Particle Accelerators, 1996, Vol. 55, pp. [429–437] / 183–191
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CORRECTION OF THE BETATRON COUPLING IN THE LHC

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(Received 20 January 1996; in final form 20 January 1996)

In LHC, the width of the systematic linear coupling resonance is of the order of 0.5. The origin is to be found in the field imperfections inherent to super-conducting magnets and in the size of the machine. Albeit very strong, the coupling is shown to be still well modelled by the resonance theory; it can thus be efficiently suppressed by only two families of skew quadrupoles. In practice, the correction should be maintained to very high accuracy. A robust correction is shown to be obtained if the betatron tunes are split by one to three units. Such tune splits can be obtained by breaking the basic antisymmetry of the LHC optics.

Keywords: Optics; betatron coupling.

1 INTRODUCTION

Even-though the position of the dipole coils are controlled to a few micrometers, the integrated skew quadrupolar component of the guide field reaches an unprecedented value. It confuses the standard technique to correct betatron coupling in the numerical model of LHC and perturbs all optical parameters. The question of the theoretical approach to be used to represent strong coupling was raised and is discussed here. Designing an efficient correction scheme is not sufficient. The optics must be robust against a possible imbalance between perturbation and correction. This situation arises typically from the decay of the persistent currents which are neither exactly predictable nor identical from magnet to magnet. This is the second issue discussed in this paper.

TABLE I Comparison between the sources of betatron coupling

Source	*Term*	*Strength*
CMS Solenoid, 24 Tm, at injection	$\frac{B_s l}{B\rho}$	0.003
Vertical orbit in sextupoles	$K'l y_{co}$	0.017 mm^{-1}
Vertical orbit in random b_3 of dipoles	$\frac{2b_3}{R_r^2}\theta_B y_{co}$	0.005 mm^{-1}
Random a_2=1.210^{-4} in the dipoles	$\frac{a_2}{R_r}\theta_B$	0.03
Random tilt of quads	$2Kl\phi$	0.05 mrad^{-1}
Systematic vertical misalignment of b_3 correctors versus average dipole position	$\frac{2b_3}{R_r^2}\theta_B \Delta y$	0.5 mm^{-1}
Systematic a_2=0.7610^{-4} in the dipoles	$\frac{a_2}{R_r}\theta_B$	0.54

2 SOURCES OF COUPLING

The hierarchy of the coupling sources is rather different from other accelerators, both due to the super-conducting technology, size of the machine and high energy. The strength of the most important sources are summarized in Table I. For each source, the term driving the resonance is specified. The resonance strength is computed by multiplying this term by $\sqrt{\beta_x \beta_y}/2\pi$ and by a phase term following.[1] The resonance strength, in general equal to the closest tune approach, is expressed in tune units. The symbols used are: $B_s l$ integrated solenoidal field, $B\rho$ magnetic rigidity, $K'l$ integrated sextupole strength, y_{co} vertical closed orbit, a_2, b_3 skew quadrupolar and sextupolar field imperfections in the dipole, R_r reference radius, θ_B bending angle of a dipole, Δy vertical misalignment, Kl integrated quadrupole strength, ϕ tilt angle of a quadrupole (except low-β triplets).

This calculation is made for the nominal working point. Due to the antisymmetry of the LHC optics, which is heavily relied upon to minimize the number of independent quadrupoles in the insertions, the horizontal and vertical tunes are almost equal ($Q_x - Q_y \approx 0.03$).

The overwhelming source of coupling is the systematic a_2 in the dipoles, while the strong CMS solenoid is already negligible at injection. The resonance width is of the order of one tune unit (now 0.23 with the latest optimization of the dipole).[2] Odd effects such as a possible systematic misalignment of the sextupole corrector with respect to the geometry of persistent currents in the dipole show up as a noticeable source of coupling. On the other hand, random effects are comparable to those observed in other machines and should be easy to correct.

3 DECOUPLING IN STATIC CONDITIONS

3.1 LHC Version 1

In spite of the strength of the a_2 perturbation, the decoupling of LHC version 1[3] was straightforward. The correction scheme was made of four families of skew quadrupoles to be able to correct two coupling resonances. Decoupling was achieved by enforcing the off-diagonal elements of the one-turn linear map to vanish. After coupling correction, the dynamic aperture was found only insignificantly reduced with respect to a machine without coupling.

3.2 LHC Version 2

When the number of cells in the LHC arc was decreased from 25 to 24 (version 2)[4] to allow a higher filling factor, the coupling became suddenly overwhelming. The approach followed so far to coupling compensation did not allow to cancel the strength of the near-by difference coupling resonance. A very large β-beating arose due to the correction and the dynamic aperture decreased.

We had in fact faced such a situation during the commissioning of LEP, tried various techniques based on resonance compensation. Only a tune-split solved the problem. In LHC, a tune-split is in principle inconsistent with the antisymmetry of the optics (antisymmetry about the interaction point and antisymmetry between the two rings at the same azimuth). It was thus an incentive to find out whether the resonance model was still valid for such a strong perturbation. In a first numerical experiment, the closest tune approach is calculated for an optics perturbed by a_2 and not corrected (Figure 1). As can be seen, the dependence of the tune split versus a_2 is purely linear. In a second experiment, the β-beating produced by the coupling correction was

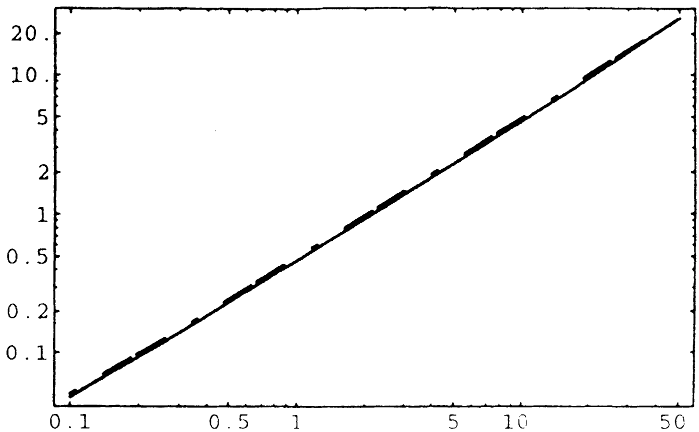

FIGURE 1 Dependence of the tune split $|Q_I - Q_{II}|$ on a_2 expressed in unit of nominal a_2.

calculated as a function of the strength of the initial coupling perturbation.[2] The β-beating increases quadratically with the perturbation. This behaviour can be interpreted[5] from the excitation terms of the difference and sum coupling resonances.

$$c_{\mp}^{p} = \frac{1}{2\pi} \oint \sqrt{\beta_x \beta_y} K_s e^{i[\mu_x \mp \mu_y - \theta \Delta^p]} ds \tag{1}$$

where K_s is the normalized skew gradient and the azimuthal angle (0 to 2π). $\Delta^l = Q_x \mp Q_y - l$ is the distance to the resonance and s the azimuthal position.

Before coupling correction, the constant skew quadrupolar perturbation can only excite significantly the coupling resonances for which the phase term in (1) is stationary. For almost equal betatron tunes, only one difference resonance is excited ($Q_x = Q_y$) and no sum resonance. Such a resonance is expected to yield a tune separation but no significant β-beating.

After coupling correction, the 64 skew quadrupole correctors are strongly excited. There are thus liable to excite significantly other coupling resonances depending on phase terms. The observed consequence was a confusion of the numerical minimization algorithm used for decoupling the one-turn map.

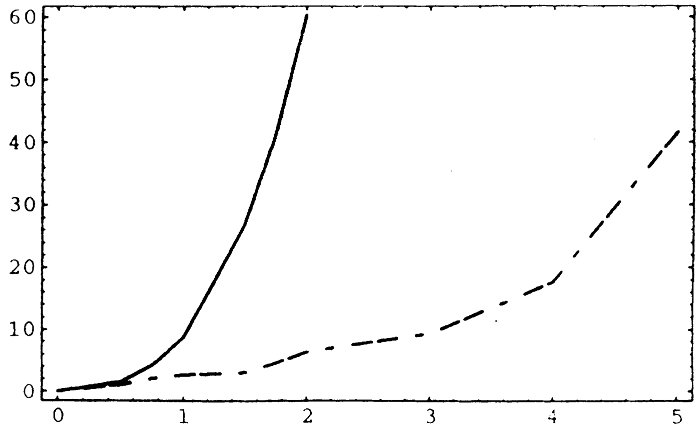

FIGURE 2 Dependence of β_x (plain) and β_y (dotted) beatings in % on the compensated a_2 in LHC version 2 (expressed in unit of nominal a_2).

The strong excitation of the near-by sum resonance in version 2 and not in version 1 is due to a peculiarity of the LHC optics induced by its antisymmetry: although the super-periodicity of the focusing is 4, the super-periodicity of the sum resonance ($Q_x + Q_y$) is 8 because the betatron phases advance by the same amount in each cell and each insertion (antisymmetry). In LHC version 1 ($Q_x + Q_y \approx 140$), the sum resonance is not super-periodic. In LHC version 2, ($Q_x + Q_y \approx 17 \times 8$), it is (Figure 3). One expect from this resonance a contribution to the tune separation and a quadratic β-beating, as observed in the numerical studies.

3.3 Correction Scheme

Since a two resonance model seems to explain well the situation, it can be used to define the correction strategy: rather than cancelling the off-diagonal terms of the one-turn linear map, the only coupling resonance significantly excited by the systematic a_2 is cancelled by a single family of skew quadrupoles placed in phase with the dipoles. The skew quadrupoles are furthermore arranged in pairs spaced by a betatron phase shift of $\pi/2$, i.e. one cell in LHC. In this way, it can be verified in (1) that the sum resonance is not excited. To demonstrate the efficiency of the scheme, various

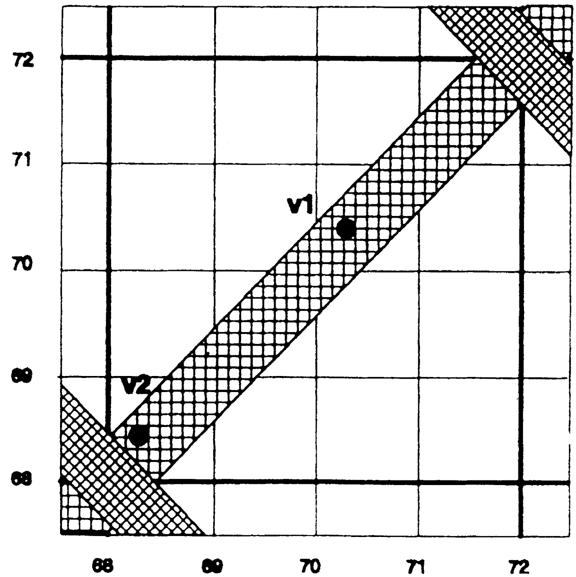

FIGURE 3 Systematic coupling resonances in LHC in Q_x, Q_y plane.

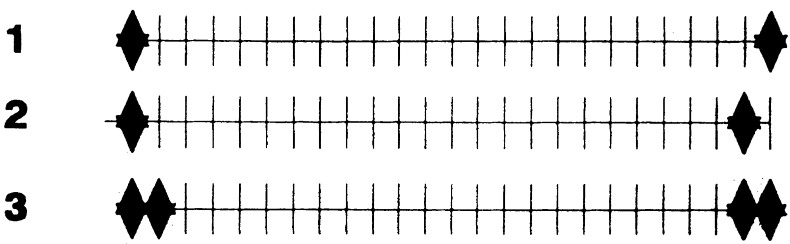

FIGURE 4 Skew quadrupole schemes tried for LHC version 2 (unit spacing is one cell).

TABLE II Decoupling results for LHC version 2

Case	*Decoupling scheme*	$\lvert Q_I - Q_{II} \rvert$	$\Delta\beta_x/\beta_x$	$\Delta\beta_y/\beta_y$
0	before decoupling	0.496	3.4%	0.3%
1	one skew at each arc end	0.065	64.0%	41.0%
2	(1) with one skew shifted by $\pi/2$	0.0027	7.6%	3.6%
3	one pair at each arc end	0.0026	1.04%	0.96%

TABLE III Sensitivity of various machines to coupling

Machine	$|c|$ *natural*	$|Q_x - Q_y|$ *nominal*	*ratio*
ISR	0.01	0.025	0.4
SPS	0.01	0.005	2.0
LEP (tune split of 2)	0.025	0.1	0.25
HERA (tune split of 1)	0.06	0.01	6.0
LHC version 4	0.5	0.01	50.0

solutions were experimented. They are shown on Figure 4 and Table II. With pairs, the β-beating disappears. The minimum tune separation may be brought to any arbitrarily small value by adding one weak skew quadrupole in quadrature with the first family. Tracking runs[6] have shown no loss of dynamic aperture after this correction, as compared to a machine without coupling. Further studies on the dynamic aperture[7] show that the correction scheme, found to be sufficient, is necessary to avoid a loss of dynamic aperture should the coupling increase due to ramp induced eddy currents.

4 DYNAMIC EFFECTS

The problem would be overcome if the perturbation was exactly known and thereby correctable. The ratio of the nominal tune separation to the tune separation enforced by coupling before any correction (Table III) is a measure of the sensitivity of the lattice to the finite precision of the correction. In LHC, the nominal tune separation is only 2% of the perturbation which must therefore be controlled to about 1%, i.e., the integral of a_2 should be known with an accuracy of $0.01 \cdot 10^{-4}$ which looks indeed very tight. A lattice less sensitive to betatron coupling is therefore necessary for LHC. Three possibilities are contemplated:

- the coherent contribution of each arc to coupling can be suppressed if the horizontal and vertical betatron phases advance differently in each insertion.
- Another way is a difference between the horizontal and vertical phase advances per cell in the arc.
- If this difference is increased to $2\pi/n_{\text{cell}}$ each arc is self-compensated with respect to systematic coupling.

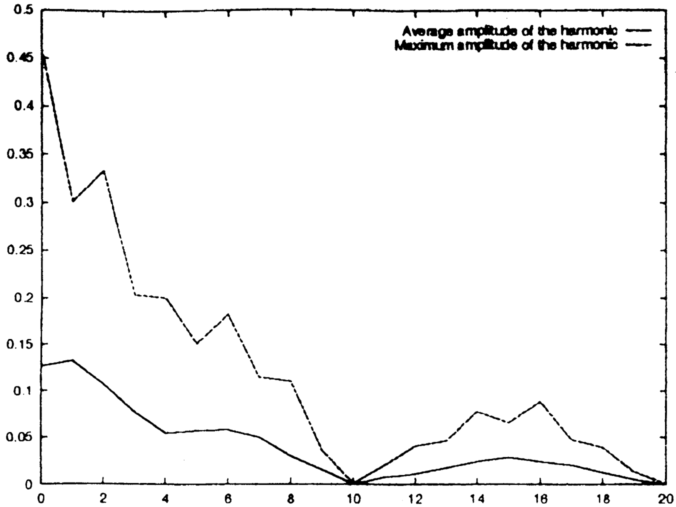

FIGURE 5 Amplitude a systematic a_2 versus harmonic number, average and worst case.

Each solution has a significant impact on the LHC optics: a differential phase shift in the insertions favours symmetric insertions with the consequence that the two LHC rings would be different; a differential phase shift in the arcs breaks the exact antisymmetry and requires more parameters for matching the insertions; the third solution respects the exact antisymmetry but causes the optical functions to be modulated in the arcs in a significant way.

The method being explored is the tune split in the arcs. Modelling a_2 by a systematic and a random component yields the obvious result that a tune split of one unit is sufficient. A simulation was done with a scenario where the systematic a_2 is assumed to be different for each of four manufacturers and from the inner to the outer channels. One furthermore assumes that each arc is assembled from magnets of the same manufacturer or production line. Each a_2 is drawn from a rectangular distribution; the maximum is the systematic a_2 used earlier. The results over 1000 cases are plotted on Figure 5. Computing the closest tune approach from these cases yields that the tune split required to decrease the coupling strength (before correction) by one order of magnitude varies between 1 and 3 tune units.

5 CONCLUSION

Albeit ten times larger than in other accelerators, the betatron coupling in LHC is well represented by the the excitation of one difference and one sum linear coupling resonance. This model allows the design of a simple scheme mainly based on one family of skew quadrupoles. The distribution of skew quadrupoles follows the general symmetries of the machine (one pair at each end of each arc). Dynamic aperture studies show that this distribution is both necessary and sufficient. This correction is both perfect and fragile as it relies on the knowledge of the field perturbation to an accuracy of some 10^{-6} relative to the main field. A tune split varying between 1 and 3 units, depending on the harmonic content of the perturbation, allows reducing the perturbation to the required level. To achieve the tune splits, the exact antisymmetry of the optics and of the rings must be abandoned thereby requiring a larger number of independently powered quadrupoles in the insertions. Lattice studies are going along these lines.

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