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Confining Strings with Topological Term

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We consider several aspects of 'confining strings', recently proposed to describe the confining phase of gauge field theories. We perform the exact duality transformation that leads to the confining string action and show that it reduces to the Polyakov action in the semiclassical approximation. In 4D we introduce a 'θ-term' and compute the low-energy effective action for the confining string in a derivative expansion. We find that the coefficient of the extrinsic curvature (stiffness) is negative, confirming previous proposals. In the absence of a θ-term, the effective string action is only a cut-off theory for finite values of the coupling e, whereas for generic values of θ, the action can be renormalized and to leading order we obtain the Nambu-Goto action plus a topological 'spin' term that could stabilize the system.

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1. Introduction

Despite many efforts the formulation of a consistent string theory for the confining phase of gauge theories remains an open problem. In order to cure the diseases of the bosonic string in 4D Polyakov [1] [2] and Kleinert [3] proposed to add to the Nambu-Goto action a term proportional to the *extrinsic curvature* of the world-sheet. However, the so obtained *rigid string* is not problem-free. The inverse of the coupling constant of the new term is asymptotically free [1] [3], so that, in absence of a non-trivial fixed point, the extrinsic curvature becomes irrelevant in the infrared. Moreover, the rigid string suffers from the stability problems common to all higher-derivative actions and can be viewed at most as a long-distance expansion of a non-local action [4].

A related problem regards the sign of the rigidity term. Clearly, this was originally proposed to enter the (Euclidean) action with a positive coefficient in order to suppress strongly creased surfaces. However, most computations of the extrinsic curvature term for Nielsen-Olesen vortices [5] produced the opposite, negative sign. No rigidity term was found in [6], while a positive coefficient was obtained in [7].

Recently there were new developments in this field. First, Kleinert [8] proposed that the extrinsic curvature *should* indeed enter the (Euclidean) action with a negative coefficient and that this actually improves the stability of the theory. Secondly, Polyakov [9] proposed a new string action, which he called the *confining string.*

The (Euclidean) action of the confining string is given by

$$e^{-S_{\rm CS}} = \frac{G}{Z(B_{\mu\nu})} \int \mathcal{D}B_{\mu\nu} \exp\left[-S(B_{\mu\nu}) + i\int d^Dx\ B_{\mu\nu}T_{\mu\nu}\right] \ , \tag{1.1}$$

where $B_{\mu\nu}$ is an antisymmetric Kalb-Ramond field [10] and

$$\begin{aligned} T_{\mu\nu}(\mathbf{x}) &= \frac{1}{2}\int d^2\sigma\ X_{\mu\nu}(\sigma)\ \delta^D(\mathbf{x}-\mathbf{x}(\sigma)) \ , \\ X_{\mu\nu} &= \epsilon^{ab}\frac{\partial x_\mu}{\partial\sigma^a}\frac{\partial x_\nu}{\partial\sigma^b} \ , \end{aligned} \tag{1.2}$$

with $\mathbf{x}(\sigma)$ parametrizing the world-sheet. G is a group factor which takes the value $G=1$ for compact Abelian gauge theories and reduces at long distances to $G=N^{-\chi}$, with χ the Euler characteristic, for SU(N) gauge theories. The action for the Kalb-Ramond tensor is given by

$$S(B_{\mu\nu}) = \int d^Dx\ f(H_{\mu\nu\alpha}) + \frac{1}{4e^2}B_{\mu\nu}B_{\mu\nu} \ , \tag{1.3}$$

where f takes the form

$$
\begin{aligned}
f(H_\mu) &= \frac{2}{3}\ \Lambda_0 H_\mu \ \sinh^{-1}\left(\frac{2}{3z\Lambda_0^3}\ H_\mu\right) - z\Lambda_0^4\sqrt{1+\frac{4}{9z^2\Lambda_0^6}H_\mu^2}\ , \\
H_\mu &\equiv \epsilon_{\mu\nu\alpha\beta}H_{\nu\alpha\beta}\ , \\
H_{\mu\nu\alpha} &= \partial_\mu B_{\nu\alpha} + \partial_\nu B_{\alpha\mu} + \partial_\alpha B_{\mu\nu}\ ,
\end{aligned}
\tag{1.4}
$$

in 4D. Here e^2 is the dimensionless coupling of the original gauge theory, Λ_0 is the cutoff, which is needed in 4D, and $z \propto \exp(-\mathrm{const}/e^2)$.

The action of the confining string was motivated by the facts that it can be explicitly derived for compact U(1) theories, where it arises due to the condensation of topological defects [2], and that, up to as yet uncontroled contact terms, it satisfies the string loop equations [2]. At long distances (and weak fields) the Kalb-Ramond action can be approximated by

$$
\begin{aligned}
S(B_{\mu\nu}) &= \int d^D x\ \frac{1}{12\Lambda^2}\ H_{\mu\nu\alpha}H_{\mu\nu\alpha} + \frac{1}{4e^2}B_{\mu\nu}B_{\mu\nu}\ , \\
\Lambda &= \frac{\Lambda_0}{4}\sqrt{z}\ .
\end{aligned}
\tag{1.5}
$$

In this form, the confining string action was independently proposed in [11], where it was viewed as a special case of the generic Julia-Toulouse mechanism for the confinement of $(h-1)$-branes by the condensation of $(D-h-3)$-branes in a compact antisymmetric tensor field theory of rank h. Eq.(1.5) corresponds to $h=1$ and Λ is the new scale generated by the condensation of $(D-4)$-branes, as explained in [11]. For example, in D=3, Λ^3 is proportional to the average density of instantons in Euclidean space. The mass of the Kalb-Ramond field in (1.5) is then given by $m = \Lambda/e$.

The purpose of the present note is threefold. First we woud like to present a full derivation of the confining string action by performing an exact duality transformation; this was done only at the semiclassical level in [9]. Secondly, we would like to investigate the issue of the extrinsic curvature term in the low-energy limit of the confining string. Finally, using the formalism developed in [11], we would like to study how the confining string action is modified by the presence of a θ-term,

$$
\begin{aligned}
S &= \int d^4 x\ \frac{1}{4e^2}F_{\mu\nu}F_{\mu\nu} + i\frac{\theta}{64\pi^2}F_{\mu\nu}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}\ , \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu\ ,
\end{aligned}
\tag{1.6}
$$

in the action for the Coulomb phase of a 4D compact U(1) gauge theory.

We shall find that, indeed, the extrinsic curvature term enters the (Euclidean) action with a *negative coefficient*, thereby confirming the observation of Kleinert [8]. Note, however, that in the confining string the extrinsic curvature term appears only in a long distance (low-energy) derivative expansion. The full action is an induced action and thus automatically non-local as suggested in [4]. Moreover, we shall find that only for $\theta \neq 0$ one can take the limit $\Lambda_0 \to \infty$, in which case e is driven to infinity, $e \to \infty$. The resulting action is the Nambu-Goto action plus a topological term measuring the self-intersection number of the world-sheet. This is in accordance with the original observation of Polyakov [1], [2] that a θ-term might stabilize the rigid string.

2. Duality Transformation

Starting with Euclidean $4D$ compact QED, the condensation of monopoles for $e > e_{cr}$ induces an effective action for the dual gauge field φ_μ. Following Polyakov [2] and Orland [12] , the corresponding partition function can be written as:

$$
\begin{aligned}
Z &= \int \mathcal{D}\varphi_\mu \ \exp\left\{-S_{\rm conf}\right\} , \\
S_{\rm conf} &= \int d^4x \left\{ 4e^2 f_{\mu\nu} f_{\mu\nu} + z\Lambda_0^4 \left(1 - \cos\frac{\varphi_\mu}{\Lambda_0}\right)\right\} ,
\end{aligned}
\tag{2.1}
$$

with $f_{\mu\nu} \equiv \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$. This is the 4D extension of the well known 3D case[1]. However, in 4D, this partition function has to be understood in terms of a lattice regularization, as emphasized in [9]:

$$
\begin{aligned}
Z_l &= \int \mathcal{D}\varphi_\mu \ \exp\left(-S_l\right) \ , \\
S_l &= \sum_{\mathbf{x}} \left(4e^2 l^4 f_{\mu\nu} f_{\mu\nu} + z\left(1 - \cos(l\varphi_\mu)\right)\right) \ ,
\end{aligned}
\tag{2.2}
$$

with $l \equiv 1/\Lambda_0$, $\mathcal{D}\varphi_\mu \equiv \prod_{\mathbf{x},\mu} d\varphi_\mu(\mathbf{x})$ and $f_{\mu\nu} \equiv d_\mu\varphi_\nu - d_\nu\varphi_\mu$, with d_μ the (forward) lattice derivative. The UV cut-off $\Lambda_0 = 1/l$ is needed in 4D since the coupling e is large.

In order to get the action $S(B_{\mu\nu})$, we should perform a duality transformation starting with the path integral

$$
Z = \int \mathcal{D}B_{\mu\nu}\mathcal{D}\varphi_\mu \ \exp\left(-S(B_{\mu\nu}, \varphi_\mu)\right) \ , \tag{2.3}
$$

[1] Everything we will say in this section applies for different numbers of dimensions but we write it explicitly in 4D for concreteness.

with the first-order Euclidean action

$$S(B_{\mu\nu},\varphi_\mu)=\int d^4x\,\frac{1}{4e^2}B_{\mu\nu}B_{\mu\nu}+i\varepsilon_{\mu\nu\alpha\beta}B_{\mu\nu}f_{\alpha\beta}+z\Lambda_0^4\left(1-\cos\frac{\varphi_\mu}{\Lambda_0}\right). \tag{2.4}$$

Integration over the field $B_{\mu\nu}$, being a Gaussian, can be easily performed and brings back $S_{\rm conf}$ as it was explicitly shown by Polyakov. To find the dual action, which is the one describing the confining string, we would have to integrate out the field φ_μ. Notice that the non-linear dependence on this field makes the integration seem essentially untractable. Polyakov proceeded by eliminating φ_μ by its field equation in Minkowski space and substituting back into the action. In our 4D case this would give

$$f_{\rm Mink}\left(H_\mu\right)=\frac{2}{3}\Lambda_0 H_\mu {\rm arcsin}\left(\frac{2}{3z\Lambda_0^3}H_\mu\right)+z\Lambda_0^4\sqrt{1-\frac{4}{9z^2\Lambda_0^6}H_\mu^2}\ , \tag{2.5}$$

which is the 4D extension of Polyakov's 3D result. The (Minkowski) partition function implies a sum over the branches of the *multivalued* function $f_{\rm Mink}\left(H_\mu\right)$. Polyakov [9] showed how this sum over branches can be traded for a summation over surfaces.

Here we shall instead derive the exact action in Euclidean space. We start by rewriting the (lattice-regularized) path integral (2.3) as follows

$$Z_l=\int\mathcal{D}B_{\mu\nu}\sum_{\{n_\mu\}}\exp\left(2\pi i\sum_{\mathbf{x}}l^3B_{\mu\nu}t_{\mu\nu}\right)\int_{-\pi/l}^{+\pi/l}\mathcal{D}\varphi_\mu\ \exp\left(-S\left(B_{\mu\nu},\varphi_\mu\right)\right)\ , \tag{2.6}$$

with $t_{\mu\nu}\equiv\epsilon_{\mu\nu\alpha\beta}\left(d_\alpha n_\beta-d_\beta n_\alpha\right)$.

In going from (2.3) to (2.6) we have restricted the φ_μ integrations to the fundamental domain at the price of introducing an additional set of link variables n_μ which take into account all other periods of the cos function.

At this point we can perform the integrations over φ_μ. To this end we use the following result [13]:

$$\mathrm{e}^{a\cos x}=\sum_{k\in Z}I_k(a)\ \mathrm{e}^{ikx}\ , \tag{2.7}$$

with $I_k(a)$ a modified Bessel function. Using this we obtain the following contribution to the partition function from the φ_μ integrations (up to an irrelevant factor):

$$\begin{aligned}
Z_\varphi &= \prod_{\mathbf{x},\mu} \int_{-\pi/l}^{+\pi/l} d\varphi_\mu(\mathbf{x}) \ \exp\left(z\cos(l\varphi_\mu) + i\frac{2}{3} l^4 \varphi_\mu H_\mu \right) \\
&= \prod_{\mathbf{x},\mu} \sum_{n_\mu(\mathbf{x})} I_{n_\mu}(z) \int_{-\pi}^{+\pi} d\varphi_\mu \ \exp i \left(\frac{2}{3} l^3 \varphi_\mu H_\mu - n_\mu \varphi_\mu \right) \\
&= \prod_{\mathbf{x},\mu} \sum_{n_\mu(\mathbf{x})} I_{n_\mu}(z) \ \delta_{n_\mu, \frac{2}{3} l^3 H_\mu} \\
&= \prod_{\mathbf{x},\mu} I_{\frac{2}{3} l^3 H_\mu}(z) \ ,
\end{aligned} \tag{2.8}$$

where we have absorbed l in the definition of φ_μ. Note that Kronecker delta conditions imply the quantization condition $(2/3) l^3 H_\mu(\mathbf{x}) =$ integer for all $\mathbf{x}$ and μ, which means that the unit of magnetic charge is quantized. In the continuum limit we also get the total condition $\int_V d^3x \mathbf{H} \cdot \mathbf{n} = 0$ where V is any 3D hypervolume in 4D with unit normal $\mathbf{n}$. This is an expression of the neutrality and isotropy of the underlying monopole condensate.

For strong coupling, only small values of φ_μ contribute to the partition function, as can be seen from (2.2) (note that an overall shift of φ_μ by $2\pi n/l$ is irrelevant since the potential is periodic). In this case it is a good approximation to restrict to values $|l t_{\mu\nu}| \leq 1$. In this 'dilute gas approximation' the sum over configurations $\{n_\mu\}$ reduces to a sum over *closed surfaces* on the lattice. Restoring the continuum notation with the appropriate factors of the UV cut-off Λ_0 we obtain

$$\begin{aligned}
Z &= \int_{\substack{\text{closed} \\ \text{surfaces}}} \int \mathcal{D} B_{\mu\nu} \exp\left(-S\left(B_{\mu\nu}\right) + i \int_{\text{surface}} B_{\mu\nu} d\sigma_{\mu\nu} \right) \ , \\
S\left(B_{\mu\nu}\right) &= \int d^4x \ \left(-\Lambda_0^4 \log I_{\left(\frac{2H_\mu}{3\Lambda_0^3}\right)}(z) + \frac{1}{4e^2} B_{\mu\nu} B_{\mu\nu} \right) \ ,
\end{aligned} \tag{2.9}$$

i.e. the partition function of a string theory with action induced by the Kalb-Ramond tensor field $B_{\mu\nu}$.

In the semiclassical approximation the modified Bessel function $I_p(a)$ behaves like [14]

$$\log I_p(a) \sim (p^2 + a^2)^{1/2} - p \sinh^{-1}(p/a) \ . \tag{2.10}$$

Using this in (2.9) we obtain the Euclidean version of Polyakov's result presented in the previous section. Furthermore, for small p/a the modified Bessel functions satisfy

$$\frac{I_p(a)}{I_0(a)} \sim e^{-p^2/2a} \ . \tag{2.11}$$

Therefore, the partition function reduces to the standard gaussian result (1.5) (after the overall factor of $I_0(a)$ is absorbed in the integral).

An analogous computation in Minkowski space leads to (2.5). Note, however, that only in Minkowski space the sum over surfaces is equivalent to a sum over branches of a multivalued function, as stressed in [9]. Our result indicates that the key feature of the dynamical generation of strings is the periodic structure of the potential induced by the condensation of the topological defects. Strings arise from the summation over the periods of this potential in the duality transformation.

Note that this computation can be generalized. Indeed, the integration over φ_μ in equation (2.3) is a particular case of the following path integral in D dimensions:

$$Z = \int \mathcal{D}A\mathcal{D}F \ \exp\left(i \int d^D x \, (A \cdot (\partial F)) - \int d^D x \left\{ g(A^2) + h(F^2) \right\} \right) \ , \tag{2.12}$$

where A and F are antisymmetric tensors of rank r and $D - r - 1$ respectively and h and g are arbitrary functions, also $A \cdot (\partial F) \equiv \varepsilon_{\mu_1 \cdots \mu_D} A_{\mu_1 \cdots \mu_r} \partial_{\mu_{r+1}} F_{\mu_{r+2} \cdots \mu_D}$. Integration over A gives precisely the Fourier transform of e^{-g}, therefore we can write:

$$Z = \int \mathcal{D}F \ \exp\left(- \int d^D x \left\{ \tilde{g}\left((\partial F)^2 \right) + h(F^2) \right\} \right) \ , \tag{2.13}$$

where $e^{-\tilde{g}}$ is the Fourier transform of e^{-g} [15]. For instance, since the Fourier transform of e^{-x^2} is $e^{-p^2/4}$, we can easily recover the well known result that a massive tensor of rank r is dual to a massive tensor of rank $D - r - 1$ in D dimensions ($g(A^2) = A^2$ and $h(F^2) = F^2$). We can then expect that for a theory of a massless antisymmetric tensor of rank r, the condensation of topological defects of dimension $D - r - 2$ gives rise to a phase defined by 'confining r-branes'.

3. Low-Energy Effective Action and θ-Term

In a *compact* U(1) theory the θ-term produces non-trivial effects, notably it assigns an electric charge $q = e\theta/2\pi$ to elementary magnetic monopoles [16]. The confining phase arises thus due to the condensation of dyons. Since we are mostly interested in the low-energy limit of the confining string we shall use the quadratic expansion (1.5) of the Kalb-Ramond action. Following [11] the change induced by the θ-term is given by

$$S(B_{\mu\nu}) = \int d^4 x \ \frac{1}{12\Lambda^2} H_{\mu\nu\alpha} H_{\mu\nu\alpha} + \frac{1}{4e^2} B_{\mu\nu} B_{\mu\nu} \ + i \frac{\theta}{64\pi^2} B_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} B_{\alpha\beta} \ . \tag{3.1}$$

Correspondingly, the mass of the Kalb-Ramond field is modified from $m = \Lambda/e$ to

$$\begin{aligned} m_\theta &= \frac{e\Lambda}{4\pi}\sqrt{\left(\frac{4\pi}{e^2}\right)^2 + t^2}\ , \\ t &\equiv \frac{\theta}{2\pi}\ . \end{aligned} \tag{3.2}$$

This mass is determined by the same modular parameter $\tau = (\theta/2\pi) + i\left(4\pi/e^2\right)$ which enters the mass formula for the dyons in the BPS limit [17]. Note that, in the limit $\Lambda \to 0$ in which the density of monopoles (dyons) vanishes, the action (3.1) simply reduces to the action (1.6) of pure QED, since only configurations for which $H_{\mu\nu\alpha} = 0$ contribute to the partition function in this case. Correspondingly, the confining string action $S_{\rm CS}$ in (1.1) reduces to the Coulomb interaction of the end-points of open strings:

$$\begin{aligned} \lim_{\Lambda\to 0} S_{\rm CS} &= -\ln\langle W(C)\rangle_{\rm Coul} = \frac{e^2}{2}\int d^4x\ j_\mu \frac{1}{-\nabla^2} j_\mu\ , \\ j_\mu(\mathbf{x}) &\equiv 2\partial_\nu T_{\mu\nu}(\mathbf{x}) = \int_{\rm C} d\tau\ \frac{dx_\mu}{d\tau}\ \delta^4\left(\mathbf{x} - \mathbf{x}(\tau)\right)\ , \end{aligned} \tag{3.3}$$

where C denotes the closed world-line bounding the original open world-sheet.

Performing the Gaussian integration over the field $B_{\mu\nu}$ we obtain the confining string action in the form

$$S_{\rm CS} = \int d^4x\ T_{\mu\nu}\frac{\Lambda^2}{m_\theta^2 - \nabla^2}T_{\mu\nu} + 2e^2\partial_\nu T_{\mu\nu}\frac{1}{m_\theta^2 - \nabla^2}\partial_\alpha T_{\mu\alpha} + i\frac{e^2\Lambda^2\theta}{16\pi^2}T_{\mu\nu}\frac{\epsilon_{\mu\nu\alpha\beta}}{m_\theta^2 - \nabla^2}T_{\alpha\beta}\ . \tag{3.4}$$

To further analyze the low-energy limit of (3.4) we first introduce the representation (1.2) in (3.4), and then perform a derivative expansion of the resulting action. Notice that this is equivalent to sending the cut-off or, equivalently, m_θ, to ∞; in other words, the derivative expansion is equivalent to an expansion in powers of $1/m_\theta$.

We start by noting that the 4D Yukawa Green function in (3.4) is given by

$$G(\mathbf{x}) \equiv \frac{1}{m_\theta^2 - \nabla^2}\delta^4(\mathbf{x}) = \frac{m_\theta^2}{4\pi^2}\frac{1}{m_\theta r}K_1(m_\theta r)\ , \tag{3.5}$$

with $r \equiv |\mathbf{x}|$ and K_1 a modified Bessel function [13]. In computing (3.4) using the representation (1.2) we encounter the expression $G\left(\sqrt{g_{ab}(\sigma)\epsilon^a\epsilon^b}\right)$ with

$$g_{ab} = \frac{\partial x_\mu}{\partial\sigma^a}\frac{\partial x_\mu}{\partial\sigma^b}\ , \tag{3.6}$$

the *induced metric* on the world-sheet. The derivative expansion of this Green function on the world-sheet is obtained by expanding $\int d^2\epsilon \ G\left(\sqrt{g_{ab}\epsilon^a\epsilon^b}\right) f(\epsilon)$ in powers of $1/m_\theta$ for any test function $f(\epsilon)$. Naturally, the coefficients of this expansion may diverge and must then be regularized by the ultraviolet cut-off Λ_0. We obtain

$$
\begin{aligned}
G\left(\sqrt{g_{ab}\epsilon^a\epsilon^b}\right) &= \frac{1}{2\pi} g^{-1/2} K_0\left(\frac{m_\theta}{\Lambda_0}\right) \delta^2(\epsilon) + \frac{1}{4\pi m_\theta^2} g^{-1/2} g^{ab} \partial_a \partial_b \delta^2(\epsilon) + \dots \ , \\
g = \det g_{ab} &= \frac{1}{2} X_{\mu\nu} X_{\mu\nu} \ .
\end{aligned}
\tag{3.7}
$$

Inserting this expression in (3.4) we obtain the desired derivative expansion of the confining string action $S_{\rm CS}$ up to terms of $O(m_\theta^0)$:

$$
\begin{aligned}
S_{\rm CS} = & \frac{\Lambda^2}{4\pi} K_0\left(\frac{m_\theta}{\Lambda_0}\right) \int d^2\sigma \ \sqrt{g} - \frac{\Lambda^2}{16\pi m_\theta^2} \int d^2\sigma \ \sqrt{g} g^{ab} \partial_a t_{\mu\nu} \partial_b t_{\mu\nu} + \frac{\Lambda^2}{16\pi m_\theta^2} \int d^2\sigma \ \sqrt{g} R \\
& - i \frac{\pi t}{\left(\frac{4\pi}{e^2}\right)^2 + t^2} \nu + \frac{e^2 m_\theta}{8\pi^2} f\left(\frac{m_\theta}{\Lambda_0}\right) \int_{\rm boundary} d\tau \ \sqrt{\frac{dx_\mu}{d\tau}\frac{dx_\mu}{d\tau}} + \dots \ ,
\end{aligned}
\tag{3.8}
$$

where

$$
t_{\mu\nu} \equiv g^{-1/2} X_{\mu\nu} \ , \tag{3.9}
$$

and R is the scalar curvature of the world-sheet. The function $f\left(\frac{m_\theta}{\Lambda}\right)$ is defined as:

$$
f(x) = \int_x^\infty \frac{dz}{z} K_1(z) \ .
$$

The first term in the expansion (3.8) is the Nambu-Goto term. The second and third term represent the *extrinsic* and intrinsic *curvature* terms, respectively. Note that the extrinsic curvature terms enters the Euclidean action with a *negative coefficient*, as anticipated. The fourth term is a topological term since

$$
\nu \equiv \frac{1}{4\pi} \int d^2\sigma \ \sqrt{g} \ \epsilon^{\mu\nu\alpha\beta} g^{ab} \partial_a t_{\mu\nu} \partial_b t_{\alpha\beta} \tag{3.10}
$$

is the analytic expression for the (signed) *self-intersection number* of the world-sheet. Finally, the fifth term represents a boundary correction of $O(m_\theta)$. All higher order terms are suppressed by negative powers of m_θ and are thus infrared irrelevant.

Case with $\theta = 0$

In the following we would like to study the renormalized action when the cut-off Λ_0 is removed. Let us begin with the case $\theta = 0$. In this case, (3.8) becomes:

$$S_{\rm CS} = \frac{\Lambda^2}{4\pi} K_0\left(\frac{\sqrt{z}}{4e}\right) \int d^2\sigma\ \sqrt{g} - \frac{e^2}{16\pi}\int d^2\sigma\ \sqrt{g} g^{ab}\partial_a t_{\mu\nu}\partial_b t_{\mu\nu} + \frac{e^2}{16\pi}\int d^2\sigma\ \sqrt{g}R$$
$$+ \frac{e\Lambda}{8\pi^2} f\left(\frac{\sqrt{z}}{4e}\right)\int d\tau \sqrt{\frac{dx_\mu}{d\tau}\frac{dx_\mu}{d\tau}}\ . \tag{3.11}$$

This result is however obtained by a derivative expansion and is thus valid only on scales much bigger than $1/m$, with $m = \Lambda/e$. Moreover, because we are in the phase in which the monopoles (dyons) are condensed, our result is valid only in the strong coupling limit $e > e_{\rm cr}$. In the limit $\Lambda_0 \to \infty$ we can use it only if $e \to$ const or $e \to \infty$ such that $m \to$ const or $m \to \infty$. However, in all these cases the string tension diverges and the strings are suppressed since $\lim_{e\to\infty} K_0(\sqrt{z}/4e) = \lim_{e\to\infty} K_0({\rm const}/e) = \infty$. If we take the limit $e \to \infty$ for fixed cut-off, or if $\Lambda_0 = o(e)$, the mass $m \to 0$. In this case we have to resort to the original expression (3.4). In the case of fixed cut-off, and on distance scales much larger than $1/\Lambda$ this reduces to the action

$$S_{\rm CS} = \int d^4x\ \Lambda^2 T_{\mu\nu}\frac{1}{-\nabla^2}T_{\mu\nu} \tag{3.12}$$

for *closed strings* (boundary terms are suppressed). If we compute this expression for a sphere of radius $R \gg 1/\Lambda$ we obtain

$$S^R_{\rm CS} \propto \Lambda^2 R^2 \ln R\Lambda\ ,$$

which means that strings are logarithmically confined on scales larger than $1/\Lambda$. In the case in which the cut-off $\Lambda_0 \to \infty$, also (3.12) diverges and again strings are completely suppressed. For $\theta = 0$ the confining string in 4D makes thus sense only as a cut-off theory for finite e.

Case with $\theta \neq 0$

In the case in which $\theta \neq 0$, instead, we have $m_\theta = \frac{e\Lambda}{4\pi}\sqrt{\left(\frac{4\pi}{e^2}\right)^2 + t^2}$, and

$$m_\theta \simeq \frac{e\Lambda t}{4\pi} \qquad \text{for } e \gg 1\ .$$

Since $m_\theta \to \infty$ for $\Lambda_0 \to \infty$, we can use the derivative expansion which takes the form

$$S_{\rm CS} = \frac{\Lambda^2}{4\pi}K_0\left(\frac{et}{4\pi}\right)\int d^2\sigma\ \sqrt{g} - \frac{1}{16\pi}\left(\frac{4\pi}{et}\right)^2\int d^2\sigma\ \sqrt{g}g^{ab}\partial_a t_{\mu\nu}\partial_b t_{\mu\nu} +$$
$$+ \frac{1}{16\pi}\left(\frac{4\pi}{et}\right)^2\int d^2\sigma\ \sqrt{g}R - i\frac{\pi t}{\left(\frac{4\pi}{e^2}\right)^2 + t^2}\nu + \frac{e^3 t\Lambda}{32\pi^3} f\left(\frac{et}{4\pi}\right)\int d\tau\sqrt{\frac{dx_\mu}{d\tau}\frac{dx_\mu}{d\tau}}\ , \tag{3.13}$$

for $e \gg 1$. We now use the asymptotic behaviours for large x:

$$K_0(x) \simeq \frac{\mathrm{e}^{-x}}{\sqrt{x}} \ , \quad f(x) \simeq \frac{\mathrm{e}^{-x}}{x^{3/2}} \ , \tag{3.14}$$

and we take the simultaneous limit $\Lambda_0 \to \infty$ and $e \to \infty$ so that

$$\frac{\Lambda^2}{4\pi} \frac{\mathrm{e}^{\frac{-et}{4\pi}}}{\sqrt{\frac{et}{4\pi}}} = T \ , \tag{3.15}$$

with T the physical string tension. In this limit only closed surfaces survive and the action becomes

$$S_{\mathrm{CS}} = T \int d^2\sigma \ \sqrt{g} - i\frac{\pi}{t} \ \nu \ . \tag{3.16}$$

The topological correction term to the Nambu-Goto action can be considered as a "*spin term*" for the string, analogous to the topological spin term of 2D point particles (self-intersection number of the world-line) [18]. Note that $t = 1$ for $\theta = 2\pi$ i.e. for a dyon condensate of elementary charge e. In this case the weight of self-intersections in the partition function is $(-1)^\nu$, which is the mechanism originally advocated by Polyakov for the stabilization of rigid strings [1][2]. The periodicity in θ is presumably recovered for Z_{N} theories which contain also electric excitations with no magnetic charges [19].

The expression (3.15) for the string tension can be rewritten as

$$\ln \frac{\Lambda^2}{4\pi T} = \frac{et}{4\pi} + \frac{1}{2} \ln \frac{et}{4\pi} \ . \tag{3.17}$$

For $e \gg 1$ the logarithm can be neglected with respect to the linear function and we can write

$$e = \frac{4\pi}{t} \ln \frac{\Lambda_0^2}{4\pi T} \ . \tag{3.18}$$

As usual we can express the cut-off in terms of the physical string tension T and a reference scale μ as $\Lambda_0 = \sqrt{4\pi} T/\mu$ so that $\Lambda_0 \to \infty$ corresponds to the infrared limit $\mu \to 0$. This gives us the running coupling constant

$$e(\mu) = \frac{4\pi}{t} \ln \frac{T}{\mu^2} \ , \tag{3.19}$$

In the infrared $\mu \to 0$ we have $e \to \infty$. At a finite scale this equation determines the perturbative corrections, like the extrinsic curvature, to the free string with topological term.

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