Extremality Versus Supersymmetry in Stringy Black Holes

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Abstract

We study general black-hole solutions of the low-energy string effective action in arbitrary dimensions using a general metric that can describe them all in a unified way both in the extreme and non-extreme cases.

We calculate the mass, temperature and entropy and study which relations amongst the charges and the mass lead to extremality. We find that the temperature always vanishes in the extreme limit and we find that, for a set of n charges (no further reducible by duality) there are $2^{(n-1)}$ combinations of the charges that imply extremality. Not all of these combinations can be central charge eigenvalues and, thus, there are in general extreme black holes which are not supersymmetric (or "BPS-saturated").

In the $N = 8$ supergravity case we argue that the existence of roughly as many supersymmetric and non-supersymmetric extreme black holes suggests the existence of an underlying twelve-dimensional structure.

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1   Introduction

Black holes are some of the most interesting objects that string theory seems to describe and are also the natural testing ground for many of the new ideas in this field. The classical black-hole solutions of the low-energy string effective action play a central role in these recent developments. Particularly interesting are supersymmetric (or "BPS-saturated") black holes. They saturate one or several supersymmetry bounds and describe the gravitational, electromagnetic and other fields of states which in the quantum theory have masses and charges protected from renormalization. The non-renormalization theorems ensure the validity of the results obtained in their study when they are extrapolated to the strong coupling regime by using duality.

Since unbroken supersymmetry relates the different bosonic fields, supersymmetric black-hole solutions are also simpler, given in terms of a smaller number of independent functions and easier to obtain. It seems that all supersymmetric black holes can be described in similar terms irrespectively of the dimensionality and the number of Abelian vector fields and scalars involved, at least in the *irreducible case* which we are going to describe now. We will say that a family of black hole solutions is irreducible when the number of independent electric or magnetic charges cannot be reduced by duality transformations. The biggest irreducible family of solutions is a *generating solution* in the language of Ref. [1]. Then, the n vector fields of an irreducible family of solutions can always be written in this way[2]:

$$A_{(i)\ t} = \alpha_i H_i^{-1}\,, \qquad \alpha_i = \pm 1\,, \tag{1}$$

where the H_is are arbitrary harmonic functions in the relevant Euclidean transverse space. If they are to describe black holes, we have to choose them with point-like singularities which correspond to the different black-hole horizons (these solutions can describe many (N) black holes in static equilibrium, another sign of supersymmetry) and so, in d dimensions

$$H_i = 1 + \sum_{a=1}^{N} \frac{h_{i,a}}{|\vec{x} - \vec{x}_a|^{d-3}}\,. \tag{2}$$

The constants $h_{i,a}$ are all taken to be positive to avoid naked singularities (why this is so will become clear when we say how these functions enter the metric). They are otherwise arbitrary. Then, since black holes can carry

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[2]Here we consider the electric case since it is the only relevant in arbitrary number of dimensions for point-like objects. In four dimensions magnetic charges can be described similarly in terms of the dual vector field t-component $\tilde{A}_t^{(i)}$.

positive or negative charge for each $U(1)$, the constants α_i must be allowed to be positive or negative. The ith $U(1)$ charge of the ath black hole is proportional to the product $\alpha_i h_{i,a} = \pm h_{i,a} = q_{i,a}$ and so all black holes have charges with the same sign for each $U(1)$ and $\alpha_i = \text{sign}\,(q_{i,a})$. The harmonic functions are normalized to be 1 at infinity so we do not need to introduce any other constants in order to have an asymptotically flat metric.

The Einstein-frame metric is always of this form:

$$ds^2 = \left(\prod_{i=1}^{i=n} H_i^{-2r_i}\right) dt^2 - \left(\prod_{i=1}^{i=n} H_i^{-2r_i}\right)^{-\frac{1}{d-3}} d\vec{x}^{\,2}\,, \tag{3}$$

where $\vec{x} = (x^1, \ldots, x^{d-1})$.

It is a experimental fact that the coefficients r_i always satisfy

$$\sum_{i=1}^{i=n} r_i = 1\,, \tag{4}$$

and when they do not, we can consider that there is an additional vector field with respect to which the black holes are uncharged, so the corresponding $h_{i,a} = 0$ and the harmonic function is simply unity and cannot be "seen" in Eq. (3) even though it is actually there[3]. So we can assume that Eq. (4) always holds true in a certain coordinate system when all possible charges are switched on.

In most cases one also has that all the r_is are equal. If they are not, one can assume that there are more harmonic functions but they happen to be equal for the particular solution one is considering. For instance, if there are two harmonic functions with $r_1 = 1/3$, $r_2 = 2/3$ one can consider that in the most general case there are three harmonic functions with all r_is equal to 1/3 but that $H_2 = H_3$. This assumption is not crucial in what follows and we will not use it, although it should be kept in mind that in most cases it is true.

If we take harmonic functions with a single pole in the origin, the ADM mass m of the single black hole they describe, which we define by

$$g_{tt} \sim 1 - \frac{2m}{\rho^{d-3}}\,, \tag{5}$$

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[3]This is what happens, for instance, in the $U(1)$ dilaton black holes with $a = 1$ of Refs. [2, 3]. The extreme black holes of this family, which are also supersymmetric [4] can be described in terms of a single harmonic function with coefficient $r = 1/2$. However they can be considered as a particular case of extreme dilaton $U(1) \times U(1)$ black holes which can be described in terms of two harmonic functions with $r_1 = r_2 = 1/2$ so $\sum_i r_i = 1$. The $U(1)$ black holes are obviously those for which one charge vanishes and the corresponding harmonic function is just 1.

is given by

$$m = \sum_{i=1}^{i=n} r_i |q_i| \,.$$ (6)

Thus, the mass of all the supersymmetric black holes of the above form is related to their charges. A relation of this kind is expected whenever a solution saturates a supersymmetry (or Bogomol'nyi) bound. However, as we are going to see, a relation of this kind does not guarantee the supersymmetry of the corresponding solution.

In all known cases, *supersymmetric* (static) black holes are also *extreme* black holes. This means that they can be obtained by taking the appropriate limit of a family of charged black hole solutions. These, in general have two horizons: an event horizon and a Cauchy horizon. When the mass diminishes these horizons get closer, eventually coinciding in a single event horizon. This is the extreme limit.

All extreme black holes of the low-energy effective string theory can be written in the above form but not all of them are supersymmetric. A extreme black hole which is not supersymmetric has been known for some time[4] [6] (see also [7]). One of our goals in this letter will be to show that this is actually a very general phenomenon.

It turns out that the general (non-extreme) black holes of the low-energy string theory can be found by modifying the above solutions with the introduction of an extra factor W in the metric which we henceforth refer to as the *Schwarzschild factor*[5]:

$$ds^2 = \left(\prod_{i=1}^{i=n} H_i^{-2r_i} \right) W dt^2 - \left(\prod_{i=1}^{i=n} H_i^{-2r_i} \right)^{-\frac{1}{d-3}} \left[W^{-1} d\rho^2 + \rho^2 d\Omega_{(d-2)}^2 \right] \,,$$ (7)

where the Schwarzschild factor is given by

$$W = 1 - \frac{2r_0}{\rho^{d-3}} \,, \qquad r_0 > 0 \,,$$ (8)

and the old harmonic functions are now forced to be of the form

$$H_n = 1 + \frac{h_n}{\rho^{d-3}} \,.$$ (9)

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[4]Obviously there are many extreme dilaton black hole which are extreme but not supersymmetric because they cannot be considered solutions of any supergravity theory, to start with [5].

[5]This procedure was first used in Ref. [8] and further exploited in Refs. [9, 10]

There are two more modifications: the constants α_i are no longer simply ± 1 but are related to the h_is and to r_0. The relation is always such that taking into account that the charges are $q_i \sim \alpha_i h_i$ one can write

$$h_i = -r_0 + \sqrt{r_0^2 + q_i^2}\,. \tag{10}$$

Observe that when the Schwarzschild factor $W = 1$ we recover the extremal single black hole solutions. For this reason r_0 is called the *extremality parameter*. In fact, all is used in checking that the above are solutions is the equation

$$\partial_\rho \left(\rho^{d-3} \partial_\rho H_i \right) = 0\,, \tag{11}$$

so when $W = 1$ one can go to Cartesian coordinates in which the above equation tells us simply that the H_is are arbitrary harmonic functions. Therefore, the above solutions include the extreme black-hole solutions in the $r_0 \to 0$ limit.

Our goal is to study general black-hole solutions of this kind trying to derive as many general consequences about their physical properties as possible.

2 Some Examples

It is worth seeing how all this is realized in a few simple examples. The four-dimensional Reissner-Nordström black-hole solution is usually written in this form:

$$
\begin{aligned}
ds^2 &= \frac{(r - r_+)(r - r_-)}{r^2} dt^2 - \frac{r^2}{(r - r_+)(r - r_-)} dr^2 - r^2 d\Omega^2\,, \\[4pt]
A_\mu &= -\delta_{\mu t} \frac{Q}{r}\,,
\end{aligned}
\tag{12}
$$

where the integration constants $r_\pm$ are given in terms of the electric charge Q and the ADM mass M by

$$r_\pm = M \pm r_0\,, \qquad r_0^2 = M^2 - Q^2\,. \tag{13}$$

If we shift the radial coordinate by $r = \rho + r_-$ and we make a gauge transformation of the gauge potential that results into the addition of the constant Q/r_-, the it takes the following form:

$$ds^2 = \left(1 + \frac{r_-}{\rho}\right)^{-2} \left(1 - \frac{2r_0}{\rho}\right) dt^2$$

$$- \left(1 + \frac{r_-}{\rho}\right)^2 \left[\left(1 - \frac{2r_0}{\rho}\right)^{-1} d\rho^2 + \rho^2 d\Omega^2\right] ,$$

$$(14)$$

$$A'_\mu = \delta_{\mu t} \frac{Q}{r_-} \left(1 + \frac{r_-}{\rho}\right)^{-1} .$$

Obviously here we have

$$H = 1 + \frac{r_-}{\rho} , \qquad \alpha = \frac{Q}{r_-} . \tag{15}$$

The higher-dimensional Reissner-Nordström solutions can be also written in this form. Other solutions are the dilaton black-hole solutions of the a-model (which include the Reissner-Nordström solutions) in d dimensions with action

$$S = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g|} \left[R + 2(\partial\varphi)^2 - e^{-2a\varphi} F^2\right] , \tag{16}$$

which can be written as follows:

$$\left\{ \begin{array}{rcl} ds^2 & = & \left(e^{-2a(\varphi - \varphi_0)} H^{-2}\right) W dt^2 \\[2mm] & & - \left(e^{-2a(\varphi - \varphi_0)} H^{-2}\right)^{\frac{-1}{d-3}} \left[W^{-1} d\rho^2 + \rho^2 d\Omega_{d-2}^2\right] , \\[2mm] A_\mu & = & \alpha \delta_{\mu t} H^{-1} , \\[2mm] e^{-2a\varphi} & = & e^{-2a\varphi_0} H^{2k} , \\[2mm] H & = & 1 + \frac{h}{\rho^{d-3}} , \qquad W = 1 - \frac{2r_0}{\rho^{d-3}} , \\[2mm] 2r_0 & = & -h \left[1 - \frac{a^2}{x} \alpha^2\right] , \\[2mm] k & = & \frac{a^2}{2} c / (1 + \frac{a^2}{2} c) , \qquad c = (d-2)/(d-3) . \end{array} \right. \tag{17}$$

Although our assumption for the constants r_i does not always seem to be true in this case, one should not forget that most of the solutions of this model are not solutions of any supergravity theory.

A case in which there are two harmonic functions is given by the solutions to the d-dimensional $a_1 - a_2$-model with action

$$S = \frac{1}{16\pi G_N^{(d)}} \int d^d x \sqrt{|g|} \left[R + 2(\partial\varphi)^2 - e^{-2a_1\varphi} F_{(1)}^2 - e^{-2a_2\varphi} F_{(2)}^2 \right] . \qquad (18)$$

When a_1 and a_2 are related by

$$a_1 a_2 = -2 \left(\frac{d-3}{d-2} \right) . \qquad (19)$$

one finds the following solutions [2]:

$$\begin{cases}
ds^2 = \left(e^{-2a_1(\varphi-\varphi_0)} H_1^{-2} \right) W dt^2 \\[2mm]
\qquad - \left(e^{-2a_1(\varphi-\varphi_0)} H_1^{-2} \right)^{-\frac{1}{d-3}} \left[W^{-1} d\rho^2 + \rho^2 d\Omega_{(d-2)}^2 \right] , \\[2mm]
e^{-2\varphi} = e^{-2\varphi_0} \left(H_1/H_2 \right)^{\frac{2}{(a_1-a_2)}} . \\[2mm]
A_{(i)\mu} = \delta_{\mu t} \alpha_i e^{a_i\varphi_0} H_i^{-1} , \\[2mm]
H_i = 1 + h_i/\rho^{d-3} , \quad W = 1 - 2r_0/\rho^{d-3} , \\[2mm]
2r_0 = -h_1 \left[1 - a_1(a_1 - a_2)\alpha_1^2 \right] , \quad 2r_0 = -h_2 \left[1 + a_2(a_1 - a_2)\alpha_2^2 \right] .
\end{cases} \qquad (20)$$

There are only two cases (known to us) in which this model really arises as a truncation of a string theory action: in $d = 5$ dimensions, where $F_{(2)}$ would be the dual of the NS-NS 3-form (the axion field strength) and we would have $a_1 = 2/\sqrt{6}, a_2 = -4/\sqrt{6}$, and in $d = 4$ dimensions where $F_{(2)}$ would be a RR vector field and we would have $a_1 = +1, a_2 = -1$. The problem in higher dimensions is that $F_{(2)}$ would have to be the dual of a 4-form, 5-form etc. and all these fields are RR-type fields and, in the string frame, where the dilaton is an overall factor for the NS-NS fields, the RR fields do not couple directly to it and, in the Einstein frame, the constraint (19) is not satisfied. Of course, other possibilities arise if φ is not considered to be the dilaton but some other moduli fields or combinations of them. In any case, in the two stringy cases of interest the solution nicely fits into the general metric written in the previous section.

More black-hole solutions with this structure can be found elsewhere [8, 9].

3 Physical Properties

3.1 Extremality and Supersymmetry

The independent parameters of the charged black-hole solutions are the ADM mass m and the charges q_i which are, up to factors that we absorb in their definition, $h_i \alpha_i$. In these factors are included all moduli.

The first thing we would like to do is to express all the parameters that appear in the solution in terms of the physical parameters. The h_is are expressed in terms of r_0 and the charges q_i through the equations (11). Then we only need to express r_0 in terms of the physical parameters. If we calculate the mass of the black-hole metric (7) we get

$$m = \sum_{i=1}^{i=n} r_i \sqrt{r_0^2 + q_i^2} \, . \tag{21}$$

This equation implicitly gives r_0 as a function of the mass and charges. It is impossible to solve for r_0 in the general case but we can extract very interesting information from the above equation.

First, let us see how it can be solved in the simplest cases. We set $r_i = 1/n$. For $n = 1$ it is trivial to get

$$r_0^2 = m^2 - q^2 \, . \tag{22}$$

r_0 vanishes when $m = |q|$ and, in principle, we could identify the central charge of $N = 2$ supergravity (or any of the central charge matrix skew eigenvalues in higher $N = 4, 8$ supergravity [11]) with q so $|z| = |q|$. This, indeed seems to be always the case.

For $n = 2$ the above equation can be transformed in a quadratic algebraic equation for r_0^2 whose solution is

$$r_0^2 = \frac{1}{m^2} \left[m^2 - \left(\frac{q_1 + q_2}{2} \right)^2 \right] \left[m^2 - \left(\frac{q_1 - q_2}{2} \right)^2 \right] \, . \tag{23}$$

We see that r_0 vanishes when m^2 equals one of the combinations

$$\begin{aligned}
|z_1| &= \left| \frac{q_1 + q_2}{2} \right| \, , \\
|z_2| &= \left| \frac{q_1 - q_2}{2} \right| \, ,
\end{aligned} \tag{24}$$

which can be identified with the two central charge skew eigenvalues of $N = 4$ supergravity or two of the four skew eigenvalue of the central charge matrix of $N = 8$ supergravity.

For $n = 3$ one gets a quartic algebraic equation for r_0^2. The general solution is complicated but we can learn the same from a simplified case: $q_2 = q_3$. (This is the case that applies to black-hole solutions of pure $d = 5, N = 4$ supergravity, which is a consistent truncation of the low-energy string effective action.) The solution is

$$r_0^2 = m^2 + \frac{q_1^2 - q_2^2}{3} + 4m^2 - q_2^2 - 4m \left(m^2 + \frac{q_1^2 - q_2^2}{3} \right)^{\frac{1}{2}}. \qquad (25)$$

We immediately notice that the expression for r_0 does not factorize into "Bogomol'nyi factors" $(m^2 - |z_i|^2)$ as it happened in the $n < 3$ cases. It is easy to see that r_0 vanishes when m^2 equals either of the two combinations

$$
\begin{aligned}
|z_1| &= \left| \tfrac{q_1 + 2q_2}{3} \right|, \\[2mm]
|z_2| &= \left| \tfrac{q_1 - 2q_2}{3} \right|,
\end{aligned}
\qquad (26)
$$

which can be naturally identified with the skew eigenvalues of the central charge matrix of $N = 4$ supergravity. These two combinations do not appear in Eq. (25), which is understandable because r_0 is not a polynomial on m^2.

For $n \geq 4$ Eq. (21) cannot be rewritten as a polynomial on r_0^2. It is, therefore, impossible to write r_0 as a product of Bogomol'nyi factors in all the cases with $n \geq 3$. This applies, in particular, for the black-hole solutions of the heterotic string effective action in four and five dimensions ($n = 4, 3$ respectively) and those of the type II theory in the same dimensions.

In spite of the difficulty of finding a expression for the extremality parameter in terms of the mass and charges, it is easy to find from the defining equation (21) the zeroes of r_0. When $r_0 = 0$, the mass and charges are related by

$$m = \sum_{i=1}^{i=n} r_i |q_i|. \qquad (27)$$

We would like to rewrite this relation in a Bogomol'nyi-like fashion, $m = |z|$ which is a necessary condition for the corresponding extreme black holes to be supersymmetric[6]. Is is easy to see that the above equation can be rewritten in $2^{(n-1)}$ different Bogomol'nyi-like ways:

$$m = \left| \sum_{q_i > 0} r_i q_i - \sum_{q_i < 0} r_i q_i \right|, \qquad (28)$$

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[6]It is not a sufficient condition because we would have to prove that the $|z|$s so obtained are the actual central charge skew eigenvalues, but it is very reasonable to expect it.

which correspond to the $2^{(n-1)}$ possible relative signs among the charges. In general, this number of Bogomol'nyi-like identities is clearly larger than the number of skew eigenvalues of the central charge matrix. To be specific, in pure $N = 2, 4$ supergravity in $d = 4, 5$ dimensions this problem does not occur. However, in $N = 8$ supergravity we would have found 8 of these Bogomol'nyi-like identities while there are only 4 supersymmetry bounds. Then, in this theory, there are extreme black holes which satisfy the identity (27) by satisfying one of the four Bogomol'nyi-like identities which are not supersymmetry bounds and therefore are not supersymmetric.

This can be better seen in an example. The black-hole generating solutions of the heterotic compactified in a six-torus can be found as solutions of the truncated action

$$
\begin{aligned}
S \;=\; & \int dx^4 \; \sqrt{|g|} \Big\{ R + 2 \left[(\partial \phi)^2 + (\partial \sigma)^2 + (\partial \rho)^2 \right] \\
& -\tfrac{1}{4} e^{-2\phi} \left[e^{-2(\sigma+\rho)} (F_1)^2 + e^{-2(\sigma-\rho)} (F_2)^2 \right. \\
& \left. + e^{2(\sigma+\rho)} (F_3)^2 + e^{2(\sigma-\rho)} (F_4)^2 \right] \Big\} \; .
\end{aligned}
\tag{29}
$$

These solutions were found in Ref. [12], further discussed in Refs. [13] and later rediscovered in Ref. [14].

The extreme solution is given in terms of four independent harmonic functions H_i, $i = 1, \ldots, 4$

$$
ds^2 \;=\; \left(\Pi_{i=1}^{i=4} H_i^{-\frac{1}{2}} \right) dt^2 - \left(\Pi_{i=1}^{i=4} H_i^{-\frac{1}{2}} \right)^{-1} d\vec{x}^{\,2} \, ,
$$

$$
e^{-4\phi} \;=\; \frac{H_1 H_3}{H_2 H_4} \, , \qquad e^{-4\sigma} = \frac{H_1 H_4}{H_2 H_3} \, , \qquad e^{-4\rho} = \frac{H_1 H_2}{H_3 H_4} \, ,
\tag{30}
$$

$$
A_{(i)\ t} \;=\; \alpha_i H_i^{-1} \, , \; i = 1, 3 \, , \quad \tilde{A}_{(i)\ t} = \alpha_i H_i^{-1} \, , \; i = 2, 4
$$

where $\alpha_i = \text{sign}\,(q_i) = \pm 1$ and

$$
\tilde{F}_2 = e^{-2(\phi+\sigma-\rho)\star} F_2 \, , \qquad \tilde{F}_4 = e^{-2(\phi-\sigma+\rho)\star} F_4 \, ,
\tag{31}
$$

and $\star F$ is the Hodge dual of F. The harmonic functions, for a single black hole are

$$
H_i = 1 + \frac{|q_i|}{|\vec{x}|} \, .
\tag{32}
$$

10

(The charges q_2 and q_4 are here magnetic charges.) These solutions fit our general metric for extreme string black holes with $n = 4$ and $r_i = 1/4$. The ADM mass is, thus, given by

$$m = \tfrac{1}{4} \sum_{i=1}^{i=4} |q_i| \,. \tag{33}$$

The four central charge skew eigenvalues z_i of $N = 8$ supergravity (where we can embed this solution too) are given in terms of the charges q_i by [15]

$$
\begin{aligned}
|z_1| &= \tfrac{1}{4}|q_1 + q_2 + q_3 + q_4| \,, \\[4pt]
|z_2| &= \tfrac{1}{4}|q_1 - q_2 + q_3 - q_4| \,, \\[4pt]
|z_3| &= \tfrac{1}{4}|q_1 + q_2 - q_3 - q_4| \,, \\[4pt]
|z_4| &= \tfrac{1}{4}|q_1 - q_2 - q_3 + q_4| \,.
\end{aligned}
\tag{34}
$$

Now, it is easy to see that there are extreme black holes which satisfy the mass formula but do not saturate any Bogomol'nyi bound (essentially half of the total). Taking unit charges, so $m = |q_i| = 1$, we have the following eight possibilities:

$$\vec{q} = \pm(1, 1, 1, 1)\,, \qquad m = |z_1| = 1\,, \tag{35}$$

and the black hole has one unbroken supersymmetry. The next case is

$$\vec{q} = \pm(1, 1, 1, -1)\,, \qquad m = \tfrac{1}{4}|q_1 + q_2 + q_3 - q_4| = 1\,, \tag{36}$$

while

$$|z_1| = |z_2| = |z_3| = |z_4| = \tfrac{1}{2} < m\,, \tag{37}$$

and the black hole has no unbroken supersymmetries in spite of being extreme. Observe that this black hole satisfies the four supersymmetry bounds without saturating any of them. The same happens in the other three cases in which one charge has sign different to the other three. The next case is

$$\vec{q} = \pm(1, 1, -1, -1)\,, \qquad m = |z_3|\,, \tag{38}$$

and the black hole has one unbroken supersymmetry, etc.

3.2 Temperature

It is straightforward to find that the temperature is in general given by

$$
T = \frac{(d-3)}{2^{\frac{1}{d-3}}4\pi} \frac{r_0^{2-\frac{1}{d-3}}}{\prod_{i=1}^{i=n}\left[r_0 + \sqrt{r_0^2 + q_i^2}\right]^{2r_i}} \ ,
\tag{39}
$$

which shows that it always vanishes in the extreme limit. However, some care has to be taken when some of the charges vanish because, then, there is an ambiguity in the calculation of the temperature: there are additional r_0 factors in the denominator which may cancel those of the numerator and then the temperature may not vanish. The resolution of this paradox is the same as in the four-dimensional $U(1) \times U(1)$ dilaton black holes [2, 4].

3.3 Entropy

The general formula for the entropy is

$$
S = \frac{\omega_{d-2}}{4} \left\{ 2 \prod_{i=1}^{i=n}\left[r_0 + \sqrt{r_0^2 + q_i^2}\right]^{r_i} \right\}^{\frac{d-2}{d-3}} \ ,
\tag{40}
$$

where ω_{d-2} is the volume of the $(d-2)$-sphere. In the extreme limit S is always proportional to a product of charges

$$
S = \frac{\omega_{d-2}}{4} 2^{\frac{d-2}{d-3}} \prod_{i=1}^{i=n} |q_i|^{r_i\frac{d-2}{d-3}} \ .
\tag{41}
$$

This formula generalizes the formulae obtained in Ref. [16].

Only in four dimensions there is a simple relation between the extremality parameter r_0 and the product ST:

$$
ST = \frac{(d-3)\omega_{d-2}}{16\pi} 2r_0^{\frac{2d-7}{d-3}} \left\{ 2 \prod_{i=1}^{i=n}\left[r_0 + \sqrt{r_0^2 + q_i^2}\right]^{r_i} \right\}^{-\left(\frac{d-4}{d-3}\right)} \ .
\tag{42}
$$

4 Conclusion

We have studied general black hole solutions of the low-energy string effective action and computed their mass, temperature and entropy. we have also studied the extremality conditions and have found that in $N = 2, 4$ supergravity plus matter as well as in $N = 8$ supergravity there are many extreme black holes which do not saturate any Bogomol'nyi bound.

In fact, in $N = 8$ supergravity roughly a half of the extreme black holes do not saturate any of the four Bogomol'nyi bounds of this theory and are not supersymmetric. Instead they saturate one of a set of four different Bogomol'nyi-like bounds.

An intriguing possibility, previously suggested in Ref. [7] is that $N = 8$ can be considered as a consistent truncation of a $N = 16$ supergravity theory which would not be consistent before the truncation is done. A concrete scenario was proposed there which may have a twelve-dimensional origin, in a sense similar to the one proposed by Kutasov and Martinec's $N = (2, 1)$ string scenario [17, 18, 19, 20] and Bars' "S theory" scenario [21, 22][7].

Such an $N = 16$ theory would have 8 central charge skew eigenvalues and the $N = 8$ would "remember" that four combinations of the charges were central charge eigenvalues even though after the truncation they are not any more. This proposal could eventually be checked in the framework of the $N = (2, 1)$ string or S theory scenarios.

Acknowledgements

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