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Neutral Heavy Leptons and Electroweak Baryogenesis

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Abstract

We investigate the possibility that baryogenesis occurs during the weak phase transition in a minimal extension of the Standard Model which contains extra neutral leptons and conserves total lepton number. The necessary CP-violating phases appear in the leptonic Yukawa couplings. We compute the CP-asymmetries in both the neutral and the charged lepton fluxes reflected in the unbroken phase. Using present experimental bounds on the mixing angles and Standard Model estimates for the parameters related to the scalar potential, we conclude that it seems unlikely to produce the observed baryon to entropy ratio within this kind of models. However, we comment on the possibility that the constraints on the mixings might be naturally relaxed due to small finite temperature effects.

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1 Introduction

The baryon number to entropy ratio in the observed part of the Universe is required to be $n_B/s \sim (4\text{–}6) \times 10^{-11}$ by nucleosynthesis constraints [1]. In 1967, Sakharov [2] established the three basic requirements for obtaining this baryon asymmetry as a result of particle interactions in the early universe: a) Baryon number violation, b) C and CP violation, c) departure from thermal equilibrium. These conditions may be fulfilled at weak scale temperatures [3], if the electroweak phase transition is first order.

In a strongly first order electroweak transition, bubbles of the true ground state (broken phase) nucleate and expand until they fill the Universe; local departure from thermal equilibrium occurs in the vicinity of the expanding bubble walls. The other two Sakharov conditions are also satisfied, since C and CP are known to be violated by the electroweak interactions and anomalous baryon number violation is fast at high temperatures in the symmetric phase. As a bubble expands, particles in the unbroken phase will reflect off the advancing wall. CP-violating interactions result in a different reflection probability for fermions with a given chirality and the corresponding antifermions, leading to a CP asymmetry in the reflected chiral number flux [4]. In the symmetric phase, anomalous $B+L$ violating interactions are in thermal equilibrium and the reflected current induces a net baryon number. An important survival requirement for the produced baryon asymmetry is that the sphaleron processes inside the bubble are slow enough and this in turn is directly related to the strength of the phase transition.

In principle the Standard Model (SM) contains all the necessary ingredients for electroweak baryogenesis, but it has two problems: the CP asymmetry induced by the Kobayashi-Maskawa phase is far too small to account for the observed n_B/s ratio [5, 6], and the phase transition appears too weakly first order for the Higgs mass experimentally allowed [7]. However, these two problems may be absent in several simple extensions of the SM, which contain additional sources of CP violation and more scalars than the SM. The larger parameter space in the scalar sector allows for a stronger first-order phase transition without such a light Higgs [8]. Several such possibilities have been considered in the literature: two Higgs models with a strong CP phase [9]–[12], heavy Majorana neutrinos [13], and supersymmetric models [14].

In the present paper, we consider models with an extended lepton sector, which conserves total lepton number. The model provides a viable alternative to the see-saw mechanism for explaining the lightness of the known neutrinos, in those extensions of the SM where there are no scalars carrying lepton number, which could generate Majorana masses. They arise in several contexts, such as GUTs [15] and E(6) superstring-inspired models [16]. Similar patterns of lepton masses have also been obtained in the context of models of Extended Technicolor with a GIM mechanism [17].

The relevant features for baryogenesis are twofold. First, the lepton Yukawa interactions contain additional CP-violating phases, which can lead to a much larger CP asymmetry than the CKM phase in the SM. In contrast to the models with Majorana neutrinos considered in [4], the CP-violating effects in this case are not suppressed by the light neutrino masses. Second, the presence of an additional singlet scalar may help in getting a stronger first-order phase transition.

An interesting issue that may be relevant in this type of models is whether finite temperature corrections can produce an enhancement of the CP asymmetry, as was found in the first detailed calculation of this quantity in the SM [18] [1]. In [5][6] it was shown that this enhancement disappears

[1] To be more precise, the typical suppression with quark masses expected in a flavour-blind CP-violating process was

in the SM when one properly includes the incoherence effects induced by the interaction of the quarks with the plasma. However, the question remained that if the particles involved were much more weakly interacting, as leptons instead of quarks, maybe this enhancement would be at work.

In section 2 we describe the model, and the order of magnitude estimates of the CP-violating asymmetries are obtained in section 3. We find that the leading effects are different for the reflection of the neutral and the charged leptons. Naively the later is smaller since it is suppressed by the charged lepton masses; however, an enhancement of the type of ref. [18] could imply no such suppression. In section 4 we compute the contribution to the asymmetry due to the reflection of the neutral leptons, which turns out to be the leading effect, as expected. In close analogy with the SM case, we consider in section 5 the lepton asymmetry generated by the charged lepton reflection on the bubble wall. As we will see, no enhancement with respect to the naive estimate is found. In section 6 we compute the baryon number induced by the CP asymmetries in the neutral sector, and we conclude in section 7.

2 The Model

The phenomenology of this type of models has been extensively studied in [19][17]. Here we briefly describe the essential features relevant for baryogenesis.

The gauge group is the standard $SU(2) \times U(1)$, with minimal quark sector. The lepton sector is extended with two electroweak singlet two-component leptons in each generation, i.e.,

$$\Psi^i_L = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix}, \; e^i_R, \; \nu^i_R, \; s^i_L. \tag{1}$$

Unlike the minimal standard model, total lepton number conservation is not an automatic symmetry. It has to be imposed, and it restricts the form of the Yukawa terms that lead to the neutral fermion masses, while the Yukawa terms involving charged leptons are completely standard:

$$\mathcal{L}_Y = \bar{\Psi}_L f H e_R + \bar{\Psi}_L f_D H \nu_R + \bar{s}_L f_S \sigma \nu_R + h.c. \tag{2}$$

where f_i are the Yukawa matrices, H is the standard Higgs doublet and σ is a new singlet scalar field. Due to the presence of σ, the weak phase transition can be quite strongly first order for a significant range of parameters [8].

For simplicity, we will assume that the singlet σ acquires a vacuum expectation value at the same scale as the standard Higgs doublet, and the theory undergoes a single phase transition at a critical temperature near the weak scale. Although this does not need to be the case, we expect that the CP asymmetry will not depend much on this choice (at least if power counting arguments give a correct estimate). Then, in the broken phase the lepton mass terms are

$$\bar{e}_R m e_L + \bar{\nu}_R D \nu_L + \bar{\nu}_R S s_L + h.c., \tag{3}$$

where

$$m = f^\dagger v \quad D = f_D^\dagger v \quad S = f_S^\dagger u \tag{4}$$

v and u being the vevs of the doublet and singlet scalar fields, respectively.

not found, while the suppression with the CP-conserving angles was explicit.

The structure of the neutral sector mass matrix (3) ensures the existence of three massless Weyl neutrinos ν^0, regardless of the relative value of D and S. The other six Weyl fermions pair up into three heavy neutral Dirac fermions n, whose masses are essentially determined by S. Note that in this model the ratio D/S is expected to be less constrained than the corresponding parameter of any model that invokes the see-saw mechanism to understand the smallness of neutrino masses. This is because in such models the neutrino masses are only suppressed by D/S, which is therefore very constrained. In the present case, this ratio is not related to the light neutrino masses. Nevertheless, we will see in the next section that it can also be constrained from the strong bounds on charged lepton mixing.

The mass matrix can be diagonalized by multiplying on the right and the left by unitary matrices:

$$\begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} D & S \\ 0 & 0 \end{pmatrix} U = \begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix} \tag{5}$$

with

$$\begin{pmatrix} \nu_L \\ s_L \end{pmatrix} = U \begin{pmatrix} \nu^0 \\ n_L \end{pmatrix} \tag{6}$$

where ν^0 are the massless neutrinos and n are the neutral heavy leptons (NHL).

The unitary matrix V diagonalizes $DD^\dagger + SS^\dagger$ to give M^2. The unitary matrix U can be written as

$$U = \begin{pmatrix} K_L & K_H \\ K_{SL} & K_{SH} \end{pmatrix} \tag{7}$$

where $K_L, K_{SH} \sim 1 - \mathcal{O}[(D/S)^2]$ and $K_H, K_{SL} \sim \mathcal{O}(D/S)$.

Using (6) we get the following form for the charged current leptonic weak interaction:

$$\mathcal{L}_{cc} = \frac{g}{2} W^\mu \sum_{a=1}^{3} \sum_{\alpha=1}^{6} \bar{e}_a \gamma_\mu L \left[K_L \nu^0 + K_H n_L \right]_{a\alpha} + h.c. \tag{8}$$

One can see that the charged current coupling of the mass eigenstates charged leptons to the massless as well as the heavy neutrinos is non-trivial, making possible the violation of individual lepton numbers L_e, L_μ and L_τ.

Due to the admixture of fermions of different weak isospin, there is no GIM mechanism in the neutral fermion couplings to the Z boson, which are given by

$$\mathcal{L}_{nc} = \frac{g}{2c_w} Z^\mu \sum_{\alpha,\beta} \bar{N}_\alpha \gamma_\mu L P_{\alpha\beta} N_\beta, \tag{9}$$

where $N_\alpha = (\nu_a^0, n_a)$, and $P = K_L^\dagger K_L + K_H^\dagger K_H$ is in general a non-diagonal projection matrix. The neutral couplings involving the massless neutrinos are diagonal but flavour-dependent.

It has been shown [20] that for n generations the total number of physical parameters describing the Yukawa sector is n^2 angles and $(n-1)^2$ phases. Thus, for three families there are four independent CP-violating phases. If the charged lepton Yukawas are neglected, it is easy to show, using the method of ref. [21], that the number of phases is $(n-1)(n-2)/2$. So one CP-violating phase still remains in this limit and consequently the associated CP invariant is not suppressed by the charged lepton masses. In contrast, the CP invariants corresponding to the other three phases are necessarily suppressed by the small differences of charged lepton masses.

3 Order of Magnitude Estimates

The observable CP asymmetry results from the interference of pure CP-violating phases with CP-even phases, equal for particles and antiparticles. These are the reflection coefficients, which become complex when the particle energy is smaller than its mass in the true vacuum. Unremovable CP-odd phases appear in the mass matrices due to either

a) CP-violating interactions in the thermal loops that correct the dispersion relations of the particles propagating in the plasma [18];

b) non-trivial space-time dependence of the scalar vevs inside the bubble wall (for more than one Higgs field), which induces space dependent CP-violating phases. These phases cannot be rotated away at two adjacent points, x and $x+dx$, by the same set of unitary transformations, i.e. $U_x^{-1}U_{x+dx} \neq 1$ [14].

Whenever mechanism b) is present, it generically will dominate over a), since in a) there are suppression factors coming from loops ($1/4\pi$). Mechanism b) is the one that generates the baryon asymmetry in all the extensions of the SM proposed in the literature for electroweak baryogenesis. In contrast, in the SM the quark mass matrix has only an overall dependence on the Higgs vev and can be diagonalized by space-independent unitary matrices; hence the CP asymmetry can only be generated through mechanism a).

In the model considered here we have to distinguish between the charged and the neutral sectors. Charged leptons get their masses only from the doublet scalar vev, so the situation is completely analogous to the SM: CP-violating phases appear in the thermal corrections to the dispersion relations. In the neutral sector the situation is different because the mass matrix has a non-trivial dependence on both singlet and doublet scalar vevs. Since generically this ratio is not constant within the wall, mechanism b) is also present.

The size of the leading CP asymmetries in the reflection of both charged and neutral leptons can be estimated by simple power counting arguments. To do so, we construct a measure of the CP violation, invariant under flavour and phase redefinitions of the lepton fields, i.e. under transformations of the type

$$\begin{aligned} \Psi_L &\to U\Psi_L \\ e_R &\to V e_R \\ \nu_R &\to W\nu_R \\ s_L &\to X s_L. \end{aligned} \tag{10}$$

One can show that the following expression is invariant under such transformations, and vanishes if CP is conserved [20]:

$$\mathrm{Im}\mathrm{Tr}\left[D^\dagger D m^\dagger m D^\dagger S S^\dagger D\right]. \tag{11}$$

Notice that this effect cannot be tree level in the reflection amplitude, since it involves the couplings of both charged and neutral leptons. Therefore it is typically down by loop factors ($1/4\pi$).

This invariant is given by

$$\delta_{CP}^2 = \sum_{a<b} M_a^2 M_b^2 (M_a^2 - M_b^2) \sum_{i<j} (m_i^2 - m_j^2) \mathrm{Im}(K_{Hia} K_{Hja}^* K_{Hib}^* K_{Hjb}), \tag{12}$$

where M_i and m_a are the NHL and charged lepton masses, respectively. The natural scale in the problem is of the order of the electroweak phase transition temperature, $T \sim 100$ GeV. Therefore, to obtain a dimensionless quantity δ_{CP}^2 should be divided by T^8. Typically $M_i \sim T$, but the small charged lepton masses give a suppression of order $(m_\tau/T)^2 \sim 10^{-4}$ at least. We expect from eq. (12) that the leading CP asymmetry in the reflection of charged leptons will appear at fourth order in the mixing $K_H \sim \mathcal{O}(D/S)$.

On the other hand, the leading effect coming from the neutral sector appears at tree level. The leading CP measure, invariant under the transformations of (11) involving only the neutral fields, is given by

$$\mathrm{Im}\mathrm{Tr}\left[DD^\dagger SS^\dagger (DD^\dagger)^2 (SS^\dagger)^2\right], \tag{13}$$

which gives

$$\delta_{CP}^3 = M_1^2 M_2^2 M_3^2 (M_1^2 - M_2^2)(M_1^2 - M_3^2)(M_2^2 - M_3^2)\mathrm{Im}\left[(K_H^\dagger K_H)_{12}(K_H^\dagger K_H)_{23}(K_H^\dagger K_H)_{31}\right]. \tag{14}$$

In this case, the asymmetry appears at sixth order in the mixing, $\mathcal{O}[(D/S)^6]$. To obtain a dimensionless quantity, we should divide δ_{CP}^3 by T^{12} but there is no suppression in the masses here since $M_i \sim T$. There is also an additional contribution of the form (12) due to loop corrections, but it would be suppressed at least by $(m_\tau/T)^2 \sim 10^{-4}$ and by loop factors, which considering the experimental bounds, is a larger suppression than the extra $(D/S)^2$.

If we assume that the asymptotic value of the ratio of scalar vevs is the same as at zero temperature $v(T)/u(T) = v(0)/u(0)$, there are quite strong experimental bounds on the elements of the submatrix $K_H^\dagger K_H$. These bounds depend on the NHL mass [22, 23]. For 3 GeV $\leq M \leq M_Z$ the strongest limits come from LEP and they are very stringent [2]: $|\mathrm{Im}(K_{Hia}K_{Hja}^* K_{Hib}^* K_{Hjb})| \leq 3\times(10^{-9}\text{–}10^{-7})$. If $M \geq M_Z$, there are low-energy constraints that arise both from the non-observation of lepton flavour violation and from universality, as well as limits from the invisible width of the Z boson [23]. The limits are slightly weaker for the mixing of the third family with any of the first two,

$$|\mathrm{Im}(K_{Hia}K_{H3a}^* K_{Hib}^* K_{H3b})| \leq 5\times 10^{-5}\,(10^{-5}) \quad i = e, \mu. \tag{15}$$

The first number corresponds to the so-called ‘joint’ bounds in ref. [23], for which cancellations among the different possible fermion mixings are allowed, while the number in brackets corresponds to the ‘single’ limits, obtained when the remaining mixing parameters are set to zero.

For the invariant (13), we get

$$\mathrm{Im}\left[(K_H^\dagger K_H)_{12}(K_H^\dagger K_H)_{23}(K_H^\dagger K_H)_{31}\right] \leq 10^{-7}\,(3\times 10^{-8}). \tag{16}$$

Based on the bounds (15) and taking into account the loop factor expected in that case, plus the further suppression in the charged lepton masses, typically of order $(m_\tau/M)^2 \sim 10^{-4}$, the CP asymmetry in the reflection of charged leptons (12) is expected to be too small to generate a significant baryon asymmetry. However, a similar enhancement as the one found in [18] could imply that there is no suppression coming from the lepton mass, in which case the effect could be important. This is the reason why we decided to do a detailed calculation in this case.

In the case of the reflection of the neutral leptons, the bound (16), together with the fact that there is no power suppression in the light masses or loop factors, implies that the effect could be of roughly the right order of magnitude.

[2]These bounds have been obtained using inequalities of the form $|\mathrm{Im}(K_{Hia}K_{Hja}^* K_{Hib}^* K_{Hjb})| \leq \frac{1}{8}[(K_H^\dagger K_H)_{ii}^2 + (K_H^\dagger K_H)_{jj}^2]$.

4 CP Asymmetry in the Neutral Sector

In this section we will compute the CP asymmetry in the number current of ν_L that get reflected into the unbroken phase. This asymmetry in the rest frame of the wall is simply given by

$$j_{CP} = j^{trans}_{s^b_L \to \nu^u_L} + j^{trans}_{\nu^b_L \to \nu^u_L} + j^{ref}_{\nu^u_R \to \nu^u_L} =$$

$$\int \frac{d^3p'}{(2\pi)^3} \sum_{i,j} [|A^t_{s^{i\,b}_L \to \nu^{j\,u}_L}(p'_z, \sqrt{p'^2_z + M_i^2})|^2 - |\bar{A}^t_{s^{i\,b}_L \to \nu^{j\,u}_L}(p'_z, \sqrt{p'^2_z + M_i^2})|^2] \, \frac{p'_z}{E} f^b_i(p')$$
$$+ \int \frac{d^3p}{(2\pi)^3} \sum_{i,j} [|A^t_{\nu^{i\,b}_L \to \nu^{j\,u}_L}(p_z, p_z)|^2 - |\bar{A}^t_{\nu^{i\,b}_L \to \nu^{j\,u}_L}(p_z, p_z)|^2] \, \frac{p_z}{E} f^b_0(p)$$
$$+ \int \frac{d^3p}{(2\pi)^3} \sum_{i,j} [|A^r_{\nu^{i\,u}_R \to \nu^{j\,u}_L}(-p_z, p_z)|^2 - |\bar{A}^r_{\nu^{i\,u}_R \to \nu^{j\,u}_L}(-p_z, p_z)|^2] \, \frac{p_z}{E} f^u_0(p), \quad (17)$$

with $\sum_{i,j}$ being the sum over flavours and

$$f^u_0(p) = \frac{1}{e^{(|\vec{p}| + v_w \, p_z)/T} + 1}, \quad (18)$$

$$f^b_0(p) = \frac{1}{e^{(|\vec{p}| - v_w \, p_z)/T} + 1}, \quad f^b_i(p') = \frac{1}{e^{(\sqrt{|\vec{p'}|^2 + M_i^2} - v_w \, p'_z)/T} + 1}, \quad (19)$$

being the thermal distributions of the different particles in the unbroken (u) and broken (b) phases as seen from the rest frame of the wall; v_w is the wall velocity, which is estimated to be $v_w \sim 0.1$–0.4 in the SM [24].

In the present case, thermal corrections to the propagation are negligible. The thermal masses are $\leq 0.25 M_i$ for the heavy leptons and thus considerably smaller than the energies at which the effect will be significant, $\omega \geq min\{M_i\}$. Furthermore, the mean free path of these weakly interacting particles is expected to be large compared both to the expected width of the bubble wall and to the reflection time of the leptons $\sim M_i^{-1}$; in the scattering with the wall the neutral leptons will therefore be assumed to be free. The transmission and reflection amplitudes will thus be computed at zero temperature, using LSZ reduction formulae in terms of the propagator in the presence of the wall:

$$\mathcal{A} = \int d^4x \int d^4y \; e^{-iq_i x} e^{iq_f y} \; \bar{u}(q_f)(i\vec{\partial} - m) S(y,x)(-i\overleftarrow{\partial} - m) u(q_i)$$
$$= (2\pi)^3 \delta(q^x_f - q^x_i) \; \delta(q^y_f - q^y_i) \; \delta(E_f - E_i) \; A(q^z_i, q^z_f), \quad (20)$$

with

$$S(y,x) \equiv \langle 0 | T[\Psi(y) \bar{\Psi}(x)] | 0 \rangle. \quad (21)$$

An analogous expression holds for antiparticles. The spinors in formula (20) are on-shell and normalized to unit flux in the z direction, i.e.

$$\bar{u} \; \gamma_z \; u = 1. \quad (22)$$

Since the potential created by the bubble wall is only dependent on the coordinate z, momenta in the x and y directions are conserved. The transmission and reflection amplitudes only depend on the momenta in the z direction and can be computed in a much simpler way by first boosting to a frame where $q_x, q_y = 0$. With the proper normalization chosen for the spinors (22), the amplitude in the boosted frame is simply given by (20), with the propagator and incoming and outcoming momenta substituted by the boosted ones.

We can further simplify the expression for j_{CP} by using CPT symmetry and unitarity constraints, which imply

$$\sum_i |A^t_{\nu^i{}^b_L \to \nu^j{}^u_L}|^2 = 1 - \sum_i |A^r_{\nu^i{}^u_R \to \nu^j{}^u_L}|^2 - \sum_i |A^t_{s^i{}^b_L \to \nu^j{}^u_L}|, \tag{23}$$

and substituting (23) in eq. (17):

$$j_{CP} = \int \frac{d^3p}{(2\pi)^3} \sum_{i,j} (|A^r_{\nu^i{}^u_R \to \nu^j{}^u_L}|^2 - |\bar{A}^r_{\nu^i{}^u_R \to \nu^j{}^u_L}|^2) \, \frac{p_z}{E} (f_0^u(p) - f_0^b(p))$$
$$+ \int \frac{d^3p}{(2\pi)^3} \sum_{i,j} (|A^t_{s^i{}^b_L \to \nu^j{}^u_L}|^2 - |\bar{A}^t{}_{s^i{}^b_L \to \nu^j{}^u_L}|^2) \, \frac{p_z}{E} (f_i^b(p') - f_0^b(p)). \tag{24}$$

Finally, by expanding the Fermi distributions for small wall velocities:

$$j_{CP} = \frac{v_w}{T} \int \frac{d^3p}{(2\pi)^3} \left\{ -2 \sum_{i,j} (|A^r_{\nu^i{}^u_R \to \nu^j{}^u_L}|^2 - |\bar{A}^r_{\nu^i{}^u_R \to \nu^j{}^u_L}|^2) \right.$$
$$\left. + \sum_{i,j} (\frac{\sqrt{p_z^2 - M_i^2} - p_z}{p_z})(|A^t_{s^i{}^b_L \to \nu^j{}^u_L}|^2 - |\bar{A}^t_{s^i{}^b_L \to \nu^j{}^u_L}|^2) \right\} \frac{p_z^2}{E} f_F(p)[1 - f_F(p)], \tag{25}$$

where $f_F(p) = 1/(e^{|\vec{p}|/T} + 1)$ is the unboosted Fermi distribution.

In order to compute the amplitudes in eq. (20) we would need the exact propagator in the presence of the wall. The potential created by the wall in the weak basis (ν_L, s_L, ν_R) is

$$M(z) = \begin{pmatrix} 0 & 0 & V(z) K_H M \\ 0 & 0 & U(z) K_{SH} M \\ V(z) M K_H^\dagger & U(z) M K_{HS}^\dagger & 0 \end{pmatrix}, \tag{26}$$

where $V(z) \equiv v(z)/v$ is the ratio of the vevs of the doublet scalar H in the wall and the asymptotic vev in the broken phase; $U(z)$ is the ratio corresponding to the singlet σ field and $M = (M_i)$ is the diagonal mass matrix of the Dirac neutrinos.

The simplest approach would be to do perturbation theory in $M(z)$ [6, 14], which is effectively an expansion in $M(z)/\omega$. Although this approximation makes the calculation much simpler, it is not justified since the region of interest is always $\omega \sim M_i$. Instead, we will perturb in the mixing, that is in $K_H = \mathcal{O}(D/S)$ and $K_{HS} - 1 = \mathcal{O}[(D/S)^2]$. We can write the mass matrix as

$$M(z) = M_0(z) + \delta M(z), \tag{27}$$

with

$$M_0(z) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & U(z) M \\ 0 & U(z) M & 0 \end{pmatrix}, \quad \delta M(z) = M(z) - M_0(z). \tag{28}$$

Our strategy will be to solve the scattering problem with potential $M_0(z)$ exactly and perturb only in $\delta M(z)$. This can be done for several forms of $U(z)$. In order to simplify the problem we consider here the thin wall approximation for the singlet field i.e., $U(z) = \theta(z)$. The result will be more important for a singlet width as different from the doublet width as possible, so we will keep the singlet width to its minimum value and vary the doublet one. There is no reason to expect that any other choice would give very different results. The perturbation in δM then gives

$$S(x_2, x_1) = \int \prod_i dz_i \; S^{(0)}(x_2, z_1)\delta M(z_1) S^{(0)}(z_1, z_2)\delta M(z_2) \ldots S^{(0)}(z_i, x_1), \tag{29}$$

where the integration is done over all $z_i(-\infty, \infty)$ and $S^{(0)}$ is the exact propagator in the potential $M_0(z)$. It is a matrix with spin structure

$$S^{(0)} = \begin{pmatrix} S^{(0)}_{\nu_L} & 0 & 0 \\ 0 & S^{(0)}_{LL} & S^{(0)}_{LR} \\ 0 & S^{(0)}_{RL} & S^{(0)}_{RR} \end{pmatrix}. \tag{30}$$

We can then use the zero temperature propagator in the presence of a thin wall that has been computed in [25]. The formulae are given in appendix A.

We approximate the doublet field wall profile, $V(z)$, as

$$V(z) = \begin{cases} 0 & z < 0 \\ \delta_H^{-1} z & 0 < z < \delta_H \\ 1 & z > \delta_H \end{cases} \tag{31}$$

so that the wall thickness is parametrized by δ_H. We expect that this simple form is enough to give a reasonable estimate of the CP asymmetry.

The calculation is straightforward. In the case of three families, the result turns out to be non-zero at sixth order in (D/S) as expected from the invariant (13). The contribution coming from the reflection is

$$\sum_{i,j} (|A^r_{\nu^i{}^u_R \to \nu^j{}^u_L}|^2 - \text{antiparticles}) = \; J_{ijk} \; F^r(M_i, M_j, M_k), \tag{32}$$

and the transmission one

$$\sum_{i,j} (|A^t_{s^i{}^b_L \to \nu^j{}^u_L}|^2 - \text{antiparticles}) = \; J_{ijk} \; F^t(M_i, M_j, M_k), \tag{33}$$

where

$$\begin{aligned} J_{ijk} &= \mathrm{Im}[(K_H^\dagger K_H)_{ij}(K_H^\dagger K_H)_{ki}(K_H^\dagger K_H)_{jk}] \\ F^r(M_i, M_j, M_k) &= 2M_i^2 M_j^2 M_k^2 \{Im[A^*_{ik} A_{ij}] - 2\mathrm{Im}[I^{i*}_{1a} \frac{I^{ijk}_{5a}}{4p_j p_k})]\} \\ F^t(M_i, M_j, M_k) &= -M_i^2 M_j^2 M_k^2 \frac{|M_i + \omega - p_i|^2}{2p_i\,(\omega + M_i)} \mathrm{Im} \left\{ \frac{I^{ij*}_{3b}}{2p_j^*} \frac{I^{ik}_{3b}}{2p_k} + 2I^{i*}_{1b} \frac{I^{ijk}_{5b}}{4p_j p_k}) \right\}, \end{aligned} \tag{34}$$

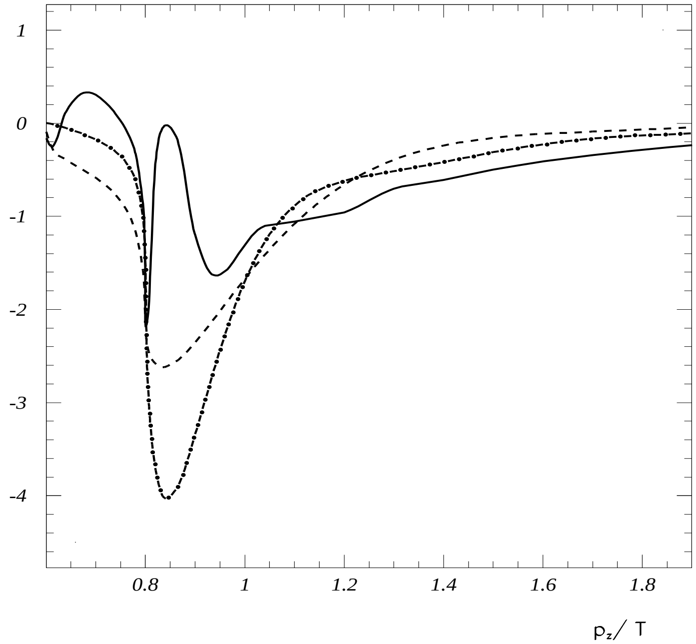

Figure 1: $\epsilon_{ijk}F^t(M_i, M_j, M_k)$ as a function of p_z for NHL masses $M_i = (.6, .8, 2.)$ and $\delta_H^{-1} = .1$ (solid), $\delta_H^{-1} = .2$ (dashed-dotted) and $\delta_H^{-1} = .3$ (dashed). All in units of the temperature.

and

$$A_{ij} \equiv \frac{1}{2p_j}(I_{3a}^{ij} - iI_{2a}^{ij}) \quad p_i \equiv \sqrt{\omega^2 - M_i^2}. \tag{35}$$

The integrals $I_{1a}, \ldots, I_{5a}$ are defined in appendix B.

It can be easily checked that whenever two masses are degenerate the result vanishes. We have also checked that in the thin wall approximation, i.e. $\delta_H \to 0$, the effect disappears, as it should happen since in this limit both the singlet and doublet wall profiles become the same.

As we saw in the previous section, for three generations there is only one CP-violating phase and $J_{ijk} = J_{123}\ \epsilon_{ijk}$. In this case, the phase J_{123} factorizes in the asymmetries of eqs. (32) and (33). We find that the contribution from the transmission amplitude is the dominant one, while the reflection amounts to a small correction. In figs. 1 and 2 we plot $\epsilon_{ijk}F^t(M_i, M_j, M_k)$ as a function of p_z, for different values of the NHL masses and δ_H. We will consider masses of order $\sim T$, because for heavier neutral leptons the current will be strongly suppressed by the Fermi distribution in eq. (25).

Figure 1 shows the dependence on δ_H for a fixed NHL spectrum. We find that the effect is more important when the mass differences of at least two NHLs are $(M_i - M_j) \leq \delta_H^{-1}$. When all mass differences are larger than δ_H^{-1}, the asymmetry oscillates rapidly (we expect the oscillation period to be related to δ_H) and the integrated result is suppressed. Also we observe that the effect is smaller for smaller δ_H, in agreement with the fact that it vanishes in the limit $\delta_H \to 0$. Thus we expect that the largest effect will occur for δ_H, satisfying $(M_i - M_j) \leq \delta_H^{-1}$, as large as possible.

In fig. 2 we fix δ_H and keep $M_2 - M_1 \leq \delta_H^{-1}$, while varying the masses M_1 and M_3. The peak appears after the second threshold (i.e. $p_z > M_2$); thus, as M_2 changes, the position of the peak moves accordingly. Nevertheless, the integrated result is not very sensitive to the particular values of the NHL masses, provided the relation $M_2 - M_1 \leq \delta_H^{-1}$ is satisfied.

The current j_{CP} we obtain for generic values of the masses, with the only restriction that $M_i \sim T$

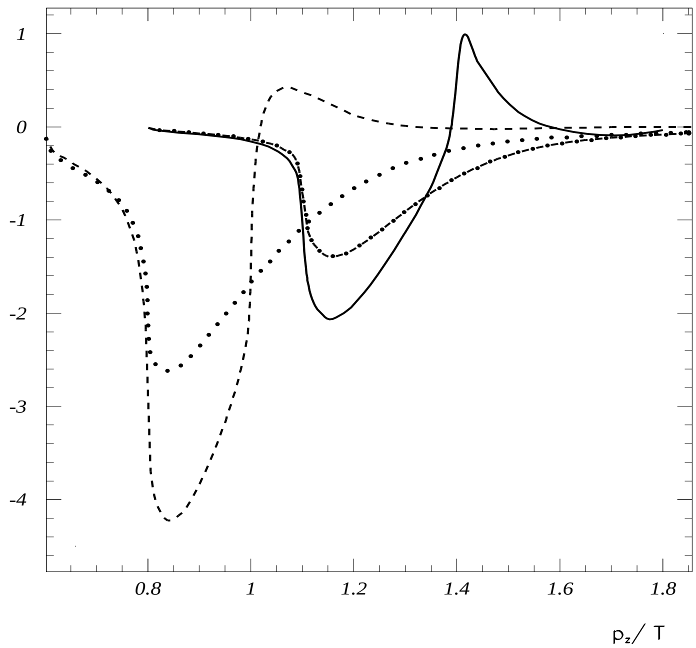

Figure 2: $\epsilon_{ijk}F^t(M_i, M_j, M_k)$ as a function of p_z for $\delta_H^{-1} = .3$ and NHL masses: i) (.8, 1.1, 1.4) solid line, ii) (.8, 1.1, 2.) dashed-dotted, iii) (.6, .8, 1.) dashed and iv) (.6, .8, 2.) dotted. All in units of the temperature.

and $M_i - M_j \sim \delta_H^{-1}$ for at least two NHL, is typically $(0.5\text{–}1) \times 10^{-2} J_{123}\ v_w$.

5 CP Asymmetry in the Charged Sector

In this section we compute the CP asymmetry in the flux of charged left-handed leptons, l_L, reflected in the unbroken phase. The charged lepton mass matrix has just an overall dependence on the doublet scalar vev (as occurs in the SM); therefore only mechanism a) as defined in section 3 is present in this case.

The calculation of the charged lepton CP asymmetry is completely analogous to the computation done for quarks in the SM [18, 5, 6], and we refer the reader to these works for further details. Contrary to the case of the NHL, the one-loop thermal corrections are much larger than the tree-level masses of the charged leptons. The resummation of the thermal self-energies considerably modifies the dispersion relations and the correct asymptotic states are now quasi-particles.

Following the notation of ref. [26], the thermal one-loop contribution to the charged lepton self-energy in the broken phase can be written as

$$\mathrm{Re}(\Sigma(k)) = -a\ \not{k} - b\ \not{u}, \tag{36}$$

where a and b are matrices in flavour space, u is the four-velocity of the plasma and $k = (\omega, \mathbf{k})$ is the external momentum. We have neglected the contribution proportional to the masses of the charged leptons. In the plasma rest frame and the mass basis,

$$\mathrm{Re}(\Sigma(\omega, \mathbf{k}))\gamma_0 = -h(\omega, \mathbf{k}) - a(\omega, \mathbf{k})\alpha \cdot \mathbf{k}, \tag{37}$$

where $h(\omega, \mathbf{k}) = a(\omega, \mathbf{k})\omega + b(\omega, \mathbf{k})$ and is given by

$$h_{ji} = -f_\gamma H(m_i, 0)\delta_{ji} - \frac{g^2}{2}\left\{[f_Z H(m_i, M_Z) + f_H H(m_i, M_H)]\delta_{ji} + \sum_\alpha f_{W,\alpha} H(M_\alpha, M_W)\right\}, \tag{38}$$

with

$$\begin{aligned} f_\gamma &= Q_i^2 g^2 s_W^2 (L+R) && (39)\\ f_{W,\alpha} &= \left(1 + \frac{\lambda_\alpha^2}{2}\right) L K_{i\alpha}^* K_{j\alpha} + \frac{\lambda_i \lambda_j}{2} R K_{i\alpha}^* K_{j\alpha} && (40)\\ f_Z &= \frac{1}{2}\left[\frac{4}{c_W^2}(T_i^3 - Q_i s_W^2)^2 + \frac{\lambda_i^2}{2}\right] L + \frac{1}{2}\left[\frac{4}{c_W^2}(Q_i s_W^2)^2 + \frac{\lambda_i^2}{2}\right] R && (41)\\ f_H &= \frac{\lambda_i^2}{4}(L+R) && (42) \end{aligned}$$

where L, R are the chiral projectors, $\lambda_i = m_i/M_W$, m_i are the masses of the external flavours, and M_α are the masses of the neutral leptons inside the loop. The function $H(M_F, M_B)$ can be found in [5].

The dispersion relations of the quasi-particles are then given by

$$\not{k} - \mathrm{Re}(\Sigma(k)) = 0. \tag{43}$$

Since these no longer are Lorentz invariant, it is not possible to simplify the calculation of the reflection amplitudes by boostings them to the frame where $k_x, k_y = 0$, as we did in section 4. The realistic computation in three dimensions then becomes very involved. However, since our main interest is to study whether the enhancement found in [18] is present, and this can already be seen in the one-dimensional problem, we restrict our discussion to this simpler case.

Our objective is to compute the number current of l_L reflected on the wall, which for small wall velocities and using unitarity and CPT, is given by

$$j_{CP} = -2\,\frac{v_w}{T}\int \frac{d\omega}{2\pi}\omega\; v_g^2\; f_F(w)[1 - f_F(w)]\Delta_{CP}(\omega), \tag{44}$$

where $v_g \equiv \frac{\partial\omega}{\partial k} = 1/3$ is the group velocity,

$$\Delta_{CP}(\omega) = \sum_{i,j}(|A^r_{l^i{}^u_R \to l^j{}^u_L}|^2 - |\bar{A}^r_{l^i{}^u_R \to l^j{}^u_L}|^2), \tag{45}$$

with A^r the reflection amplitudes on the wall, and $f_F(\omega)$ is the unboosted Fermi distribution of quasi-particles in the plasma rest frame:

$$f_F(\omega) = 1/(e^{\omega/T} + 1). \tag{46}$$

To lowest order in the wall velocity, the scattering problem can be approximately solved in the rest frame of the wall, neglecting the corrections to the dispersion relations of the quasi-particles due to the small boost (which are proportional to v_w and are negligible at lowest order). In this frame, to leading order in T and neglecting the flavour-non-diagonal corrections in (38), the quasi-particles propagate according to the effective Hamiltonian

$$H^0_{eff} = \theta(-z)\begin{pmatrix} -\frac{i}{3}\sigma_z\partial_z + \omega^u_R & 0 \\ 0 & \frac{i}{3}\sigma_z\partial_z + \omega^u_L \end{pmatrix} + \theta(z)\begin{pmatrix} -\frac{i}{3}\sigma_z\partial_z + \omega^b_R & \frac{m}{2} \\ \frac{m}{2} & \frac{i}{3}\sigma_z\partial_z + \omega^b_L \end{pmatrix}, \tag{47}$$

where $\omega_{L,R}^{u(b)}$, satisfying

$$\omega_{L,R}^{u(b)} + \bar{h}_{L,R}^{u(b)}(\omega_{L,R}^{u(b)}, 0) = 0, \tag{48}$$

are the thermal masses in the unbroken and broken phases respectively (the functions $\bar{h}$ contain only the leading T flavour-diagonal corrections in (38)). This effective Hamiltonian is only valid for low momentum compared to the thermal masses $\omega_{L,R}^{u(b)}$. Since the reflection of quasi-particles on the wall will occur for $k_z \leq m << \omega_{L,R}^{u(b)}$, this approximation is justified.

In order to obtain a non-vanishing CP asymmetry, both the subleading corrections in T (which introduce the dependence on the NHL masses) and the flavour-non-diagonal terms (which contain the mixings) in (38) are needed. After including these corrections, we get the following effective Hamiltonian:

$$H_{eff} = H_{eff}^0 + \frac{1}{2}\begin{pmatrix} \theta(-z)\delta h_R^u(\omega^0, 0) + \theta(z)\delta h_R^b(\omega^0, 0) & 0 \\ 0 & \theta(-z)\delta h_L^u(\omega^0, 0) + \theta(z)\delta h_L^b(\omega^0, 0) \end{pmatrix} \tag{49}$$

where $\delta h \equiv h - \bar{h}$, contain the subleading effects in T and the flavour-non-diagonal electroweak corrections. The reflection amplitudes of quasi-particles on the wall can then be obtained by first solving for eigenstates of the unperturbed Hamiltonian (47), which are superpositions of incoming, reflected and transmitted plane waves, and then perturbing in the extra terms of (49).

Up to now, we have neglected the imaginary part of the one-loop self-energy (36). This contribution is proportional to the damping rate of the quasi-particles, i.e. their inverse lifetime. There is no calculation of the damping rate γ of leptons in the SM, but from the result for pure $SU(2)$ at zero momentum [27], we can estimate $\gamma \sim \alpha_W T$, i.e. $\gamma \sim 1$ GeV at $T = 100$ GeV. In refs. [5, 6] it was shown that the damping effects for quarks in the SM lead to a sizeable suppression of the CP asymmetry, because the lifetime $\sim 1/(2\gamma)$ of the quasi-quarks in the plasma, was much smaller than their reflection time on the wall $\sim 1/m$ (for the down quarks, which gave the leading contribution). In the present case, we expect that the main effect will come from the reflection of the τ lepton and, according to the previous rough estimate, the lifetime of the quasi-tau would be of the same order of magnitude as its reflection time. In this situation, it is not clear whether the damping will have an important effect or not. We will first compute the asymmetry neglecting the damping completely and at the end of this section we will estimate its effect. As we will see, it leads to a suppression that varies rapidly with the exact value of γ around the region $\gamma \sim m_\tau$.

Similarly to the SM case, we find that the first effect in the asymmetry appears at $\mathcal{O}(\alpha_W^2)$. Defining $r_0 \equiv A_0^r$ (the unperturbed reflection amplitude), we get

$$\begin{aligned} \Delta_{CP}^{(2)}(\omega) \;\sim\; & \sum_{i,j} \mathrm{Im}[(\delta h_L^b)_{ji}(\delta h_R^b)_{ij}] \\ \times\; & \mathrm{Im}\left\{ r_{ii}^{0*}\left[\frac{r_{jj}^0}{|d_{ij}|^2} + \frac{m_j[(r_{ii}^0)^2 - (r_{jj}^0)^2]}{2d_{ii}d_{ij}d_{ji}} + \frac{r_{jj}^0}{d_{ii}}\left(\frac{1}{d_{ij}} + \frac{1}{d_{ji}}\right)\right]\right\}, \end{aligned} \tag{50}$$

where δh^b are flavour-dependent and $d_{ij} \equiv \omega_L^i + \omega_R^j - 2\omega + \frac{m_i r_{ii}^0}{2} + \frac{m_j r_{jj}^0}{2}$. From eq. (50) we see that, just as in the SM, the effect comes from the interference of the $\mathcal{O}(\alpha_W)$ terms in δh_L^b and δh_R^b, and there is no effect at leading order in T, because at this order $\delta h_R^b = 0$.

Substituting the expressions for $\delta h_{L,R}^b$, $\Delta_{CP}^{(2)}(\omega)$ can be written as

$$\Delta_{CP}^{(2)}(\omega) = \alpha_W^2 \sum_{i,j}\sum_{a<b} \mathrm{Im}(K_{Hia}K_{Hja}^* K_{Hib}^* K_{Hjb}) f(m_i, m_j) F(M_a, M_b), \tag{51}$$

where

$$f(m_i, m_j) = \lambda_i \lambda_j \mathrm{Im} \left\{ r_{ii}^{0*} \left[\frac{r_{jj}^0}{|d_{ij}|^2} + \frac{m_j[(r_{ii}^0)^2 - (r_{jj}^0)^2]}{2d_{ii}d_{ij}d_{ji}} + \frac{r_{jj}^0}{d_{ii}} \left(\frac{1}{d_{ij}} + \frac{1}{d_{ji}} \right) \right] \right\} \tag{52}$$

and

$$\begin{aligned} F(M_a, M_b) &= \Big[(2+\lambda_a^2)I(M_a) - 2I(0)\Big]_{\omega_L} [I(M_b) - I(0)]_{\omega_R} \\ &- \Big[(2+\lambda_b^2)I(M_b) - 2I(0)\Big]_{\omega_L} [(I(M_a) - I(0)]_{\omega_R}. \end{aligned} \tag{53}$$

$I(M_a) = \frac{\pi}{2} H(M_a, M_W)$ and the subscript $\omega_{L,R}$ indicates at which value of ω the function H is evaluated. Equation (51) shows explicitly the GIM cancellation for both external and internal lepton masses.

Of all the terms in eq. (51), the one corresponding to the pair of external flavours (μ, τ) gives the largest contribution, because $f(m_\mu, m_\tau)$ is a few orders of magnitude larger than for the other combinations, while the experimental bounds on the mixings are of the same order. We restrict to this leading term for which the 'joint' bound described in section 3 is

$$|\mathrm{Im}(K_{H2a} K_{H3a}^* K_{H2b}^* K_{H3b})| \leq 5 \times 10^{-5}, \tag{54}$$

independently of the flavour of the heavy leptons a, b. Thus, if we assume that all the mixings (54) are of the same order of magnitude, the size of the various terms in the sum over the heavy flavours depends only on the function $F(M_a, M_b)$. We consider just one of these terms as a prototype, i.e.

$$\Delta_{CP}^{23ab} = \alpha_W^2 \mathrm{Im}(K_{H2a} K_{H3a}^* K_{H2b}^* K_{H3b}) f(m_2, m_3) F(M_a, M_b), \tag{55}$$

where there is no sum over a, b. Since we will allow the two heavy masses M_a, M_b to vary arbitrarily, the largest value obtained for the integrated asymmetry, considering only (55), is also an upper bound for the other terms. Thus the final result will be at most three times larger, if the terms add coherently.

In fig. 3 we show the contribution to the CP asymmetry, $\Delta_{CP}^{23ab}(\omega)$. We have taken the following values for the masses at the phase transition temperature ($T \sim 100$ GeV): $m_\mu = 69$ MeV, $m_\tau = 1.176$ GeV, $M_W = 50$ GeV, and the weak coupling is $\alpha_W = 0.035$. We have fixed the mass of one NHL to $M_a = 80$ GeV, and we plot the result for different values of the other NHL mass.

The peaks are situated in regions where the τ lepton reflects completely, while the μ does not. The amplitude of the peaks is larger than one would expect from naive power counting, implying that the suppression in the charged lepton masses is not at work, as found in [18]. However, in contrast to what was obtained in the SM, the two peaks tend to cancel each other, and there is a big suppression in the integrated result, since the Fermi factors in eq. (44) are approximately constant. For $M_b = 140$ GeV the contribution of this term to the integrated result in (44) is $\sim 10^{-12} v_w$ (which turns out to be of the same order as the naive estimate). Whether the peaks come with equal or opposite signs seems to be very dependent on the relative position of the thermal masses of the different flavours. In this case the thermal masses are almost flavour-independent, while in the down sector of the SM there is a big shift in the third family thermal masses compared to the other two, due to the top Yukawa. This is why there is no such cancellation in that case. The conclusion is that the enhancement found in [18] is rather model-dependent and it seems to require large flavour-dependent thermal corrections.

Finally, we want to estimate the effect of the damping rate which, as discussed before, is not negligible compared to the reflection time of the τ lepton. As shown in refs. [5, 6], the decoherence effects induced by the damping rate can be taken into account by including the imaginary part of the

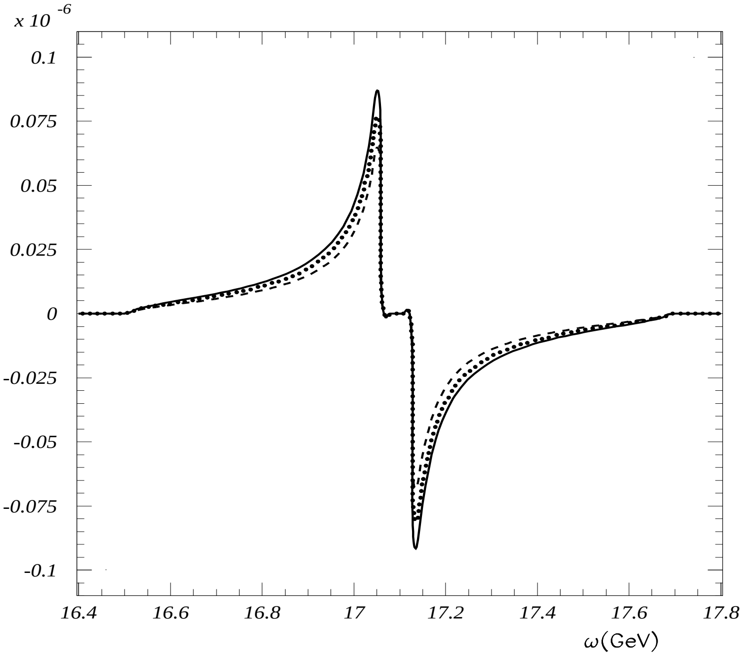

Figure 3: CP asymmetry, $\Delta_{CP}^{23ab}(\omega)$, for zero damping rate and NHL masses $M_a = 80$ GeV and $M_b = 110$ GeV (dashed), 140 GeV (solid) and 200 GeV (dotted).

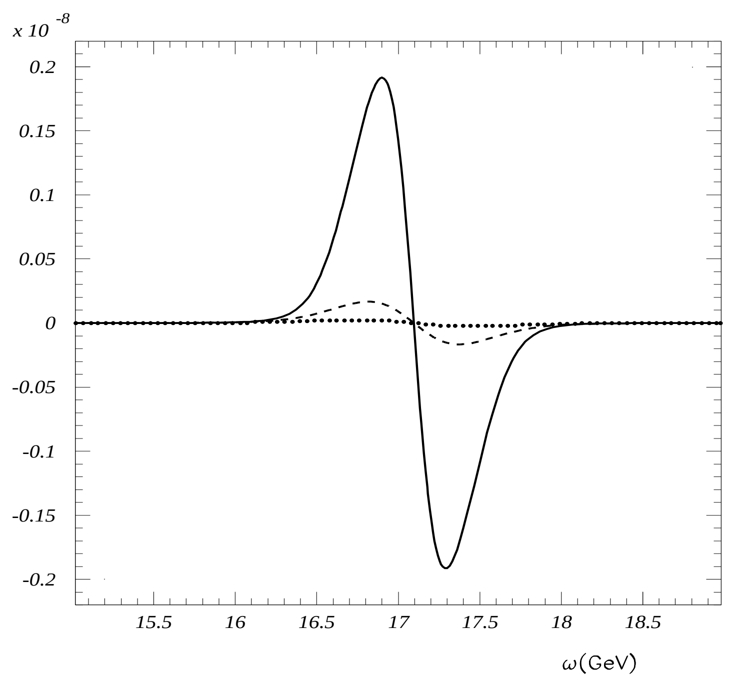

Figure 4: $\Delta_{CP}^{23ab}(\omega)$ for $M_a = 80$ GeV, $M_b = 140$ GeV and damping rate $\gamma = .5$ GeV (solid), 1. GeV (dashed) and 1.5 GeV (dotted).

self-energy in the effective Hamiltonian (47) and solving it for spatially damped waves. Since the exact value of the damping rate is not known, we have computed $\Delta_{CP}^{23ab}(\omega)$ for different values of γ, namely $\gamma = 0.5$, 1., and 1.5 GeV. The result is shown in fig. 4. As is clear from the curves, the suppression due to the damping increases rapidly when $\gamma \sim m_\tau$. Without a precise determination of this quantity, it is thus impossible to estimate the actual suppression, although it is clear that neglecting this effect is not justified.

To summarize, we have found that the CP asymmetry in the reflection of the charged leptons is at most of $\mathcal{O}(10^{-12})$, even neglecting decoherence effects due to interactions in the plasma.

6 Baryon Asymmetry

In this section, we calculate the baryon asymmetry induced by the CP asymmetries computed in the previous sections. This is a very difficult problem since a microscopic treatment is no longer possible and we have to match somehow the microscopic result of j_{CP} with the thermodynamic treatment of transport of the chiral lepton number generated at the wall. Strictly speaking, the two problems, reflection and transport, are completely coupled and should be solved at the same time. This however implies treating a many-body non-equilibrium quantum system, and some approximations are necessary.

We will consider here only the effect obtained from the reflection of the neutral leptons, since the CP asymmetry of the charged leptons computed in the last section is far too small. In the neutral sector, we have completely neglected the thermal incoherence effects in the reflection. This has been shown to be a very bad approximation when the damping rate is comparable to both the height and/or width of the wall [5][6][14]. However, this is not the case here as we discussed in section 3. We believe that, because of this, reflection can be treated independently of transport [3].

The picture is then that near the wall in the unbroken phase, a local density of ν_L lepton number is generated due to reflection. This local density generates a diffusion current in the plasma and decays due to sphaleron processes that take place in the unbroken phase as we go away from the wall, generating a baryon number density. This picture is only consistent if the reflected particles have enough time to diffuse before the wall catches up. This will be true for small velocities of the wall. In this case also the incoming flux of particles in the calculation of j_{CP} can be taken to be the thermal one, as we assumed in the previous sections.

The diffusion equations in the wall rest frame read

$$\begin{pmatrix} \partial_t \, n_B \\ \partial_t \, n_L \end{pmatrix} = \begin{pmatrix} D_B \partial_z^2 - v_w \partial_z - 3\Gamma\theta(-z) & -\Gamma\theta(-z) \\ -3\Gamma\theta(-z) & D_L \partial_z^2 - v_w \partial_z - \Gamma\theta(-z) \end{pmatrix} \begin{pmatrix} n_B \\ n_L \end{pmatrix} \tag{56}$$

where $\Gamma \equiv 9\frac{\Gamma_{WS}}{T^3}$ and $\Gamma_{WS} = \kappa(\alpha_W T)^4$ is the weak sphaleron rate with κ a coefficient of $\mathcal{O}(1)$ [28] [4]. We have made the further approximation that the sphaleron rate in the broken phase is zero. This drastic approximation can only be justified if the phase transition is strongly first order; v_w is the velocity of the wall. The constants $D_{L,B}$ are the diffusion coefficients for leptons and quarks respectively. Since quarks suffer strong interactions, it is clear that

$$D_B \ll D_L. \tag{57}$$

[3] If the damping rate is not small compared to other scales in the problem, we do not think one can separate the problems of reflection and transport, and a detailed calculation is much more complicated.

[4] However, there is a recent claim that damping effects in the plasma suppress the sphaleron rate to $\mathcal{O}(\alpha_W^5 T^4)$ [29].

We take the values of the diffusion constants estimated in the SM in ref. [10], namely $D_L \sim 110/T$ and $D_B \sim 6/T$. These estimates are obtained from the elastic scattering, t-channel vector boson exchange diagrams, which are expected to dominate the scattering process. Yukawa interactions are neglected. We have not included here any other possible ν_L number decay process than the sphalerons. Other decays through Higgs interactions are obviously possible, but we find that their rate is smaller than the sphaleron rate in the unbroken phase, so we can safely neglect them.

We look for stationary solutions, i.e. $\partial_t n_{B,L} = 0$. In order to solve the equations for $n_{L,B}$ (56), we need to impose boundary conditions on the densities and their derivatives (diffusion currents). We will require that $n_{L,B}(-\infty) = 0$, since there is no asymmetry in the incoming thermalized flux seen by the wall. At $z \to \infty$, we require that the solutions be constants. These would be precisely the values of L and B in the broken phase that will survive the phase transition. At the interphase $z = 0$, we impose continuity of the diffusion current,

$$D_{L,B}\partial_z n_{L,B} - v_w n_{L,B} \,|_-^+ \; = \; 0 \tag{58}$$

and the existence of the reflected flux is taken into account in a constraint on the lepton density in the unbroken phase near the wall,

$$n_L|_{z=0^-} \; = \; n^0 \tag{59}$$

where $n^0 = j_{CP}/\langle v_i \rangle$ and $\langle v_i \rangle$ is the average velocity of the particles in the reflected flux that we define as,

$$\langle v_i \rangle \equiv \frac{\int \frac{d^3p}{(2\pi)^3} \sum_{i,j} J_{ijk}\{ -2F^r(M_i,M_j,M_k) + (\frac{\sqrt{p_z^2-M_i^2}-p_z}{p_z})F^t(M_i,M_j,M_k) \}\frac{p_z^3}{E^2} f_F(p)[1-f_F(p)]}{\int \frac{d^3p}{(2\pi)^3} \sum_{i,j} J_{ijk}\{ -2F^r(M_i,M_j,M_k) + (\frac{\sqrt{p_z^2-M_i^2}-p_z}{p_z})F^t(M_i,M_j,M_k) \}\frac{p_z^2}{E} f_F(p)[1-f_F(p)]}$$

It is straightforward to obtain the most general solutions [18] of (56) in the approximation,

$$\frac{3D_{B,L}\Gamma}{v_w^2} \ll 1, \tag{60}$$

which is expected to be of $\mathcal{O}(10^{-1})$. The solution is

$$\begin{aligned} n_B &= C_1 a_{11} e^{k_1 z} + C_2 a_{21} e^{k_2 z} & \\ n_L &= C_1 a_{12} e^{k_1 z} + C_2 a_{22} e^{k_2 z} & z < 0 \\ n_B &= B & \\ n_L &= L & z > 0 \end{aligned}$$

with

$$\begin{aligned} k_1 &\equiv \frac{v_w}{D_B}\left(1 + \frac{3\Gamma D_B}{v_w^2}\right) \\ k_2 &\equiv \frac{v_w}{D_L}\left(1 + \frac{\Gamma D_L}{v_w^2}\right) \end{aligned} \tag{61}$$

and

$$\begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{3\Gamma D_B}{v_w^2(D_L/D_B-1)} \end{pmatrix} \quad , \begin{pmatrix} a_{21} \\ a_{22} \end{pmatrix} = \begin{pmatrix} \frac{\Gamma D_L}{v_w^2(D_B/D_L-1)} \\ 1 \end{pmatrix} . \tag{62}$$

Now, the constants $C_{1,2}$ and B, L can be determined from (58) and (59). In the limit $D_B \ll D_L$ the result for B is

$$B = \frac{\Gamma D_L}{v_w^2} n^0. \tag{63}$$

Although the dependence on the wall velocity seems to have a singular limit when $v_w \to 0$ (there is only one power of the wall velocity in n^0), this is only because we have made the approximation that the ratio $\frac{\Gamma D_L}{v_w^2} \ll 1$. In the limit $v_w \to 0$, this approximation is obviously not valid and indeed the solution of the diffusion equations in this case gives $B = 0$, as expected.

Finally, in order to compare this result with the experimental one $B/s \sim (4\text{–}6) \times 10^{-11}$, we need to divide by the entropy at the temperature of the phase transition, $s = \frac{2\pi^2}{45} g_* T^3$, where $g_* \sim 110$ counts the degrees of freedom of the relativistic particles at the electroweak phase transition. Putting everything together we find generically a effect of the order of

$$B/s \sim \frac{\Gamma D_L}{v_w} \; J_{123} \times 10^{-4}. \tag{64}$$

If we assume $v(T)/u(T) = v(0)/u(0)$, we can use the experimental bound $J_{123} < 10^{-7}$ (16). Considering the values quoted in the literature for the ratio $\Gamma D_L/v_w \sim 10^{-2}$ (within the SM) [10] [24] [28], we get a baryon to entropy ratio two orders of magnitude smaller than required. However, the bounds on J_{123} only hold if the ratio of the scalar vevs does not vary with the temperature, which is not necessarily true. For instance, a variation by a factor of 2 in the right direction (i.e. a larger ratio at T), increases the result by two orders of magnitude. This is because the CP asymmetry goes like $O(D/S)^6$ which, up to Yukawa couplings, is $\sim O(v(T)/u(T))^6$. An enhancement due to this effect has been suggested in the context of two-Higgs models in [10]. In order to establish whether this enhancement could take place, a detailed study of the scalar potential is required, which is beyond the scope of this paper.

7 Conclusions

We have considered the possibility that baryogenesis occurs during the weak phase transition in a minimal extension of the Standard Model, which contains extra neutral leptons and conserves total lepton number. The leading CP asymmetries come from the reflection of both neutral and charged leptons on the bubble wall. Due to the large mean free path of the leptons as compared to the typical values of the wall thickness, the calculation is done in the thin wall regime. The CP-violating phases come from two sources. For the NHL there are unremovable CP phases due to the non-trivial space dependence of the mass matrix inside the bubble wall. The effect turns out to be tree level and in agreement with naive estimates. It is only suppressed by the mixing angles. For the charged leptons there is no tree-level contribution and the CP-violating phases appear in the one-loop thermal corrections to the lepton propagation in the plasma. The naive estimate gives a suppression in the charged lepton masses and in loop factors $(1/4\pi)$, besides that in the mixing angles. The result of [18] suggests that the suppression on the charged lepton masses could be absent; however, we find agreement with the naive estimate. We argued that the effect found in [18] requires a large flavour dependence of the leading T thermal corrections, which is not the case in this type of models.

Using the present constraints on the mixing angles, we obtain that the leading effect comes from the neutral sector and gives $B/s \leq \frac{\Gamma D_L}{v_w} \, 10^{-11}$. Assuming SM estimates for the lepton diffusion constant

D_L, the wall velocity v_w and the sphaleron rate Γ, we get $B/s \leq 10^{-13}$, which even though the errors involved are very large, seems too small to account for the observed baryon asymmetry. However, the constraints on the mixing angles apply only if the ratio of the scalar vevs at the temperature of the phase transition is the same as today, which is not necessarily true. It would interesting to study a realistic scalar potential to determine whether this possibility is realized.

Finally, we want to comment on other scenarios where the baryon asymmetry is also generated at the electroweak phase transition, through lepton reflection on the bubble wall. In ref. [13] the singlet majoron model was considered. The CP asymmetry in that case was also due to the reflection of neutrinos. However, the relevant phase space was around the mass of the τ-neutrino $\sim O(10 \text{ MeV})$. Although the asymmetry obtained was roughly of the correct order of magnitude, we think that thermal corrections to the dispersion relation of the ν_τ in the plasma, which were neglected in [13], should be taken into account. In particular, from the calculation of the damping rate of neutrinos in this model [30], it is clear that the typical reflection time of the light neutrinos is much larger than the lifetime of the quasi-particles in the plasma. In this situation, we expect a considerable suppression in the CP asymmetry. In refs. [10][12], the reflection of τ leptons was considered as the leading contribution to the baryon asymmetry in the two-Higgs model, in the thin wall regime. The effects of the damping rate have also been neglected in this case. The results for the charged lepton contribution to the asymmetry in the present work show that this effect could be important.

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Appendix A.

In the calculation of the CP asymmetry in the neutral sector (section 4) we have used the exact propagator in the presence of a wall in position space. We give here the expression for $S^{(0)}$ (eq. (30)) in the boosted frame, $p_x = p_y = 0$:

$$S^{(0)} = S^{(0)}_{left} + S^{(0)}_{across} + S^{(0)}_{right} \tag{A. 1}$$

with

$$\begin{aligned} S^{(0)}_{left}(z_2, z_1)\gamma^0 &= -\Theta(-z_1)\Big\{\Theta(z_2 - z_1)\Theta(-z_2)e^{iE(z_2-z_1)}\frac{1+\alpha_z}{2} \\ &+ \Theta(z_1 - z_2)e^{-iE(z_2-z_1)}\frac{1-\alpha_z}{2} + \Theta(-z_2)e^{-iE(z_2+z_1)}\frac{1-\alpha_z}{2}\frac{m\gamma^0}{E+p'}\Big\} \end{aligned} \tag{A. 2}$$

$$S^{(0)}_{across}(z_2,z_1)\gamma^0 = -\Theta(-z_1)\Theta(z_2)e^{-iEz_1}e^{ip'z_2}\left(1+\frac{m\gamma^0}{E+p'}\right)\frac{1+\alpha_z}{2} -\Theta(z_1)\Theta(-z_2)e^{ip'z_1}e^{-iEz_2}\frac{1-\alpha_z}{2}\left(1+\frac{m\gamma^0}{E+p'}\right) \quad \text{(A. 3)}$$

$$S^{(0)}_{right}(z_2,z_1)\gamma^0 = \Theta(z_1)\left\{-\Theta(z_2-z_1)e^{ip'(z_2-z_1)}\frac{1}{2}\left(\frac{E}{p'}+\alpha_z+\frac{m}{p'}\gamma^0\right) - \Theta(z_1-z_2)\Theta(z_2)e^{-ip'(z_2-z_1)}\frac{1}{2}\left(\frac{E}{p'}-\alpha_z+\frac{m}{p'}\gamma^0\right) + \Theta(z_2)e^{ip'(z_2+z_1)}\frac{1}{2}\left(\frac{E}{p'}+\alpha_z+\frac{m}{p'}\gamma^0\right)\frac{m\gamma^0}{p+p'}\right\} \quad \text{(A. 4)}$$

and $p'=\sqrt{E^2-m^2}$.

The propagator in position space for the massless left-handed neutrino is

$$S^{(0)}_{\nu_L}(z_2,z_1) = \Theta(z_2-z_1)e^{iE(z_2-z_1)}. \quad \text{(A. 5)}$$

Appendix B.

$$I^i_{1a} = \int_0^\infty dz V(z)e^{i(E+p_i)z} = \frac{1}{\delta_H(E+p_i)^2}(e^{i(E+p_i)\delta_H}-1) \quad \text{(B. 1)}$$

$$I^{ij}_{2a} = \int_0^\infty dz_1\int_0^\infty dz_2 V(z_1)e^{iEz_1}e^{ip_iz_2}\left\{\Theta(z_1-z_2)e^{ip_j(z_1-z_2)}+\Theta(z_2-z_1)e^{ip_j(z_2-z_1)}-e^{ip_j(z_1+z_2)}\right\} = i\frac{2p_j}{M_i^2-M_j^2}[I^i_{1a}-I^j_{1a}] \quad \text{(B. 2)}$$

$$I^{ij}_{3a} = \int_0^\infty dz_1\int_0^\infty dz_2\int_{z_2}^\infty dz_3 V(z_1)V(z_2)V(z_3)e^{iE(z_1-z_2+z_3)}e^{ip_iz_3} \left\{(E+p_j)\Theta(z_1-z_2)e^{ip_j(z_1-z_2)}+(E-p_j)[\Theta(z_2-z_1)e^{ip_j(z_2-z_1)}-e^{ip_j(z_1+z_2)}]\right\} \quad \text{(B. 3)}$$

$$I^{ijk}_{5a} = \int_0^\infty dz_1\int_0^\infty dz_2\int_{z_2}^\infty dz_3\int_0^\infty dz_4\int_{z_4}^\infty dz_5 V(z_1)\dots V(z_5)e^{iE(z_1-z_2+z_3-z_4+z_5)}e^{ip_iz_5} \left\{(E+p_j)\Theta(z_1-z_2)e^{ip_j(z_1-z_2)}+(E-p_j)[\Theta(z_2-z_1)e^{ip_j(z_2-z_1)}-e^{ip_j(z_1+z_2)}]\right\} \left\{(E+p_k)\Theta(z_3-z_4)e^{ip_k(z_3-z_4)}+(E-p_k)[\Theta(z_4-z_3)e^{ip_k(z_4-z_3)}-e^{ip_k(z_3+z_4)}]\right\} \quad \text{(B. 4)}$$

$$I^i_{1b} = I^i_{1a}(-p_i)-I^i_{1a}(p_i) \quad \text{(B. 5)}$$
$$I^{ij}_{3b} = I^{ij}_{3a}(-p_i)-I^{ij}_{3a}(p_i) \quad \text{(B. 6)}$$
$$I^{ijk}_{5b} = I^{ijk}_{5a}(-p_i)-I^{ijk}_{5a}(p_i). \quad \text{(B. 7)}$$

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