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On the Space-Time Uncertainty Relations of Liouville Strings and D Branes

G. Amelino-Camelia[a], **John Ellis**[b], **N.E. Mavromatos**[a,⋄]
and
D.V. Nanopoulos[c,d]

Abstract

Within a Liouville approach to non-critical string theory, we argue for a non-trivial commutation relation between space and time observables, leading to a non-zero space-time uncertainty relation $\delta x \delta t > 0$, which vanishes in the limit of weak string coupling.

[a] University of Oxford, Dept. of Physics (Theoretical Physics), 1 Keble Road, Oxford OX1 3NP, United Kingdom,
[b] Theory Division, CERN, CH-1211 Geneva 23, Switzerland,
[c] Center for Theoretical Physics, Dept. of Physics, Texas A & M University, College Station, TX 77843-4242, USA
[d] Astroparticle Physics Group, Houston Advanced Research Center (HARC), The Mitchell Campus, Woodlands, TX 77381, USA.

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⋄ P.P.A.R.C. Advanced Fellow

One important aspect of recent developments [1] in the understanding of non-perturbative structures in string theory is renewed interest in the study of modified uncertainty relations. The conventional *critical string theory* modification of Heisenberg's uncertainty principle [2] was codified by the *Enlarged Uncertainty Principle* [3]

$$\delta x \, \delta p \ \geq \ 1 + L_s^2 \, \delta p^2 \ , \tag{1}$$

where L_s is the string length, *i.e.*, the square root of the Regge slope: $L_s = \sqrt{\alpha'}$, which leads to an absolute lower bound on the measurability of distance:

$$min \, [\delta L] \ = \ L_s \ . \tag{2}$$

Similar phase-space uncertainty relations and measurability bounds for distances are believed to hold in any theory involving a minimum length. In particular, an equation of the form of (2), but with L_s replaced by the Planck length L_P, has been proposed in the context of some non-stringy approaches to quantum gravity[4]. In addition, there has been discussion [5] in the string-theory literature of a possible non-trivial space-time uncertainty relation of the type

$$\delta x \delta t \geq L_s^2 \ . \tag{3}$$

Both (1) and (3) have been reanalyzed in recent studies [6, 7, 8, 9] of the non-perturbative solitonic structures in string theory known as D branes [1]. In particular, evidence has been found [6] in support of the idea that "D particles" (Dirichlet 0 branes) could probe the structure of space-time down to scales shorter than the string length, raising the possibility that (1) might have to be modified in the D-brane context.

In parallel with these developments in the literature on critical string theories, there have recently been studies of novel uncertainty relations and measurability bounds in the context of models of quantum gravity theories which include intrinsic microscopic mechanisms for quantum decoherence. The emergent general expectation is that decoherence effects should cause the uncertainties involved in a measurement procedure to grow with the time needed for the measurement. In particular, as discussed in ref.[10], the fact that gravitational effects prevent one from relying on the availability of *classical* agents for the measurement procedure, since the limit of infinite mass leads to inconsistencies associated with the formation of horizons, leads to the following bound[1] for the measurability of a distance L:

$$min \, [\delta L] \sim \sqrt{\frac{T L_P^2}{s}} \sim \sqrt{\frac{L L_P^2}{s}} \ , \tag{4}$$

where s is a length scale characterizing the spatial extension of the devices (*e.g.*, clocks) used in the measurement, T is the time needed to complete the procedure of measuring L, and on the right-hand side we have taken into account the fact that T is typically proportional [10] to L. The bound (4) is always larger than L_P for acceptable values [10] of s, *i.e.* $L_P \lesssim s \lesssim L$, and is maximal in the idealized scenario $s \sim L_P$, in which case

$$min \, [\delta L] \ = \ \sqrt{L L_P} \ . \tag{5}$$

[1]Related work is reported in ref. [11].

A candidate modified space-momentum uncertainty relation that leads to the bound (4) is given by [10, 12]

$$\delta x \, \delta p \; \geq \; 1 + L_P^2 \, \frac{T}{s} \, \delta p^2 \; , \tag{6}$$

but it has recently become clear [12] that the space-time uncertainty relations

$$\delta x \, \delta t \geq x \frac{L_P^2}{s} \tag{7}$$

or

$$\delta x \, \delta t \geq t \frac{L_P^2}{s} \tag{8}$$

would lead to the same bound.

The analysis of relations of the type (4)-(7) has often remained at a rather heuristic level. However, it has recently been realized [13] that Liouville (non-critical) string theories [14], in which the target time is identified with the Liouville mode [14, 15], and there is an intrinsic microscopic mechanism for decoherence, provide a natural framework for the investigation of such relations. Evidence in support of the validity of a measurability bound (5) in Liouville Strings has been provided in ref. [13]. The analysis relied on the observation that the propagation of massless probes in the stochastic quantum-gravitational environment of Liouville stringy modes leads to a modification of the dispersion relation of the massless probe. In the case of a scalar-field prototype, the resulting equation is a deformation of the Klein-Gordon equation, with energy-dependent speeds for massless particles, and this deformation leads to the bound (5) for the measurement of distances using such massless probes.

In this paper, we present some new evidence for a possible non-trivial space-time uncertainty relation, based on the formulation of the sum over genera in Liouville string theory, which entails the quantization of theory space, as discussed in [16, 17]. As we discuss in more detail below, this induces non-trivial quantum behaviour in the time variable, which we identify with the Liouville field. Within the context of D-brane dynamics, in which there is a departure from criticality related to the D-brane velocity [18], this leads to a non-trivial space-time commutation relation of the form (3). We find, however, that this depends on the string coupling.

The canonical quantization of string theory space has been discussed in [16, 17]. Motion in this space is characterized as a renormalization-group flow in the space of effective σ-model couplings g^i on the lowest-genus world sheet, with the flow variable identified in turn with the Liouville field and the target time variable. This flow is classical in the absence of higher-genus effects. However, in their presence, the renormalization-group flow has been shown to obey the relevant Helmholtz conditions which are necessary and sufficient for the canonical quantization of the σ-model couplings g^i:

$$[g^i, p_l] = i\delta^i_l \; : \; p_l \equiv G_{lj}\dot{g}^j \; , \tag{9}$$

with an effective action related to an appropriate version of the Zamolodchikov $\mathcal{C}$ function, where $G_{ij} =< V_i V_j >$ is the Zamolodchikov metric in theory space, and the dot denotes differentiation with respect to the Liouville mode.

This summation over genera and the ensuing quantization clearly entail a modification of the conventional conception of space-time coordinates, though the exact nature of the resulting picture is still to be understood. Some aspects are, however, already apparent. The normalization of the Liouville field ϕ is such that the target time t is given by [14, 19]

$$t = Q[g^i]\,\phi_0 \qquad Q[g^i]^2 \equiv \frac{1}{3}(\mathcal{C}[g^i] - 25) \tag{10}$$

where ϕ_0 is the world-sheet zero mode of ϕ, and $\mathcal{C}[g^i]$ the Zamolodchikov $\mathcal{C}$-function. In the presence of the sum over genera, the quantity $Q[g^i]$ becomes a q number, as a result of the quantization of the σ-model couplings, and hence t also becomes a quantum operator.

Concerning the target-space coordinates, we observe that their tree-level description as zero modes of the σ-model fields X implies in non-critical strings an interdependence between these coordinates and the background $\{g^i\}$. For example, in the case of the bosonic string that we consider here for simplicity, for tachyonic backgrounds the following relation holds at tree-level

$$< G_{lj}\dot{g}^j >=< [e^{ikX}] > \tag{11}$$

where $[\ldots] = 1 + O(g)$ denotes a normal-ordered (renormalized) product, which depends in general on the background $\{g^i\}$. Eq. (11), which has trivial content in critical string theory, due to the on-shell conformal-invariance conditions $\beta^i = 0$, implies - upon appropriate inversion [2] - a non-trivial relation between the zero modes of the target space coordinates and the backgrounds $G_{lj}\dot{g}^j$. We expect that upon summation over genera, which implies an appropriate path integration over the g^i [16, 17], this c-number relation would be turned into a (modified) relation between the operator $G_{lj}\dot{g}^j$ and the target-space coordinates. Although the precise nature of the (correspondingly) quantized target-space coordinates remains to be fully understood, this suggests that the eqs. (9) and (10) may lead to non-trivial space-time commutation and uncertainty relations.

The emergence of non-trivial space-time commutation and uncertainty relations can be seen somewhat more directly in the dynamical context of D branes treated as non-relativistic heavy objects. In a σ-model formalism, the collective center of mass spatial coordinates of these soliton-like string backgrounds, as well as the associated momentum coordinates, can be described [1, 7, 17, 8] by couplings of relevant σ-model deformations. Specifically, in describing D-brane recoil induced by string-matter scattering in a bosonic σ-model framework [7], one introduces the two deformations

$$\int_{\partial\Sigma} y_i C(z) = y_i \int_{\partial\Sigma} d\tau \epsilon \Theta_\epsilon(X^0)\partial_n X^i \tag{12}$$

$$\int_{\partial\Sigma} u_{R,i} D(z) = u_{R,i} \int_{\partial\Sigma} d\tau \epsilon X^0 \Theta_\epsilon(X^0)\partial_n X^i \tag{13}$$

and the D-brane spatial coordinates are identified with the couplings y^i, whereas the couplings $u_{R,i}$ correspond to the 'renormalized' [18, 8] Galilean recoil velocity of the D

[2] In order to establish this relation, one would have to separate out the world-sheet zero mode in $X = y + \xi(z, \bar{z})$, and perform the path integration over X involved in $< \ldots >$ by first integrating over the quantum fluctuations ξ, using the background-field method. The resulting relation between zero-mode integrals also entails a relation between the respective integrands, which however can only be established up to the usual kernel ambiguities.

brane [7, 18, 8]. The D-brane mass is given [1] by the inverse of the string coupling g_R, and therefore its momentum coordinates are given by $p_i = u_{R,i}/g_R$. In (12, 13), the $X^i, i = 1, 2, \ldots$ are understood to satisfy fixed Dirichlet boundary conditions on the boundary $\partial\Sigma$, while for X^0 standard Neumann boundary conditions are maintained. Also, in (12, 13), the quantity ϵ is an infrared world-sheet scale related to the world-sheet size L and the world-sheet ultraviolet cut-off a by $\epsilon \equiv (\ln|L/a|)^{-1/2}$.

Since the phase-space coordinates of the D brane are themselves described by σ-model couplings, after summation over genera the Liouville time[3] is given by a q-number function of the generic form

$$t = Q[y^i, p^i, g^i]\,\phi_0 \ , \tag{14}$$

where we have reserved the notation g^i for couplings that are not related to the D-brane kinematics. Moreover, the analysis in ref. [8] indicates that the D-brane coordinates do not commute with their associated momenta. This, together with (14), suggests the appearance of non-trivial commutation relations between the space and time variables, and associated space-time uncertainty relations. The determination of the exact form of these uncertainty relations still requires substantial technical developments, along the lines set out in refs. [7, 17, 8], in order to understand better the structure of the function $Q[y^i, p^i, g^i]$ and the structure of the commutation relations between the D-brane coordinates and momenta. At present, the best estimate of Q comes from the analysis of Liouville dressing in the context of the non-critical (if $\epsilon \neq 0$) σ-models describing D-brane recoil induced by string-matter scattering. The recoil operators C and D, involved in (12) and (13) respectively, are known [7] to have a small negative anomalous dimension $-\frac{1}{2}\epsilon^2$. This non-criticality leads to Liouville dressing, and via an application of Zamolodchikov's $\mathcal{C}$-theorem one finds in this D-brane case that [18, 8]

$$Q^2 \sim \int_{\phi^*}^{\phi} d\phi' \frac{\partial}{\partial\phi'}\mathcal{C}[\phi'] = -\int_{\phi_*}^{\phi} d\phi' \beta^C G_{CC} \beta^C = -\int_{\phi_*}^{\phi} \frac{d\phi'}{(\phi')^2}\frac{u_R^2}{g_R} \sim \frac{u_R^2}{g_R \phi} \tag{15}$$

where ϕ^* is an infrared fixed point on the world-sheet, and we have used the results of ref. [7] to express the two-point function of the C recoil operator in terms of the local renormalization-group scale on the world sheet (Liouville mode):

$$G_{CC} \sim 2|z|^4 < C(z)C(0) > \sim \frac{1}{g_R \phi^2} \tag{16}$$

Using the relations (14) and (15), we can obtain some preliminary evidence on the structure of the commutation relations for the Liouville D-brane space-time coordinates. Considering for simplicity the case of a 1+1-dimensional space-time, it follows from (14) and (15) that

$$[y, t] \sim [y, p]\,\sqrt{g_R \phi} L_s^2 \ . \tag{17}$$

We use the preliminary (lowest-order) results of ref. [8] for the phase-space coordinates of the D-brane, which indicate the following form of commutation relation

$$[y^i, p_j] = i\delta^i_j + \text{stringy corrections} \ , \tag{18}$$

[3] Note that, in the D-brane case, the zero mode ϕ_0 of the Liouville field can be identified [18] with $\epsilon^{-1/2}$.

i.e., the space and momentum coordinates of the D brane form, as a first approximation, an ordinary quantum-mechanical phase space.

Combining (17) and (18), one finally obtains the non-trivial space-time commutation relation

$$[y,t] \sim i\sqrt{g_R\phi}L_s^2 \; . \tag{19}$$

In order to evaluate the corresponding space-time uncertainty relation, by analogy with the uncertainty relations characterizing ordinary quantum mechanics, we need to estimate the minimum value of the expectation value $<\sqrt{g_R\phi}>$ in a physical state. At present, the space of physical string states, and especially its measure, is not well understood. However, if one interprets a world-sheet σ-model partition function as a target-space string wave function, the quantity $<\sqrt{g_R\phi}>$ could be evaluated as a σ-model vacuum expectation value. In the Liouville approach discussed here, such σ-model vevs include the summation over world-sheet topologies, which represents a path integration over stringy backgrounds [16, 17], as well as the integration over the σ-model fields X^i and ϕ. In the string-theory language, such vevs are string field-theory vevs. With this interpretation in mind, we estimate

$$\delta y \delta t \; \geq <\sqrt{g_R\phi}> L_s^2 \; \sim \; \sqrt{g_R}L_s^2 \; , \tag{20}$$

where the approximation on the right-hand side follows if we estimate $<\sqrt{\phi}>$ to be of $O(1)$.

We observe that the uncertainty relation (20) is y- and t-independent, just like (3). However, we also note that modifications are to be expected from effects beyond our leading-order analysis, because of non-trivial contributions coming from terms on the right-hand-side of eq.(18) that we ignored. Even at the present level of analysis, there is the important difference of a factor of $\sqrt{g_R}$ between eq. (20) and eq. (3). As a result, we find no space-time uncertainty relation in the weak-coupling limit $g_R \to 0$. This difference between the uncertainty relations characterizing the dynamics of the Liouville D branes [17, 18] considered here and the corresponding relations of [1, 6, 9] originates from the structure of the relevant recoil operators [7] and the associated [8] space-momentum uncertainty relation (18).

It would be of great interest to understand more deeply the group-theoretical structure of the non-trivial space-time commutation relation (19) and uncertainty relation (20) that we have found above. Whilst one might expect the underlying structure to be $O(d)$ invariant, we see no reason to expect that Lorentz invariance would be retained, since this is a property derived from critical string theory. It is attractive to speculate that some sort of quantum group structure, such as κ-Poincaré [20], might be relevant, but we have no formal evidence for this hypothesis. A proper setting for the preliminary investigation of these issues might be provided by field theories based on such uncertainty relations, in analogy with the analysis given in ref. [21] of field theories with modified phase-space uncertainties. However, such an analysis goes beyond the scope of this note.

Our essential point here has been to observe that as soon as one broadens the scope of string theory to include non-critical configurations, as is necessary to accommodate fluctuations in the classical string background, e.g. the case of D-brane quantum recoil [17, 18], time becomes a q number related to the Liouville field, and as such acquires non-trivial commutation relations with spatial coordinate observables, implying in turn non-trivial space-time uncertainty relations.

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