Two-dimensional Yang–Mills theory in the leading 1/N expansion revisited

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Abstract

We obtain a *formal* solution of an integral equation for $q\bar{q}$ bound states, depending on a parameter η which interpolates between 't Hooft's ($\eta = 0$) and Wu's ($\eta = 1$) equations. We also get an explicit approximate expression for its spectrum for a particular value of the ratio of the coupling constant to the quark mass. The spectrum turns out to be in qualitative agreement with 't Hooft's as long as $\eta \neq 1$. In the limit $\eta = 1$ (Wu's case) the entire spectrum collapses to zero, in particular no rising Regge trajectories are found.

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I. THE $Q\bar{Q}$ BOUND STATE EQUATION AND ITS FORMAL SOLUTION

In 1974 G. 't Hooft [1] proposed a very interesting model to describe mesons, starting from an SU(N) Yang–Mills theory in 1+1 dimensions in the large-N limit.

Quite remarkably in this model quarks look confined, while a discrete set of quark-antiquark bound states emerges, with squared masses lying on rising Regge trajectories.

The model is solvable thanks to the "instantaneous" character of the potential acting between quark and antiquark.

After that pioneering investigation, many interesting papers followed 't Hooft's approach, pointing out further remarkable properties of his theory and blooming into the recent achievements of two-dimensional QCD [2], [3], [4].

Three years later such an approach was criticized by T.T. Wu [5], who replaced the instantaneous 't Hooft potential by an expression with milder analytical properties, allowing for a Wick rotation without extra terms. Actually this expression is nothing but the (1+1)-dimensional version of the Mandelstam-Leibbrandt (ML) [6] propagator, a choice which is mandatory in order to achieve gauge invariance and renormalization in 1+3 dimensions [7], [8], [9].

Unfortunately this modified formulation led to a quite involved bound state equation. An attempt to treat it numerically in the zero bare mass case for quarks [10] led only to partial answers in the form of a completely different physical scenario. In particular no rising Regge trajectories were found.

The integral equation for the quark self-energy in the Minkowski momentum space is

$$\Sigma(p;\eta) = i\frac{g^2}{\pi^2}\frac{\partial}{\partial p_-}\int dk_+ dk_- \Big[P\Big(\frac{1}{k_- - p_-}\Big) + i\eta\pi \ \mathrm{sign}(k_+ - p_+)\delta(k_- - p_-)\Big] \cdot \frac{k_-}{k^2 + m^2 - k_-\Sigma(k;\eta) - i\epsilon}, \tag{1}$$

where P denotes the Cauchy principal value prescription (CPV), $g^2 = g_0^2 N$, and η is a parameter that is used to interpolate between 't Hooft's ($\eta = 0$) and Wu's equation ($\eta = 1$).

Its exact solution reads

$$\Sigma(p;\eta) = \frac{1}{2p_-}\Bigg(\Big[p^2 + m^2 + (1-\eta)\frac{g^2}{\pi}\Big] -$$
$$- \sqrt{\Big[p^2 + m^2 - (1-\eta)\frac{g^2}{\pi}\Big]^2 - \frac{4\eta g^2 p^2}{\pi}}\ \Bigg)\ , \tag{2}$$

where the boundary condition has been chosen in such a way that $p_-\Sigma(p^2 = +\infty) = g^2/\pi$. When continuing in p^2, care is to be taken in the choice of the square-root determination.

One can immediately realize that 't Hooft's and Wu's solutions are recovered for $\eta = 0$ and $\eta = 1$, respectively.

The dressed quark propagator turns out to be

$$S(p;\eta) = -\frac{ip_-}{m^2 + 2p_+p_- - p_-\Sigma(p;\eta)} \tag{3}$$

and the equation for a $q\bar{q}$ bound state in Minkowski space, using light-cone coordinates, is

$$\psi(p,r) = \frac{-ig^2}{\pi^2} S(p;\eta)S(p-r;\eta)\int dk_+ dk_- \Big[P\Big(\frac{1}{(k_- - p_-)^2}\Big) -$$
$$- i\eta\pi\, sign(k_+ - p_+)\delta'(k_- - p_-)\Big]\psi(k,r). \tag{4}$$

We are here considering for simplicity the equal mass case.

The 't Hooft potential ($\eta = 0$) exhibits an infrared singularity, which was handled, in the original formulation, by introducing an infrared cutoff; a quite remarkable feature of this theory is that bound state wave functions and related eigenvalues turn out to be cutoff-independent. Actually, in ref. [11], it has been pointed out that the singularity at $k_- = 0$ can also be regularized by the CPV without altering gauge invariant quantities. Then, the difference between the two cases $\eta = 1$ and $\eta = 0$ is represented by the following distribution

$$\Delta(k) \equiv \frac{1}{(k_- - i\epsilon\ \mathrm{sign}(k_+))^2} - P\Big(\frac{1}{k_-^2}\Big) = -i\pi\ \mathrm{sign}(k_+)\delta'(k_-). \tag{5}$$

both in the equation for the self-energy and in the one for $q\bar{q}$ bound states.

In ref. [12], it has been shown that, starting from 't Hooft's solutions, no correction affects 't Hooft's spectrum when the difference in eq. (5) is treated as a single insertion both in the "potential" and in the propagators. Wu's equation for colourless bound states, although much more involved than the corresponding 't Hooft one, might still apply, according to the heuristic lesson one learns from a single insertion.

It is the purpose of this paper to show that unfortunately this conclusion is unlikely to persist beyond a single insertion. This should not come as a surprise, since Wu's equation is deeply different from 't Hooft's and might be related to a different physical scenario (see for instance [13]).

It is useful to introduce dimensionless variables x, y and α

$$\begin{aligned} p_- &= x\, r_- \\ p_+ &= y\, r_+ \\ 2\, r_+ r_- &= -\alpha m^2, \end{aligned}$$

so that

$$\frac{p^2}{m^2} = \frac{2\, p_+ p_-}{m^2} = xy\, \frac{2\, r_+ r_-}{m^2} = -\alpha\, xy.$$

In these notations the quark propagators are

$$\begin{aligned} S(p) &= -\frac{i r_-}{m^2} \frac{x}{1-\alpha xy - \Sigma(\alpha xy)} = -\frac{i r_-}{m^2}\, S(x,y), \\ S(p-r) &= \frac{i r_-}{m^2} \frac{\overline{x}}{1-\alpha\overline{x}\overline{y} - \Sigma(\alpha\overline{x}\overline{y})} = \frac{i r_-}{m^2}\, S(\overline{x},\overline{y}), \end{aligned}$$

where

$$\overline{x} \equiv 1-x, \quad \overline{y} \equiv 1-y,$$

$$\begin{aligned} \Sigma = \frac{1}{2} \Bigg(&[1-\alpha xy + (1-\eta)\frac{g^2}{\pi m^2}] + \\ &+ \sqrt{[1-\alpha xy - (1-\eta)\frac{g^2}{\pi m^2}]^2 + 4\eta \frac{g^2 \alpha xy}{\pi m^2}} \Bigg). \end{aligned}$$

Notice that the square root has changed sign as it has been continued to positive values of α.

The bound state equation takes the form

$$\psi(x,y) = -\frac{ig^2}{2\pi^2 m^2}\,\alpha\, S(x,y)S(\overline{x},\overline{y}) \int dx'dy' \Big[P\frac{1}{(x'-x)^2} + \\ + i\eta\pi\,\mathrm{sign}(y'-y)\,\delta'(x'-x)\Big]\,\psi(x',y'). \tag{6}$$

We write eq. (6) symbolically as

$$\psi(x,y) = \mathcal{S}\int_{-\infty}^{\infty} dy'[\{H\,\psi\}(x,y') - i\,\eta\pi\,\mathrm{sign}(y'-y)\,\partial_x\psi(x,y')]. \tag{7}$$

Here H denotes 't Hooft's operator

$$\{H\,\psi\}(x,y) = \int_{-\infty}^{\infty} dx'\, P\frac{1}{(x'-x)^2}\psi(x',y)$$

and $\mathcal{S} = -\frac{ig^2}{2\pi^2 m^2}\alpha S(x,y)S(\bar{x},\bar{y})$ is a multiplication operator.

After introducing the function

$$F(x,y) = \int_{-\infty}^{y} dy'\,\psi(x,y'), \qquad F(x,-\infty) = 0$$

the equation takes the form

$$\frac{\partial F}{\partial y} = \mathcal{S}\{(H - i\pi\eta\,\partial_x)F\}(x,\infty) + 2\pi i\eta\,\mathcal{S}\,\partial_x\,F(x,y). \tag{8}$$

The *formal* solution of this equation is

$$F(x,y) = P\exp\left\{2\pi i\eta\int_{-\infty}^{y} dy'\,\mathcal{S}(x,y')\,\partial_x\right\}\cdot \\ \cdot\int_{-\infty}^{y} dw\left[P\exp\left\{2\pi i\eta\int_{-\infty}^{w} dy'\,\mathcal{S}(x,y')\,\partial_x\right\}\right]^{-1}\cdot \\ \cdot\ \mathcal{S}(x,w)\{(H - i\pi\eta\partial_x)F\}(x,\infty), \tag{9}$$

where the "path-ordered" exponent appears since the operators $\mathcal{S}$ and ∂_x do not commute.

For $y = +\infty$, one gets the closed one-dimensional equation for the function $F(x,\infty)$ (which is just the analogue of 't Hooft's wave function):

$$F(x,\infty) = \int_{-\infty}^{\infty} dy\,P\exp\left\{2\pi i\eta\int_{y}^{\infty} dy'\,\mathcal{S}(x,y')\,\partial_x\right\}\cdot \\ \cdot\ \mathcal{S}(x,y)\{(H - i\pi\eta\partial_x)F\}(x,\infty). \tag{10}$$

An equivalent form is

$$
\begin{aligned}
F(x,\infty) \; &= \; i\int_{-\infty}^{\infty} dy\, \frac{1}{2\pi\eta}\frac{\partial}{\partial y}\, P\exp\left\{2\pi i\eta\int_{y}^{\infty} dy'\,\mathcal{S}(x,y')\,\partial_x\right\}\, \cdot \\
&\cdot\; \frac{1}{\partial_x}\,\{(H-i\pi\eta\partial_x)F\}(x,\infty),
\end{aligned}
\tag{11}
$$

or, finally,

$$
\begin{aligned}
F(x,\infty) \; &= \; \frac{i}{2\pi\eta}\left[I \;-\; P\exp\left\{2\pi i\eta\int_{-\infty}^{\infty} dy'\,\mathcal{S}(x,y')\,\partial_x\right\}\right]\, \cdot \\
&\cdot\; \frac{1}{\partial_x}\,\{(H-i\pi\eta\partial_x)F\}(x,\infty).
\end{aligned}
\tag{12}
$$

We notice that, in the limit $\eta = 0$, 't Hooft's equation is correctly reproduced. Once eq. (12) is solved, eq. (9) provides us with the solution of the original two-variable equation (6).

II. AN ANALYTICAL APPROXIMATE SOLUTION

Equations (12)and (6) are by far too difficult to be concretely solved by means of analytical procedures (and probably also numerically). There is, however, a particular case in which they can be successfully tackled, at least heuristically. This case is realized when $\Sigma(\alpha xy) \simeq 1$, which in turn means a peculiar value of the ratio $\frac{g^2}{\pi m^2} \simeq 1$.

In such a case the operator $\mathcal{S}$ takes the form

$$
\mathcal{S}(x,y) = -\frac{ig^2}{2\pi^2\alpha m^2} \qquad \frac{1}{y + i\epsilon\ \mathrm{sign}(x)} \qquad \frac{1}{\bar{y} + i\epsilon\ \mathrm{sign}(\bar{x})}. \tag{13}
$$

From here on we shall rely on this particular form of $\mathcal{S}$. Such a form can also be approximately obtained for a generic value of the coupling in the limit of large values of α.

In future developments the following integral will be needed:

$$
\int_{-\infty}^{+\infty} dy \mathcal{S}(x,y) = -\lambda\theta(x)\theta(\bar{x}),
$$

θ being the usual step distribution and $\lambda = \frac{g^2}{\alpha\pi m^2}$.

The commutator between $\mathcal{S}$ and ∂_x is a measure concentrated at the values $x = 0$ and $x = 1$. In the Appendix we shall argue that, whenever the product $\eta\lambda$ is not pure imaginary, the contribution from that commutator vanishes: "path-ordered" exponentials can be turned into "normal-ordered" ones.

Then eq. (12) can be written as

$$\begin{aligned} F(x,\infty) &= \frac{i}{2\pi\eta}\,[I\; - \;N\exp\{-2\pi i\eta\lambda\theta(x)\theta(\bar{x})\,\partial_x\}]\;\cdot \\ &\cdot\;\frac{1}{\partial_x}\,\{(H - i\pi\eta\partial_x)F\}(x,\infty). \end{aligned} \tag{14}$$

This equation can be approximately diagonalized by a Fourier transform

$$\tilde{F}(k) = \int_{-\infty}^{+\infty} e^{ikx}F(x,\infty)\,dx. \tag{15}$$

Equation (14) indeed becomes

$$\tilde{F}(k) = \frac{1}{4\pi\eta}\int_{-\infty}^{+\infty} dq\,\tilde{F}(q)e^{i\frac{k-q}{2}}\,\frac{\sin\frac{k-q}{2}}{\frac{k-q}{2}}\,(\mathrm{sign}\,(q) + \eta)\,(1 - e^{-2\pi q\eta\lambda}), \tag{16}$$

where the well-known relation

$$\exp\{\Lambda\partial_x\}f(x) = f(x + \Lambda)$$

has been used.

Taking the approximation $\frac{\sin x}{x} \simeq \pi\,\delta(x)$ into account, we get

$$\tilde{F}(k)\cdot\Big[(\mathrm{sign}(k) + \eta)\exp(-2\pi\eta\lambda k) - (\mathrm{sign}(k) - \eta)\Big] \simeq 0. \tag{17}$$

In order to get non-vanishing solutions, the equation

$$\lambda|k| = \frac{1}{2\pi\eta}\mathrm{Log}\frac{1+\eta}{1-\eta} \tag{18}$$

has to be satisfied. Here the Log has to be interpreted as a multivalued function.

The condition $F(0,\infty) = 0$ entails the choice

$$\tilde{F}(k) = C(k)\cdot\delta[|k| - \frac{1}{2\pi\eta\lambda}\mathrm{Log}\frac{1+\eta}{1-\eta}], \tag{19}$$

where $C(k)$ is an odd function of k.

Finally the condition that $F(1,\infty)=0$ induces the eigenvalues

$$|k_n| = n\pi, \qquad\qquad n > 0, \tag{20}$$

namely

$$\alpha_n = n\,\frac{2\pi\eta g^2}{m^2\,\mathrm{Log}\frac{1+\eta}{1-\eta}}. \tag{21}$$

Each eigenvalue is thereby infinitely degenerate; but only the principal determination of the Log reproduces 't Hooft's spectrum in the limit $\eta = 0$. The vanishing of the first-order corrections in η found in ref. [12] is also confirmed.

Equation (21) nicely exhibits the interpolating role of the parameter η; eigenvalues stay discrete until the value $\eta = 1$ is reached; at that value, which is by the way the value pertinent to Wu's equation, the entire spectrum collapses to zero.

One should remember that expression (13) is exact for the particular value of the ratio $\frac{g^2}{\pi m^2} = 1$. For such a tuning of the coupling constant to the quark mass, the eigenvalues

$$\alpha_n = n\,\frac{2\pi^2\eta}{\mathrm{Log}\frac{1+\eta}{1-\eta}}, \tag{22}$$

give the (approximate) spectrum of the theory at the "critical" coupling $g^2 = \pi m^2$. This spectrum collapses to zero at $\eta = 1$.

The problem of studying the behaviour in a full neighbourhood of $\eta = 1$ is a difficult one. It might be that different limits are related to possible different phases of the theory. This interesting issue will be deferred to future investigation.

III. CONCLUSIONS

We have shown that Wu's equation for $q\bar{q}$ bound states, although much more involved than the corresponding 't Hooft's one, can nevertheless be explored, at least *formally*. We then obtained in a heuristic way approximate explicit results for a particular value of the ratio $\frac{g^2}{\pi m^2}$.

To go beyond our treatment seems an arduous task. One perhaps needs mathematical theorems controlling the spectrum of the involved operators. One could try numerical solutions of the Wick-rotated equation and/or try to develop a perturbation theory around the "critical" value $g^2 = \pi m^2$. In so doing, great care has to be taken when continuing the square root in the exact self-energy expression.

In the light of the results we have found, it seems unlikely that Wu's equation describe the $q\bar{q}$ bound state spectrum. In spite of the singular character of its potential, 't Hooft's equation looks in a much better shape on this point.

The way of handling SU(N) Yang–Mills theories in 1+1 dimensions by canonically quantizing them on the light-front, thereby reproducing 't Hooft's formulation, might be acceptable. After all, in so doing, no contradiction occurs with causality as, in strictly $1+1$ dimensions, there are no propagating vector degrees of freedom. Unitarity in turn is trivially satisfied, at least in the large-N (i.e. planar) approximation.

However, one should bear in mind that 't Hooft's theory cannot be considered as the limiting case in 1+1 dimensions of Yang–Mills theories in higher dimensions. There, equal-time quantization becomes compulsory in order to perform a consistent renormalization procedure [7], [8]. On the other hand, a Wilson loop calculation in $1+(d-1)$ dimensions, performed either in the Feynman gauge or in the light-cone gauge quantized at equal time, leads to a result which, in the limit $d = 2$, cannot be reconciled with 't Hooft's [8], [9].

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Appendix

The $P\exp$ in eq. (9) can be expressed through the normal form

$$P\exp\left\{2\pi i\eta\int_{-\infty}^{y} dy'\,\mathcal{S}(x,y')\,\partial_x\right\} \;=\; N_{x\partial_x}[\Phi(y,x,\partial_x)], \tag{23}$$

where the label $N_{x\partial_x}$ means that the operators x stand to the left of ∂_x.

Now we introduce the symbol of the operator $\Phi(y,x,\partial_x)$, which we denote by $\Phi(y,q,p)$, q and p being "dual" variables [14].

From the equation defining the $P\exp$, it is easy to derive, in terms of the function $\Phi(y,q,p)$, the relation

$$\frac{\partial\Phi}{\partial y} \;=\; 2\pi i\eta\mathcal{S}(q,y)\,\frac{\partial\Phi}{\partial q} \;-\; 2\pi\eta\mathcal{S}(q,y)\,p\Phi \tag{24}$$

with the initial condition

$$\Phi(y=-\infty) \;=\; 1.$$

One can search for solutions of the form

$$\Phi(y,q) \;=\; \theta(-q)\,\Phi_{-\infty 0} \;+\; \theta(q)\theta(1-q)\,\Phi_{01} \;+\; \theta(q-1)\,\Phi_{1\infty}. \tag{25}$$

Beginning with the second term and using the relations

$$\frac{1}{y+i\epsilon\ \mathrm{sign}(q)}\frac{1}{1-y+i\epsilon\ \mathrm{sign}(1-q)}\,\theta(q)\theta(1-q) \;=$$
$$=\;\frac{1}{y+i\epsilon}\frac{1}{1-y+i\epsilon}\,\theta(q)\theta(1-q)$$
$$\frac{\partial}{\partial q}\,\theta(q)\theta(1-q) \;=\; 2\theta(q)\theta(1-q)\,[\delta(q)\;-\;\delta(1-q)]$$

($\theta(0)=1/2$ is supposed here) one gets the equation for Φ_{01}:

$$\frac{\partial\Phi_{01}}{\partial y} \;-\; \varphi(y)\,\frac{1}{i}\,\frac{\partial\Phi_{01}}{\partial q} \;=\; \varphi(y)[p\;-\;2i(\delta(q)-\delta(1-q))]\,\Phi_{01}, \tag{26}$$

with

$$\varphi(y) \;=\; a\,\frac{1}{y+i\epsilon}\frac{1}{1-y+i\epsilon}, \qquad\qquad a=i\eta\lambda.$$

After introducing the variable

$$\xi(y) \;=\; a[\log(y+i\epsilon) \;-\; \log(1-y+i\epsilon) \;-\pi i]$$

which is chosen in such a way that $\xi(-\infty)=0$, the solution of eq.(26) is

$$\begin{aligned}\log\Phi_{01} \;&=\; p\xi \;+\; 2\theta(-q) \;+\; 2\theta(q-1) \;+\; \frac{1}{\pi i}\,[\log(q-i\xi-i\epsilon) \;-\; \log(q-i\xi+i\epsilon)]\\ &+\; \frac{1}{\pi i}\,[\log(1-q+i\xi-i\epsilon) \;-\; \log(1-q+i\xi+i\epsilon)].\end{aligned} \tag{27}$$

The $P\exp$ is actually needed only when the upper limit is $y=\infty$, namely $\xi=-2\pi i a$, and, whenever a has an imaginary part, the $i\epsilon$ term in the logarithms can be omitted.

In this case we obtain

$$\log\Phi_{01}(-2\pi i a, q) \;=-\,2\pi i a p. \tag{28}$$

which reproduces the expression in eq. (14).

The first and the third pieces of F in eq. (25) result in the same type of equations:

$$\frac{\partial\Phi_{-\infty 0}}{\partial y} \;-\; \varphi(y)\,\frac{1}{i}\,\frac{\partial\Phi_{-\infty 0}}{\partial q} \;=\; \varphi(y)[p\;+\;2i\delta(q)]\,\Phi_{-\infty 0}$$

and

$$\frac{\partial\Phi_{1\infty}}{\partial y} \;-\; \varphi(y)\,\frac{1}{i}\,\frac{\partial\Phi_{1\infty}}{\partial q} \;=\; \varphi(y)[p\;-\;2i\delta(q-1)]\,\Phi_{1\infty},$$

with the functions

$$\varphi(y) \;=\; a\,\frac{1}{y-i\epsilon}\,\frac{1}{1-y+i\epsilon}$$

in the equation for $\Phi_{-\infty 0}$ and

$$\varphi(y) \;=\; a\,\frac{1}{y+i\epsilon}\,\frac{1}{1-y-i\epsilon}$$

in the equation for $\Phi_{1\infty}$. The solutions to these equations are expressed by formulae analogous to (27) through the variable ξ:

$$\xi(y) \;=\; a[\log(y-i\epsilon) \;-\; \log(1-y+i\epsilon) \;+\pi i]$$

for $\Phi_{-\infty 0}$ and

$$\xi(y) \;=\; a[\log(y+i\epsilon) \;-\; \log(1-y-i\epsilon) \;-\pi i]$$

for $\Phi_{1\infty}$.

According to these definitions, $\xi(-\infty)=0$ but, owing to the signs of $i\epsilon$, $\xi(+\infty)=0$ too. Then $\log\Phi_{-\infty 0}=0$ and $\log\Phi_{1\infty}=0$.

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