



# GUIDE TO THE EXPRESSION OF UNCERTAINTY IN MEASUREMENT

This Guide establishes general rules for evaluating and expressing uncertainty in measurement that are intended to be applicable to a broad spectrum of measurements. The basis of the Guide is Recommendation 1 (CI-1981) of the Comité International des Poids et Mesures (CIPM) and Recommendation INC-1 (1980) of the Working Group on the Statement of Uncertainties. The Working Group was convened by the Bureau International des Poids et Mesures (BIPM) in response to a request of the CIPM. The CIPM Recommendation is the only recommendation concerning the expression of uncertainty in measurement adopted by an intergovernmental organization.

This Guide was prepared by a joint working group consisting of experts nominated by the BIPM, the International Electrotechnical Commission (IEC), the International Organization for Standardization (ISO), and the International Organization of Legal Metrology (OIML).

The following seven organizations supported the development of this Guide, which is published in their name:

BIPM	Bureau International des Poids et Mesures
IEC	International Electrotechnical Commission
IFCC	International Federation of Clinical Chemistry
ISO	International Organization for Standardization
IUPAC	International Union of Pure and Applied Chemistry
IUPAP	International Union of Pure and Applied Physics
OIML	International Organization of Legal Metrology

Users of this Guide are invited to send their comments and requests for clarification to any of the seven supporting organizations, the mailing addresses of which are given on the inside front cover.

# Contents

	Page		Page
<b>Foreword</b>	v		
<b>0 Introduction</b>	vii		
<b>1 Scope</b>	1		
<b>2 Definitions</b>	2		
2.1 General metrological terms	2		
2.2 The term "uncertainty"	2		
2.3 Terms specific to this <i>Guide</i>	3		
<b>3 Basic concepts</b>	4		
3.1 Measurement	4		
3.2 Errors, effects, and corrections	5		
3.3 Uncertainty	5		
3.4 Practical considerations	7		
<b>4 Evaluating standard uncertainty</b>	9		
4.1 Modelling the measurement	9		
4.2 Type A evaluation of standard uncertainty	10		
4.3 Type B evaluation of standard uncertainty	11		
4.4 Graphical illustration of evaluating standard uncertainty	14		
<b>5 Determining combined standard uncertainty</b>	19		
5.1 Uncorrelated input quantities	19		
5.2 Correlated input quantities	20		
<b>6 Determining expanded uncertainty</b>	23		
6.1 Introduction	23		
6.2 Expanded uncertainty	23		
6.3 Choosing a coverage factor	24		
<b>7 Reporting uncertainty</b>	25		
7.1 General guidance	25		
7.2 Specific guidance	25		
<b>8 Summary of procedure for evaluating and expressing uncertainty</b>	28		
		<b>Annexes</b>	
		<b>A Recommendations of Working Group and CIPM</b>	29
		A.1 Recommendation INC-1 (1980)	29
		A.2 Recommendation 1 (CI-1981)	30
		A.3 Recommendation 1 (CI-1986)	30
		<b>B General metrological terms</b>	31
		B.1 Source of definitions	31
		B.2 Definitions	31
		<b>C Basic statistical terms and concepts</b>	35
		C.1 Source of definitions	35
		C.2 Definitions	35
		C.3 Elaboration of terms and concepts	38
		<b>D "True" value, error, and uncertainty</b>	40
		D.1 The measurand	40
		D.2 The realized quantity	40
		D.3 The "true" value and the corrected value	40
		D.4 Error	41
		D.5 Uncertainty	41
		D.6 Graphical representation	42
		<b>E Motivation and basis for Recommendation INC-1 (1980)</b>	45
		E.1 "Safe," "random," and "systematic"	45
		E.2 Justification for realistic uncertainty evaluations	45
		E.3 Justification for treating all uncertainty components identically	46
		E.4 Standard deviations as measures of uncertainty	48
		E.5 A comparison of two views of uncertainty	49

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<b>F</b>	Practical guidance on evaluating uncertainty components . . . . .	51	<b>H</b>	Examples . . . . .	67
<b>F.1</b>	Components evaluated from repeated observations: Type A evaluation of standard uncertainty . . . . .	51	<b>H.1</b>	End-gauge calibration . . . . .	67
<b>F.2</b>	Components evaluated by other means: Type B evaluation of standard uncertainty . . . . .	53	<b>H.2</b>	Simultaneous resistance and reactance measurement . . . . .	72
<b>G</b>	Degrees of freedom and levels of confidence . . . . .	59	<b>H.3</b>	Calibration of a thermometer . . . . .	76
<b>G.1</b>	Introduction . . . . .	59	<b>H.4</b>	Measurement of activity . . . . .	79
<b>G.2</b>	Central Limit Theorem . . . . .	60	<b>H.5</b>	Analysis of variance . . . . .	83
<b>G.3</b>	The <i>t</i> -distribution and degrees of freedom . . . . .	60	<b>H.6</b>	Measurements on a reference scale: hardness . . . . .	88
<b>G.4</b>	Effective degrees of freedom . . . . .	61	<b>J</b>	Glossary of principal symbols . . . . .	91
<b>G.5</b>	Other considerations . . . . .	63	<b>K</b>	Bibliography . . . . .	94
<b>G.6</b>	Summary and conclusions . . . . .	64		Alphabetical index . . . . .	96