Type IIB instanton
as a wave in twelve dimensions

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Abstract

0-brane of type IIA string theory can be interpreted as a dimensional reduction of a gravitational wave in 11 dimensions. We observe that a similar interpretation applies also to the D-instanton background of type IIB theory: it can be viewed as a reduction (along one spatial and one time-like direction) of a wave in a 12-dimensional theory. The instanton charge is thus related to a linear momentum in 12 dimensions. This suggests that the instanton should play as important role in type IIB theory as the 0-brane is supposed to play in type IIA theory.

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Recently, it was suggested [1,2] that 0-branes [3] may be considered as basic building blocks of M-theory. This is related to the fact that the 0-brane charge can be interpreted as a 11-dimensional momentum. At the level of the classical solutions, the $D = 10$ 0-brane background is just a dimensional reduction of a gravitational wave propagating in 11 dimensions, $ds_{11}^2 = -dt^2 + dx_{11}^2 + \frac{q}{r^7}(dt - dx_{11})^2 + dx_i dx_i$. The fundamental nature of the 0-brane is also indicated by the fact that other extended objects in the theory can be 'constructed' out of arrays of 0-branes by duality transformations.

It seems important to understand the type IIB theory analog of this picture. The object of minimal dimensionality here is the D-instanton [4,3,5]. It is related to D0-brane by formal T-duality in the time direction. This is a hint that the instanton should play a central role in (a 12-dimensional reformulation of) type IIB theory. In fact, the recent proposal of a matrix model behind type IIB theory [6] which is based on a large N limit of the zero-dimensional reduction of SU(N) 10-d super Yang-Mills theory may be interpreted in this way $(L = \text{tr}([A_\mu, A_\nu]^2 + 2i\bar{\psi}\gamma^\mu[A_\mu, \psi])$ is the leading term in the action for N D-instantons [7,5]).[1]

Below we shall provide a new evidence of a fundamental nature of the type IIB instanton, and, at the same time, of an existence of a 12-dimensional structure behind type IIB theory: just like the type IIA 0-brane corresponds to a gravitational wave in 11 dimensions, the type IIB instanton is an 'image' of a gravitational wave in 12 dimensions. In particular, the instanton charge is this identified with the 12-dimensional momentum.

Our discussion of the 12-dimensional interpretation of the type IIB instanton solution of [9] (see also [10]) will be in the spirit of the F-theory proposals in [11] and, especially, in [12].[2] There will be an important new point: we will need to consider the *euclidean* version of type IIB theory (with signature (0,10)), and thus will compactify the $D = 12$ theory with signature (1,11) on a 2-space of (1,1) signature.

While a hypothetical 12-dimensional theory which leads to type IIB theory upon dimensional reduction can not be of the standard supergravity type, it may not be that different, assuming certain additional constraints are imposed. The $SL(2, R)$ structure of the field equations [14] or the action [15] of type IIB theory provide strong hints about its 12-dimensional counterpart. Let us first consider the type $IIB_{(1,9)}$ theory with Minkowski signature. The 12-dimensional action should contain at least the Einstein term, probably supplemented with certain conditions on the $D = 12$ metric. We shall adopt the following ansatz for the metric $(\mu, \nu = 0, 1, ..., 9; \ p, q = 1, 2)$

$$ds_{12}^2 = ds_{10E}^2 + ds_2^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + M_{pq}(x)dy^p dy^q \tag{1}$$

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[1] In view of T-duality between instantons and 0-branes, it is not surprising that this action can be formally related [6] to the 0-brane action used in [2]. A similar action but in 12 dimensions was suggested in connection with the action of [2] in [8].

[2] For other suggestions about 12-dimensional origin of type IIB theory see [13].

$$= g_{\mu\nu}(x)dx^\mu dx^\nu + e^{-\phi(x)}dy_1^2 + e^{\phi(x)}[dy_2 + C(x)dy_1]^2 \ ,$$

where $g_{\mu\nu}, \phi$ and C are the Einstein-frame metric, dilaton and R-R scalar of IIB theory. The metric M of an internal 2-torus with the complex structure modulus τ is

$$M_{pq} = e^\phi \begin{pmatrix} e^{-2\phi} + C^2 & C \\ C & 1 \end{pmatrix} \ , \quad \tau = C + ie^{-\phi} \ , \quad \det M_{pq} = 1 \ . \tag{2}$$

Note that in contrast to the similar relation between the $D = 11$ supergravity and type IIA theory metrics, i.e. $ds_{11}^2 = e^{-\phi/6}ds_{10E}^2 + e^{4\phi/3}(dy + A_\mu dx^\mu)^2$, the ansatz (1) is not of the most general type: the volume of the internal 2-torus is assumed to be non-dynamical ($\det M = 1$) [12] as there are only two scalars in type IIB action.[3] Dimensional reduction then gives

$$S = \int d^{12}x \sqrt{g_{(12)}} R_{(12)} = \int d^{10}x \sqrt{g} [R + \tfrac{1}{4}\mathrm{Tr}(\partial_\mu M \partial^\mu M^{-1})] \tag{3}$$

$$= \int d^{10}x \sqrt{g} [R - \tfrac{1}{2}\tau_2^{-2}|\partial\tau|^2] = \int d^{10}x \sqrt{g} [R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}e^{2\phi}(\partial C)^2] \ . \tag{4}$$

The metric (1) and the action (3) are covariant under the $SL(2, R)$ transformations, explaining the corresponding symmetry of type IIB theory [11,12]. Other terms in the bosonic part of the type IIB action can be understood by making a bold assumption that the $D = 12$ theory should contain also the 3-rank and 4-rank antisymmetric tensors C_3 and C_4.[4] Then the two 2-rank tensors of type IIB theory B_p $(p = 1, 2)$ appear as $C_{\mu\nu p}$ components of C_3 and, moreover, the natural kinetic term $F^2(C_3)$ reduces to another important $SL(2, R)$ covariant structure in the type IIB action, $M^{pq}dB_p dB_q$. The $D = 12$ Chern-Simons coupling $\int C_4 \wedge dC_3 \wedge dC_3$ produces the needed $\int C_4 \wedge dB^{(1)} \wedge dB^{(2)}$ term in IIB theory action [16]. Obviously, there should be other magical constraints that should (i) rule out various extra terms which appear from $\int [R_{(12)} - F^2(C_3) - F^2(C_4) - C_4 \wedge dC_3 \wedge dC_3 + ...]$ upon direct dimensional reduction, (ii) imply self-duality of the field strength of C_4 in $D = 10$ and, of course, (iii) ensure the existence of supersymmetry.

Assuming the existence of such 12-dimensional theory, it should be possible to relate type IIB p-brane solutions to certain 12-dimensional field configurations. It is natural to expect that the $D = 12$ theory should have 3-brane and 5-brane solutions (which are 'electro-magnetic' dual in $D = 12$). The $SL(2, Z)$ family of type IIB strings [17] then may

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[3] This restriction may be related to some extra symmetry (conformal invariance?) of the $D = 12$ theory.

[4] It may be more natural to assume that the fundamental field of $D = 12$ theory is only C_4 (which already has enough components) while C_3 is related to it by some constraint. That would also make it clear that the $D = 12$ theory should contain only the 3-brane and 5-brane extended objects (see also below).

appear as wrappings of the 12-dimensional 3-brane around the internal 2-torus. To understand such relation in detail one first needs to clarify the structure of the antisymmetric tensor field couplings in the $D = 12$ theory (in particular, the relative roles of C_3 and C_4).[5]

In what follows we shall concentrate on purely gravitational $D = 12$ backgrounds which do not depend on unknown details of the structure of the antisymmetric tensor sector. Like the 0-brane and the 6-brane of type IIA theory which correspond to the gravitational solutions in $D = 11$ theory (plane wave and euclidean Taub-NUT or Kaluza-Klein monopole), the instanton and the 7-brane of type IIB theory also have purely gravitational counterparts in $D = 12$ theory. The 7-brane case was already discussed in [12]. The solution corresponding to a collection of n type IIB 7-branes [9,18] is given by (1) of the following special form ($z = x_8 + i x_9$)

$$ds_{12}^2 = -dt^2 + dx_1^2 + ... + dx_7^2 + H^2(z,\bar{z})dzd\bar{z} + H^{-1}(z,\bar{z})|dy_2 + \tau(z)dy_1|^2 \ , \qquad (5)$$

where $H = e^{-\phi} = \tau_2$, and $j(\tau(z)) = P_n(z)/P_{n-1}(z)$. The regular case of $n = 24$ 7-branes on a compact $(z, \bar{z})$ 2-space in type IIB theory can be interpreted [12] as a special K3 compactification [18] of the 12-dimensional theory.

Our aim here is to give a similar interpretation to the type IIB D-instanton. The instanton is a solution [9] of the *euclidean* type IIB theory (which has a well-defined euclidean supersymmetry) with the action (4) where $g_{\mu\nu}$ is assumed to have euclidean signature and the scalar C is replaced by iC. This rotation of C has a $D = 10$ explanation if type IIB theory is defined in terms of the dual F_9 field strength [9]. At the same time, it has also an alternative natural $D = 12$ explanation if the *euclidean* type IIB$_{(0,10)}$ theory corresponds to a compactification of the same 12-dimensional theory of the signature $(1,11)$ but now on a 2-space of the signature $(1,1)$. If $g_{\mu\nu}$ in (1) is taken to be euclidean, the coordinate y_1 should become time-like, $y_1 = -it$. To preserve the reality of the metric (1) one should then rotate $C \to iC$. The result is ($y \equiv y_2; \ m, n = 1, 2, ..., 10$)

$$ds_{12}^2 = -e^{-\phi(x)}dt^2 + e^{\phi(x)}[dy + \mathcal{C}(x)dt]^2 + g_{mn}(x)dx^m dx^n \ . \qquad (6)$$

The type IIB$_{(0,10)}$ theory is then obtained by dimensional reduction in the spatial direction y and the *time-like* direction t (cf. [12,13]).

One of the simplest examples of such a gravitational background is a spherically symmetric pp-wave,

$$ds_{12}^2 = -dt^2 + dy^2 + [H(x) - 1](dt - dy)^2 + dx_m dx_m \qquad (7)$$

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[5] Naive wrapping of 3-brane with C_4 charge does not seem to give the charges of the B_p-fields related to C_3 as discussed above. One also needs to understand how to connect the $SL(2, Z)$ family of type IIB 5-branes to the $D = 12$ 5-brane: a puzzle here is that the internal 2-torus should not be part of the 5-brane (see also below).

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$$= -H^{-1}dt^2 + H[dy + (H^{-1} - 1)dt]^2 + dx_m dx_m , \qquad H = 1 + \frac{q}{x^8} .$$

It solves the vacuum Einstein equations and should be supersymmetric (as is always the case in lower dimensions), provided a supersymmetric $D = 12$ theory can be defined. Comparing (6) to (7) we learn that the corresponding type $\text{IIB}_{(0,10)}$ background is exactly the instanton solution of [9]

$$e^\phi = H(x) , \qquad \mathcal{C} = H^{-1}(x) - 1 , \qquad ds^2_{10E} = dx_m dx_m . \qquad (8)$$

We assumed that the fields have trivial asymptotic values $\phi_\infty = 0$, $\mathcal{C}_\infty = 0$, i.e. that the vacuum '2-torus' $T^{(1,1)}$ is trivial. The analog of the $SL(2, R)$ symmetry in type $\text{IIB}_{(0,10)}$ theory acts only on the constant parameters ($g = e^{\phi_\infty}$, $\mathcal{C}_\infty$) of the generic solution. The constant q is related to the instanton charge $Q_{-1} = 8\omega_9 q = \frac{2}{3}\pi^{5/2}q$ [9], which can now be interpreted as a linear momentum carried by the wave in the 12-th dimension. As in the case of the '0-brane charge – 11-dimensional momentum' correspondence, this provides another reason for the quantisation of Q_{-1}.

Let us now discuss some implications of the above observation. Suppose that the metric (7) has an extra spatial isometry in one of the x_m directions, e.g., in $x_{10} \equiv z$ (then $H = 1 + \frac{q}{x^7}$). The reduction along (t, y) then connects it to a type $\text{IIB}_{(0,10)}$ background produced by a periodic array of instantons in z direction. If we also assume that the 12-dimensional theory is somehow related to M-theory by a reduction in z (more generally, that a compactification of the (2,10) theory on $T^{(1,1)} \times S^1$ corresponds to a compactification of (1,10) theory on $S^1 \times S^1$, cf. [12]) then the resulting 11-dimensional background is again a similar plane wave. Further reduction along y leads to the 0-brane solution of type IIA theory with the following string-frame metric, dilaton and vector field: $ds^2 = H^{1/2}(-H^{-1}dt^2 + dx_k dx_k)$, $e^\phi = H^{3/4}$, $A_t = H^{-1} - 1$. The latter background is related to the above type type $\text{IIB}_{(0,10)}$ solution (the one which is 'smeared' in z-direction) by formal T-duality in t ($A_t \to \mathcal{C}$, etc.) and the identification of the dual t-coordinate with iz (T-duality in time direction transforms a real background in IIA theory into a complex one in type $\text{IIB}_{(1,9)}$ theory but again a real background in type $\text{IIB}_{(0,10)}$ theory). The consistency of this picture seems to suggest that like the $SL(2, Z)$ symmetry of type IIB theory, the T-duality between type IIB and type IIA theories may have a simple origin in the 12-dimensional theory, being related to a coordinate transformation interchanging y and z compactification directions.[6]

Finitely boosting 0-brane in one extra isometric direction x_9 (this corresponds to a wave along generic cycle of 2-torus (x_{11}, x_9) in $D = 11$ theory [19]) and doing T-duality in

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[6] This is also implied by the fact that the $D = 9$ theory now appears as a reduction of the $D = 12$ theory on the (t, z, y) space of signature (1,2), thus suggesting an explanation for the $D = 9$ U-duality symmetry $SL(2, R) \times SO(1, 1)$ [15].

x_9 (i.e. performing $O(2,2)$ duality on the 0-brane background) leads to the $SL(2,Z)$ family of strings [17] in type $\text{IIB}_{(1,9)}$ theory. Its counterpart in type $\text{IIB}_{(0,10)}$ theory, which may be interpreted as a mixture of a 'smeared' instanton and a string, should correspond to a reduction of a 3-brane configuration (with non-trivial antisymmetric tensor background) in $(1,11)$ theory.

The T-duality between the 'smeared' type IIB instanton and the 0-brane implies also the existence of a non-supersymmetric (non-BPS) generalisation of the 'smeared' instanton ('black instanton'). Applying T-duality to the non-extremal 0-brane (which is a dimensional reduction of the $D = 11$ Schwarzschild background finitely boosted in an additional isometric direction, with the extremal case corresponding to the infinite boost limit) we find the following type $\text{IIB}_{(0,10)}$ solution (cf. (8))

$$e^\phi = \hat{H}(x) \,, \quad \mathcal{C} = \coth\beta\,[\hat{H}^{-1}(x) - 1] \,, \quad ds^2_{10E} = f^{-1}(r)(dz^2 + dr^2) + r^2 d\Omega_8^2 \,, \qquad (9)$$

$$f = 1 - \frac{\mu}{r^7} \,, \qquad \hat{H} = 1 + \frac{\hat{q}}{r^7} \,, \qquad \hat{q} = \mu\,\sinh^2\beta = q\,\tanh\beta \,,$$

where μ is the non-extremality and β is the boost parameter. For zero charge $q = 0$ this metric is T-dual to the $D = 10$ Schwarzschild metric in the euclidean time direction ($t = iz$).

The relation between the instanton charge and the 12-momentum suggests also an interpretation of instantons bound to euclidean 3-brane world-volume in type $\text{IIB}_{(0,10)}$ theory[7] as corresponding to 3-branes boosted in the twelfth dimension. This is similar to the 11-dimensional interpretation of analogous 0-brane bound states in type IIA theory [22,19].

Finally, let us note that the above discussion of type $\text{IIB}_{(0,10)}$ theory based on compactification of $D = 12$ theory in one spatial and one time-like direction suggests that a similar interpretation should be possible also for type $\text{IIB}_{(1,9)}$ theory: one is just to assume that the metric g_{mn} in (6) has Minkowski signature while still reducing in the (t,y) directions. The $D = 12$ theory then has the $(2,10)$ signature as in some of the proposals in [12,13]. Its 3-brane solution will then have to have $(2,2)$ world-volume signature (to be related to type IIB strings upon compactification in (t,y)) while the 5-brane may still have the usual $(1,5)$ signature, possibly resolving the problem mentioned in footnote 5.

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[7] By considering a 3-brane probe in the D-instanton background one finds that there is no non-trivial potential term in the 3-brane action. The 1/4 supersymmetric solution of type $\text{IIB}_{(0,10)}$ theory corresponding to a combination of a euclidean 3-brane and an instanton is easy to construct explicitly; for example, the string-frame metric is $ds^2_{10} = H_{-1}^{1/2} H_3^{1/2}(H_3^{-1} dx_k dx_k + dx_i dx_i)$, where $k = 1, ..., 4$, $i = 5, ..., 10$, and H_{-1} and H_3 are the instanton and the 3-brane harmonic functions depending on x_i. An alternative approach is to assume that the instanton charge is generated by the gauge field instanton of the 3-brane world-volume theory due to the presence of the $\int \mathcal{C} F \wedge F$ coupling in the 3-brane action [20] (this may have a relation to a discussion of D-instantons in [21]).

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