

Sezione di Trieste

INFN/AE-96/31

09 Ottobre 1996



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Lorenzo Vitale

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SIS - Pubblicazioni Laboratori Nazionali di Frascati

### SCALING VIOLATIONS, FRAGMENTATION FUNCTIONS AND q/g JETS AT LEP

#### Lorenzo VITALE

#### DELPHI speaker, for DELPHI and OPAL

#### INFN, Area di Ricerca, Padriciano 99, 34012 Trieste, Italy email: vitale@trieste.infn.it

The recent progress in the identification of quark and gluon jets in Z hadronic decays has allowed a new set of precise measurements and quantitative tests of the QCD predictions.

The hadronic fragmentation functions of the various quark flavors and gluons have been measured with the DELPHI detector at Z energy and have been compared with the fragmentation functions measured at lower energies down to 14 GeV. The large scaling violation observed has been used to extract the strong coupling constant, by means of a numerical integration of the second order DGLAP evolution equations. The result is  $\alpha_s(M_Z) = 0.124^{+0.006}_{-0.007}(exp) \pm 0.009(theory)$ , where the theory error is mainly due to the scale dependence. Scaling violation has also been observed for the first time in identified gluon jets measured at different energies within the same detector. The ratio of the energy dependence of the fragmentation functions in gluon to quark jet at large hadron energy has been found to be  $2.8 \pm 0.8(stat)$ , consistent with the colour factor ratio  $C_A/C_F$ . QCD analytical predictions for the mean multiplicity value is based on rare hadronic Z decays in which a high energetic gluon jet recoils against the two identified b quark jets and it is applied to a sample of data collected by the OPAL detector. The result is  $r = 1.552 \pm 0.041(stat) \pm 0.060(syst)$  and is only weakly dependent of a jet definition.

#### Introduction

Three analysis are presented, all based on the identification of quark and gluon jets in Z hadronic decays. The first two are performed by DELPHI, the third by OPAL.

In the first<sup>1</sup>, described in section 1, a value for the strong coupling constant is extracted from the energy dependence of the hadronic fragmentation functions (scaling violation).

A first observation of scaling violation in identified gluon jets  $^2$  is presented in section 2.

Section 3 describes the mean multiplicity ratio between gluon and quark jets at 39 GeV as measured by OPAL with a new technique<sup>3</sup>.

## 1 $\alpha_s$ from Scaling Violation in $e^+e^-$ Annihilation

Hadron production in  $e^+e^-$  annihilation originates from the production of quark-antiquark pairs which can radiate gluons, the quanta of the field theory of the strong interactions, Quantum ChromoDynamics (QCD). Gluon radiation depends logarithmically on the centre of mass energy Q due to the increasing phase space with increasing energy and the energy dependence of  $\alpha_s(Q)$  the running coupling constant of QCD. These effects lead to logarithmic variations of the momentum spectra of the produced hadrons as a function of the centre of mass energy, even if the momenta are scaled to that energy. These scaling violations can be used to determine  $\alpha_s$ .

DELPHI's previous analysis of this type<sup>4</sup> was based on the exact second order matrix-element and string or independent fragmentation, with a relatively small number of free parameters. This analysis extends the previous by including new data on gluon and heavy quark fragmentation and using a numerical integration of the second order DGLAP evolution equations<sup>5</sup> employing the program by Nason and Webber<sup>6</sup>.

The inclusive production of charged hadrons in the reaction  $e^+e^- \rightarrow h + X$  can be described by two kinematic variables,  $Q^2$  and x, where  $Q^2$ is defined as the square of the four-momentum transferred from the leptons to the hadrons and  $x = P_h/P_{beam}$  is the fraction of the beam momentum  $P_{beam}$  carried by the hadron h. In  $e^+e^$ annihilation  $Q^2$  equals s, the total centre of mass energy squared.

The cross section can be factorized into two regimes: the short distance regime, in which hard gluons can be radiated and the long distance regime of the hadronization process. The short distance regime is perturbatively calculable in terms of the so-called coefficient functions, while the non-calculable long distance regime has to be parametrized by phenomenological fragmentation functions.

Combining the two regimes according to the factorization theorem yields the following expression for the cross section  $^{6}$ :

$$\frac{d\sigma}{dx} = \sum_{i} \int_{x}^{1} \frac{dz}{z} C_i(z, \alpha_s(\mu_R), \frac{\mu_F^2}{Q^2}) D_i(\frac{x}{z}, \mu_F^2) \quad (1)$$

where  $C_i$  are the coefficient functions for the creation of a parton with flavour i and momentum fraction  $z = P_{parton}/P_{beam}$ , while the fragmentation function  $D_i$  represents the probability that this parton fragments into the hadron h with momentum fraction x/z. In leading order the  $C_i$  are given by the flavour specific weights of the electroweak theory and the  $\alpha_s$  dependence of  $C_i$  represents the next-to-leading order QCD corrections to the primary quark pair production. Both the coefficient and fragmentation functions depend on an arbitrary factorization scale  $\mu_F$ , in such a way that the dependence of the physical cross section on  $\mu_F$  would cancel if the calculation could be carried out to all orders in perturbation theory. In principle the renormalization scale  $\mu_R$  used for  $\alpha_s$ in (1) could be chosen differently from the factorization scale  $\mu_F$ ; for simplicity, the two scales have been taken to be the same.

The scaling violation in the fragmentation function is described by the coupled integrodifferential evolution equations  $^5$ , which can be written as:

$$\frac{\frac{dD_j(x,Q^2)}{d\ln Q^2}}{\sum_i \int_x^1 \frac{dz}{z} P_{ij}(z,\alpha_s(\mu_R),\frac{\mu_R^2}{Q^2}) D_i(\frac{x}{z},Q^2)}$$
(2)

with the splitting functions

$$P_{ij}(z, \alpha_s(\mu_R), \frac{\mu_R^2}{Q^2}) = \frac{\alpha_s(\mu_R)}{2\pi} P_{ij}^{(0)}(z) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 P_{ij}^{(1)}(z, \frac{\mu_R^2}{Q^2}) + O(\alpha_s^3).$$
(3)

The indices i and j run over all active quarks, antiquarks and the gluon. The splitting functions  $P_{ij}(z)$  are the probabilities for finding parton *i* with momentum fraction *z* after splitting from its parent parton *j*. The  $P_{ij}(z)$  are known to nextto-leading order in perturbation theory <sup>7</sup>. Thus the fragmentation functions at a given energy  $Q_0$ can be fitted to experimental data using equation (1) and then evolved to a different energy using the evolution equations (2). The strong coupling constant can then be extracted from a simultaneous fit of these calculated spectra to the data at different centre-of-mass energies.

In the analysis presented here data collected with the DELPHI detector <sup>8</sup> in the years 1991 to 1993 have been used. Samples enriched in bottom, charm and light (uds) quarks were used to determine the scaled momentum distributions for each quark flavour separately. In addition, the scaled momentum distribution for charged particles from gluon fragmentation was obtained from 3-jet events.

Hadrons coming from bottom quark fragmentation were selected with the help of the large impact parameter for the decay products of Bhadrons. Due to the long lifetime of B-hadrons and their large mass and momentum, the computed probability P for all the decay products to come from the primary interaction point is small. By requiring P < 0.01 a sample of 217,000 events with a purity of 83% is obtained. The technique used is described in detail in<sup>9</sup>.

The same method has been used to obtain a light quark sample, using a cut of P > 0.5 for the probability that all charged particles come from the interaction point which resulted in a sample of 357,000 events containing 79% light, 14% charm and 7% bottom quark events.

The c-quark fragmentation function was determined by the following method:  $D^{*+}$ -mesons were reconstructed as described in <sup>10</sup>. 2580  $D^{*+}$ candidates have been selected. An additional enrichment in charm quarks was achieved by requiring 0.005 < P < 0.015, which suppresses parts of the D-mesons originating from bottom quarks. The resulting sample consists of 50% charm events, 18% light and 32% bottom-quark events.

The gluon fragmentation function was determined from tagged gluon jets in b-quark induced 3-jet events. Both bottom quark jets were tagged by requiring  $P_j < 0.01$  and the remaining jet was taken as a gluon jet if the value of its probability variable  $P_j$  exceeded 0.1. A total of 6791 gluon jets have been identified with a purity of 94%. For these gluon jets the fraction of the jet momentum  $x_j = P_h/P_{jet}$  of the charged particles in the jet has been measured. This technique has been described in detail in <sup>11</sup> and the results have been taken from this analysis.

Detector effects and the effects of selection cuts have been corrected by computing correction factors for each bin of the distributions from a detailed Monte Carlo simulation of the detector. The corrected x distributions are plotted in fig.1.



Figure 1: Corrected  $x_p$  distributions and results of the fit.

To extract the scaling violation, we used all mixed-flavour spectra from the following experiments: DELPHI 91 GeV<sup>4</sup>, ALEPH 91 GeV<sup>12,13</sup>, AMY 54 GeV<sup>14</sup>, TASSO 44, 35, 22, 14 GeV<sup>15</sup>, CELLO 35 GeV<sup>4</sup>, MARK II 29 GeV<sup>16</sup>.

The fragmentation functions cannot be derived in perturbative QCD. Only the energy evolution (equation (2)) is known. Therefore a parametrization of the fragmentation functions is needed at one energy where the evolution starts and these parameters have to be the same at all energies, so that the differences between the momentum spectra at different energies are only dependent on  $\alpha_s$ .

In our analysis the parametrization from Ref.<sup>12</sup> was used:

$$xD_{i}(x,Q_{0}) = N_{i} \frac{x^{a_{i}}(1-x)^{b_{i}} \exp(-c\ln^{2}x)}{\int_{0,1}^{0.8} dx \ x^{a_{i}}(1-x)^{b_{i}} \exp(-c\ln^{2}x)}$$
(4)

where the coefficients  $N_{i}, a_{i}$  and  $b_{i}$  were assumed to take different values for light (uds) quarks, c-, b-quarks and gluons;  $Q_{0}$  is the "starting energy" which was taken to be 91.2 GeV. The exponential term is inspired by the Modified Leading-Log Approximation (MLLA).

The strong coupling constant was extracted in the following way. A simultaneous fit of the QCD scale  $\Lambda_{\overline{MS}}^{(5)}$  and the fragmentation parameters  $a_i$ ,  $N_i$ ,  $b_i$  and c defined in (4) was made by minimizing the following  $\chi^2$  function:

$$\chi^2 = \Delta^T V^{-1} \Delta. \tag{5}$$

Here  $\Delta$  is a column vector containing the residuals between the inclusive momentum spectrum in a given x bin and the theoretical prediction which can be computed from equations (1) and (2) by numerical integration <sup>6</sup>. The matrix V is the N by N covariance matrix between N measurements. In the range 0.10 < x < 0.80 and  $14^2 < Q^2 < 91.2^2$  GeV<sup>2</sup> a total of 14 parameters including  $\Lambda_{\overline{MS}}^{(5)}$  were fitted to the 13 distributions of the data listed above. The  $\chi^2$  per degree of freedom of the fit was 174.5/175 and the fit results are shown in fig.2.



Figure 2:  $Q^2$  dependence of the inclusive momentum cross section for various x bins and results of the fit.

If all higher orders are known one expects no dependence of  $\alpha_s$  on the renormalization scale  $\mu_R$  and the factorization scale  $\mu_F$ . However, the scaling violation is only known up to order  $\alpha_s^2$  with a resummation of the leading logs. Therefore one expects a scale dependence of at most  $O(\alpha_s^3)$ . The



Figure 3: Renormalization scale dependence of  $\alpha_s$ .

scale dependence of  $\alpha_s$  is shown in fig.3. Surprisingly, it did not decrease with respect to the  $O(\alpha_s^2)$ result, which is shown in fig.3 too.

The theoretical error due to the renormalization and factorization scale uncertainty was estimated as half the maximum variation for scales in the range  $0.5\sqrt{s} < \mu < 2\sqrt{s}$ . An additional check for non-perturbative corrections has been carried out by changing the lowest  $Q^2$ - value in the fit between 14<sup>2</sup> and 29<sup>2</sup> GeV<sup>2</sup>.

Combining the errors in quadrature and symmetrizing the dominant theoretical error yields:

$$\alpha_s(M_Z) = 0.124^{+0.006}_{-0.007}(exp) \pm 0.009(theory)$$
(6)

which is in agreement with other  $\alpha_s$  determinations<sup>17</sup> and with a similar analysis of ALEPH<sup>13</sup>.

#### 2 Scaling Violation in Gluon Jets



Figure 4: Y and Mercedes events topologies.

Scaled momentum distributions of charged hadrons for quark and gluon jets at jet energies of about 23 and 30 GeV have been measured by using Y and Mercedes events (fig.4) in a sample of 3 millions hadronic Z decays collected by DELPHI (the technique is described in detail in  $^{11}$ ).



Figure 5: Scaled momentum distributions.

These fragmentation function, showed in fig.5, are in good agreement with those measured from all events at centre of mass energies  $^{14,15}$  of similar hardness (largest possible transverse momentum  $E_{2,3} \cdot \sin \theta_{2,3}$ ,  $\theta$  defined in fig.4), supporting the theoretical presumption that the scale of the partonic processes is indeed the hardness defined above.



In fig.6b the ratios  $S_{q,g}$  of the fragmentation functions at the two jet energies are showed for quark and gluon jets. Also for gluon jets the typical scaling violation pattern is observed and it's even more pronounced, due to higher gluon splitting probability. Fig.6c shows the ratio  $S_g/S_q$ , which averaged for x > 0.2 yields

$$2.8 \pm 0.8(stat),$$
 (7)

which is consistent with the colour factor ratio  $C_A/C_F = 9/4$ .

#### 3 Multiplicity Difference between Quark and Gluon Jet

The difference between the multiplicity of quark and gluon jets has been studied by OPAL with a new method, proposed in Ref. <sup>18</sup>. In this method gluon jets are identified by using an inclusive definition, similar to that used for the analytical calculations. It is based on rare events (fig.7) of the



Figure 7: Topology of studied events.

type  $e^+e^- \rightarrow q\bar{q}g_{incl}$  in which the q and  $\bar{q}$  are identified b quark jets which appear in the same hemisphere defined by the plane perpendicular to the thrust axis of the event. The quantity  $g_{incl}$ , taken to be the gluon jet, is defined by the sum of all the particles observed in the opposite hemisphere.

The present analysis is based on a sample of about 3,044,000 hadronic events collected by OPAL from 1991 to 1994. In total 278  $g_{incl}$ gluon jets are selected, with  $(83.0 \pm 1.7)\%$  estimated purity. The mean gluon jet energy is determined to be  $39.2 \pm 0.3(stat) \pm 1.8(syst)$  GeV. The uds quark jets are defined inclusively by using the charged particles observed in the hemisphere opposite to identified uds jets. A hemisphere is tagged as uds hemisphere if the are no tracks with impact parameter normalized by its error bigger than 1.5 and the maximum scaled momentum of the charged particles is bigger than 0.5. This selection tags 28,000 hemispheres allowing an uds estimated purity of  $(93.2 \pm 0.2)\%$ . Purities for gluon and uds selections are estimated from a sample of simulated events. After correcting for detector response, initial-state photon radiation and background the following mean charged multiplicities are obtained:

$$\langle n_{ch} \rangle_g (39.2) = 14.63 \pm 0.38 \pm 0.60$$
 (8)

$$\langle n_{ch} \rangle_{uds}(45.6) = 10.05 \pm 0.04 \pm 0.23,$$
 (9)

where the first error is the statistical and the second the systematical. The results are showed in fig.8, where also the predictions of HERWIG



Figure 8: Charged particle multiplicity for gluon and uds jets compared to predictions of HERWIG and JETSET.

and JETSET models are visible as the solid and dashed lines respectively.

To account for the different energies of the two samples, before forming the ratio  $r_{ch}$  between the gluon and uds jet multiplicities,  $\langle n_{ch} \rangle_{uds}$  (45.6) has been evolved to 39.2 GeV by employing the QCD analytical prediction <sup>19</sup>, yielding  $\langle n_{ch} \rangle_{uds}$  (39.2) = 9.43 ± 0.06 ± 0.22.

Finally the ratio obtained is:

$$r_{ch} = 1.552 \pm 0.041(stat) \pm 0.060(syst)$$
(10)

shown in fig.9 together with the predictions of HERWIG and JETSET models, and of the analyt-



Figure 9: Analytic and Monte Carlo predictions for r and OPAL experimental result.

ical calculations. The result agrees well with the predictions of HERWIG and JETSET at hadron level and with HERWIG at parton level. JETSET prediction at parton level is 10% lower. The result is in agreement with the prediction of an analytic calculation<sup>20</sup> which incorporates energy conservation into the parton shower branching processes, but it is considerably smaller than analytic predictions which don't incorporate energy conservation.

#### Acknowledgments

I thank A. De Angelis, W. De Boer, J. Furster and K. Hamaker for help in preparing this talk.

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