

2. Limits due to the physical message

Let us first see which are the physical limits to the accuracy at the generation of the position message, i.e. what is the correspondence between the ionized track left in the gas and the original trajectory of the ionizing particle. We will briefly consider the cases of a minimum ionizing particle, and of soft x-rays.

A high energy charged particle interacts with the gas molecules of the detector in a discrete number of points, generating ion-electron pairs. Because of its small mass, the electron may be ejected with considerable energy (δ rays); events of this kind have however a probability very quickly decreasing with the ejected electron energy¹⁾. In Fig. 1 we show the number of electrons produced by a 1 GeV/c proton in a cm of argon, at normal conditions, as a function

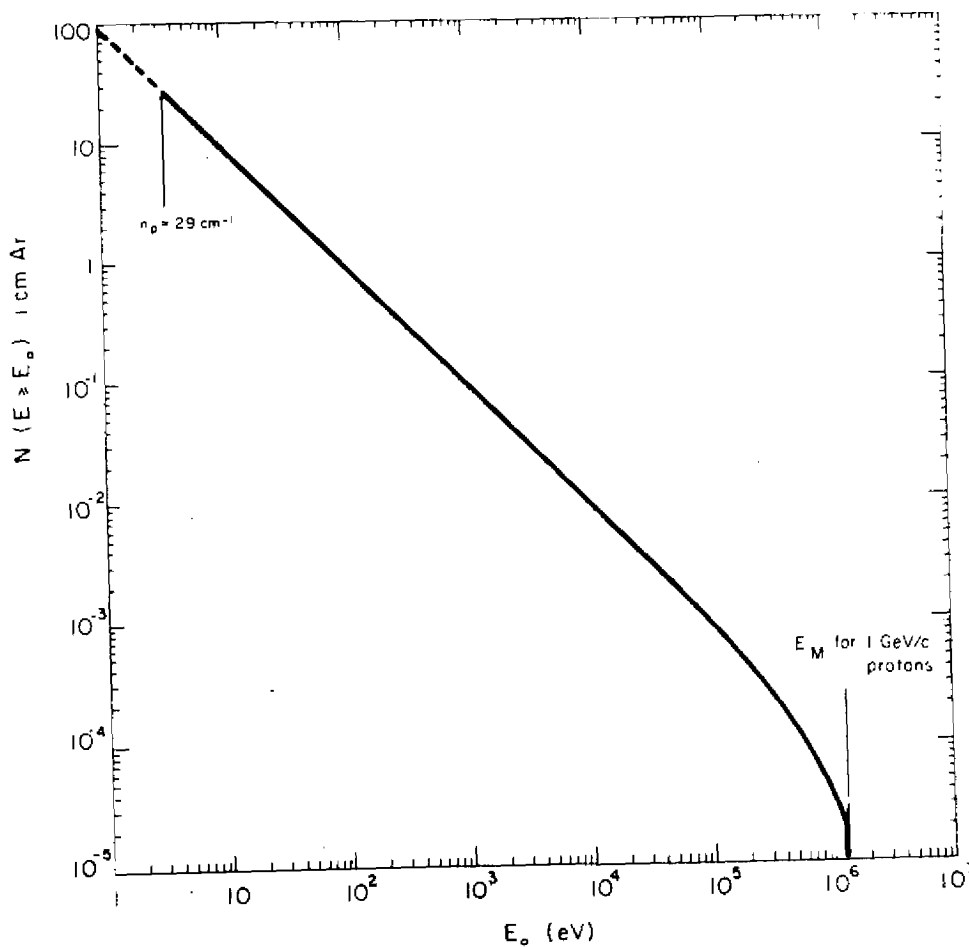


Fig. 1. Number of δ -electrons ejected with energy greater or equal to E_0 , in one cm of argon at normal conditions by a 1 GeV/c Proton.

of their energy. For example, one can see that about one out of ten tracks will contain at least one electron with energy equal or larger than one keV. Since the practical range of such an electron in the gas is of around ten microns, this implies an asymmetry in the distribution of produced charges and is a source of error in the localization. Fig. 2 shows the computed number of tracks that, in one cm of argon, contain at least one electron of range R_0 ; because of the double exponential dependence of the number of δ electrons and of their range on energy, the probability distribution shown in the figure is not at all Gaussian, and have substantial tails: 5% of tracks have an asymmetry of about 50 μm . Obviously, the tails are more pronounced for large detection volumes, and this explains the tendency, for high accuracy drift chambers, of using relatively narrow detection gaps (five to ten millimetres²). One can easily see that the method used to measure the position of the charges, either edge or centre-of-gravity, has little influence on the measurement error introduced by the quoted asymmetry. In fact, when localising the track in a drift chamber by detection of the first electrons reaching the anode, the maximum error corresponds to the δ -ray range; if the average time of the drifting charge is measured, since the total 'normal' energy loss is a few KeV, the extra

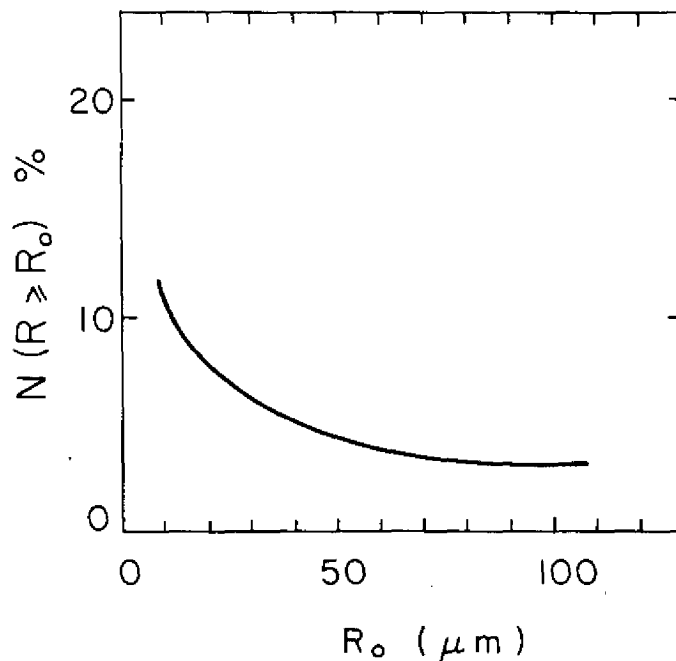
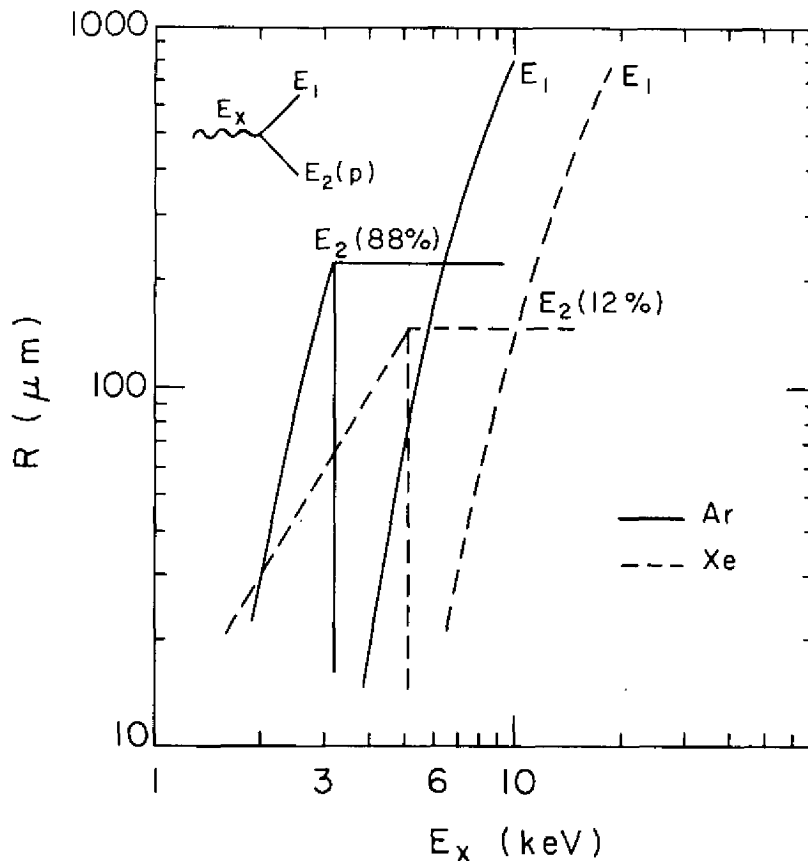


Fig. 2. Number of ejected electrons having in argon, at normal conditions, a range equal or larger than R_0 .

asymmetric few keV δ -electron will offset the centroid of the charge by half of its range, on either side of the central charge distribution. Also, for a given thickness, increasing the gas pressure with the aim of decreasing the δ -rays range does not help, since their number at a given energy is correspondingly increased.

Let us see now which are the physical limits in the localisation of soft x-rays; this is rather important since x-ray sources of this kind are currently used in laboratory tests (for example the 5.9 keV ^{55}Fe emission). Let E_x be the energy of the photon; by photoelectric effect, the quantum is absorbed in a shell of energy E_i , with emission of a photoelectron of energy $E_1 = E_x - E_i$. The excess energy can either escape as fluorescent radiation, or appear as an Auger electron of energy E_2 (very close to E_i^3). In Fig. 3 we have represented, as a function of the incoming photon energy, the range in argon and xenon at normal conditions of the photoelectron (E_1) and the Auger electron (E_2 , emitted with probability p). For example, the range in argon of both electrons produced by a 5.9 keV source is around 200 μm ; this sets a clear limit to the width of the physical message, and it is useless to collimate more the source.



We will see later that in some particular chamber geometries a focussing effect of the electric field allows to obtain physical messages much narrower than the previous calculations have shown: but it should be kept in mind that this is not the general case.

3. Accuracy limits in positioning by a drift time measurement

In a drift chamber the space coordinates of a track are reconstructed from a measurement of drift time of the electrons produced in the gas, and migrating under the influence of a suitable electric field until they reach an anode wire. In this case, and neglecting both the systematic effects and the track asymmetry effects discussed above, the limiting accuracy will depend on the electron diffusion in the gas. An initially point-like charge, migrating in the gas, spreads with a Gaussian distribution of standard deviation given by:

$$\sigma_x = \sqrt{2 Dt}$$

where t is the drift time and D the diffusion coefficient. Using the definition of characteristic energy of electrons $\epsilon_k = eDE/wP$, where w is the drift velocity of electrons migrating in the field E at a pressure P , one gets:

$$\sigma_x = \sqrt{\frac{2 \epsilon_k Px}{eE}}$$

where x is the drift length. One can see therefore that the diffusion in a given gas depends on E/P (since ϵ_k does), and is smaller in a 'cool' gas (a gas in which the characteristic energy is small even at high E/P values). Figs. 4 and 5 show the characteristic energy in several gases and gas mixtures, at $P = 1$ atmosphere, as a function of E^4). The normal range of fields used in drift chambers lays between 500 and 1500 V/cm, where the drift velocity is high enough or saturated. Carbon dioxide is the coolest known gas, but unfortunately its use in high-gain proportional counters is not to be recommended (high

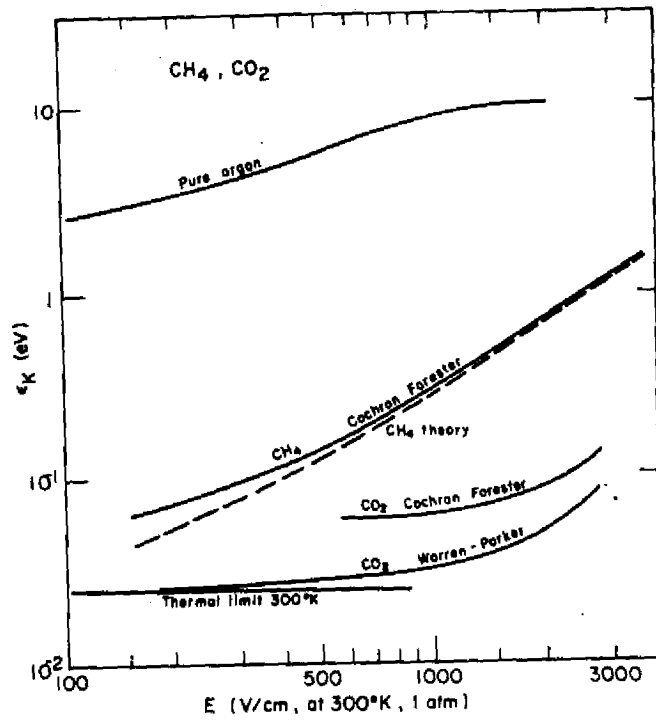


Fig. 4. Characteristic energy of electrons in different gases, as a function of electric field⁵⁾.

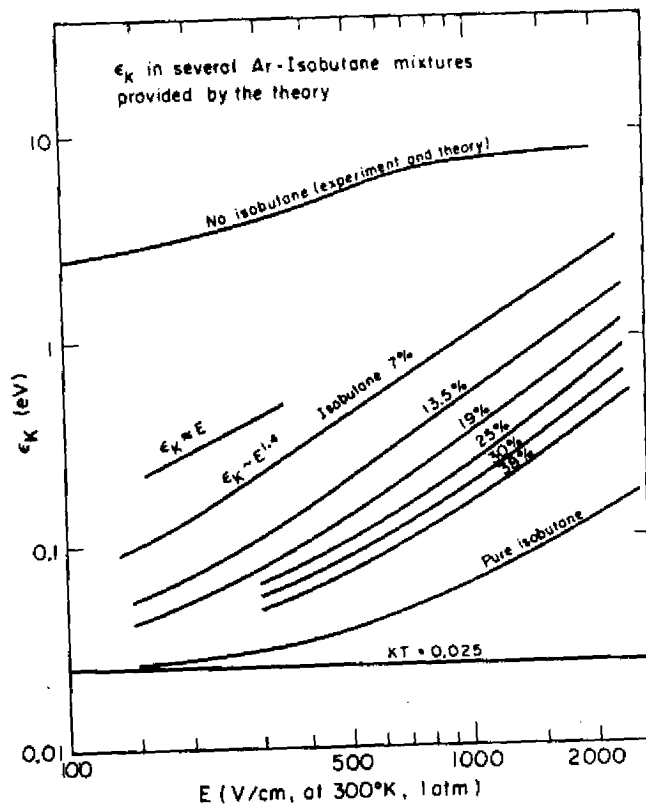


Fig. 5. Characteristic energy of electrons in several gas mixtures⁵⁾.

sparking probability). Fig. 6 shows the computed standard deviation due to diffusion, after one cm of drift in several gases⁴⁾; the argon-isobutane mixture, of a rather popular use, gives $\sigma_x = 200 \mu\text{m}$ for one cm drift. Is this the limiting accuracy of space localisation? No, since it represents the actual width of the charge cluster while the error of localisation depends on its dispersion. Schultz⁵⁾ has computed the dispersion in the position of the electron i in a cluster of n to be:

$$\sigma_i = \frac{\sigma_x}{\sqrt{2 \ln n}} \sqrt{\sum_{j=1}^n \frac{1}{j^2}}$$

for the first electron reaching the anode, this reduces to:

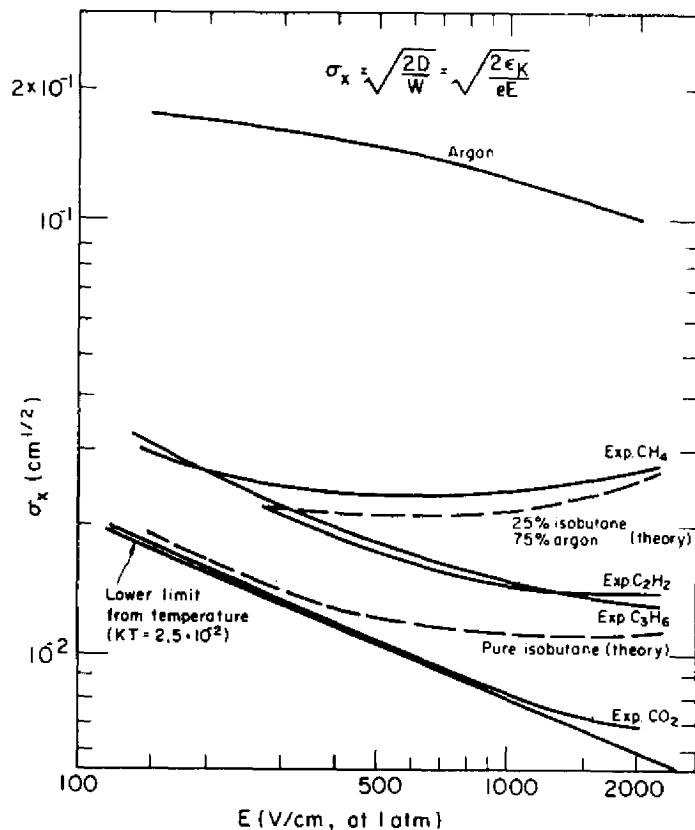


Fig. 6. Standard deviation of the charge distribution due to diffusion, after one cm drift in several gases at normal conditions, as a function of the electric field.

$$\sigma_1 = \frac{\sigma_x}{\sqrt{2 \ln n}} \sqrt{\frac{\pi^2}{6}}$$

The best information about the position of the cluster is obviously in its center-of-gravity; in this case the error would be:

$$\sigma_G = \frac{\sigma_x}{\sqrt{n}}$$

For $n = 70$, average number of electrons produced by a minimum ionizing track in 6 mm of gas, one gets therefore $\sigma_1 \approx 0.44 \sigma_x$ and $\sigma_G \approx 0.12 \sigma_x$. In the argon-isobutane mixture whose dispersion was shown in Fig. 5, and around one KV/cm one gets therefore, for 1 cm of drift, $\sigma_1 \approx 90 \mu\text{m}$ and $\sigma_x \approx 25 \mu\text{m}$. What does the experiment say? In Fig. 7, we have reproduced the best known measured accuracies in a set of drift chambers⁶⁾ as a function of the drift space; simple threshold discriminators were used, essentially detecting the arrival of the first electron (therefore with a dispersion σ_1). One can separate in the measurement the effect of the diffusion, the electronics dispersion and the primary ions statistics (important only close to the anode wires, $x = 0$), as shown in the figure.

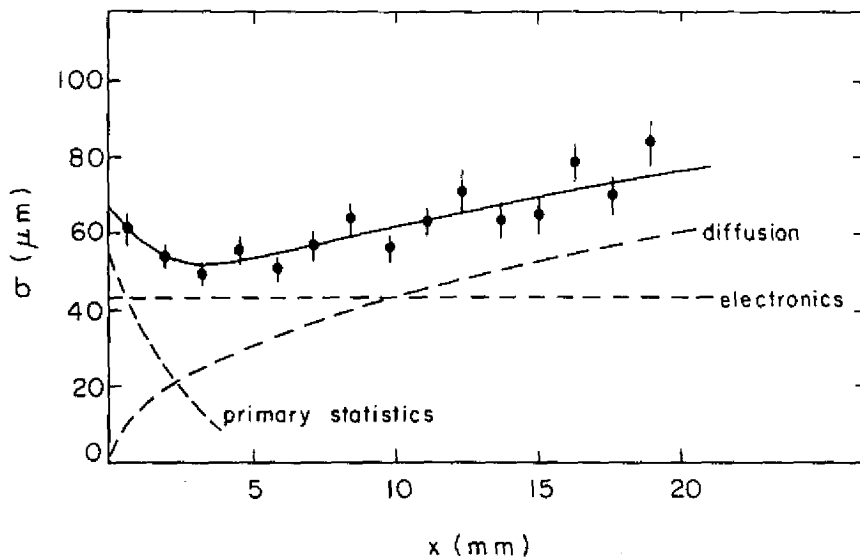


Fig. 7. Measurement of accuracy in a drift chamber as a function of the drift space⁶⁾. Assuming a constant spread due to the recording electronics, and taking into account the primary ion pairs statistics, one can identify the contribution due to the diffusion alone.

At 10 mm, the diffusion alone accounts for a standard deviation of about 45 μm : about half of what we had computed before! A possible explanation of the discrepancy based on the assumption that the electronics operates a kind of average over the timing of the successive avalanches has been put forward, but is not very convincing. The true explanation is probably to be found elsewhere. All characteristic energy measurements, that are at the base of the calculation of σ_x , are at the origin measurements of transverse diffusion, i.e. of increase in the size of a migrating charge in a direction perpendicular to the electric field (Townsend's method⁷); the assumption that the diffusion is symmetric in space is implicit. Recently however it has been found⁸) that the longitudinal diffusion, i.e. in the direction of the field, can be in some gases much smaller than the transverse diffusion. This is shown in Fig. 8⁹) for argon. At the fields we are concerned with, around 1 Volt/cm Torr, the longitudinal diffusion is about a factor of 6 smaller than the transverse one. One should therefore reconsider the theory of gaseous diffusion that allowed to obtain the curves of Fig. 6, putting in the correct value for the longitudinal diffusion.

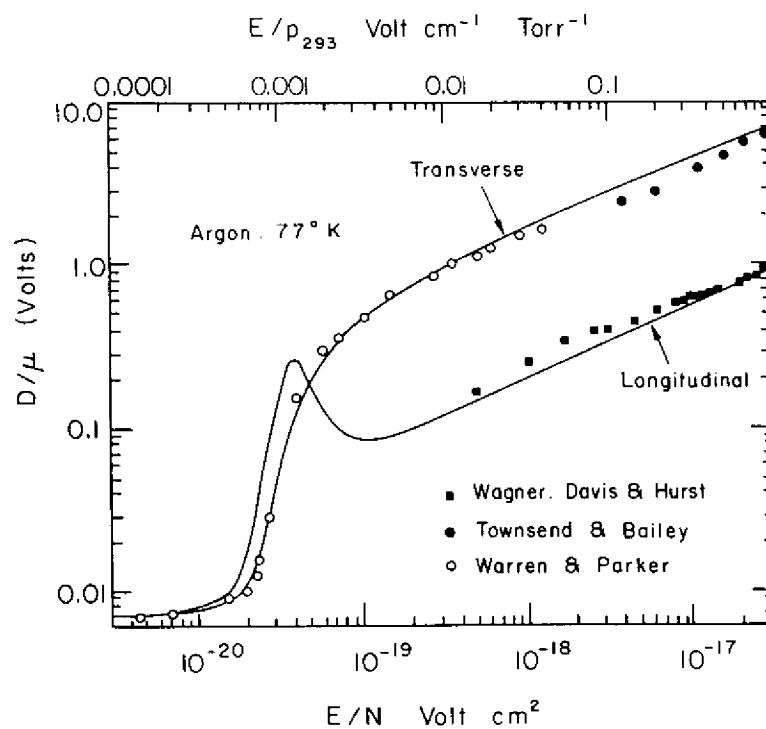


Fig. 8. Experimental and computed results showing the ratio between diffusion coefficient and mobility (or characteristic energy) for the transverse as well as for the longitudinal diffusion, in argon⁹).

Unfortunately, measurements of this parameter have been realised only for a restricted number of gases. If one assumes that, in the argon-isobutane mixture, there is a factor of four between the two coefficients of diffusion, this would explain the quoted factor of two discrepancy between the computed and the measured accuracies.

To conclude this section, which are the limits of intrinsic accuracy one can think to obtain in a drift chamber? The discussed arguments prove that, for one cm drift, accuracies around 50 μm have been obtained with simple threshold discriminators in argon-isobutane; a constant fraction or zero crossing discrimination should allow to approach the theoretical limit of the center-of-gravity, or about ten microns*. And there is, of course, always the possibility of using a 'cooler' gas to reduce diffusion. In this case, however, systematics and electronics dispersions, as well as the physical message asymmetries would dominate the limiting accuracy.

4. Limiting accuracy in the localization by center-of-gravity of induced signals

The method of localization by center-of-gravity of the induced signals has been originally developed to measure the coordinates of conversion of neutral radiation, since it is intrinsically bi-dimensional¹⁰⁾. Recent measurements have shown, however, that this method could be, in the domain of high precision accuracies, very competitive with drift methods for the detection of charged particles¹¹⁾.

* Notice however that until now, and to our knowledge, all measurements of position by center-of-gravity have not shown any substantial improvement in accuracy over the conventional threshold detection method. It is possible that at the rather high chamber gains at which people operate, a quick development of a positive space charge on the first amplified electrons prevents a uniform detection of all charges.

The principle of the method is shown in Fig. 9; in a conventional multiwire proportional chamber, the movement of positive ions (produced in the avalanches around the anode wires) induces positive signals on the cathode planes, with a distribution centered around the position of the avalanches. A measurement of the center-of-gravity of the induced pulses on the two planes provides the bi-dimensional coordinates of the ionizing event.

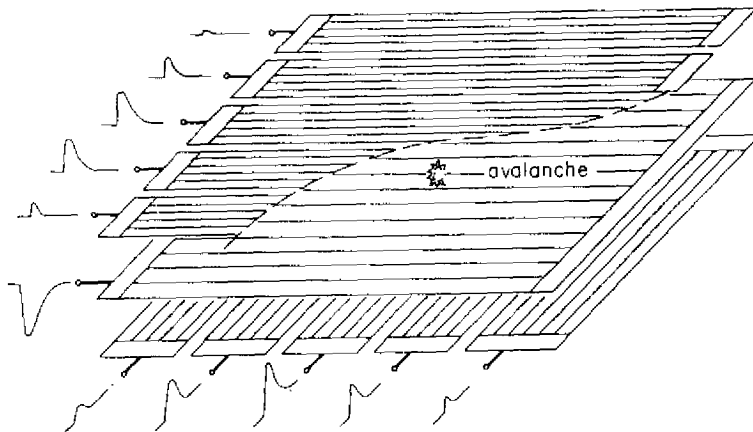


Fig. 9. Principle of the localization by center-of-gravity of the induced pulses in a multiwire proportional chamber¹¹⁾.

Localization has been obtained using several analogic methods, by a delay line technique or by direct measurement of the pulse heights on suitably stripped cathode planes. It is this last method that, despite its obvious practical complications, allows to obtain the best results in terms of accuracy. As an example, Figs. 10a) and b) show the localization accuracy obtained on a ^{55}Fe collimated source in the directions parallel and perpendicular to the anode wires respectively¹¹⁾. The full width half maximum of the distribution in Fig. 10a) is $250\ \mu\text{m}$, compatible with the range of the photoelectrons in the gas (see the discussion in section 2). In the direction perpendicular to the anode wires, on the other hand, the avalanches are localized on the wires, whatever the conversion point and the space

distribution of the original charges; one gets therefore a measurement of the overall instrument resolution (in the case of Fig. 10b), a FWHM of 60 μm).

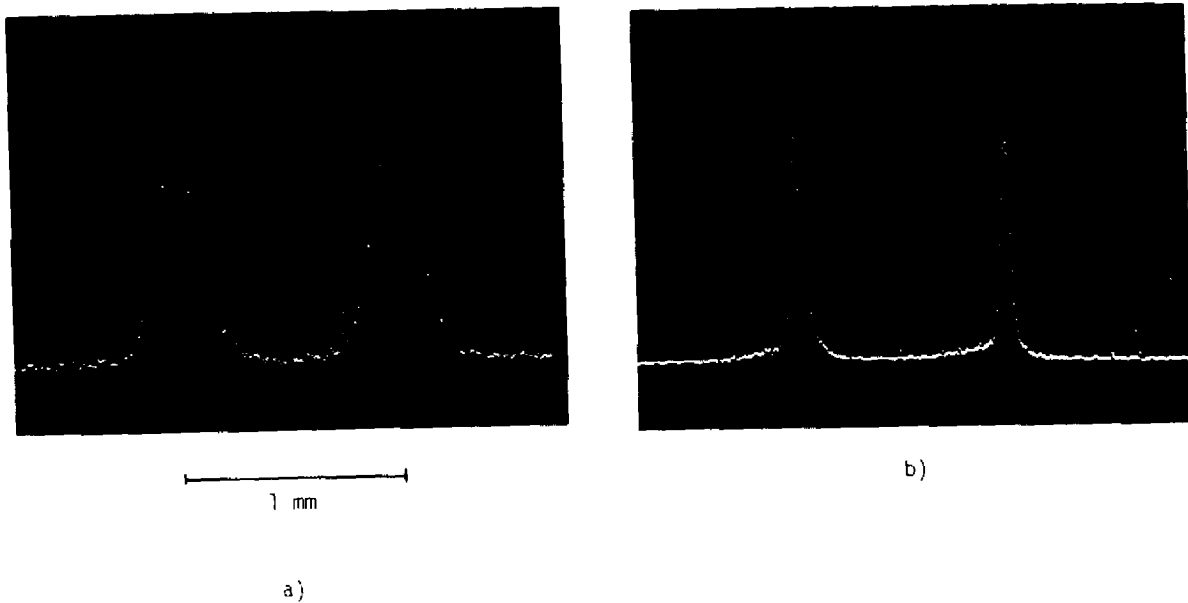


Fig. 10. Precision of localization by center-of-gravity, with a collimated ^{55}Fe source in the direction parallel (a) and perpendicular (b) to the anode wires. The full width half maximum of the distributions are 250 μm and 60 μm , respectively.

The same measurement has been repeated in a minimum ionizing beam, and allowed to verify that, for charged particles crossing the chamber at an angle, an interpolation effect occurs that allows to obtain, in the direction perpendicular to the anode wires, accuracies better than the wire spacing. This is shown in Figs. 11a), b) and c) for incidence angle of 0° , 10° and 45° with the normal to the wire planes, respectively. The 1 mm scale, corresponding to the anode wires spacing, is shown below each picture; the interpolating effect (that appears in the chosen chamber geometry above 7° of incidence) is evident. When, using a collimated beam, one measures the center-of-gravity in the chamber versus position of the beam, a linear interpolation is observed between wires as shown in Fig. 12 (for 16° of incidence). One can

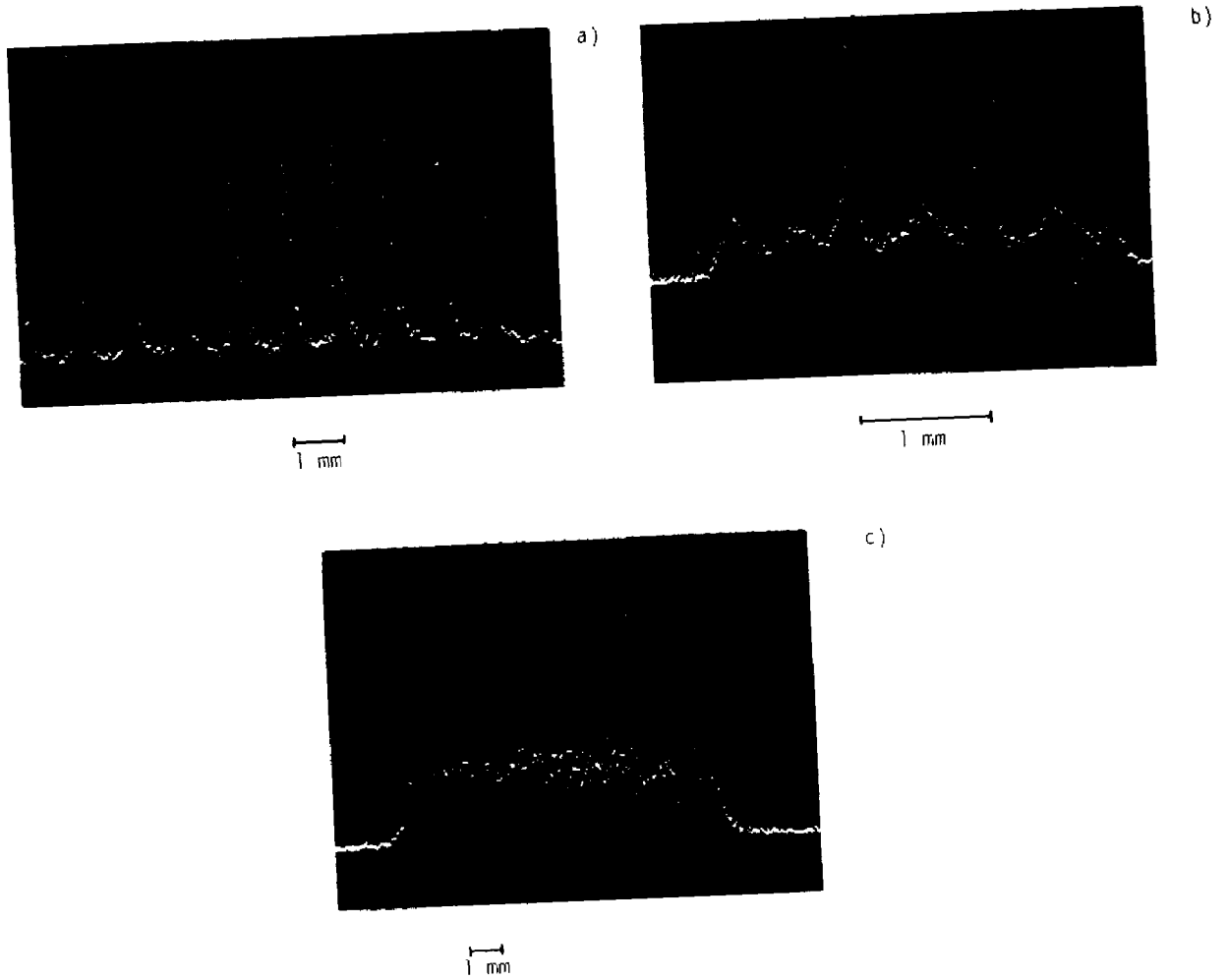


Fig. 11. Center-of-gravity distribution, in a uniform minimum ionizing beam, in the direction perpendicular to the anode wires, showing the interpolation effect as soon as the incidence angle (angle between the track and the normal to the wire planes) exceeds a critical value (7° in the adopted geometry). At 0° (a) the spectra are well separated and correspond to the wire spacings, while at 10° (b) the interpolation effect appears and is complete at 45° (c).

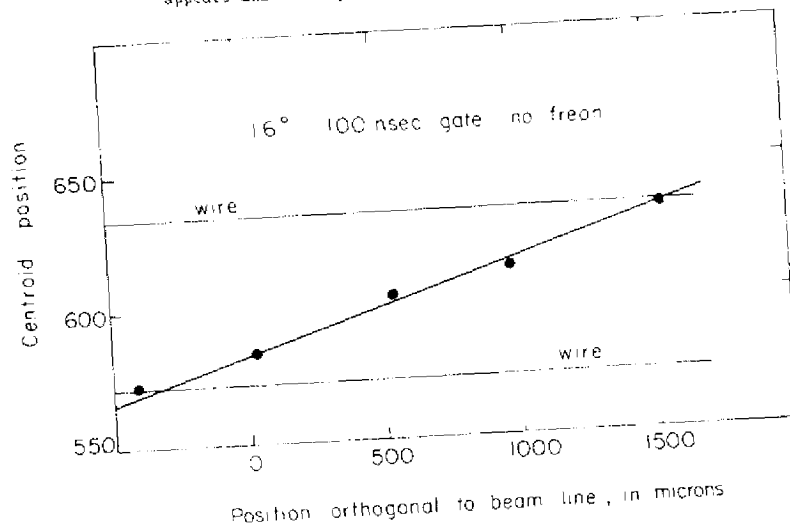


Fig. 12. Measured relationship between the position of an inclined collimated minimum ionizing beam, and the center-of-gravity of the induced charge, showing the linearity of interpolation.

therefore expect that, whenever the beam is slightly inclined in respect to the normal to the wire planes, accuracies around $60 \mu\text{m}$ FWHM can be obtained in both directions. Because of the poor collimation of the beam, in the described experiment, this expected accuracy was not measured yet.

In this the limiting accuracy by the centre-of-gravity method? The intrinsic resolution of the pulse height measurements (about 0.5%) results in an electronics spread of around $10 \mu\text{m}$; why the measurement in the direction perpendicular to the wires has a FWHM of $60 \mu\text{m}$? An answer to this question has been found only very recently. Consider the field geometry around the anode wires shown in Fig. 13. If one converts an x-ray in the constant field region, the lateral extension of the charge distribution of the photoelectron (as we have seen in section 2, at 5.9 keV the typical range is around $200 \mu\text{m}$) will be strongly reduced during the drift down to the wire, due to the focussing effect of the electric field. Lateral diffusion may spread a little the charge, but for not too large drift distances its effect can be neglected. In these conditions, if one collimates an inclined x-ray beam through the chamber above and below the anode, a more or less uniform distribution around the wire of the clusters of charge reaching the anode is obtained. Assuming then that in the avalanche process the original direction of arrival is preserved (i.e. there is not too much photon

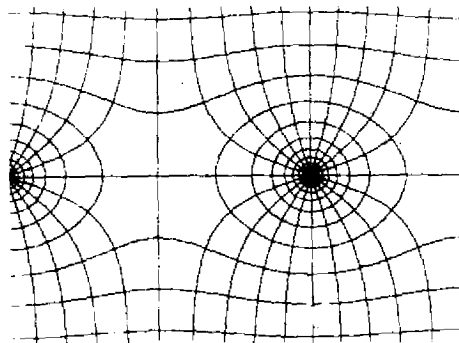


Fig. 13. Close look of the field structure around the anode wires, illustrating the focussing effect of the electric field in the neighbourhoods of the wires.

propagation of secondary avalanches around the wire), a careful measurement of the center-of-gravity and of the up-down induced charge ratio will localize the charge distribution around the anode. This is shown in Fig. 14¹²⁾, where from the measurement of the pulse height on strips on both cathodes, the horizontal coordinate has been computed as the center-of-gravity of the pulses on one cathode, and the vertical from the ratio of pulse height between each cathode and the anode signal. In other words, the figure represents the average position of the cloud of positive ions, drifting away from the anode, during the sampling time (around 100 nsec). The shape of the measured distribution, remarkably reproducing the circular shape of the equipotentials, demonstrates the very good localization of the initial charge and of

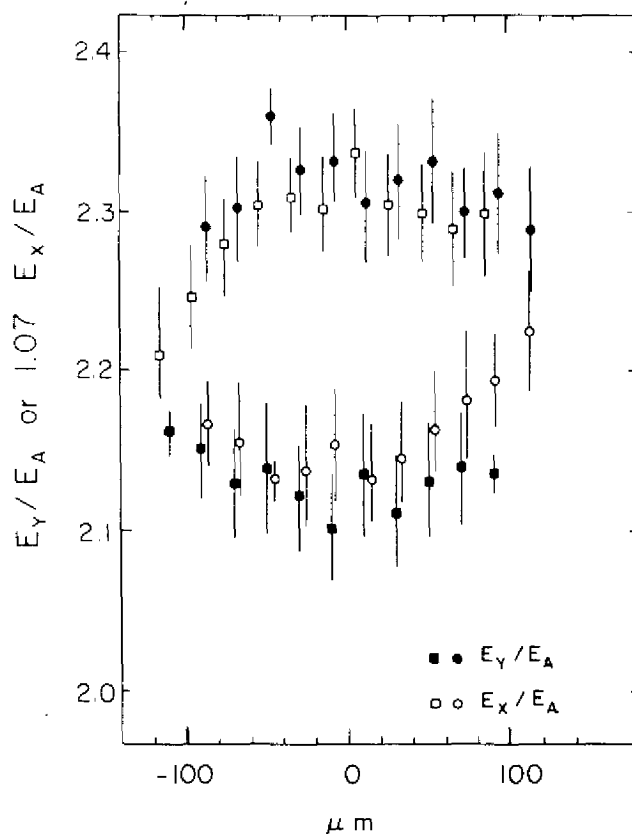


Fig. 14. Localization of avalanches in the plane perpendicular to the anode wires (plane of Fig. 13) by accurate center-of-gravity. The source used is a ^{55}Fe x-ray emitter, collimated either above or below the anode wire. Pulse height distribution are measured on both cathode planes, as well as on the anode; the horizontal coordinate is obtained from the center-of-gravity on one cathode, while the vertical coordinate is given by the ratio of the cathode to the anode pulses.

the avalanche itself; such a measurement is capable of radially identifying the point of conversion of the photon, despite of the quantizing effect of the wires. From the measurement one can infer for each avalanche an intrinsic position resolution around 10 μm . Obviously, in the general case this limiting accuracy may be spoiled by other physical effects, like avalanches asymmetry, space charge effects, electrostatic shadows due to the other wires etc: the work in this domain has just started!

References

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