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MINIMISATION OF THE RAMP-INDUCED NON-LINEAR FIELD IMPERFECTIONS IN LHC

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We consider in this study the consequences of dynamic field imperfections of b_n, $n \geq 3$ on the dynamic aperture. They may be minimised by a proper shaping of the ramp-rate $\partial B(t)/\partial t$, at the expense of the ramp time. While the beam stability criterion chosen cannot be met with the MTP1A1 prototype, it is shown that MTP1A2 and MTP1A3 prototypes are very suitable for ramping. The consequences of b_1, b_2 and a_2 will be studied separately.

Keywords: Field imperfection; ramp-rate.

1 INTRODUCTION

At any static level of energy, field imperfections arise from mechanical tolerances and from the persistent currents in the filaments, which are the memory of the former variations of field. In this study, we assume that these field errors are taken care of either by design or proper correction.

During the ramp, a new kind of eddy current develops in the loops formed by the twisted strands. This phenomenon is referred to as inter-strand coupling. The resulting field imperfections are proportional to the ramp-rate and inversely proportional to inter-strand resistance, R_c.[1] R_c depends on the cable design and seems to show a large spread from loop to loop.

2 RELATIVE NORMAL FIELD ERRORS IN LHC DIPOLES DURING RAMPING

Let us recall the usual field expansion used to define the multipole coefficients:[2]

$$B_y+iB_x = \sum_{n=1}^{\infty}(B_n+iA_n)(x+iy)^{n-1} = B_{\text{main}} \sum_{n=1}^{\infty}(b_n+ia_n)\left(\frac{x+iy}{R_r}\right)^{n-1} \tag{1}$$

where B_n is the normal coefficient and A_n is the skew coefficient of the $2n$-pole, B_{main} is the amplitude of the nominal field of the magnet, R_r is the reference radius of the expansion, b_n is the normal relative coefficient and a_n is the skew relative coefficient of the $2n$-pole.

In Table I we summarise the values of the relative normal field imperfections at injection, in static and dynamic conditions, given in references:[3,4]

- Static conditions (b_{n0}): $\dot{B}(t=0) = \dot{B}_{\text{inj}} = 0$.
- Dynamic conditions ($b_n - b_{n0}$): $\dot{B}_{\text{inj}}$ = 0.4 T/min (nominal value).

where $\dot{B}(t) = \frac{\partial B(t)}{\partial t}$ is the ramp-rate. The static values were calculated taking in account the measured multipoles of the HERA, RHIC and prototype LHC magnets (MTP1A1, MTP1A2, MTP1A3). Systematic b_3 and b_5 will be compensated by correctors at each dipole end. The b_3 values in this

TABLE I Relative normal field errors in LHC dipoles at injection, all relative field errors are at R_r = 0.01 m, in units of 10^{-4}

	Static b_{n0} Expected		Dynamic $(b_n - b_{n0})$ MTP1A1	MTP1A2		MTP1A3
			Aper. 1	Aper. 1	Aper. 2	Aper. 1
n	syst.	random				
1			87.931		10.814	
2	±0.7	0.4	–0.96	–1.109	0.585	–0.106
3	–3.4±0.3	0.5	4.6	0.945	0.683	0.095
4	±0.2	0.1	0.447	–0.129	0.034	0.024
5	0.25±0.05	0.08	2.08	0.045	–0.003	
6	–0.004	0.02				
7	–0.026	0.01				0.001
8		0.005				
9	0.006	0.003				
10		0.002				
11	0.008	0.001				

table for static conditions indicate the magnitude of the persistent current effect, the coil geometry being designed for $b_3 = 0$. The dynamic values are those measured on the actual LHC magnet prototypes MTP1A1, MTP1A2, MTP1A3.

The ramp induced field errors are of the same order as the static errors in most of these cases. At a constant ramp rate the eddy currents produce a constant field, so that the relative field errors are highest at injection. The errors are proportional to $R_c^{-1}\dot{B}$. It is probable that b_1, b_3 and b_5 would be systematic, whereas b_2 and b_4 would be random.[4] MTP1A1 has the lowest R_c ($R_c = 1.6\mu\Omega$) of all magnets measured and could therefore be considered as a worst case. On the contrary MTP1A2 ($R_c = 6.5\mu\Omega$) and MTP1A3 ($R_c = 14.0\mu\Omega$) have a value of R_c close to the design value.

3 MINIMISATION OF RELATIVE NORMAL FIELD ERRORS IN LHC DIPOLES

An increase of the field imperfections during the ramp is unavoidable. It will distort the linear optics and reduce the dynamic aperture. In this study, we only consider the second effect. It is still possible to limit it to fulfil some criterion on beam stability by initially reducing the ramp rate $\dot{B}$. At the end of this initial phase $\dot{B}$ attains its nominal value and the rest of the ramp is linear. In this study the maximum ramp rate $\dot{B}_{\max}$ is assumed to be a parameter which may allow to control the total ramp time.

3.1 Criterion for Beam Stability

There is so far no reliable law to predict the dynamic aperture for a given configuration of magnetic field imperfections. One normally resorts to tracking, which is much too heavy for this kind of study as it requires about 1 day of CPU for each and any field configuration under study.

A method often employed is to compute one parameter of the non-linear system which lends itself to a faster calculation: onset of chaos, "norm" of the non-linear part of the map, amplitude detuning, resonance strength, e.g.[5,6] and demonstrate that it carries a reasonable correlation with the dynamic aperture in the domain of interest.

In this study we rather use "loosely" an exact scaling law of the dynamic aperture. We know from the invariance of the equation of motion that, if each $2n$-pole b_n is multiplied by k^{n-2}, k being an arbitrary number, then the dynamic aperture is reduced by k. For example, the dynamic aperture is reduced by 2 if: b_3, b_4, b_5, b_6,... are respectively multiplied by 2, 4, 8, 16....[7,8] We now make the assumption that, if the dynamic aperture is to be significantly reduced by a small increase of a single multipole b_n, then this multipole must dominate; we can thus apply the scaling law to this single multipole. If we require the dynamic aperture A (expressed in amplitude) to decrease by not more than $(1 - \xi_a)$:

$$A(t) \geq A_0(1 - \xi_a) \tag{2}$$

the dominant multipole b_n shall not exceed:

$$b_n(t) \leq b_{n0}(1 + \xi_b), \quad (1 + \xi_b)^{n-2} = (1 - \xi_a) \tag{3}$$

A tight control of the dynamic aperture being required, we take $\xi_a = 5\%$.

3.2 Ramp at Constant Field Imperfections

The theory and the measurements show that the absolute field error, B_n, related to the relative field error $b_n = R_r^{n-1}\frac{B_n}{B_{\text{main}}}$, depends linearly on $\dot{B}$ like:

$$B_n(t) = B_{n0} + k_n\dot{B}(t) \tag{4}$$

where B_{n0} is the absolute static field error and k_n is a constant which depends on the cable design. In this expression we assume that B_{n0} does not depend on $B(t)$. This assumption is valid around injection, where it is most critical.

The condition for a constant perturbation equal to the maximum allowed by beam dynamics is, assuming that b_n is dominant:

$$b_n(t) = R_r^{n-1}\frac{B_n(t)}{B(t)} = b_{n0}(1 + \xi_b) = b_n, \quad \forall t \tag{5}$$

From Equations (4) and (5) we can obtain the differential equation:

$$\dot{B}(t) - \frac{b_n}{k_n R_r^{n-1}}B(t) + \frac{B_{n0}}{k_n} = 0. \tag{6}$$

The solution of this differential equation is an exponential increase of the guide field:

$$B(t) = \left(B_{\text{inj}} - \frac{B_{n0} R_r^{n-1}}{b_n}\right) \exp\left(\frac{b_n}{k_n R_r^{n-1}} t\right) + \frac{B_{n0} R_r^{n-1}}{b_n} \tag{7}$$

The constants B_{n0} and k_n are computed from observables (Table I):

$$B_{n0} = B_{\text{inj}} \frac{b_{n0}}{R_r^{n-1}} \tag{8}$$

$$k_n = \frac{B_{\text{inj}}}{R_r^{n-1} \dot{B}_{\text{inj}}} (b_n - b_{n0}) \tag{9}$$

In the following when we refer to this part of the ramp we put the subscript "exp" in the variables.

3.3 Ramp Profile and Parameters

The ramp profile and parameters are given in Figure 1.

Taking as parameter $b_{n,\text{exp}}$, knowing that $B_{\text{inj}} = 0.58$ T, $B_{\text{col}} = 8.4$ T and assuming a maximum ramp-rate, $\dot{B}_{\text{max}}$, we can calculate the parameters of the ramp from the formulas above.

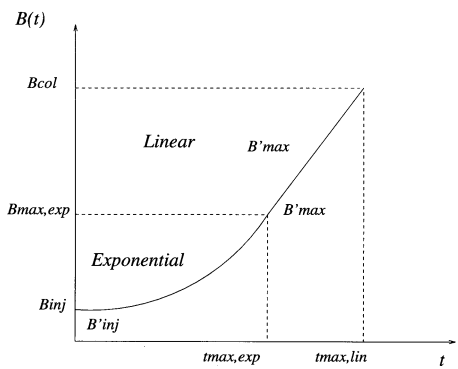

FIGURE 1 Ramp profile (B' is equivalent to $\dot{B}$).

4 BEHAVIOUR OF OTHER NORMAL FIELD ERRORS WHEN THE EFFECT OF ONE OF THEM IS MINIMISED

The aim of this section is to calculate the behaviour of the other field imperfections when we minimise the effect of one of them by adjusting the ramp parameters.

4.1 Exponential Part of the Ramp

From Equations (4) and (5), two different multipolar coefficients are related by:

$$B(t) = \frac{R_r^{n-1}(B_{n0} + k_n \dot{B}(t))}{b_n(t)} = \frac{R_r^{m-1}(B_{m0} + k_m \dot{B}(t))}{b_m(t)} \tag{10}$$

From Equation (10) and assuming that we minimise $b_n(t) = b_{n,\exp}$ adjusting the ramp parameters, one can write for $b_m(t)$ the expression:

$$b_m(t) = R_r^{m-n} b_{n,\exp} \frac{B_{m0} + \frac{k_m}{k_n}\left(\frac{b_{n,\exp} B_{\text{inj}}}{R_r^{n-1}} - B_{n0}\right)\exp\left(\frac{b_{n,\exp}}{k_n R_r^{n-1}} t\right)}{B_{n0} + \left(\frac{b_{n,\exp} B_{\text{inj}}}{R_r^{n-1}} - B_{n0}\right)\exp\left(\frac{b_{n,\exp}}{k_n R_r^{n-1}} t\right)} \tag{11}$$

$b_m(t)$ is a decreasing function if we can meet the condition:

$$\frac{(k_m B_{n0} - k_n B_{m0})}{k_n} < 0 \tag{12}$$

4.2 Linear Part of the Ramp

For the linear part of the ramp we have the same behaviour for all the coefficients.

An example of the profiles of $b_n(t)$ and $b_m(t)$ during the ramp is given in Figure 2. $b_n(t)$ and $b_m(t)$ decrease in the linear ramp if:

$$B_{n0} + k_n \dot{B}_{\max}) > 0, \quad B_{m0} + k_m \dot{B}_{\max}) > 0 \tag{13}$$

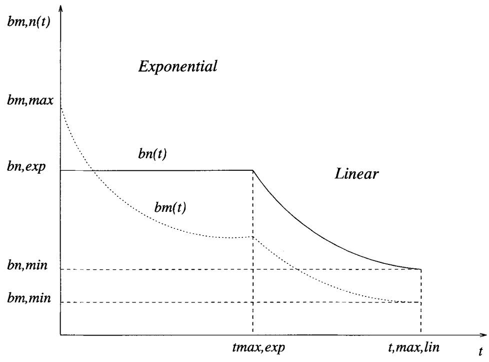

FIGURE 2 Profile of $b_n(t)$ and $b_m(t)$ during the ramp adjusted taking as parameter $b_{n,\text{exp}}$.

5 MINIMISATION OF RELATIVE NORMAL FIELD COMPONENTS IN MTP1A1, MTP1A2 AND MTP1A3 DIPOLES

All the parameters and relations mentioned so far have been implemented in a Mathematica program. It will be used for the following case studies.

Usually the larger field imperfections produced by the inter-strand coupling current in dipoles are the normal sextupole (b_3) and decapole (b_5) terms. We have three possible cases:

- b_3 is the most important imperfection, in that case from beam dynamics we know that the dynamic aperture is proportional to $1/b_3$. The criterion that we take is that b_3 should not increase by more than 5% in dynamic conditions just after t=0.
- b_5 is the most important imperfection, in that case from beam dynamics we know that the dynamic aperture is proportional to $1/(b_5)^3$. The criterion is that b_5 should not increase by more than 15% in dynamic conditions just after t=0.
- b_3 and b_5 both contribute, hence the two conditions should be satisfied simultaneously.

Sometimes the important imperfection is b_7. In that case the criterion is that b_7 should not increase by more than 27% in dynamic conditions just after t=0.

TABLE II Ramp performances for MTP1A1 dipole, adjusted taking as parameter $b_{5,\mathrm{exp}}$

$\dot{B}_{\mathrm{max}}$	0.400	[T/min]
$\dot{B}_{\mathrm{inj}}$	0.012	[T/min]
T_{exp}	72.225	[min]
$B_{\mathrm{max,exp}}$	8.650	[T]
T	72.225	[min]
$\xi_b(n=3)$	28.676	%
$\xi_b(n=5)$	81.041	%

5.1 MTP1A1 (10 Meter Dipole)

From Table I both b_3 and b_5 show a large dynamic increase; we should try to minimise the two components at the same time. Condition (12) shows that if we adjust the ramp-parameters to minimise $b_5(t = 0) = b_{5,\mathrm{exp}}$, then $b_3(t)$ is a decreasing function in the exponential part of the ramp. Hence we take $b_5(t = 0) = b_{5,\mathrm{exp}}$ as parameter to adjust the ramp.

It may be that no exponential ramp is found which satisfies the criterion on beam stability. The minimum increase ξ_b is obtained for a purely exponential ramp (no linear part) as summarised in Table II. The increase of $b_{5,\mathrm{exp}}$ by 81% is unacceptable, as well as the ramp time of over 70 minutes.

5.2 MTP1A2 (10 Meter Dipole)

5.2.1 Aperture 1 From Table I both b_3 and b_5 show a large dynamic increase; we should try to minimise the two components at the same time. Condition (12) shows that if we adjust the ramp-parameters to minimise $b_3(t = 0) = b_{3,\mathrm{exp}}$, then $b_5(t)$ is a decreasing function in the exponential part of the ramp. Hence we take $b_3(t = 0) = b_{3,\mathrm{exp}}$ as parameter to adjust the ramp. The solution found is summarised in Table III. The ramp time T is increased from to 20 to 27 min. The nominal time may be recovered by increasing the nominal ramp-rate by 50%.

5.2.2 Aperture 2 From Table I we can see that the most important imperfection in this aperture is b_3. We should try to minimise this component without disturbing very much b_5 which in this case is lower than the static value. Condition (12) is the same than in Aperture 1. Hence we take $b_3(t = 0) = b_{3,\mathrm{exp}}$ as parameter to adjust the ramp. The solution found is

TABLE III Ramp performances for MTP1A2 (Aperture 1 and 2) dipole adjusted taking as parameter $b_{3,\exp}$, for the different values of $\dot{B}_{\max}$ considered

	Aperture 1			*Aperture 2*			
$\dot{B}_{\max}$	0.400	0.600	0.800	0.400	0.600	0.800	[T/min]
$\dot{B}_{\text{inj}}$	0.011	0.011	0.011	0.015	0.015	0.015	[T/min]
$T_{\exp}$	9.480	10.539	11.289	6.239	7.004	7.547	[min]
$B_{\max,\exp}$	1.596	2.118	2.640	1.307	1.684	2.061	[T]
T	27.114	21.425	18.802	24.597	18.614	15.783	[min]
$\xi_b(n=3)$	5.000	5.000	5.000	5.000	5.000	5.000	%
$\xi_b(n=5)$	1.488	1.488	1.488	–0.137	–0.137	–0.137	%

summarised in Table III. The increase in ramp time for the nominal $\dot{B}$ seems acceptable.

5.3 MTP1A3 (10 Meter Dipole)

From Table I we can see that the most important imperfection is b_3. We do not have a contribution from b_5 but we have a contribution from b_7. Then we should try to minimise b_3 without disturbing very much b_7. Condition (12) shows that if we adjust the ramp-parameters to minimise $b_3(t = 0) = b_{3,\exp}$, then $b_7(t)$ decreases.

The solution, which only requires 20 seconds of exponential ramp is shown on Figures 3 and 4 and Table IV.

6 CONCLUSION

In this study, we considered the case of three individual magnets. What matters is really the spread of imperfections for the whole population of dipoles; the average b_3 and b_5 can indeed be corrected with the spool pieces. A spread comparable to the imperfections of MTP1A1 would significantly reduce the LHC dynamic aperture during the ramp. A spread comparable to the imperfections of MTP1A2 or even better MTP1A3 would not degrade significantly the dynamic aperture. MTP1A2 might be seen as an acceptance limit, as higher imperfections could be manageable, but at the expense of a significant increase of the already long ramp time. These conclusions should be confirmed by tracking studies.

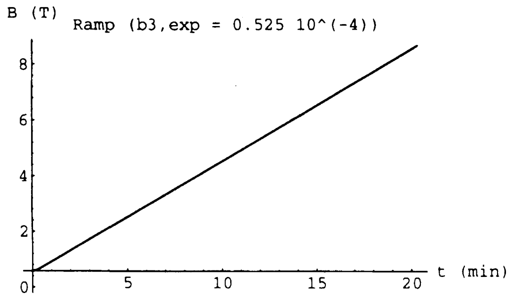

FIGURE 3 B as a function of t, $b_{3,\mathrm{exp}} = 0.525\ 10^{-4}$ ($\dot{B}_{\mathrm{max}} = 0.4$ T/min), for MTP1A3 dipole.

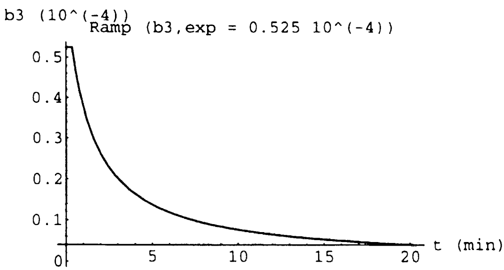

FIGURE 4 Profile of b_3 and b_7 as a function of t, for MTP1A3 dipole adjusted taking as parameter $b_{3,\mathrm{exp}}$.

TABLE IV Ramp performances for MTP1A3 dipole adjusted taking as parameter $b_{3,\mathrm{exp}}$

$\dot{B}_{\mathrm{max}}$	0.400	[T/min]
$\dot{B}_{\mathrm{inj}}$	0.105	[T/min]
T_{exp}	0.350	[min]
$B_{\mathrm{max,exp}}$	0.657	[T]
T	20.332	[min]
$\xi_b(n{=}3)$	5.000	%
$\xi_b(n{=}7)$	–1.645	%

Acknowledgements

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