CERN-TH/96-298
UND-HEP-96-BIG 03
hep-ph/9611259

Theoretical Aspects of the Heavy Quark Expansion

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Abstract

I give a brief outline of the theoretical framework for the modern treatment of the strong interaction effects in heavy quark decays, based on first principles of QCD. This model-independent approach is required to meet the precision of current and future experiments. Applications to a few problems of particular practical interest are reviewed, including the precise determination of V_{cb} and V_{ub}. I emphasize the peculiarities of simultaneously accounting for the perturbative and power-suppressed effects necessary for accurate predictions.

Invited lecture presented at the Workshop
Beauty '96
Rome, 17–21 June 1996

CERN–TH/96–298
October 1996

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1 Introduction

A key role in exploring the Standard Model is played by studying electroweak heavy flavor decays. It was realized 20 years ago that the strong interaction effects in heavy flavor hadrons can be treated within QCD. Yet the full power of theoretical methods acquired in QCD was applied here only recently. They were developed along two main directions, 'symmetry-based' and 'dynamical'. These two lines in the heavy quark theory were the counterparts of the basic theoretical strategy in studying strong interactions: *isotopic invariance* and *chiral symmetry* on the one hand, and *asymptotic freedom* on the other.

In heavy quark physics, the early period to the end of the 80s saw mostly the dynamical approach applied at a simplified 'intuitive' level. The nonperturbative effects were often thought to be small even in the decays of charm particles. The following few years were dominated by 'symmetry' considerations; the operating language for those analyses was the so-called Heavy Quark Effective Theory (HQET), which incorporated some basic elements of the general heavy quark expansion in QCD (HQE) but was limited only to certain classes of processes.

Finally, over the last few years a consistent well-defined dynamical approach has been developed, which automatically respects the heavy quark symmetries in a manifest way. Here the most precise determinations of $|V_{cb}|$ and $|V_{ub}|$ were made.

The main effects in a weak decay of heavy quarks Q originate from distances $\sim 1/m_Q \ll 1/\Lambda_{\rm QCD}$. Since $\alpha_s(m_Q) \ll 1$ they are tractable through perturbation theory. The QCD interaction becomes strong only when the momentum transfer is much smaller than the heavy quark mass, $k \ll m_Q$. Two basic ingredients of HQE are thus elucidated:

• The *nonrelativistic expansion*, which yields the effects of 'soft' physics in the form of a power series in $1/m_Q$.

• The treatment of the strong interaction domain based on the *Operator Product Expansion* (OPE).

Unless an analytic solution of QCD is at hand, these two elements appear to be indispensable for heavy quark theory.

The general idea of separating the two domains and applying different theoretical tools to them was formulated long ago by K. Wilson [1] in the context of problems in statistical mechanics; in the modern language, applied to QCD it is similar to lattice gauge theories. The novel feature we face in the theoretical analysis of beauty decays is that they often allow – and even demand by virtue of the existence of precise experimental measurements – rather accurate predictions, requiring a *simultaneous* treatment of perturbative and nonperturbative QCD effects with enough precision in both. This problem is not new; the theoretical framework has been elaborated more than 10 years ago [2], but its phenomenological implementation was not mandatory until recently. Failure to incorporate it properly leads to certain theoretical paradoxes and, unfortunately, some superficial controversy in the numerical estimates in the literature.

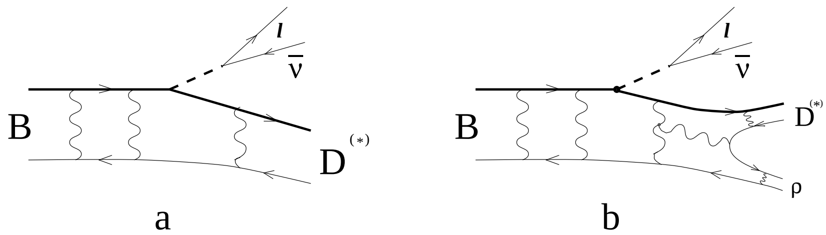

Figure 1: Exclusive $B \to D^{(*)}$ (**a**) and generic (**b**) semileptonic decays.

With significant progress made over the last years, the theory of the heavy flavors is still not a completed field and is undergoing to further extensive development. I will focus on a few selected topics that illustrate the theoretical framework, and review the overall status of the heavy quark expansion, with the main emphasis on the qualitative features. Some important theoretical applications are presented in the lectures by C. Sachrajda [3] (exclusive decays), A. Ali [4] (rare b decays) and M. Gronau [5] (CP violation) (these Proceedings). Additional theoretical aspects are covered in the summarizing contribution by G. Martinelli [6], where recent experimental data are also discussed.

2 Semileptonic decays

The QCD-based heavy quark expansion can equally be applied to all types of heavy flavor transitions. Semileptonic decays are the simplest case and I shall devote most of the attention to them; for practical reasons I focus on $b \to c$ transitions. A brief discussion of the $b \to u$ decays will be given later.

A typical semileptonic decay is schematically shown in Figs. 1. Generally, two types of decay rates can be singled out: inclusive widths where any combination of hadrons is allowed in the final state, and exclusive decays, when a transition into a particular charmed hadron is considered, usually D or D^*.

2.1 Inclusive semileptonic width

The semileptonic width of a heavy quark has the form

$$\Gamma_{\rm sl} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \cdot z_0\left(\frac{m_c^2}{m_b^2}\right) \cdot æ \ \ , \tag{1}$$

where z_0 is the known phase space suppression factor and æ generically includes all QCD corrections. In the heavy quark limit the difference resulting from using the quark mass m_b and the meson mass M_B in eq. (1) disappears:

$$(M_B - m_b)/m_b \ \sim \ 1/m_b \ \ . \tag{2}$$

For the actual b quark M_B^5/m_b^5 amounts to a factor of 1.5–2, which formally constitutes a power-suppressed effect. This demonstrates the necessity of a systematic control of nonperturbative corrections even in decays of beauty particles.

The central result obtained by direct application of OPE to the inclusive decay widths in QCD is the absence of $1/m_Q$ corrections [7] – in contrast with the presence of such terms in the hadron masses. The physical reason behind this fact is the conservation of the color flow in QCD, which leads to the cancellation of the effects of the color charge (Coulomb) interaction in the initial and final states. In terms of nonrelativistic quantum mechanics (QM), it is the cancellation between the phase space suppression caused by the Coulomb binding energy in the initial state, and the Coulomb distortion of the final state quark wavefunctions. The inclusive nature of the total widths ensures that they are sensitive only to the interaction on the time scale $\sim 1/\Delta E \sim 1/m_b$. The final-state-interaction effect is thus not determined by the actual behavior of the strong forces at large distances, but only by the potential in the close vicinity of the heavy quark. The cancellation therefore occurs universally, whether or not a nonrelativistic QM description is applicable.

The leading power corrections start with terms $1/m_b^2$; they were calculated in [7] and are expressed in terms of the expectation values of two operators of dimension 5, which have a transparent QM interpretation:

$$\mu_G^2 \simeq \frac{1}{2M_B}\langle B|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle \leftrightarrow \langle B|\vec{\sigma}_b\cdot g_s\vec{\mathcal{H}}_g|B\rangle \simeq \frac{3}{4}(M_{B^*}^2-M_B^2)\simeq 0.35\,\mathrm{GeV}^2 \quad (3)$$

$$\mu_\pi^2 \simeq \frac{1}{2M_B}\langle B|\bar{b}(i\vec{D}\,)^2 b|B\rangle \leftrightarrow \langle B|\vec{p}^{\,2}|B\rangle \quad . \quad (4)$$

The value of μ_π^2 is not yet known directly; a model-independent lower bound was established in [8, 9]: $\mu_\pi^2 > \mu_G^2$; this puts an essential constraint on its possible values. This bound is in agreement with QCD sum rule calculations [10], yielding a value of about $0.5\,\mathrm{GeV}^2$ and with a more phenomenological estimate [11]. In the absence of gluon corrections, as in simple QM models, the expectation value μ_π^2 would coincide with the HQET parameter $-\lambda_1$; they are different, however, in the actual field theory, where both μ_π^2 and μ_G^2 depend on the normalization point.

Including the nonperturbative corrections, the semileptonic width has the following form [7, 12, 13, 14]:

$$\Gamma_{\rm sl} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2\left\{z_0\left(1-\frac{\mu_\pi^2-\mu_G^2}{2m_b^2}\right)-2\left(1-\frac{m_c^2}{m_b^2}\right)^4\frac{\mu_G^2}{m_b^2}-\frac{2}{3}\frac{\alpha_s}{\pi}z_0^{(1)}+...\right\} \quad . \quad (5)$$

The $1/m_b^2$ corrections to $\Gamma_{\rm sl}$ are rather small, about -5%, and increase the value of $|V_{cb}|$ by 2.5%; the impact of the higher order power corrections is negligible.

Good control of the QCD effects in the inclusive semileptonic widths provides the most accurate direct way to determine $|V_{cb}|$ in a truly model-independent way. It sometimes faces a traditional scepticism: which numerical value must be used

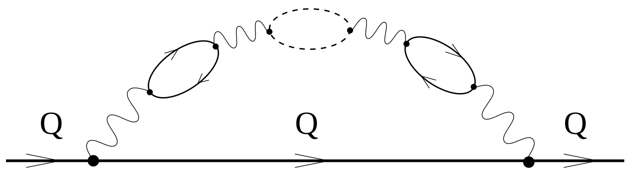

Figure 2: Perturbative diagrams leading to the IR renormalon uncertainty in $m_Q^{\rm pole}$ of the order of $\Lambda_{\rm QCD}$. The contribution of the gluon momenta below m_Q expresses the classical Coulomb self-energy of the colored particle. The number of bubble insertions into the gluon propagator can be arbitrary.

for m_b and m_c? This practical problem has deep roots; failure to understand them is the major source of controversy about masses and inclusive widths found in the literature. It will be briefly discussed below. In reality, the precise value of m_b is not too important, since the $b \to c$ width depends to a large extent on the difference $m_b - m_c$ rather than on m_b itself; the former is constrained in the HQE:

$$m_b - m_c = \frac{M_B + 3M_{B^*}}{4} - \frac{M_D + 3M_{D^*}}{4} + \mu_\pi^2 \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) + ... \approx 3.50\,\mathrm{GeV}\,. \quad (6)$$

It also independently enters lepton spectra in semileptonic decays [14] and can be extracted from the data [15]. Numerically [8, 16], a change in m_b by 50 MeV leads only to a 1% shift in $|V_{cb}|$.

Heavy quark masses

The controversy about m_b is due to the fact that HQET was popularly based on the so-called 'pole' mass of the heavy quarks. Not only was it a starting parameter of the HQET-based expansions, it is this pole mass that one always attempted to extract from the experimental data. It turns out, however, that the pole mass of the heavy quark is not a direct observable and its definition suffers from an irreducible intrinsic theoretical uncertainty of order $\Lambda_{\rm QCD}$ [17].

At first sight this looks paradoxical and counter-intuitive: for example, the value of m_e quoted in the tables of physical constants is just the *pole* mass of the electron. In QCD there is no 'free heavy quark' particle in the physical spectrum, and its pole mass is not well defined. The problems facing the possibilities to extract the pole mass from typical measurements were illustrated in Refs. [18] and [19].

The physical origin of the uncertainty $\delta m_Q^{\rm pole} \sim \Lambda_{\rm QCD}$ is the gluon Coulomb self-energy of the static colored particle. The energy stored in the chromoelectric field inside a sphere of radius $R \gg 1/m_Q$ is given by

$$\delta E_{\rm Coulomb}(R) \propto \int_{1/m_b \sim |x| < R} \vec{E}_c^{\,2}\, d^3x \;\propto {\rm const} - \frac{\alpha_s(R)}{\pi}\frac{1}{R}\,. \quad (7)$$

The pole mass assumes that all energy is counted, i.e. $R \to \infty$. Since in QCD the interaction becomes strong at $R_0 \sim 1/\Lambda_{\rm QCD}$, the domain outside R_0 would yield an uncontrollable and physically senseless contribution to the mass $\sim \Lambda_{\rm QCD}$ [17].

Being a classical effect originating at a momentum scale well below m_Q, this uncertainty can be traced in the usual perturbation theory, where it manifests itself in higher orders as a so-called $1/m_Q$ infrared (IR) renormalon singularity in the perturbative series for the pole mass [20, 21], see Fig. 2.

Nonetheless, the inclusive widths can be theoretically calculated since they are governed, instead, by well-defined short-distance running masses $m_Q(\mu)$ with the Coulomb energy originating from distances $\gtrsim 1/\mu$ peeled off. It is precisely this short-distance running mass that can be extracted from experiment with, in principle, unlimited accuracy: the pole mass does not enter any genuine short-distance observable at the level of nonperturbative corrections [20].

Applied to the inclusive widths, it suggests certain information about the importance of higher order perturbative corrections: if masses entering eq. (5) are the pole masses, the perturbative series

$$\Gamma_{\rm sl}^{\rm pert} = \Gamma_0 æ^{\rm pert} = \Gamma_0 \left(1 + a_1(\alpha_s/\pi) + a_2\,(\alpha_s/\pi)^2 + ...\right) \tag{8}$$

is poorly behaved, with coefficients a_k factorially growing, which makes the radiative correction factor uncalculable in principle with an accuracy $\sim \Lambda_{\rm QCD}/m_b$. In contrast, if one uses the short-distance masses, the higher-order corrections become smaller and the factor $æ^{\rm pert}$ becomes calculable with the necessary precision [20, 22].

This seemingly academic observation, in reality proved to underlie the pattern of the corrections from the very first terms. Remarkably, the actual model-independent calculations of $\Gamma_{\rm sl}$ through observables measured in experiment are very stable against perturbative corrections. Including $\mathcal{O}(\alpha_s^2)$ terms in the extraction of the b pole mass from, say, the $\mathrm{e^+e^-} \to \mathrm{\bar{b}b}$ threshold region [23] noticeably increases its value. However, the parallel perturbative improvement in calculating the width yields an essential suppression of the perturbative factor, æ, so that the two effects offset each other almost completely [16].

This conspiracy is not unexpected: the appearance of large corrections at both stages is an artefact of using the ill-defined pole mass in the intermediate calculations. The situation is peculiar since the actual nonperturbative effects appear only at the level $1/m_Q^2$, whereas the pole mass is infrared ill-defined already at an accuracy of $1/m_Q$. The failure to realize this fact led to the superficial suggestion [24] that even in beauty particles the perturbative corrections may go out of theoretical control; a more careful analysis [16, 25] showed that this is not the case.

Moreover, the OPE requires using short-distance running masses $m_{b,c}(\mu)$ normalized at $\mu \sim 1\,\mathrm{GeV}$ [20]. It has been done in [16] and demonstrated that neither these masses nor the perturbative corrections to the width æ show significant contributions from higher orders.

To summarize, the idea that the perturbative corrections in the extraction of $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ are large comes from an inconsistent usage of ill-defined pole masses:

• It is 'difficult to extract' accurately $m_b^{\rm pole}$ from experiment; in any given calculation it is easy to identify the effects that were left out, which can change its value by $\sim 200\,\mathrm{MeV}$. This uncertainty leads to a 'theoretical error' δ_I in $\Gamma_{\rm sl}(B)$ of $\sim 10\%$.

• When routinely calculating $\Gamma_{\rm sl}(B)$ in terms of the pole masses, there are significant higher order corrections $\delta_{II} \approx 10\%$.

The naive conclusion drawn from such experience [26] is that one cannot reliably calculate the width without $\sim 20\%$ uncertainty:

$$\delta\Gamma_{\rm sl}/\Gamma_{\rm sl} = \delta_I + \delta_{II} \simeq 20\% \quad \leftrightarrow \delta|V_{cb}|/|V_{cb}| \simeq 10\% \;\; .$$

On the contrary, theory predicts a strong anticorrelation between δ_I and δ_{II} in a consistent perturbative calculation, and that was explicitly checked in [16, 25]. The net impact of the calculated (presumably dominant) second-order $\mathcal{O}(\alpha_s^2)$ corrections on the value of $|V_{cb}|$ appeared to be less than 1%! Moreover, just neglecting *all* perturbative corrections altogether, both in the semileptonic width and in extracting m_b from experiment, yields a $|V_{cb}|$ smaller by less than 5% [27].

Recently, all-order corrections associated with the running of α_s in one-loop diagrams (referred to as BLM approximation) were calculated in [25]. Using the most accurate model-independent determination of m_b [23], one gets [16]

$$|V_{cb}| = 0.0413 \left(\frac{\mathrm{BR}(B\to X_c\ell\nu)}{0.105}\right)^{\frac{1}{2}} \left(\frac{1.6\,\mathrm{ps}}{\tau_B}\right)^{\frac{1}{2}} \times$$
$$\left(1 - 0.012\frac{(\tilde\mu_\pi^2 - 0.4\,\mathrm{GeV}^2)}{0.1\,\mathrm{GeV}^2}\right)\cdot\left(1-0.006\frac{\delta m_b^*}{30\,\mathrm{MeV}}\right) \;\; . \tag{9}$$

The main source of theoretical uncertainty is the exact value of μ_π^2 (marked with a tilde in eq. (9), indicating that a particular field-theoretic definition is assumed), which enters through the value of $m_b - m_c$, eq. (6). A dedicated analysis of the lepton spectra will reduce this uncertainty. At the moment a reasonable estimate of the uncertainty in μ_π^2 is about $0.2\,\mathrm{GeV}^2$, leading to a 2.5% uncertainty in $|V_{cb}|$.

The dependence on m_b is minor; since we rely here on the well-defined short-distance mass m_b^*, there is no intrinsic uncertainty in it. The analysis [23] estimated $\delta m_b^* \simeq 30\,\mathrm{MeV}$; even for $\delta m_b^* \simeq 60\,\mathrm{MeV}$, the related uncertainty in $|V_{cb}|$ is only 1.2%.

As previously explained, the actual impact of the known perturbative corrections when relating the semileptonic width to other low-energy *observables* is very moderate, and there is no reason to expect the higher-order effects to be significant. With the *a priori* dominant all-order BLM corrections calculated [25], one may be concerned only with the true two-loop effects $\mathcal{O}(\alpha_s^2)$. These have not been calculated completely yet; however, the recent $\mathcal{O}(\alpha_s^2)$ calculation [28] in the small velocity kinematics suggested that they must be small. There are some enhanced higher-order non-BLM corrections that are specific to the inclusive widths [29]. They have been accounted for in the analyses [8, 16], but went beyond those in [30, 25]. Thus it seems unlikely that as yet uncalculated second-order corrections can change the width by more than 2–3%; therefore, assigning an additional uncertainty of 2% in $|V_{cb}|$ is a quite conservative estimate.

Adding up these uncertainties we arrive at

$$(\delta|V_{cb}|/|V_{cb}|)\,|_{\rm th} \;\lesssim\; 5\% \;\; . \tag{10}$$

The theoretical accuracy in extracting of $|V_{cb}|$ appears to be better than its current experimental counterpart. This method can be improved further in a model-independent way. I think that the 2% level of a *defensible* theoretical precision can be ultimately reached here; an essential improvement beyond that is questionable, because of effects of higher-order power corrections and possible violations of duality.

Similarly, $|V_{ub}|$ is directly related to the total $b \to u$ semileptonic width [16]:

$$|V_{ub}| = 0.00458 \cdot [\mathrm{BR}(B \to X_u \ell\nu)/0.002]^{\frac{1}{2}}\; (1.6\,\mathrm{ps}/\tau_{\rm B})^{\frac{1}{2}} \;\; . \tag{11}$$

Recently, ALEPH announced [31] a model-independent measurement of the inclusive $b \to u\,\ell\nu$ width: $\mathrm{BR}(B \to X_u\ell\nu) = 0.0016 \pm 0.0004$. I cannot judge the reliability of the quoted error bars in this sophisticated analysis; it certainly will be clarified soon. Accepting this input literally, I arrive at the model-independent result

$$|V_{ub}|/|V_{cb}| \;=\; 0.098 \pm 0.013 \;\; . \tag{12}$$

The theoretical uncertainty in converting $\Gamma(B \to X_u\,\ell\nu)$ into $|V_{ub}|$ is a few times smaller.

Let me briefly comment on the literature. It is sometimes stated [26, 32] that the uncertainty in $\Gamma_{\rm sl}$ is at least 20%. The origin of such claims is ignoring the subtleties related to using the pole mass in the calculations and considering separately the perturbative corrections to the pole masses, and to the widths expressed in terms of $m_Q^{\rm pole}$. This is inconsistent on theoretical grounds [20], whose relevance was confirmed by the concrete numerical evaluations [16, 25]. The dependence on m_b and $m_b - m_c$ used to determine the uncertainty in $|V_{cb}|$ was calculated erroneously in [26] (cf. [16]), apparently because of an arithmetic mistake that led to a significant overestimate. Finally, no argument was given to justify a sevenfold boosting of the theoretical uncertainty in m_b obtained in the dedicated analysis [23].

2.2 Exclusive zero recoil $B \to D^*\,\ell\nu$ rate

Good control of all QCD effects in $\Gamma_{\rm sl}$ was due to the fact that removing constraints on the final state to which decay partons can hadronize, makes such a probability a short-distance quantity amenable to a direct OPE expansion. A similar approach to the exclusive zero-recoil decay rate $B \to D^*\,\ell\nu$ yielded quite an accurate determination of $|V_{cb}|$ as well [8, 18], though with a more significant irreducible model dependence and a larger intrinsic uncertainty. The limitation is twofold: constraining the decays to a specific final state makes the transition not a genuinely short-distance effect; it also suffers from a larger expansion parameter, namely $1/m_c$ vs. $1/m_b$.

Near zero recoil the decay is governed by the single hadronic formfactor F_{D^*}. In the infinite mass limit $F_{D^*}=1$ holds; for finite $m_{b,c}$ it acquires corrections:

$$F_{D^*} = 1 - (1-\eta_A) + \delta_{1/m^2} + ... \quad . \tag{13}$$

The effect of the nonperturbative domain starts with the terms $\sim 1/(m_c,m_b)^2$ [33, 34], but otherwise is rather arbitrary, depending on the details of the long-distance dynamics in the form of wavefunction overlap. This opened the field for speculations and controversy [35].

The situation as it existed by 1994 was summarized in reviews by Neubert [36]:

$$\eta_A = 0.986 \pm 0.006 \qquad\qquad \delta_{1/m^2} = (-2\pm 1)\% \quad , \tag{14}$$

yielding $F_{D^*}\simeq 0.97$, and was assigned the status of "one of the most important and, certainly, most precise predictions of HQET". Nowadays we believe that the actual corrections to the symmetry limit are larger, and the central theoretical value lies rather closer to 0.9 [8, 18]. Perturbative-wise, it has been pointed out [37] that the improvement [38] of the original one-loop calculation was incorrect, and the proper estimate is $\eta_A \approx 0.965 \pm 0.025$; subsequent calculations of the higher-order BLM corrections [25, 39] confirmed it: $\eta_A \approx 0.965\pm 0.02$. The purely perturbative chapter was closed recently with the complete two-loop $\mathcal{O}(\alpha_s^2)$ result [28] $\eta_A^{(2\,\mathrm{loop})} = 0.960 \pm 0.007$; however, the inherent irreducible uncertainty of the *complete* perturbative series for η_A exceeds the quoted one by a factor of three [25, 41, 42].

If the mass of the charm quark were a few times larger, in practice the two-loop calculation would have been the whole story for F_{D^*}. In reality, the power corrections originating from the domain of momenta below $\sim 0.6\,\mathrm{GeV}$ appear to be more significant. Not much can be said about them without model assumptions; they have been shown to be negative and exceed about 0.04 [8, 18] in magnitude.

The idea of this dynamical approach was to consider the sum over all hadronic states in the zero recoil kinematics; such a rate sets an upper bound for the production of D^*. This *inclusive* quantity is of a short-distance nature and can be calculated in QCD using the OPE. The result through order $1/m^2$ is

$$|F_{D^*}|^2 + \sum_{\epsilon_i<\mu} |F_i|^2 = \xi_A(\mu) - \frac{\mu_G^2}{3m_c^2} - \frac{\mu_\pi^2-\mu_G^2}{4}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}+\frac{2}{3m_cm_b}\right) \quad , \tag{15}$$

where F_i are the transition formfactors to charm states i with the mass $M_i = M_{D^*} + \epsilon_i$, and ξ_A is a perturbative factor (the role of μ will be addressed later). Considering a similar sum rule for another type of 'weak current', say $\bar c i\gamma_5 b$, yields

$$\sum_{\tilde\epsilon_k<\mu} |\tilde F_k|^2 \;=\; \left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)^2 \left(\mu_\pi^2-\mu_G^2\right) \tag{16}$$

with the tilde referring to the quantities occurring in the transitions induced by this hypothetical current. These sum rules (and similar ones at arbitrary momentum transfer), established in [8, 18], have been subjected to a critical scrutiny for two years, but are now accepted and constitute the basis for currently used estimates of F_{D^*}.

Since eq. (16) is the sum of certain transition probabilities, it results in a rigorous lower bound

$$\mu_\pi^2 \ > \ \mu_G^2 \simeq 0.4\,\mathrm{GeV}^2 \ . \tag{17}$$

The sum rule (15) then leads to the model-independent lower bound for δ_{1/m^2}:

$$-\,\delta_{1/m^2} \ > \ \left(M_{B^*}^2 - M_B^2\right)/8m_c^2 \simeq 0.035\,. \tag{18}$$

The actual estimate depends essentially on the value of μ_π^2. It was suggested in [8] to estimate the contribution of the excited states in the l.h.s. of the sum rule (15) from 0 to 100% of the power corrections in the right-hand side:

$$-\,\delta_{1/m^2} = (1+\chi)\left(\frac{M_{B^*}^2 - M_B^2}{8m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{8}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_cm_b}\right)\right), \quad 0 \le \chi \le 1\,. \tag{19}$$

If so, one arrives at [8]

$$\begin{aligned} -\delta_{1/m^2} &= (5.5 \pm 1.8)\% \qquad \text{at} \ \ \mu_\pi^2 = 0.4\,\mathrm{GeV}^2 \\ -\,\delta_{1/m^2} &= (6.8 \pm 2.3)\% \qquad \text{at} \ \mu_\pi^2 = 0.5\,\mathrm{GeV}^2 \\ -\delta_{1/m^2} &= (8.1 \pm 2.7)\% \qquad \text{at} \ \ \mu_\pi^2 = 0.6\,\mathrm{GeV}^2 \end{aligned} \tag{20}$$

The QM meaning of the sum rules is transparent [18]. The act of a semileptonic decay of the b quark is its instantaneous replacement by a c quark. In ordinary QM the overall probability of the produced state to hadronize to something is exactly unity. Why are there nonperturbative corrections in the sum rule? The answer is that the 'normalization' of the weak current $\bar{c}\gamma_\mu\gamma_5 b$ is not exactly unity and depends, in particular, on the external gluon field. Expressing the QCD current in terms of the nonrelativistic fields used in QM one has, for example, through order $1/m^2$:

$$\bar{c}\gamma_k\gamma_5 b \leftrightarrow \sigma_k - \left(\frac{1}{8m_c^2}(\vec\sigma i\vec D)^2\sigma_k + \frac{1}{8m_b^2}\sigma_k(\vec\sigma i\vec D)^2 \ - \frac{1}{4m_cm_b}(\vec\sigma i\vec D)\sigma_k(\vec\sigma i\vec D)\right). \tag{21}$$

The last term just yields the correction seen in the r.h.s. of the sum rule. Let me note that in the standard HQET analysis, the first two terms in the brackets are missing (see, e.g., Ref. [35]) and the dominant effect $\sim 1/m_c^2$ is lost; the nonrelativistic expansion was correctly done in the works by the Mainz group [43].

The inequality $\mu_\pi^2 > \mu_G^2$ in QM expresses the positivity of the Pauli Hamiltonian $\frac{1}{2m}(\vec\sigma\, i\vec D\,)^2 = \frac{1}{2m}((i\vec D)^2 - \frac{i}{2}\sigma G)$ [9]. It is interpreted as the Landau precession of a charged (colored) particle in the (chromo)magnetic field where one has $\langle p^2\rangle \ge |\vec B|$. Although the QM average of $\vec B$ in the B meson is suppressed, the chromomagnetic field is proportional to the spin of the light degrees of freedom and is thus essentially non-classical, which enhances the bound and makes up for the suppression.

The perturbative factor ξ_A is not equal to η_A^2 [18, 20] and depends on the separation scale μ. Unlike η_A, which in principle cannot be defined theoretically with better than a few percent accuracy, $\xi_A(\mu)$ is well-defined; no significant uncertainty is associated with it, $\xi_A \simeq (0.99)^2$.

Allowing a very moderate variation of μ_π^2 between $0.4\,\mathrm{GeV}^2$ and $0.6\,\mathrm{GeV}^2$ only, we see that $-\delta_{1/m^2}$ varies between 3.5% and 11%; moreover, since there are no model-independent arguments to prefer any part of the interval, the whole range must be considered equally possible. Adding small perturbative corrections we end up with the reasonable estimate $F_{D^*} \approx 0.9$. It is curious to note that at a 'central' value $\chi = 0.5$ the dependence of the zero-recoil decay rate on μ_π^2 through δ_{1/m^2} effect practically coincides with that of $\Gamma_{\rm sl}(B)$ (see eq. (9)) although they actually vary in opposite directions. The typical size of the $1/m_Q^2$ corrections to the exclusive zero-recoil decay rate is thus significant, around 15%, which is expected since they are driven by the scale $m_c \simeq 1.3\,\mathrm{GeV}$. It is evident that $1/m_c^3$ corrections in F_{D^*} not addressed so far are *at least* about $\frac{1}{2}(0.15)^{3/2} \simeq$ 2–3%.[1]

Thus, I believe that the current theoretical technologies do not allow to reliably predict the zero recoil formfactor F_{D^*} with a precision better than 5–7% in a model-independent way; its value is expected to be approximately 0.9, although a correction to the symmetry limit twice smaller, as well as larger deviations, are possible. It is encouraging that the 'educated guess' $F_{D^*} \simeq 0.9$, which emerged from the first – and so far the only – dynamical QCD-based consideration [8, 18], yielded a value of $|V_{cb}|$ close to a less uncertain result obtained from $\Gamma_{\rm sl}(B)$.

Future, more accurate data will enable us to measure F_{D^*} with a theoretically informative precision using $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$, and thus provide us with deeper insights into the dynamics of strong forces in the heavy quark system.

Certain statements in the literature deserve comments. Neubert claimed [40, 41] that the sum rule (15) cannot be correct, since $1/m^2$ renormalons allegedly mismatch in it. It was failed to realized in these papers that in Wilson's OPE the IR renormalons are always absent from any particular term. On the other hand, IR renormalon calculus can still be applied if the OPE relation is considered in the pure perturbation theory itself, and formally setting $\mu = 0$. In particular, this amounts to subtracting a 'perturbative piece' from the observable probabilities. However, in this way the perturbative terms appear in the left-hand side as well, and these terms were ignored in Refs. [40, 41].

It was suggested in Ref. [44] that the higher-order radiative corrections to the sum rules are too large and allegedly make them next to useless. Such conclusions emerged from a theoretically inappropriate treatment. The concrete numerical analysis in the OPE quoted below, on the contrary, suggests a quite moderate impact

[1] This is consistent with the fact that the $1/m_Q^3$ IR renormalon ambiguity in η_A^2 constitutes 5% at $\Lambda_{\rm QCD}^{\overline{\rm MS}} \simeq 220\,\mathrm{MeV}$ [42].

of radiative corrections. According to [44], the perturbative corrections to the sum rule of the type of eq. (16) weaken the bound for the expectation value of the kinetic operator to such an extent that it becomes non-informative. One must realize that, in the quantum field theory, the renormalized operators can be defined in different non-equivalent ways; $-\lambda_1$ addressed in [44] is known to be different from μ_π^2. Moreover, the only field-theoretic definition of the kinetic operator $\bar{Q}(i\vec{D}\,)^2Q$ given so far was made in [18] and, for it, $\mu_\pi^2 > \mu_G^2$ always holds. As for $-\lambda_1$, a parameter in HQET, its definition beyond the classical level has never been given; the procedure adopted in Ref. [44] reduces to an attempt to completely subtract the 'perturbative piece' of $\mu_\pi^2(\mu)$:

$$-\lambda_1 = \mu_\pi^2(\mu) - c_1\left(\alpha_s(\mu)/\pi\right)\mu^2 - c_2\left(\alpha_s(\mu)/\pi\right)^2\mu^2 - ... \qquad (22)$$

(the method to calculate c_i was elaborated in [18]). Yet it has been known for a long time [2] that such a program theoretically cannot be performed: the series in eq. (22) is factorially divergent and cannot be assigned a meaningful number. No wonder the second-order BLM correction calculated in [44] seemed to be dangerously large: there can be no bound established for a quantity that is not defined. Moreover, the situation is clear in the BLM approximation, where all c_i are readily calculated: the series, whose second term was discussed in [44], is divergent and sign-varying, so using merely the second term is misleading for any numerical estimate.

The above subtleties are peculiar to the field-theory analysis. Inequality $\mu_\pi^2 > \mu_G^2$ must hold in *any* QM model relying on a potential description without additional degrees of freedom, if the heavy quark Hamiltonian is consistent with QCD. Unfortunately, a failure to realize this fact is seen in a number of recent analyses.

A second-order BLM analysis of the sum rule (15) for F_{D^*} was also attempted in [44] and claimed to destroy its predictive power (the first-order calculation had been performed in [18]). This calculation, however, was not done consistently, and the actual effect is smaller [42]. Let me define $\eta_A(\mu) \equiv \xi_A(\mu)^{1/2}$; the quantity $\eta_A(\mu)$ must be added to δ_{1/m^2} in the framework of the OPE instead of η_A in the model calculations. Then, at a reasonable choice $\mu \simeq 0.5\,\mathrm{GeV}$, $\Lambda_{\mathrm{QCD}}^{(V)} = 300\,\mathrm{MeV}$, one has

$$\eta_A(\mu) = 1 \qquad \text{tree level} \qquad (23)$$

$$\eta_A(\mu) = 0.975 \qquad \text{one loop} \qquad (24)$$

$$\eta_A(\mu) = 0.99 \qquad \text{all-order BLM} \qquad (25)$$

Clearly, the effect of the calculated perturbative corrections is not drastic and $\eta_A(\mu)$ is very close to the value of 0.98 adopted in the original analysis [8].

Conceptually, the deficiency of the alternative application of the original sum rules of Refs. [8, 18] adopted in [44], is to gauge ξ_A on the value of η_A^2 as it has been defined in the HQET (the idea that ξ_A is to be identified with η_A^2 ascends to [41, 40]). However, it is η_A that is ill-defined, and only for this reason must the difference between the stable Wilson coefficient ξ_A and η_A^2 suffer from large corrections. It is worth noting that, in reality, η_A cannot be equal to a matching coefficient of $\bar{c}\gamma_\mu\gamma_5 b$ to a corresponding current in *any* effective field theory.

Smaller theoretical uncertainty, 2.5% and 3%, is now quoted by Neubert for δ_{1/m^2} and F_{D^*}, respectively. The former was obtained in Ref. [40], in what he calls a "hybrid approach", which reduces to assigning the fixed value $\mu_\pi^2 = 0.4\,\mathrm{GeV}^2$ and using it in the sum rule (15) within the same model assumption of eq. (19) as suggested in [8]: $0 \le \chi \le 1$.[2] Correspondingly, the quoted number for δ_{1/m^2} practically coincided with the first line of eqs. (20). In reality, allowing μ_π^2 to vary within any reasonable interval significantly stretches the uncertainty. Moreover, the analysis [40] was based on using η_A as a perturbative factor assuming, literally, that in the proper treatment the final result would not be changed numerically – which is just the case according to Ref. [44]. On top of that, the uncertainty in the definition of η_A due to $1/m^2$ and $1/m^3$ IR renormalons constitutes 2–3% each and is an additional one in the adopted usage of the sum rules. Altogether, the stated theoretical confidence of those estimates cannot be accepted as realistic.

Recently, Ref. [45] claimed to have established an intriguing relation between the slope and the curvature of the formfactor near the zero recoil point, using analyticity and unitarity of the amplitudes. If correct, this would reduce the experimental uncertainties in extrapolating the rate to the zero recoil. However, both the below-threshold contribution to the dispersion integral and the $1/m_c$ power corrections [46] to the heavy quark symmetry relations were grossly underestimated; therefore the relation stated in [45] rather should not be used for deriving model-independent experimental results.

3 The semileptonic branching fraction

The QCD-based HQE provides a systematic framework for calculating the total widths of heavy flavors, which are not amenable to the traditional methods of HQET. The difference between nonleptonic and semileptonic widths appears only at a quantitative level. The only assumption is that the mass of a decaying quark (actually, the energy release) is sufficiently large; for a review, see [47].

The overall semileptonic branching ratio $\mathrm{BR_{sl}}(B)$ seems to be of a particular practical interest: while the simple-minded parton estimates yield $\mathrm{BR_{sl}}(B) \simeq 15\%$ [48], experiments give smaller values $\mathrm{BR_{sl}}(B) \simeq 10.5$–$11.5\%$. The leading $1/m_Q^2$ effects in the nonleptonic widths were calculated in [7, 12, 13]; they exhibit some cancellations and one literally gets [49] a downward shift $\lesssim 0.5\%$. Estimated $1/m_b^3$ corrections do not produce a significant effect either [50]. As a result, most of the attention was paid to a more accurate treatment of the perturbative corrections,

[2] I disagree with the statements of [40], reiterated in later papers, suggesting that the original analysis [8, 18] missed some elements of the heavy quark spin-flavor symmetry; on the contrary, it was stated in the latter paper that all these relations automatically emerge from the sum rules that replace the QM wavefunction description in the quantum field theory.

including the effect of the charm mass in the final state. It was found [51, 52] that the nonleptonic width is indeed boosted up.

Since the inclusive widths are expanded in inverse powers of energy release, one expects larger corrections or even a breakdown of the expansion and violation of duality in the channel $b \to c\bar{c}s(d)$; however, this channel can be isolated via charm counting [7]. The original experimental estimate $n_c \lesssim 1.15$ did not allow one to attribute the apparent discrepancy to it, and gave rise to the so-called '$\mathrm{BR}_{\mathrm{sl}}$ versus n_c' problem.

The perturbative corrections in the $b \to c\bar{u}d$ itself cannot naturally drive $\mathrm{BR}_{\mathrm{sl}}$ below 12.5%; the calculation for the $b \to c\bar{c}s(d)$ channel is less certain and, in principle, admits increasing the width by a factor of 1.5–2, leading to $n_c \simeq 1.25$–1.3. In the latter case a value of $\mathrm{BR}_{\mathrm{sl}}$ as low as 11.5% can be accommodated.

The experimental situation with n_c does not seem to be quite settled yet: $n_c = 1.134 \pm 0.043$ (CLEO), $n_c = 1.23 \pm 0.07$ (ALEPH). In a recent analysis [53] it was argued that consistency requires a major portion of the final states in $b \to c\bar{c}s$ to appear as modes with kaons but D_s, which previously escaped proper attention. This allows for a larger value $n_c \simeq 1.3$ needed to resolve the problem with $\mathrm{BR}_{\mathrm{sl}}$. The dedicated theoretical analysis [54] shows that, indeed, the dominance of such modes is natural and does not require violation of duality. Thus, if a larger value of $n_c \simeq 1.25$ is confirmed experimentally, the problem of $\mathrm{BR}_{\mathrm{sl}}$ will not remain.

In my opinion, however, we cannot consider even this successful scenario as a complete QCD-based theoretical prediction of $\mathrm{BR}_{\mathrm{sl}}$; a strong enhancement of a tree-level-unsuppressed channel raises doubts about the trustworthiness of its one-loop calculations. The possibility to get the necessary enhancement should be rather viewed as an indication of the presence of large effects working in the right direction.

4 Lifetimes of beauty particles

A thoughtful application of the HQE to charm lifetimes demonstrated that the actual expansion parameter appeared to be too low to ensure a trustworthy accurate description, so that *a priori* one expects only emergence of the qualitative features. Surprisingly, in most cases the expansion works well enough even numerically (for a recent review, see [55]).

Applying the expansion to beauty particles one expects a decent numerical accuracy, although the overall scale of the effects is predicted to be small, making a challenge to experiment:

$$\begin{array}{rclcll}
\tau_{B^-}/\tau_{B^0} & \simeq & 1+0.04\left(\frac{f_B}{180\,\mathrm{MeV}}\right)^2 & [50] & \mathrm{EXP:} & 1.04\pm0.04 \\
\overline{\tau}_{B_s}/\tau_{B^0} & \simeq & 1+\mathcal{O}(1\%) & [50] & \mathrm{EXP:} & 0.97\pm0.05 \\
(\tau_{B_s^L}-\tau_{B_s^S})/\tau_{B_s} & \simeq & 0.18\left(\frac{f_{B_s}}{200\,\mathrm{MeV}}\right)^2 & [56] & & \\
\tau_{\Lambda_b}/\tau_{B^0} & \approx & 0.9 & & \mathrm{EXP:} & 0.78\pm0.06
\end{array} \qquad (26)$$

These differences appear mainly as $1/m_b^3$ corrections and, depending on certain four-fermion matrix elements, cannot be predicted at present very accurately, in particular in baryons. For mesons the estimates are based on the vacuum saturation approximation, which cannot be exact either. The impact of non-factorizable terms has been studied a few years ago in [57] and possibilities to directly measure the matrix elements in future experiments were suggested.

The apparent agreement with experiment is obscured by reported lower values of τ_{Λ_b}. Since the baryonic matrix elements are rather uncertain, a few model estimates have been done [58]. All seem to fall short; however, this might be attributed to deficiencies of the simple quark model. Nevertheless, it was shown [59] that irrespective of the details one cannot have an effect exceeding 10–12% while residing in the domain of validity of the standard $1/m_Q$ expansion itself; the natural 'maximal' effects that can be accommodated are $\sim 7\%$ and $\sim 3\%$ for weak scattering and interference, respectively.

Thus, if the low experimental value of τ_{Λ_b} is confirmed, it will require a certain revision of the standard picture of the heavy hadrons and of convergence of the $1/m_Q$ expansion for nonleptonic widths with the offset of duality in beauty particles.

Recently, the problem of the accuracy of the calculations of $\delta\tau_B$ based on the factorization was emphasized again in [60]. It is difficult to agree, however, with the wide intervals, up to $\pm 20\%$ allowed for the difference between τ_{B^+} and τ_{B^0}; the constraints discussed in [57, 59] were missed. One can see that the values of hadronic parameters saturating such large differences would move one beyond the domain of applicability of the whole expansion used in [60].

5 $1/m_Q$ expansion and duality violation

Duality violation attracts more and more attention in the context of the heavy quark theory; a recent extensive discussion was given in [61]. The expansion in $1/m_Q$ is asymptotic. There are basically two questions one can ask here: what is the onset of duality, i.e. *when* does the expansion start to work? The most straightforward approach was first undertaken in [62], and no apparent indication toward an increased energy scale was found. Another question, of *how* is the equality of the QCD parton-based predictions with the actual decay rates achieved, was rarely addressed. An example of such a problem is easy to give.

The OPE states that no terms $\sim 1/m_Q$ can be in the widths and the leading terms start with $1/m_Q^2$. However, the OPE *per se* cannot forbid a scenario where, for instance,

$$\delta\Gamma_{H_Q}/\Gamma_{H_Q} \;\sim\; C\,\sin{(m_Q\rho)}/(m_Q\rho)\;, \quad \rho\sim\Lambda_{\rm QCD}^{-1}\;. \qquad (27)$$

In the actual strong interaction, m_b and m_c are fixed, so from the practical viewpoint these types of corrections are not too different – but the difference is profound theory-wise! It is a specific feature of the OPE in Minkowski space, and it can hardly be addressed, for example, in lattice calculations. Their complete control requires a deeper understanding of the underlying QCD dynamics beyond the knowledge of first few nonperturbative condensates.

The literal corrections of the type of eq. (27) are hardly possible; the power of $1/m_Q$ in realistic scenarios is larger, and they must be eventually exponentially suppressed though, probably, starting at a higher scale [61]. But a theory of such effects is still in its embryonic stage and needs an experimental input as well.

The possibility has been discussed for some time [63] to have certain unidentified corrections to the (nonleptonic) widths, eventually leading to the dependence

$$\Gamma^{\rm nl}_{H_Q} \sim M^5_{H_Q} \tag{28}$$

and manifesting an explicit $1/m_Q$ effect:

$$\delta\Gamma_{H_Q}/\Gamma_{H_Q} \;\simeq\; \gamma\,\delta M_{H_Q}/M_{H_Q}\;, \quad \gamma \simeq 5\;. \tag{29}$$

Such scaling in the intermediate energy domain cannot literally contradict OPE if the offset of duality has not been passed yet. But is this possibility natural?

Leaving aside the QCD-based arguments completely, one must still account for the charm mass in the final state, and thus differentiate between M_D and M_{Λ_c} in the decays of B and Λ_b, respectively. Although M_{Λ_b} is notably larger than M_B, M_{Λ_c} exceeds M_D by almost the same amount! Counting only the phase space factors in analogy with the free quark decay one would get for the $b \to c$ transitions $\gamma \lesssim 2$ [16], which thus seems to be a more natural value in the 'poor man on the street' hypothesis [6] considered in [63]. It is worth noting that the fit of γ in charm particles does not convincingly indicate favoring $\gamma = 5$: incorporating the calculated $1/m_c^2$ and $1/m_c^3$ corrections destroys it – while discarding them as a part of the 'poor man' philosophy makes it impossible to explain the very different values of τ_D and $\mathrm{BR}_{\rm sl}(D)$. The short Λ_b lifetime thus still seems to constitute an important problem for the whole heavy quark theoretical community.

Acknowledgements

I am grateful to F. Ferroni, G. Martinelli and other organizers of Beauty '96 for the opportunity to participate in this meeting. I benefited from invaluable collaboration with I. Bigi, M. Shifman and A. Vainshtein. I am thankful to M. Voloshin for many clarifications. Informal discussions with V. Braun, G. Martinelli and C. Sachrajda were enjoyable, although short. It is a pleasure to thank P. Schlein for detailed comments on the manuscript. This work was supported in part by the NSF under the grant number PHY 92-13313.

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