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Renormalization of Velocity-Changing Dimension-Five Operators in the Heavy-Quark Effective Theory

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Abstract

We study the renormalization of operators of the type $\bar{h}_{v'}\Gamma\, G^{\mu\nu} h_v$ in the heavy-quark effective theory (HQET). We construct the combinations of such operators that are renormalized multiplicatively, and calculate their velocity-dependent anomalous dimensions at the one-loop order. We then show that the virial theorem of the HQET is not renormalized, and that in the limit of equal velocities the anomalous dimension of the chromo-electric operator vanishes to all orders in perturbation theory. This implies an exact relation between renormalization constants, which may help in a future calculation of the two-loop anomalous dimension of the chromo-magnetic operator.

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1 Introduction

The heavy-quark effective theory (HQET) is a convenient tool to explore the physics of hadrons containing a heavy quark [1]. It provides a systematic expansion around the limit $m_Q \to \infty$, in which new symmetries of the strong interactions arise, relating the long-distance properties of many observables to a small number of hadronic matrix elements. In the HQET, a heavy quark inside a hadron moving with four-velocity v is described by a velocity-dependent field h_v subject to the constraint $\not{v}\, h_v = h_v$. This field is related to the original heavy-quark field by a phase redefinition, so that it carries the "residual momentum" $k = p_Q - m_Q v$, which characterizes the interactions of the heavy quark with gluons. The effective Lagrangian of the HQET is [2]–[4]

$$\mathcal{L}_{\rm eff} = \bar h_v\, iv\cdot D\, h_v + \frac{1}{2m_Q}\, \bar h_v (iD_\perp)^2 h_v + \frac{C_{\rm mag}(\mu)}{4m_Q}\, \bar h_v\, \sigma_{\mu\nu} G^{\mu\nu} h_v + O(1/m_Q^2)\,, \qquad (1)$$

where $D_\perp^\mu = D^\mu - (v\cdot D)\, v^\mu$ contains the components of the gauge-covariant derivative orthogonal to the velocity, and $G^{\mu\nu} = i[D^\mu, D^\nu]$ is the gluon field strength tensor. The leading term in the effective Lagrangian, which gives rise to the Feynman rules of the HQET, is invariant under a global $SU(2N_Q)$ spin–flavour symmetry group. This so-called heavy-quark symmetry results from the fact that in the limit $m_Q \to \infty$ the properties of the light constituents inside a heavy hadron become independent of the spin and flavour of the heavy quark. The symmetry is explicitly broken by the higher-dimensional operators arising at order $1/m_Q$, whose origin is most transparent in the rest frame of the heavy hadron: the first operator corresponds to the kinetic energy resulting from the motion of the heavy quark inside the hadron (in the rest frame, $(iD_\perp)^2$ is the operator for $-\mathbf{k}^2$), and the second operator describes the magnetic interaction of the heavy-quark spin with the gluon field. The Wilson coefficient $C_{\rm mag}(\mu)$ results from short-distance effects and depends logarithmically on the scale at which the chromo-magnetic operator is renormalized [2, 4]. As a consequence of the so-called reparametrization invariance of the HQET (an invariance under infinitesimal changes of the velocity), the kinetic operator is not multiplicatively renormalized [5, 6].

One of the most important applications of heavy-quark symmetry concerns the analysis of semileptonic weak decays of heavy hadrons. These processes are mediated by flavour-changing currents containing two heavy-quark fields, which in the HQET are represented by velocity-changing operators of the type $\bar h_{v'}\Gamma\, h_v$, where Γ represents some Dirac matrix. In the heavy-quark limit, all weak decay form factors parametrizing the current matrix elements between two heavy mesons or baryons are proportional to a universal (Isgur–Wise) form factor $\xi(v\cdot v')$ [7]. Large logarithms of the heavy-quark masses, which arise due to quantum corrections, can be summed up to all orders in perturbation theory by calculating the anomalous dimension of the operator $\bar h_{v'}\Gamma\, h_v$ in the HQET and solving its renormalization-group equation (RGE) [8, 9].

In the analysis of the symmetry-breaking corrections to the relations between form factors, higher-dimensional current operators play an important role. At order $1/m_Q$, there appear local dimension-four operators of the type $\bar h_{v'}\Gamma\, iD^\mu\, h_v$, whose matrix elements have been studied in [10]. The renormalization of such operators is completely determined by reparametrization invariance [11]. In the present paper, we study the renormalization of dimension-five current operators containing the gluon field, i.e. operators of the form $\bar h_{v'}\Gamma\, G^{\mu\nu} h_v$. There are several motivations for this study: The matrix elements of these operators determine part of the $1/m_Q^2$ corrections to heavy-hadron weak decay form factors [12], which at the present level of accuracy already have to be included in some applications of the HQET, such as the extraction of the Cabibbo–Kobayashi–Maskawa matrix element $|V_{cb}|$ [13]. The same operators also play a role in the description of non-factorizable corrections in non-leptonic two-body decays of B mesons [14]. Our primary motivation, however, is the connection between velocity-changing dimension-five operators and the velocity-conserving operators appearing at order $1/m_Q$ in the effective Lagrangian (1). Clearly, for $\Gamma = \sigma_{\mu\nu}$ and in the limit of equal velocities, the operator $\bar h_{v'}\Gamma\, G^{\mu\nu} h_v$ reduces to the chromo-magnetic operator. More interestingly, however, there is also a connection with the kinetic operator $\bar h_v (iD_\perp)^2 h_v$. It is provided by the virial theorem of the HQET, which relates the kinetic energy of a heavy quark inside a hadron to its interactions with gluons [15]. For the ground-state mesons and baryons, this theorem can be written in the form

$$\lim_{v'\to v} \frac{\langle H(v')|\, \bar h_{v'} v_\mu v'_\nu\, iG^{\mu\nu} h_v\, |H(v)\rangle}{(v\cdot v')^2 - 1} = \frac{1}{3}\, \langle H(v)|\, \bar h_v (iD_\perp)^2 h_v\, |H(v)\rangle\,. \qquad (2)$$

It has been used to study the properties of the kinetic operator under renormalization, and to estimate its matrix element using QCD sum rules [16].

Below we study first the general structure of operator mixing of current operators containing the gluon field and show that it can be described by a 4×4 matrix of renormalization constants. We then calculate the corresponding anomalous dimensions at the one-loop order. Next, we investigate the limit of equal velocities, in which case the operator basis consists of a chromo-electric and a chromo-magnetic operator, which are renormalized multiplicatively. We show that, as a consequence of the virial theorem and reparametrization invariance, the chromo-electric operator is not multiplicatively renormalized to all orders in perturbation theory. This, in turn, implies an exact relation between renormalization constants, which may help in the calculation of the yet unknown two-loop anomalous dimension of the chromo-magnetic operator. Finally, we consider the hadronic matrix elements of velocity-changing dimension-five operators and determine, for the case of the ground-state heavy mesons, the scale dependence of the corresponding hadronic form factors.

2 Operator mixing and anomalous dimensions

Our goal is to study the renormalization of the local operator $O_1^{\mu\nu} = \bar h_{v'}\Gamma\, G^{\mu\nu} h_v$, where v and v' are the heavy-quark velocities, and Γ may be an arbitrary Dirac matrix. Since the Feynman rules of the HQET do not involve γ matrices, the structure of Γ will not be altered by radiative corrections. Under renormalization, the operator $O_1^{\mu\nu}$ mixes with other operators carrying the same global quantum numbers. We use the background-field formalism [18] and work in dimensional regularization, so that it suffices to consider gauge-invariant operators of the same dimension as $O_1^{\mu\nu}$. Moreover, we shall not consider operators that vanish by the equations of motion, since they have vanishing matrix elements between physical states. To construct the operator basis, we find it convenient to introduce the two vectors

$$v_+ = \frac{v+v'}{\sqrt{2(w+1)}}\,, \qquad v_- = \frac{v-v'}{\sqrt{2(w-1)}}\,, \tag{3}$$

where $w = v\cdot v'$ is the product of the two velocities. This definition is such that $v_+^2 = 1$, $v_-^2 = -1$, and $v_+\cdot v_- = 0$. Hence, v_+ can be regarded as a four-velocity, whereas v_- is a space-like four vector. In the Breit frame, where the two hadrons move with opposite velocities, we have $v_+^\mu = (1,\mathbf{0})$ and $v_-^\mu = (0,\mathbf{n})$, where $\mathbf{n}$ is a spatial unit vector. Since the original operator $O_1^{\mu\nu}$ is invariant under Hermitean conjugation followed by an interchange of the velocities, the basis operators must contain even powers of v_-. Four such operators can be constructed, and we define

$$\begin{aligned}
O_1^{\mu\nu} &= \bar h_{v'}\Gamma\, G^{\mu\nu} h_v\,,\\
O_2^{\mu\nu} &= v_+^{[\mu} v_{+\alpha}\, \bar h_{v'}\Gamma\, G^{\alpha\nu]} h_v\,,\\
O_3^{\mu\nu} &= v_-^{[\mu} v_{-\alpha}\, \bar h_{v'}\Gamma\, G^{\alpha\nu]} h_v\,,\\
O_4^{\mu\nu} &= (v_+^\mu v_-^\nu - v_+^\nu v_-^\mu)\, v_{+\alpha} v_{-\beta}\, \bar h_{v'}\Gamma\, G^{\alpha\beta} h_v\,.
\end{aligned} \tag{4}$$

We use a short-hand notation such that $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$. In principle, there are other operators carrying the same quantum numbers, which contain two derivatives acting on the heavy-quark fields. However, the Feynman rules of the HQET ensure that all operators that can mix with the above ones and do not involve the gluon field strength tensor vanish by the equations of motion.

We define renormalization constants Z_{ij}, which absorb the ultraviolet (UV) divergences in the matrix elements of the bare operators, by the relation

$$O_{i,\mathrm{bare}}^{\mu\nu} = \sum_{j=1}^4 Z_{ij} O_j^{\mu\nu}\,. \tag{5}$$

From the definition of the basis operators, it follows that the 4×4 matrix $\hat{\mathbf{Z}} = (Z_{ij})$ can be expressed in terms of the four entries $Z_{1j} \equiv Z_j(w)$, which in general are functions of the variable w. Substituting for Γ the appropriate expressions corresponding to the operators in (4), we find that

$$\hat{\mathbf{Z}} = \begin{pmatrix} Z_1(w) & Z_2(w) & Z_3(w) & Z_4(w) \\ 0 & Z_1(w)+Z_2(w) & 0 & Z_3(w)+Z_4(w) \\ 0 & 0 & Z_1(w)-Z_3(w) & Z_2(w)-Z_4(w) \\ 0 & 0 & 0 & Z_1(w)+Z_2(w)-Z_3(w)-Z_4(w) \end{pmatrix} . \tag{6}$$

Let us denote by $z_n(w)$ the eigenvalues of this matrix, given by the diagonal entries. Since the mixing of $O_1^{\mu\nu}$ with $O_3^{\mu\nu}$ and $O_4^{\mu\nu}$ must vanish in the limit of equal velocities, it follows that

$$Z_3(1) = Z_4(1) = 0 \,. \tag{7}$$

This implies $z_1(1) = z_3(1)$ and $z_2(1) = z_4(1)$. The eigenoperators $\mathcal{O}_n^{\mu\nu}$, which are renormalized multiplicatively according to $\mathcal{O}_{n,\mathrm{bare}}^{\mu\nu} = z_n(w)\mathcal{O}_n^{\mu\nu}$, are given by

$$\begin{aligned} \mathcal{O}_1^{\mu\nu} &= O_1^{\mu\nu} - O_2^{\mu\nu} + O_3^{\mu\nu} - O_4^{\mu\nu} \,, \\ \mathcal{O}_2^{\mu\nu} &= O_2^{\mu\nu} + O_4^{\mu\nu} \,, \\ \mathcal{O}_3^{\mu\nu} &= O_3^{\mu\nu} - O_4^{\mu\nu} \,, \\ \mathcal{O}_4^{\mu\nu} &= O_4^{\mu\nu} \,. \end{aligned} \tag{8}$$

The anomalous dimensions of these operators, which appear in the RGE

$$\left(\mu\,\frac{\mathrm{d}}{\mathrm{d}\mu} + \gamma_n(w) \right) \mathcal{O}_n^{\mu\nu}(\mu) = 0 \,, \tag{9}$$

are obtained from the relation

$$\gamma_n(w) = -2\alpha_s\,\frac{\partial}{\partial\alpha_s}\,z_n^{(1)}(w) \,, \tag{10}$$

where $z_n^{(1)}(w)$ denotes the coefficient of the $1/\epsilon$ pole in $z_n(w)$ calculated in dimensional regularization, i.e. in $d = 4 - 2\epsilon$ space-time dimensions.

At the one-loop order, the renormalization factors $Z_i(w)$ are determined by the UV divergences of the diagrams shown in Fig. 1. Z_h denotes the wave-function renormalization constant for the heavy-quark field in the HQET. A virtue of the background field formalism is that the gluon field is not renormalized, since $Z_g Z_A^{1/2} = 1$ [18]. We have performed the calculation of these diagrams in an arbitrary covariant gauge, and with arbitrary momentum assignments. The sum of the UV divergent contributions is independent of the gauge and of the external momenta of the heavy quarks. We find

$$Z_1(w) = 1 + \frac{\alpha_s}{4\pi\epsilon}\,\Big\{ (C_A - 2C_F)\Big[w\,r(w) - 1 \Big] - C_A \Big\} \,,$$

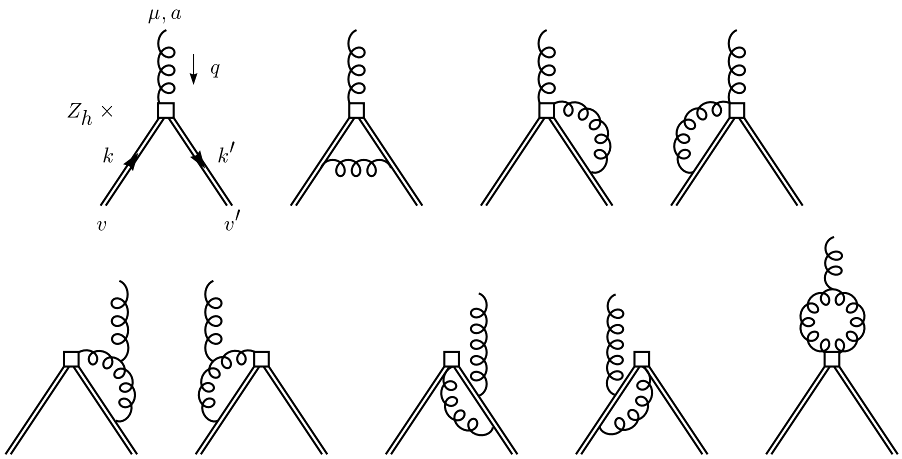

Figure 1: One-loop diagrams contributing to the calculation of the renormalization factors $Z_n(w)$. The velocity-changing operators are represented by a square, and heavy-quark propagators are drawn as double lines.

$$
\begin{aligned}
Z_2(w) &= \frac{w+1}{2}\,\frac{C_A\alpha_s}{4\pi\epsilon}\,,\\
Z_3(w) &= \frac{w-1}{2}\,\frac{C_A\alpha_s}{4\pi\epsilon}\,,\\
Z_4(w) &= 0\,,
\end{aligned}
\tag{11}
$$

where $C_F = \frac{1}{2}(N-1/N)$ and $C_A = N$ are the eigenvalues of the quadratic Casimir operator in the fundamental and the adjoint representations, and N is the number of colours. The function $r(w)$ is given by

$$
r(w) = \frac{1}{\sqrt{w^2-1}}\,\ln\Big(w+\sqrt{w^2-1}\Big)
\tag{12}
$$

and satisfies $r(1)=1$. Note that $Z_3(w)$ vanishes in the limit of equal velocities, in accordance with (7). Moreover, in that limit we find

$$
Z_1(1) + Z_2(1) = 1\,.
\tag{13}
$$

Below we shall argue that this is an exact relation, valid to all orders in perturbation theory.

Applying now the relation (10) to the combinations of renormalization factors appearing in the diagonal entries in (6), we obtain for the one-loop anomalous dimensions

$$
\begin{aligned}
\gamma_1(w) &= \gamma_4(w) + \frac{C_A\alpha_s}{2\pi}\,,\\
\gamma_2(w) &= \gamma_4(w) - \frac{w-1}{2}\,\frac{C_A\alpha_s}{2\pi}\,,
\end{aligned}
$$

$$
\begin{aligned}
\gamma_3(w) &= \gamma_4(w) + \frac{w+1}{2}\,\frac{C_A\alpha_s}{2\pi}\,, \\
\gamma_4(w) &= -(C_A - 2C_F)\,\frac{\alpha_s}{2\pi}\,\Big[w\,r(w) - 1\Big]\,.
\end{aligned}
\tag{14}
$$

In the limit of equal velocities, we find

$$
\begin{aligned}
\gamma_1(1) &= \gamma_3(1) = \frac{C_A\alpha_s}{2\pi}\,, \\
\gamma_2(1) &= \gamma_4(1) = 0\,.
\end{aligned}
\tag{15}
$$

Because of (7) and (13), the second relation is valid to all orders in perturbation theory.

Knowing the one-loop anomalous dimensions allows us to solve the RGE (9) for the eigenoperators in the leading logarithmic approximation. The solution reads

$$
\mathcal{O}_n(m_Q) = \left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right)^{\gamma_n^0(w)/2\beta_0} \mathcal{O}_n(\mu)\,,
\tag{16}
$$

where the coefficients $\gamma_n^0(w)$ are defined by $\gamma_n(w) = \gamma_n^0(w)(\alpha_s/4\pi)$, and $\beta_0 = \frac{11}{3}N - \frac{2}{3}n_f$ is the first coefficient of the β function.

3 Equal-velocity limit

The discussion of operator mixing becomes more transparent in the limit of equal velocities (i.e. $v' = v$ and $w = 1$), in which $v_+ \to v$ and $v_- \to n$, where n is an external space-like four-vector satisfying $n^2 = -1$ and $n\cdot v = 0$. Because of relation (7), the mixing of the operators $O_1^{\mu\nu}$ and $O_2^{\mu\nu}$ decouples from that of $O_3^{\mu\nu}$ and $O_4^{\mu\nu}$; indeed, the latter two operators are simply proportional to the first two: $O_3^{\mu\nu} = n^{[\mu} n_\alpha O_1^{\alpha\nu]}$ and $O_4^{\mu\nu} = n^{[\mu} n_\alpha O_2^{\alpha\nu]}$. The operators that are multiplicatively renormalized can be chosen as

$$
\begin{aligned}
O_{\rm mag}^{\mu\nu} &= O_1^{\mu\nu} - O_2^{\mu\nu} = \bar h_v \Gamma\, G_\perp^{\mu\nu} h_v\,, \\
O_{\rm el}^{\mu\nu} &= O_2^{\mu\nu} = v^{[\mu} v_\alpha\, \bar h_v \Gamma\, G^{\alpha\nu]} h_v\,,
\end{aligned}
\tag{17}
$$

where $G_\perp^{\mu\nu} = (g_\alpha^\mu - v_\alpha v^\mu)(g_\beta^\nu - v_\beta v^\nu) G^{\alpha\beta}$ contains the components of the field strength tensor in the subspace orthogonal to the velocity. In the rest frame, where $v^\mu = (1,\mathbf{0})$, the operators $O_{\rm mag}^{\mu\nu}$ and $O_{\rm el}^{\mu\nu}$ correspond to purely chromo-magnetic and chromo-electric field configurations, respectively. The anomalous dimensions of these operators are given by

$$
\begin{aligned}
\gamma_{\rm mag} &= \gamma_1(1) = \gamma_3(1) = -2\alpha_s\,\frac{\partial}{\partial\alpha_s}\,Z_1^{(1)}\,, \\
\gamma_{\rm el} &= \gamma_2(1) = \gamma_4(1) = -2\alpha_s\,\frac{\partial}{\partial\alpha_s}\,\Big[Z_1^{(1)} + Z_2^{(1)}\Big]\,,
\end{aligned}
\tag{18}
$$

where $Z_n^{(1)}$ is the coefficient of the $1/\epsilon$ pole in $Z_n(1)$. The anomalous dimension of the chromo-magnetic operator is known at the one-loop order [2, 4], and it is in agreement with our result for $\gamma_1(1)$ in (15). The virial theorem (2) implies that the chromo-electric operator has the same anomalous dimension as the kinetic operator $\bar{h}_v(iD_\perp)^2 h_v$. However, reparametrization invariance enforces that to all orders in perturbation theory the kinetic operator is not multiplicatively renormalized [5]. Therefore, we obtain the exact relation

$$\gamma_{\rm el} = \gamma_{\rm kin} = 0\,, \tag{19}$$

which is equivalent to (13). Again, this constraint is satisfied by our explicit one-loop result for $\gamma_2(1)$ in (15). Using this relation, we obtain

$$\gamma_{\rm mag} = -2\alpha_s\,\frac{\partial}{\partial\alpha_s}\,Z_1^{(1)} = 2\alpha_s\,\frac{\partial}{\partial\alpha_s}\,Z_2^{(1)}\,. \tag{20}$$

Recalling the definition of the renormalization factors,

$$[\bar{h}_v\Gamma\,G^{\mu\nu}h_v]_{\rm bare} = Z_1\,\bar{h}_v\Gamma\,G^{\mu\nu}h_v + Z_2\,v^{[\mu}v_\alpha\,\bar{h}_v\Gamma\,G^{\alpha\nu]}h_v\,, \tag{21}$$

we observe that relation (20) provides for two complementary ways to calculate the anomalous dimension of the chromo-magnetic operator. This relation may be useful in a future calculation of $\gamma_{\rm mag}$ beyond the one-loop order.

4 Hadronic matrix elements

In the HQET, hadronic matrix elements of the operators $O_n^{\mu\nu}$ in (4) can be parametrized in terms of invariant functions $\phi_i(w,\mu)$, which are generalizations of the Isgur–Wise form factor. These functions are most conveniently introduced using a covariant formalism, in which heavy hadrons are classified in multiplets of the heavy-quark spin symmetry and described by spin wave functions with the appropriate transformation properties [8, 17]. In particular, the ground-state pseudoscalar and vector mesons, $P(v)$ and $V(v)$, are described by a matrix

$$\mathcal{M}(v) = \sqrt{m_M}\,\frac{1+\rlap{/}v}{2}\,\Big[\gamma_5\,P(v) + \rlap{/}e\,V(v)\Big]\,, \tag{22}$$

which satisfies $\rlap{/}v\mathcal{M}(v) = \mathcal{M}(v) = -\mathcal{M}(v)\,\rlap{/}v$. Here m_M is the hadron mass, v the hadron velocity, and e the polarization vector of the vector meson. The matrix elements of the operators $O_n^{\mu\nu}$ between meson states can be obtained from the relation [12]

$$\langle M(v')|\,\bar{h}_{v'}\Gamma\,iG^{\mu\nu}h_v\,|M(v)\rangle = -\mathrm{Tr}\Big\{\phi^{\mu\nu}(v,v',\mu)\,\overline{\mathcal{M}}(v')\,\Gamma\mathcal{M}(v)\Big\}\,, \tag{23}$$

where μ is the scale at which the operators are renormalized. The tensor form factor $\phi^{\mu\nu}(v, v', \mu)$ can be decomposed in terms of three scalar functions, i.e.

$$\begin{aligned}\phi^{\mu\nu}(v, v', \mu) &= (v^\mu v'^\nu - v^\nu v'^\mu)\,\phi_1(w, \mu) \\ &\quad + \Big[(v - v')^\mu \gamma^\nu - (v - v')^\nu \gamma^\mu\Big]\,\phi_2(w, \mu) + i\sigma^{\mu\nu}\,\phi_3(w, \mu)\,. \end{aligned} \tag{24}$$

Evaluating (23) for the eigenoperators $\mathcal{O}_n^{\mu\nu}$ in (8), we obtain the combinations of scalar functions that are renormalized multiplicatively. They are

$$\begin{aligned}\psi_1(w, \mu) &= \phi_3(w, \mu)\,, \\ \psi_3(w, \mu) &= \phi_3(w, \mu) - (w - 1)\,\phi_2(w, \mu)\,, \\ \psi_4(w, \mu) &= (w + 1)\,\phi_1(w, \mu) - 2\phi_2(w, \mu) - \phi_3(w, \mu)\,, \end{aligned} \tag{25}$$

and the corresponding anomalous dimensions are $\gamma_1(w)$, $\gamma_3(w)$, and $\gamma_4(w)$, respectively. The operator $\mathcal{O}_2^{\mu\nu}$ has a vanishing matrix element between the ground-state mesons.

In leading logarithmic approximation, the scale dependence of the functions $\psi_n(w, \mu)$ is the same as that of the operators $\mathcal{O}_n^{\mu\nu}$ shown in (16). The theoretical predictions for, e.g., weak decay form factors in the HQET involve these functions renormalized at the large scale set by the mass of the heavy quark that decays. Our results can then be used to rewrite the results in terms of functions renormalized at a low scale $\mu \ll m_Q$, which may be identified with the scale at which a non-perturbative evaluation of these functions is performed. As an example, consider the combination $f(w, m_b) = \psi_1(w, m_b) + 2\psi_3(w, m_b)$, which parametrizes a class of non-factorizable contributions in non-leptonic weak decays such as $\bar{B}^0 \to D^+\pi^-$ [14]. Using our results, we can relate the function $f(w, m_b)$ with the functions $\psi_n(w, \mu)$ renormalized at a low scale $\mu \approx 1$ GeV, which have been calculated recently using QCD sum rules [16]. The result is

$$f(w, m_b) = \left(\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right)^{\gamma_1^0(w)/2\beta_0} \psi_1(w, \mu) + 2\left(\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right)^{\gamma_3^0(w)/2\beta_0} \psi_3(w, \mu)\,. \tag{26}$$

The functions $\psi_n(w, \mu)$ obey non-trivial normalization conditions at $w = 1$. They are given by [12]

$$\psi_1(1, \mu) = \psi_3(1, \mu) = \lambda_2(\mu)\,, \qquad \psi_4(1, \mu) = -\frac{2}{3}\,\lambda_1\,, \tag{27}$$

where λ_1 and $\lambda_2(\mu)$ parametrize the forward matrix elements of the kinetic and chromo-magnetic operators present in the effective Lagrangian (1). The anomalous dimension determining the scale dependence of the parameter $\lambda_2(\mu)$ is $\gamma_{\rm mag}$, whereas the parameter λ_1 has no (logarithmic) scale dependence because of reparametrization invariance [5]. Thus, the relations (27) are in accordance with our exact results in (18) and (19).

5 Conclusions

We have discussed the general structure of the mixing and renormalization of velocity-changing operators of the type $\bar{h}_{v'}\Gamma\, G^{\mu\nu} h_v$ in the HQET. The primary motivation for this study is the connection between these operators and the velocity-conserving operators appearing at order $1/m_Q$ in the effective Lagrangian of the HQET. Besides, the hadronic matrix elements of such operators determine part of the $1/m_Q^2$ corrections to heavy-hadron weak decay form factors, as well as a class of non-factorizable corrections in non-leptonic two-body decays of B mesons.

We have constructed the combinations of operators that are renormalized multiplicatively, and calculated the corresponding velocity-dependent anomalous dimensions at the one-loop order. We have also considered the hadronic matrix elements of the operators $\bar{h}_{v'}\Gamma\, G^{\mu\nu} h_v$ and determined, for the case of the ground-state heavy mesons, the scale dependence of the corresponding hadronic form factors. In the limit of equal velocities, we find that the chromo-electric and the chromo-magnetic operators do not mix under renormalization. Moreover, as a consequence of the virial theorem and reparametrization invariance, the anomalous dimension of the chromo-electric operator vanishes to all orders in perturbation theory. This implies an exact relation between renormalization constants, which may help in the calculation of the yet unknown two-loop anomalous dimension of the chromo-magnetic operator.

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