta}{\sqrt{2}}\left(|M_0\rangle - |\bar{M}_0\rangle\right)$$

$$\simeq \frac{1}{\sqrt{2}}\left(e^{-i\theta}|M_0\rangle + e^{i\theta}|\bar{M}_0\rangle\right)$$

$$\equiv \frac{1}{\sqrt{2}}\left(|M_0\rangle_\theta + |\bar{M}_0\rangle_\theta\right)$$

eigenstate of $(C.P)_\theta$

Same for $|M_L\rangle$

- $\Im m\, \varepsilon$ is phase dependent (arbitrary)
- CP violation $\Longleftrightarrow$ $Re\, \varepsilon \neq 0$

<u>check</u>: $Re\, \varepsilon \propto \left(\frac{\Im m\, M_{21}}{Re\, M_{21}} - \frac{\Im m\, \Gamma_{21}}{Re\, \Gamma_{21}}\right)$

$$= \arg\left(\frac{M_{21}}{\Gamma_{21}}\right)$$

Note 2

From 2nd order perturbation theory:

$$M_{21} = \sum_n \mathcal{P}\left(\frac{\langle \bar{K}^0 | H_{weak} | n \rangle \langle n | H_{weak} | K^0 \rangle}{m_K - m_n}\right)$$

$+ \ldots$

$\mathcal{P}$ = principal part

$$\Gamma_{21} = \sum_n (2\pi)\, \delta(E_n - M_K) \langle \bar{K}^0 | H_{weak} | n \rangle \langle n | H_{weak} | K^0 \rangle$$

$M_{21} \longleftrightarrow$ - virtual intermediate states
- If dominated by large momentum states $\Rightarrow$ asympt. freedom reliable
- box diagram 
- Sensitive to superweak interactions !!

$\Gamma_{21} \longleftrightarrow$ - real intermediate states
- much harder to estimate theoretically - case by case -

5. Parameters for mixing and oscillations, Intensity rules

Typical effect : oscillations in lepton charge in semileptonic decay

in our convention

$$M_0 \equiv (\bar{s}\, d) \;,\; (\bar{c}\, \bar{u}) \;\ldots$$
$$\qquad K^0 \qquad D^0 \quad \ldots$$

$$\left.\begin{array}{l} M_0 \to e^+ \nu + \ldots \\ \not\to e^- \nu + \ldots \end{array}\right\} \text{ if no mixing}$$

Mixing is signalled by wrong-sign leptons

ex.
$$K^- p \to \bar{K}^0 n$$
$$\quad \hookrightarrow e^- \nu \ldots$$
$$\quad \xrightarrow[\text{mixing}]{} e^+ \bar{\nu} \ldots$$

$$K^+ n \to K^0 p$$
$$\quad \hookrightarrow e^+ \nu \ldots$$
$$\quad \hookrightarrow e^- \bar{\nu} \ldots$$

(More complicated cases, later on).

At $t=0$ we start with $|M_0\rangle$, or $|\bar{M}^0\rangle$

at time t, we have:

$$|M_0(t)\rangle \simeq \frac{(1-\varepsilon)}{\sqrt{2}}\Big[\big(\overbrace{e^{-im_L t - \frac{\Gamma_L}{2}t}}^{D_L(t)}\big)|M_L\rangle + \\ +\big(\underbrace{e^{-im_S t - \frac{\Gamma_S}{2}t}}_{D_S(t)}\big)|M_S\rangle\Big]$$

$$|\bar{M}_0(t)\rangle = \frac{1+\varepsilon}{\sqrt{2}}\big[-D_L(t)|M_L\rangle + D_S(t)|M_S\rangle\big]$$

$$\langle e^-\ldots|M_0(t)\rangle = \left(\frac{1-\varepsilon}{1+\varepsilon}\right)\frac{\bar{A}}{2}(D_S - D_L)(t)$$

wrong sign

$$\langle e^+\ldots|M_0(t)\rangle = \frac{A}{2}(D_S + D_L)(t)$$

right sign

$$A \equiv \langle e^+\ldots|M_0\rangle$$

$$\bar{A} \equiv \langle e^-\ldots|\bar{M}_0\rangle$$

$$\text{Rate}(M_0 \to e^{\pm}\ldots) = \begin{cases} \dfrac{|A|^2}{4}\displaystyle\int_0^\infty |D_S + D_L|^2\,dt \\ \left|\dfrac{1-\varepsilon}{1+\varepsilon}\right|^2 \dfrac{|\bar{A}|^2}{4}\displaystyle\int_0^\infty |D_S - D_L|^2\,dt \end{cases}$$

<u>an easy calculation</u> ...

Final results

$$R_{WS/RS} \equiv \frac{\text{Rate}(M_0 \to e^- \ldots)}{\text{Rate}(\bar{M}_0 \to e^+ \ldots)} =$$

$$= \left|\frac{1-\varepsilon}{1+\varepsilon}\right|^2 \underbrace{\left|\frac{\bar{A}}{A}\right|^2}_{=1} \frac{x^2+y^2}{2+x^2-y^2}$$

$$x = \frac{M_L - M_S}{\Gamma} \qquad \Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)$$

$$y = \frac{\Gamma_S - \Gamma_L}{2\Gamma} \qquad 0 \lesssim |y| \lesssim 1$$

$$\bar{R}_{WS/RS} = \frac{\text{Rate}(\bar{M}_0 \to e^+ \ldots)}{\text{Rate}(\bar{M}_0 \to e^- \ldots)} = \left|\frac{1+\varepsilon}{1-\varepsilon}\right|^2 \left|\frac{A}{\bar{A}}\right|^2 \frac{x^2+y^2}{2+x^2-y^2}$$

- $R_{WS/RS} \neq 0$ mixing
- $R_0 \neq \bar{R}$ CP-violation ($R - \bar{R} \approx \mathrm{Re}\,\varepsilon$)

Critical parameters : x, y

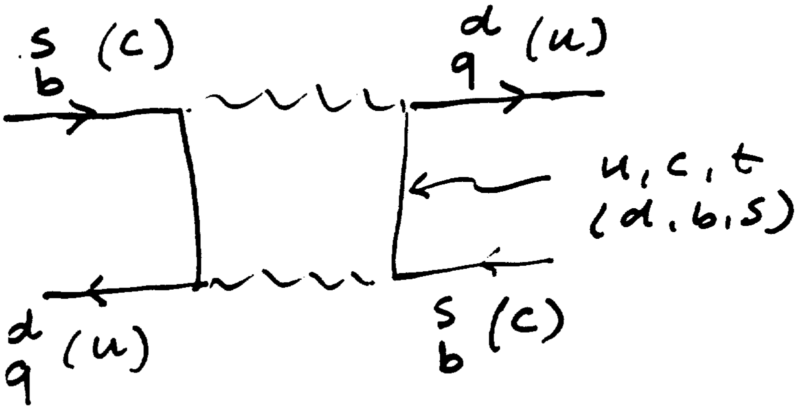
for $y \simeq 1$ $R \sim 1$ indep. of x

$y \sim 0$ $R \sim x^2$

Intensity rules

ΔM: box diagram

$$\left[\frac{x^2+y^2}{2+x^2-y^2}\right]$$



Particle	ΔM	$(\Gamma_S-\Gamma_L)/2\Gamma$	Γ	$(\Delta M/\Gamma)^2$
K^0	$G^2 m_c^2 \theta^2$	~ 1	$G^2\theta^2$	$O(1)$
D^0	$G^2 m_s^2 \theta^2$	~ 0	G^2	$O(\theta^4)$ Double cabibbo supp.
B_d^0	$G^2 m_t^2 (\underbrace{U_{tb} U_{td}^*}_{\theta^2\gamma^2(1-\frac{\beta}{\gamma\theta}e^{i\varphi})})^2$	~ 0	$G^2\gamma^2 m_b^5$	$\theta^4\left(\frac{m_t}{m_b}\right)$!!
B_s^0	$G^2 m_t^2 [\underbrace{U_{tb} U_{ts}^*}_{\gamma^2}]^2$	same	same	$1 \times \left(\frac{m_t}{m_b}\right)^4$ large anyhow

II - lecture

Summary

1. CP violation in K-decays
2. Experimental Data
3. Calculation of ϵ and ϵ' in the Standard Model
4. Hadronic Weak Parameters in lattice QCD

1. CP-violation in K decays

1. Mixing parameters

+ useful phase convention:

Recall

$$\Gamma_{21} = \sum_n (2\pi)\, \delta(E_n - M_K) \langle \bar{K}_0 | H_w | n \rangle \langle n | H_w | K_0 \rangle$$

sum is dominated by $(2\pi)_{I=0}$

$$\langle (\pi\pi)_I | H_w | K_0 \rangle = A_I \, e^{i\delta_I}$$

$$\langle (\pi\pi)_I | H_w | \bar{K}_0 \rangle = A_I^* \, e^{i\delta_I} \qquad (CPT)$$

$$\omega = \frac{|A_2|}{|A_0|} \approx \frac{1}{22} \ll 1$$

Wu-Yang convention: $A_0 = \text{real}$

$\Rightarrow \quad \Gamma_{21} \sim \text{real}$

$$\varepsilon = \frac{2\, \mathrm{Im}\, M_{21}}{\Gamma_S - \Gamma_L - 2i(M_L - M_S)}$$

now ε measures CP-violation by itself

denominator known:

$$\Gamma_S \gg \Gamma_L \,, \quad (M_L - M_S)\, 2 \sim \Gamma_S$$

$$\Rightarrow \quad \varepsilon \simeq \frac{2\,\Im m\,M_{21}}{\Gamma_S - 2i(M_L - M_S)} \approx \frac{2\,\Im m\,M_{21}}{\Gamma_S(1-i)}$$

$$\approx \sqrt{2}\,\frac{\Im m\,M_{21}}{\Gamma_S}\,e^{i\pi/4}$$

Recall : for kaons $y = \frac{\Gamma_S - \Gamma_L}{2\Gamma} \sim 1$

we can separate K_L from K_S just by waiting, and measure

$$R_L = \frac{\Gamma(K_L \to e^+) - \Gamma(K_L \to e^-)}{\Gamma(K_L \to e^+) + \Gamma(K_L \to e^-)} \simeq 2\,\mathrm{Re}\,\varepsilon \simeq \sqrt{2}\,|\varepsilon|$$

Conclusion :

CP-violation in (Mixing described by 1 parameter only :

$|\varepsilon|$ or $\mathrm{Re}\,\varepsilon$ or $(\Im m\,M_{21})$

- measured from R_L

expt. $R_L = (3.3 \pm 0.12) \times 10^{-3} > 0$

$$\Downarrow$$

$$|\varepsilon| \simeq (2.3 \pm 0.08) \times 10^{-3}$$

2. CP violation in $\Delta S = \pm 1$ Amplitudes

Recall $\langle (\pi\pi)_I | H_w | K_0 \rangle = A_I \, e^{i\delta_I}$

$A_0 =$ real

$\mathrm{Im}\, A_2 =$ measures CP-viol. in H_{weak}

This can be determined from the ratios:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | H_w | K_L \rangle}{\langle \pi^+\pi^- | H_w | K_S \rangle} \simeq \varepsilon + \varepsilon'$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | H_w | K_L \rangle}{\langle \pi^0\pi^0 | H_w | K_S \rangle} \simeq \varepsilon - 2\varepsilon'$$

$$\varepsilon' \simeq \frac{i}{\sqrt{2}} \frac{\mathrm{Im}\, A_2}{A_0} e^{i(\delta_2 - \delta_0)}$$

$$= \frac{i}{\sqrt{2}} \underbrace{\left(\frac{A_2}{A_0}\right)}_{\omega} e^{i(\delta_2 - \delta_0)} \left(\frac{\mathrm{Im}\, A_2}{A_2}\right)$$

$$\left|\frac{\eta_{+-}}{\eta_{00}}\right|^2 - 1 = \left|\frac{1 + \varepsilon'/\varepsilon}{1 - 2\varepsilon'/\varepsilon}\right|^2 - 1$$

$$\simeq 6 \; \mathrm{Re}(\varepsilon'/\varepsilon) \quad \left[\text{for } \frac{\varepsilon'}{\varepsilon} \ll 1\right]$$

- ε'/ε naturally small [if $\frac{\mathrm{Im}\, A_2}{A_2} \approx |\varepsilon|$] because of ω
- In Superweak theory $\varepsilon'/\varepsilon = 0$.

2 - Experimental data

1) $R_L = \dfrac{\Gamma(K_L \to e^+) - \Gamma(K_L \to e^-)}{\Gamma(K_L \to e^+) + \Gamma(K_L \to e^-)} = (3.3 \pm 0.12)10^{-3}$

$[|\epsilon| = (2.3 \pm 0.08)10$
$\varphi_\varepsilon \simeq 45^\circ$

2) $\eta_{+-} = \dfrac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$: $|\eta_{+-}| = (2.274 \pm 0.022)\, 10^{-3}$

$= \varepsilon + \varepsilon'$ $\qquad \varphi_{+-} = (44.6 \pm 1.2)^\circ$

3) $\eta_{00} = \dfrac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$ $\qquad |\eta_{00}| = (2.33 \pm 0.08)10^{-}$

$= \varepsilon - 2\varepsilon'$ $\qquad \varphi_{00} = (54 \pm 5)^\circ$

4) (NA31, 1987)

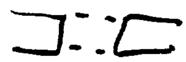
$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 - 1 = -6 * \left[(3.3 \pm 1.1) * 10^{-3}\right]$$

3 - Calculation of ϵ and ϵ' in Standard Model

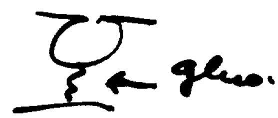
$$\begin{cases} \varepsilon \cong \sqrt{2}\, \dfrac{\operatorname{Im} M_{21}}{\Gamma_S}\, e^{i\pi/4} \\[2ex] \varepsilon' \simeq \dfrac{i}{\sqrt{2}}\, \omega\, e^{i(\delta_2-\delta_0)}\, \dfrac{\operatorname{Im} A_2}{A_2} \end{cases} \qquad \left(\omega = \frac{\operatorname{Re} A_2}{\operatorname{Re} A_0}\right) \sim \frac{1}{22}$$

$$\left(A_0 \equiv e^{-i\delta_0} \langle (\pi\pi)_0 | H | K_0 \rangle = \text{real}\right)$$

In K-M Model : CP-violation arises from

1- box diagram 

2- Penguin diagrams



3- Electropenguin diagrams

<u>Unknowns</u> : φ, m_t, $(\beta \leftrightarrow \rho \leftrightarrow V_{ub})$

(+ B factors

+ Wilson coefficients $C_i(\mu)$ in H_{eff})

Strategy

- assume we know matrix elements & coefficients

- fix $\rho = 0.6$ [or other values]

- extract $\varphi = \varphi(m_t)$ from ϵ

- see if ϵ'/ϵ is correct.

[we can do better with $B_0 - \bar{B}_0$ mixing]

Last problem about phases.

In K-M model, direct computation leads to $A_0^{K-M} = \text{complex} = A_0 e^{i\varphi_0}$

$$\varphi_0 \sim \frac{\mathrm{Im}\, A_0^{KM}}{A_0^{KM}}$$

$A_2^{K-M} \sim$ real (but for e.m. effects)

Therefore in previous expressions we have to substitute

$$\begin{cases} A_0 \Rightarrow A_0^{KM} e^{-i\varphi_0} \\ A_2 = A_2^{K-M} e^{-i\varphi_0} \\ M_{21} = M_{21}^{K-M} e^{-2i\varphi_0} \end{cases}$$

dropping the K-M superscript we get:

$$\epsilon = \sqrt{2}\, \frac{\mathrm{Im}\, M_{21}}{\Gamma_S} e^{i\pi/4} \left(1 + \frac{\Gamma_S}{\mathrm{Im}\, M_{21}} \frac{\mathrm{Im}\, A_0}{A_0}\right)$$

small correct.

$$\epsilon' = \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left(\frac{\mathrm{Im}\, A_2}{A_2} - \frac{\mathrm{Im}\, A_0}{A_0}\right)$$

Finally, we write, in ε'

$$\frac{\Im m\, A_0}{A_0} - \frac{\Im m\, A_2}{A_2} =$$

$$= \frac{\Im m\, A_0}{A_0} \left(1 - \Omega\right)$$

$$\Omega = \left(\frac{\Im m\, A_2}{A_2}\right) \frac{A_0}{\Im m\, A_0} = \frac{1}{\omega} \left(\frac{\Im m\, A_2}{\Im m\, A_0}\right)$$

notice the enhancement

$$\Omega = \Omega_{\eta,\eta'} + \Omega_{EMP}$$

mixing η, η'

electropenguins

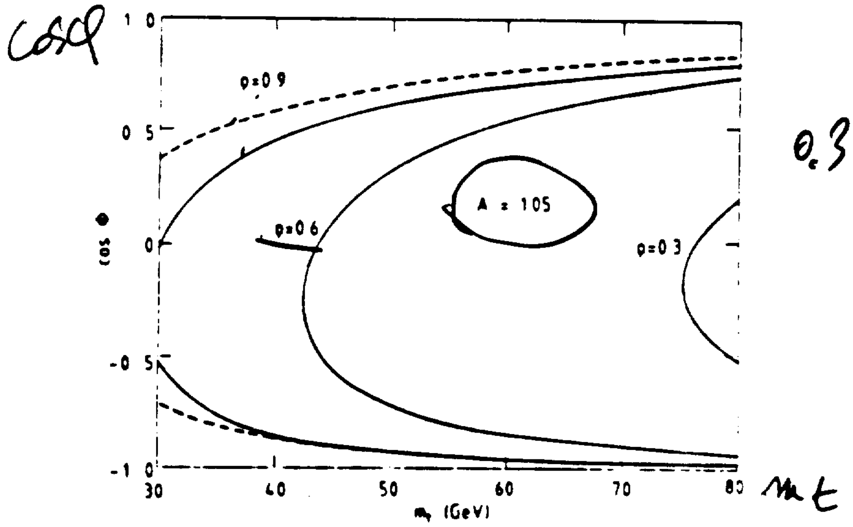
- Bijnens, Wise
- Buras, Gerard
- Sharpe
- Donoghue et al.

and (at last):

$$\epsilon' = \frac{i\omega\, e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left(\frac{\Im m\, A_0}{A_0}\right) \left(1 - \Omega\right)$$

Recent analyses: G. Altarelli, P. Franzini
CERN TH 4914/87 (quoted in Lect. 1)
J. Ellis, J. Hagelin, S. Rudaz,
D. D. Wu CERN TH 4816/8.

Fig. 1 Limits on cos ϕ obtained from the experimental value of the CP violating parameter ϵ for the kaon system as functions of the top quark mass, mt, for various values of ρ. The solid (dashed) lines include (do not include) the effect of the ξ term. Here we have taken A = 1.05 (the central value in Eq. (2.10)). The parameters ϕ, ρ and A are defined in Eq. (2.8). The indicated values of ρ = 0.9, 0.6 and 0.3 correspond to R = Γ(b→u)/Γ(b→c) ≈ 0.08, 0.04 and 0.009, respectively.



$0.3 < \rho < 1.0$

$A \sim 1.05$

- Note the lower bound to m_t, for fixed ρ (Buras)

$$\left|\frac{\varepsilon'}{\varepsilon}\right| = 1.2\, f(m_t)\, \underbrace{P}_{\substack{(0.5 \div 5)\\(0.5 \div 2)}} \left(A^2 \lambda^4 \rho \sin\varphi\right)\left(1 - \Omega_{\eta,\eta'} + \Omega_{EMP}\right)$$

taken from expt.

Altarelli. Franzini

$$= \left(1.9 \times 10^{-3}\right)\left(0.5 \div 2\right)\left(\frac{A}{1.05}\right)^2\left(\frac{\rho}{0.6}\right)\sin\varphi\left(1 - \Omega_{\eta} + \Omega_{E}\right)$$

$\Omega_{\eta,\eta'} \sim 30\%$

$\Omega_{EMP} \sim 30\%$ [e.g. lattice calcul., see below]

Magnitude & sign ~~[illegible]~~ correct

4. Calculations of hadronic weak parameters in lattice ~~approx~~ QCD

- Effort to determine matrix elements in a really non perturbative ~~set~~ set-up

- Analytical methods also studied at present:

  QCD - sum rules

  $\frac{1}{N_c}$ expansion

- Lattice QCD : in principle is a ~~non~~ first principle methods
  many approximations at present
  (- no fermion loops
  - not very small quark masses ...)
  ~~but~~ ~~very~~ different systematic errors from any other method.

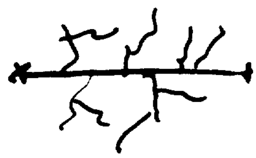
- What does it mean:

- choose a lattice of points in space-time
  $N_x \times N_y \times N_z \times N_T$

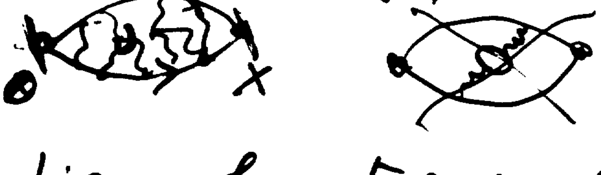
- generate a number of gauge field configurations, with probability
$$P(C) \approx e^{-\beta S(C)}$$
$$S(C) \sim \text{"}\int d^4x \; F_{\mu\nu} F^{\mu\nu}\text{"}$$
the Y-M action

$\beta = \frac{6}{g_0^2}$ $\quad g_0$ = "bare" QCD coupling.

- Compute the quark propagator $S(0,x)$ in any configurations:



- Compute correlation functions and average over configurations



- Correlation functions <u>decay</u> ~~expon~~ exponentially in time
$$\langle \mathcal{O}(t)\, \mathcal{O}(0) \rangle \sim e^{-M_n t} \langle 0|\mathcal{O}|n\rangle\langle n|\mathcal{O}|0\rangle$$

Mass of lightest particle which can be excited by $\mathcal{O}$

- Calculation of matrix elements
  e.g. $\langle \pi | O | K \rangle$
  more complicated but similar

- Who tells the energy unit (GeV?) to the lattice

<u>Dimensional transmutation</u>:

take massless QCD

$$g_0 = \frac{-1}{b \ln(\Lambda_{QCD}\, a)}$$

a = lattice spacing
a^{-1} = ultraviolet cut-off

the mass of any physical particle must be proportional to Λ_{QCD}

$$(m_X)_{phys} = C_X \Lambda_{QCD} =$$
$$= C_X a^{-1} e^{-\frac{1}{bg_0}}$$
$$= (C_X e^{-\frac{1}{bg_0}})\, a^{-1}$$
$$= m_X^L (g_0)\, a^{-1}$$

pure number, only function of g_0 is "measured" on the lattice.

Now, go the other way round

compute m_X^L, say the ρ-mass, on the lattice. Requiring

$$m_\rho^L(g_0)\, a^{-1} = (m_\rho)_{phys} \quad (= 0.75\ GeV)$$

gives $a^{-1} = f(g_0)$

- e.g. $g_0 = 1$ $\left(\beta = \frac{6}{g_0^2} = 6.0\right) \Leftrightarrow a^{-1} \simeq 2\ GeV$

- <u>Size of present calculations</u>

$16^3 \times 48$ lattice $\beta = 6.2$ $(a^{-1} \sim 3\ GeV)$

$\Rightarrow$ a block of hadronic matter

$\begin{cases} \sim 1 \text{ Fermi wide} \\ \sim \text{lattice spacing} \sim \frac{1}{15} \text{ Fermi} \end{cases}$

- Gauge field : $\sim 10^7$ real variables/conf
- quark prop. : $\sim 5.7 \cdot 10^7$ " "
- 1 experiment (large)
  $\sim 10^{15}$ floating point operations
  $\sim 1000\,h \times 4$ CPU hours of a CRAY 2.

our group: CERN - ORSAY - MADRID 46

- B. GAVELA
- L. MAIANI
- G. MARTINELLI
- O. PENE
- S. PETRARCA
- C. SACHRAJDA

Results:

- Computation of f_π, f_K, f_D
- f_B (extrapolated from the law
  $$f_M \sim \frac{1}{\sqrt{M}}$$)
- f_B (calculation on the lattice with static beauty) (Eichten)

  $\underline{f_B < 220 \text{ MeV}}$ (Boucaud, Gavela, Martinelli, Pene, Sachrajda)

- B-parameters
- $\Delta I = 3/2$ amplitude $(K^+ \to \pi^+\pi^0)$
- ϵ'/ϵ and electropenguins
- $\Delta I = 1/2$ amplitude $(K_S \to \pi^+\pi^-)$

$\left.\begin{array}{l} \Delta I = 3/2 \\ \epsilon'/\epsilon \\ \Delta I = 1/2 \end{array}\right\}$ still in infancy

f_M in MeV

Meson	Lattice calc.	QCD s. rules (Narison)	Expt.
π	140 ± 20 ± 20 [CERN-ROMA-ORSAY] ...		132
K	158 ± 13 [CRO] Univ. of Colorado 173 ± 13 160 ± Sharpe et al.		164
D	180 ± 30 [CRO] 215 ± 60 Univ. of Colo. 136 ± 50 UCLA	172 ± 15	< 290
D_s	218 ± 30 [CRO] ~220 Univ. of Colo. 200 ± 30 UCLA	~ 220	
B_d	extrapol. $f \sim \frac{1}{\sqrt{M}}$ 120 [CRO] 90 ± 30 UCLA	187 ± 24	
B_s	150 [CRO] 160 ± 40 UCLA		

B parameter [renorm. group invariant]

$(B_{LL})_K$ $\qquad \langle \bar{K} | (\bar{s}_L \gamma^\mu d_L)^2 | K \rangle$

Lattice	QCD s-rules	$\frac{1}{N_c}$ expansion
• 0.85 ± 0.20 [C-R-O]	0.33 ± 0.09 (De Rafael et al.)	0.7 ± 0.1 (Buras et
• UCLA: very similar	0.5 ± 0.2 (Ecker)	

$(B_{LL})_D$ $\qquad \langle \bar{D} | \cdots | D \rangle$ —

1.1 ± 0.1
(C-R-O)

$\Delta I = 3/2$ amplitude $K^+ \to \pi^+ \pi^0$ (Expt. $3.7 \times 10^{-8} = \frac{A(K^+ \to \pi^+ \pi^0)}{m_K}$

$(6 \pm 2) \times 10^{-8}$ [CRO]	very close to expt.	very close to expt.
UCLA: very similar		—

ϵ'/ϵ :

- Sharpe et al. (Staggered fermions)
  $(2 \div 3)\, 10^{-3}$
- C.R.O : able to compute only Ω_{EMP} $\qquad \left(\frac{\mathrm{Im}\, A_2}{A_2} \right)_{EMP}$
  $(\Omega_{EMP})_{\text{lattice}} \sim^{+} (15 \div 40)\,\%$

$\Delta I = 1/2$

CRO results (CERN TH 5032/88, ROMA 599, MADRID 88/13)

β	matrix element	$\frac{A(K_S \to \pi^+\pi^-)}{m_K}$	$\frac{A(K^+ \to \pi^+\pi^0)}{m_K}$
6.0	K-π	$\begin{cases}(1.2 \pm 1.3)\,10^{-6} \\ (2.5 \pm 1.8)\,10^{-6}\end{cases}$	$(16 \pm 6)\,10^{-8}$
6.2	K-π-π	$(2.1 \pm 1.3)\,10^{-6}$	$(5.9 \pm 1.4)\,10^{-8}$
Expt.		$0.78\ 10^{-6}$	$3.7\ 10^{-8}$

UCLA group quotes the ratio

$\beta = 5.7$ K-ππ 16 ± 9 Expt. (21.2)

Very encouraging - Needs refinement.

III - lecture

Summary -

- $B_0 - \bar{B}_0$ mixing
- Dileptons from $\Upsilon(4S)$
- Theory of opposite vs equal sign dileptons
- comparison with data
- CP-violation in equal sign dileptons
- CP-violation in exclusive B-decays
- Conclusions