ATLAS sensitivity range for the x_s measurement

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Abstract

Previous results for the prospects of B_s^0 mixing measurement in the ATLAS experiment at LHC are updated. The improved analysis method of the studied decay channels $B_s^0 \to D_s^- \pi^+$ and $B_s^0 \to D_s^- a_1^+$, combined with most recent values for the branching ratios and the B_s^0 lifetime, leads to the new ATLAS sensitivity range for the x_s measurement: $x_s^{max} = 42$. An extensive study is done in order to estimate how x_s^{max} is influenced by the B-decay proper-time resolution of the vertex detector, as well as by the number of events and by the signal-to-background ratio.

1   Measurement of x_s

CP-violation in B-meson decays can be characterized by the unitarity triangle, composed of Cabibbo-Kobayashi-Maskawa matrix elements by requiring that the matrix is unitary. In addition to the angles of the unitarity triangle, measurements of the lengths of the sides provide us with complementary information about the triangle. The length of the side $|V_{td}|$ is least well-known of the sides. The mixing parameter x_d is proportional to $|V_{td}|^2$, but inferring the value of $|V_{td}|^2$ from x_d is hampered by hadronic uncertainties. To large extent, these uncertainties cancel, when considering the ratio of the mixing parameters, $x_s/x_d \propto |V_{ts}/V_{td}|^2$.

The mixing parameter x_s has not been measured yet. The Standard Model predicts it to be in the range 10-30, and the present lower limit from LEP is $x_s > 11.1$ [1].

Principles of the x_s measurement are described in previous notes [2, 3]. A time-dependent asymmetry $A(t)$, observed in the experiment, is defined as:

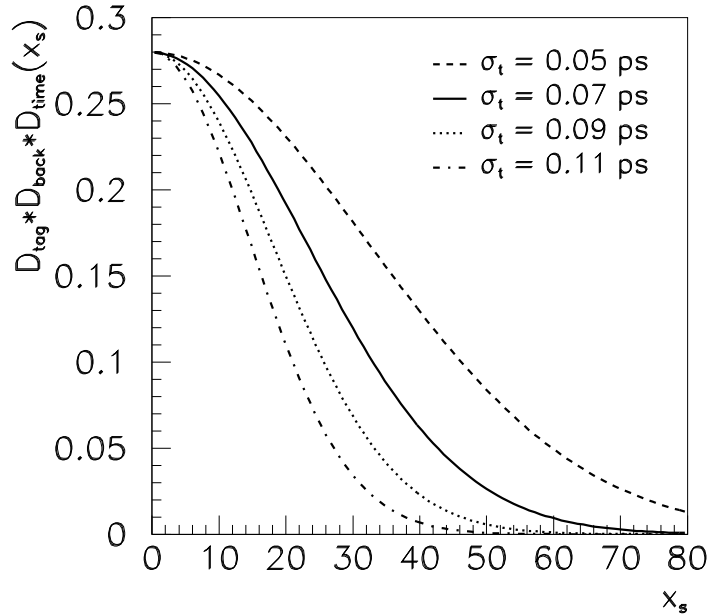$$A(t) = \frac{dn(++)/dt - dn(+-)/dt}{dn(++)/dt + dn(+-)/dt} = D\cos(x_s t/\tau_{B_s}) \qquad (1)$$

Figure 1: Expected amplitude of the asymmetry $D = D_{tag}D_{back}D_{time}(x_s)$ as a function of x_s for various proper time resolutions σ_t.

where $(++)$ refers to B-decays in which the flavour of the B appears to be the same in the production and decay, $(+-)$ refers to cases where an oscillation of the B into its anti-particle is observed, and t is the proper time. D is a product of all the dilution factors:

$$D = D_{tag}D_{back}D_{time} \tag{2}$$

$$D_{tag} = 1 - 2W \tag{3}$$

$$D_{back} = \frac{N_S}{N_S + N_B} \tag{4}$$

$$D_{time} = \exp\left(-\frac{1}{2}\left(\frac{\sigma_t x_s}{\tau}\right)^2\right) \tag{5}$$

where W is the wrong tag fraction, N_S is the number of signal events, N_B is the number of background events[1], σ_t is the proper time resolution and τ is the B_s^0 lifetime. Formula 5 can be obtained using Fourier transforms [4].

In case of ATLAS $W = 0.22$ [5] and $N_B = N_S$ is assumed (section 2.1). The lifetime of the B_s^0 is assumed to be $\tau = 1.61$ ps after [6]. Figure 1 shows the total dilution factor D estimated with the above assumptions for different values of the proper-time resolution ($\sigma_t = 0.05$, 0.07, 0.09 and 0.11 ps), as a function of x_s. The amplitude of the cosine wave which can be observed by the experiment is reduced

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[1] It is assumed for the formula 1 that background $dN_B/dt \sim exp(-t/\tau)$, with no asymmetry.

with increasing x_s because of the finite σ_t, as given by the formula 5. One can expect that for any given statistical sensitivity of the experiment, defined by the available signal statistics and by the background, there is a minimal value of the asymmetry amplitude D which still enables a reliable measurement of x_s. Therefore, from equation 5, one expects

$$x_s^{max} \sim \frac{1}{\sigma_t} \tag{6}$$

for any objective definition of x_s^{max}.

2 Definition of the ATLAS range

2.1 Input parameters

In order to estimate the sensitivity of the experiment for the x_s measurement, we use as input parameters the number of signal events N_S, the number of background events N_B, and the resolution in the proper-time of the B_s^0 decay, σ_t.

Two of the B_s^0 decay channels which can be used for the x_s measurement have been analyzed so far: $B_s^0 \to D_s^- \pi^+$ and $B_s^0 \to D_s^- a_1^+$. The original work is described in [2] and in [3], respectively. The results have been updated, for the Technical Proposal and on a few other occasions, because of minor changes of the cuts, updated branching ratios or trigger efficiencies, in particular the second level trigger sensitive to $D_s^\pm$. It is therefore useful to summarize the event counting as it was done for this work. The calculation is explained in tables 1 and 2.

The number of background events for $B_s^0 \to D_s^- \pi^+$ was also estimated in [2]. First some potentially dangerous exclusive decays ($B_d^0 \to D_s^- \pi^+$, $B_d^0 \to D^- \pi^+$ and $\Lambda_b \to \Lambda_c^+ \pi^-$ followed by $\Lambda_c^+ \to pK^- \pi^+$) were checked and found not to contribute significantly to the background. The combinatorial background can not be precisely estimated because of the very large Monte Carlo statistics required. With the set of cuts shown in table 1, no candidate event was found in the mass window $5 < M_{KK\pi\pi} < 6$ GeV within 370,000 inclusive μX events. That gives an upper limit on the level of $N_B \leq \sim 0.6 \cdot N_S$ within the final mass cut of $\pm 2\sigma$.

A similar procedure was carried out in [3] for the channel $B_s^0 \to D_s^- a_1^+$. The channels $B_d^0 \to D^- a_1^+$, $\Lambda_b \to \Lambda_c^+ \pi^-$ followed by $\Lambda_c^+ \to pK^- \pi^+ \pi^+ \pi^-$ and $B_d^0 \to D_s^- a_1^+$ were found not to contribute significantly, as compared to the limit on combinatorial background, which was $N_B \leq \sim N_S$. The latter was based on the Monte-Carlo sample of 300,000 inclusive μX events, where no event has passed cuts from table 2 in the mass window $M_{B_s^0} - 150$ MeV/c$^2 < M_{KK\pi\pi\pi} < M_{B_s^0} + 150$ MeV/c^2.

Adding the contribution from the two channels, the total number of signal events is $N_S = 4480$ for one year of low luminosity data taking ($\int Ldt = 10^4$ pb^{-1}). A conservative value of $N_B = N_S$ was used in the following. A dedicated study of the impact of background on x_s^{max} was also done, and is described in section 4.

The proper-time resolution σ_t was calculated in [2] using full GEANT simulation of the ATLAS tracker equipped with the strip 'B-physics' layer. The result was $\sigma_t = 0.069$ ps. A slightly better value $\sigma_t = 0.064$ ps was obtained with track level simulation for the second decay channel in [3]. There are many other track-level

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Table 1: Number of signal events from $B_s^0 \to D_s^- \pi^+$ channel expected in ATLAS after 1 year (10^7 s) of operation at 10^{33} cm^{-2}s^{-1}

Parameter	Value	Comment
$\mathcal{L}$ [cm^{-2}s^{-1}]	10^{33}	
t [s]	10^7	
$\sigma(b\bar{b} \to \mu X)$ [μb]	2.3	$p_T^\mu > 6$ GeV/c
		$|\eta^\mu| < 2.2$
$N(b\bar{b} \to \mu X)$	$2.3 \cdot 10^{10}$	
Br(b $\to$ B$_s^0$)	0.112	
Br(B$_s^0 \to$ D$_s^- \pi^+$)	0.003	
Br(D$_s^- \to \phi^0 \pi^-$)	0.036	
Br($\phi^0 \to$ K$^+$K$^-$)	0.491	
$N($K$^+$K$^-\pi^+\pi^-)$	136,600	
Acceptance of the cuts:		
$p_T > 1$ GeV/c		
$|\eta| < 2.0$		
$\Delta\varphi_{KK} < 10^o$		
$\Delta\Theta_{KK} < 10^o$		
$|M_{KK} - M_{\phi^0}| < 10$ MeV/c^2		
$|M_{KK\pi} - M_{D_s^-}| < 15$ MeV/c^2		
D$_s^-$ vertex fit $\chi^2 < 10.0$		
B$_s^0$ vertex fit $\chi^2 < 5.0$		
B$_s^0$ proper decay time $> +0.4$ ps		
B$_s^0$ impact parameter $< 55 \mu$m		
B$_s^0$ $p_T > 10.0$ GeV/c	8%	
$N($K$^+$K$^-\pi^+\pi^-)$ after cuts	10,900	
Trigger efficiency	0.54	
Lepton identification	0.8	
Track efficiency	$(0.95)^4$	
Mass cut $\pm 2\sigma$	0.95	
$N($K$^+$K$^-\pi^+\pi^-)$ reconstructed	3,640	

results, which were produced later, and which are in a good agreement with the latter value.

Replacing the strip 'B-physics' layer at $R = 6$ cm with a third pixel layer at $R = 4$ cm gives slightly worse performance, $\sigma_t = 0.068$ to 0.094 ps, depending on the resolution assumed for pixels [7]. Removal of the 'B-physics' layer, with the first tracking layer made of pixels at $R = 11$ cm, results in $\sigma_t = 0.146$ to 0.154 ps.

A value of $\sigma_t = 0.07$ ps, well justified by the result from [2], which is the only one so far using full simulation, is assumed in this section. In section 3 the work is repeated for values of σ_t ranging from 0.05 to 0.011 ps.

A parameter of importance which was also updated for this work is the B$_s^0$ lifetime. An up to date value $\tau = 1.61$ ps used here is higher than in case of previous

Table 2: Number of signal events from $B_s^0 \to D_s^- a_1^+$ channel expected in ATLAS after 1 year (10^7 s) of operation at 10^{33} cm^{-2}s^{-1}

Parameter	Value	Comment
$\mathcal{L}$ [cm^{-2}s^{-1}]	10^{33}	
t [s]	10^7	
$\sigma(b\bar{b} \to \mu X)$ [μb]	2.3	$p_T^\mu > 6$ GeV/c
		$|\eta^\mu| < 2.2$
$N(b\bar{b} \to \mu X)$	$2.3 \cdot 10^{10}$	
Br(b $\to B_s^0$)	0.112	
Br($B_s^0 \to D_s^- a_1^+$)	0.006	
Br($D_s^- \to \phi^0 \pi^-$)	0.036	
Br($\phi^0 \to K^+ K^-$)	0.491	
Br($a_1^+ \to \rho^0 \pi^+$)	~ 0.5	
Br($\rho^0 \to \pi^+ \pi^-$)	~ 1	
$N(K^+ K^- \pi^+ \pi^- \pi^+ \pi^-)$	136,600	
Acceptance of the cuts:		
$p_T > 1$ GeV/c		
$|\eta| < 2.5$		
$N(K^+ K^- \pi^+ \pi^- \pi^+ \pi^-)$	9,015	6.6%
$\Delta\varphi_{KK} < 10^o$		
$\Delta\Theta_{KK} < 10^o$		
$|M_{KK} - M_{\phi^0}| < 20$ MeV/c^2		
$|M_{KK\pi} - M_{D_s^-}| < 15$ MeV/c^2		
$\Delta\varphi_{\pi\pi} < 35^o$		
$\Delta\Theta_{\pi\pi} < 15^o$		
$|M_{\pi\pi} - M_{\rho^0}| < 192$ MeV/c^2		
$|M_{\pi\pi\pi} - M_{a_1^+}| < 300$ MeV/c^2		
$N(K^+ K^- \pi^+ \pi^- \pi^+ \pi^-)$	6,830	5.0%
D_s^- vertex fit $\chi^2 < 12.0$		
a_1^+ vertex fit $\chi^2 < 12.0$		
B_s^0 proper decay time $> +0.4$ ps		
B_s^0 impact parameter $< 55\mu$m		
B_s^0 $p_T > 10.0$ GeV/c	8%	
$N(K^+ K^- \pi^+ \pi^- \pi^+ \pi^-)$ after cuts	4,100	3.0%
Trigger efficiency	0.54	
Lepton identification	0.8	
Track efficiency	$(0.95)^6$	
Mass cut $\pm 2\sigma$	0.95	
$N(K^+ K^- \pi^+ \pi^- \pi^+ \pi^-)$ reconstructed	1,240	

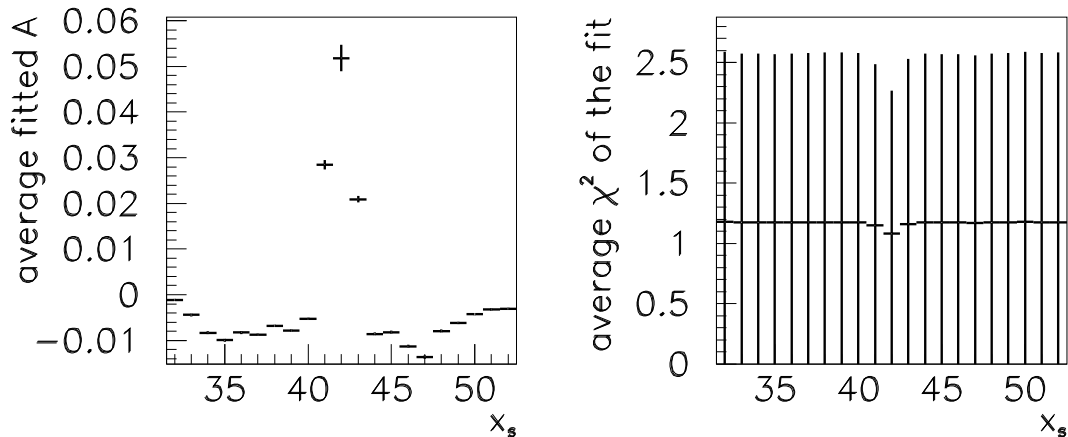presentations ($\tau = 1.54$ ps, [8]), adding to an increase of the predictions for x_s^{max}.

Figure 2: An average fitted amplitude $A(x_s)$ (left) and an average χ^2 of the fit (right), both as a function of x_s, from 1000 experiments. The error bars show the RMS.

2.2 Procedure of defining the range

The number of signal events N_S, the number of background events N_B and the proper-time resolution σ_t are used as parameters in a Monte-Carlo program, which generates "experimental" distribution of the "measured" asymmetry $A(t)$. The distribution then has D_{back}, D_{time} and Poisson fluctuations introduced with Monte-Carlo methods. Dilution from wrong tags D_{tag} is also introduced, with the wrong tag fraction $W = 0.22$ [5]. Finally, the program generates $A(t)$ distributions for $t > 0.4$ ps because of the analysis cut listed in tables 1 and 2.

Resulting 'experimental' $A(t)$ distribution is analyzed with a method called the amplitude fit, recommended in [4]. $A(t)$ distribution is fitted with the function $A_{fit} \cos(x_s t/\tau)$, where x_s is a constant value and A_{fit} is the only free parameter. The fit is repeated for different values of x_s, giving an $A_{fit}(x_s)$ distribution.

The amplitude fit method can be described as a variant of the Fourier transformation, a 'cosine transformation'. It has the power of Fourier analysis for the periodical signals, but one introduces known information about the phase in addition - we are looking explicitly for a cosine wave in the $A(t)$ distribution.

Figure 2 shows the properties of the transformation. An average fitted amplitude is shown as a function of x_s after 1000 experiments which had the true $x_s = 42$. An average χ^2 value of the fit is also shown as a function of x_s and the error bars show the RMS calculated from 1000 'experiments'. It can be observed, that $A_{fit}(x_s)$ is more useful to find the true value of the oscillation frequency.

The value of x_s giving the highest A_{fit} is the x_s measurement of the experiment. The peak in the fitted amplitude has its natural width, which can be seen in Fig. 2 (more explanation can be found in [4]). An experiment is called 'successful' here when the measured x_s value is within that width from the 'true' value defined in the Monte-Carlo.
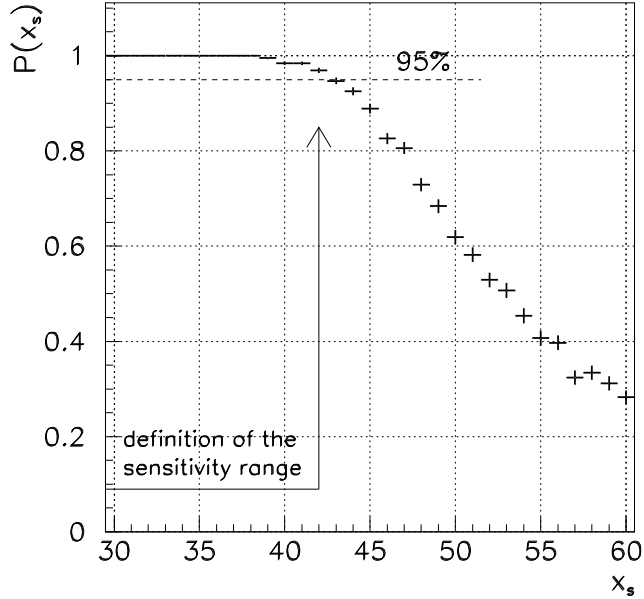
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Figure 3: Probability of a successful measurement, estimated using 1000 "experiments" with nominal ATLAS parameters: $\sigma_t = 0.07$ ps and $\int L dt = 10^4$ pb^{-1}.

With increasing 'true x_s' the amplitude of the oscillation seen in the $A(t)$ distribution decreases, as shown in figure 1. When a limit is passed the amplitude fit no longer enables to distinguish the right oscillation frequency form the noise generated by the statistical fluctuations in bins.

The probability of the experiment success calculated from 1000 simulated 'experiments' is shown in figure 3 as a function of x_s. The limit x_s^{max} is defined as the highest value of x_s for which such probability of an experimental success is above 95%.

For ATLAS, with the parameters listed in section 2.1 the limit turns out to be $x_s^{max} = 42$. A time-dependent asymmetry distribution, its transformation with the amplitude fit procedure and the results of 1000 simulated experiments are shown in Fig.4.

For the correct frequency an average amplitude should be equal to the product of all dilution factors D (see formula 1). Figure 5 shows the distribution of the fitted amplitudes. An average value $(53.15 \pm 0.49) \cdot 10^{-3}$ is in agreement with the prediction from formulas 2 to 5 equal $D = 52.85 \cdot 10^{-3}$.
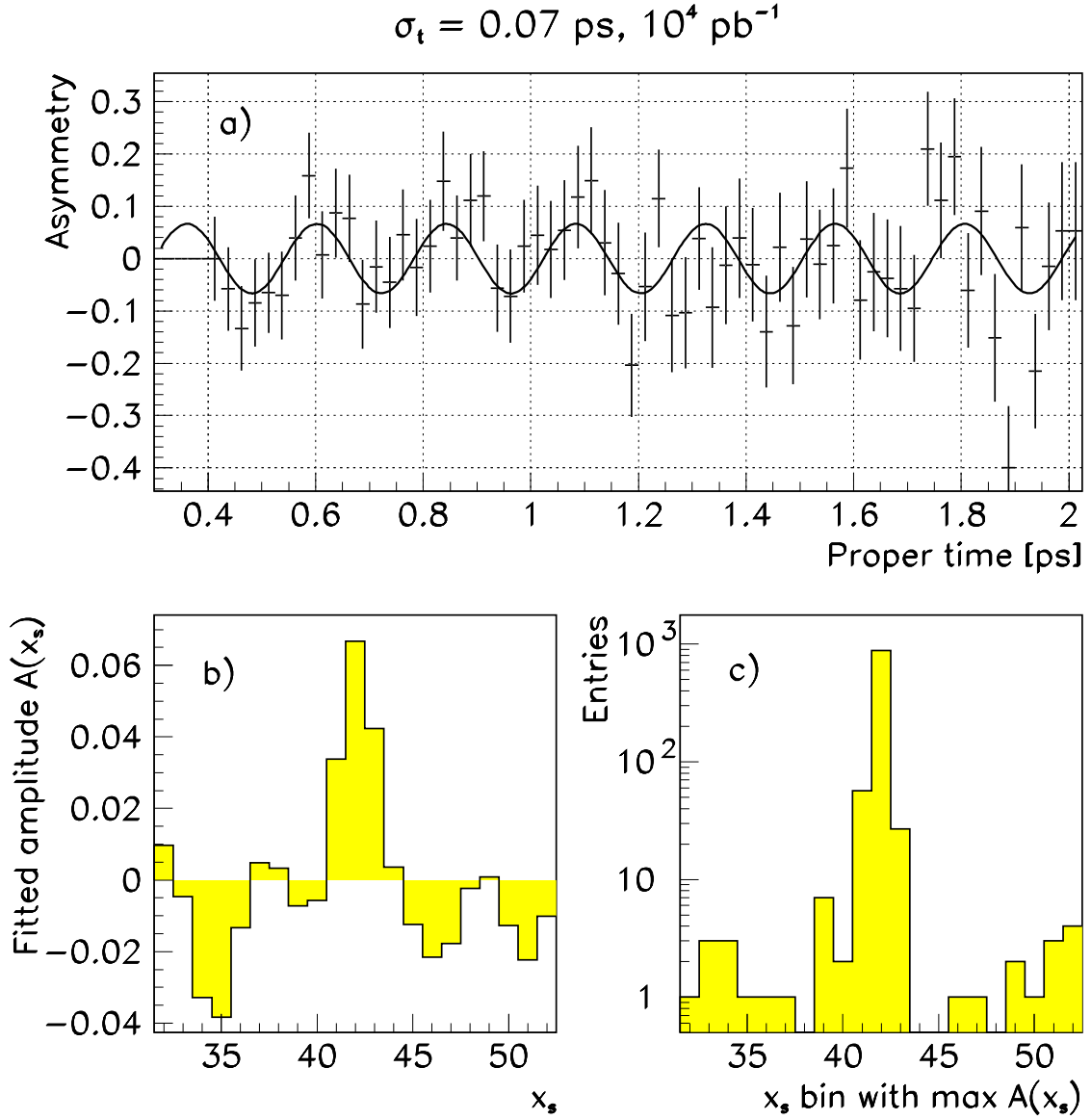
Figure 4: $x_s = 42$, "nominal" parameters: $\sigma_t = 0.07$ ps and $\int L dt = 10^4$ pb^{-1}; (a) time-dependent asymmetry distribution (single experiment), (b) amplitude $A(x_s)$ fitted to the distribution shown in (a), (c) distribution of x_s giving the highest value of $A(x_s)$ when the experiment was repeated 1000 times.
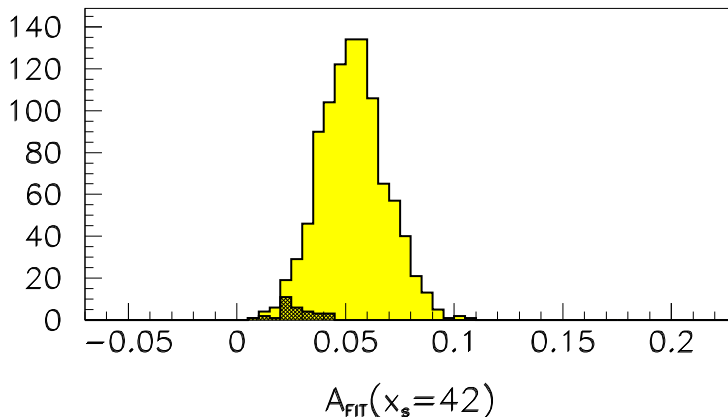
Figure 5: Distribution of the amplitudes fitted for $x_s = 42$ for 1000 'experiments' (generated with the 'true' $x_s = 42$). Dark histogram shows amplitudes for 31 'experiments' which 'fail'.

3 Dependence of the range on σ_t and on integrated luminosity

In order to evaluate the importance of the proper time resolution σ_t and of the integrated luminosity, the analysis was repeated for different values of these parameters. All combinations given by σ_t values of 0.05, 0.07, 0.09 and 0.11 ps, together with $\int Ldt$ values of 0.5, 1.0, 2.0, 5.0 and 10.0×10^4 pb^{-1} were tried.

Deteriorating the proper time resolution form the 'nominal' 0.07 ps to 0.11 ps reduces the x_s^{max} to 27, as could be expected from formula 6. The corresponding summary plot is shown in figure 6.

If σ_t has the nominal value and the integrated luminosity is reduced to $0.5 \cdot 10^4$ pb^{-1} the x_s^{max} is less reduced: $x_s^{max} = 38$ (see figure 7).

A full scan of the two parameters also gives a set of values: $\sigma_t = 0.05$ ps and $\int Ldt = 10^5$ pb^{-1}. Such a combination gives $x_s^{max} = 77$ (Fig. 8).

A summary plot showing the x_s^{max} versus σ_t for various integrated luminosities is shown in figure 9. The expected dependence given by the formula 6, shown with solid lines in the figure, is well confirmed.

The same summary in another projection, x_s^{max} versus $\int Ldt$ for different values of σ_t, is shown in figure 10. It is found, that the ATLAS range for the x_s measurement can be parameterized in the following way:

$$x_s^{max} \simeq \frac{2.90}{\sigma_t} \left[1 + 0.15 \ln \left(\int Ldt \right) \right] \tag{7}$$

where σ_t is given in ps and $\int Ldt$ in units of 10^4 pb^{-1} . The solid lines in figure 10 show the prediction of the above formula.
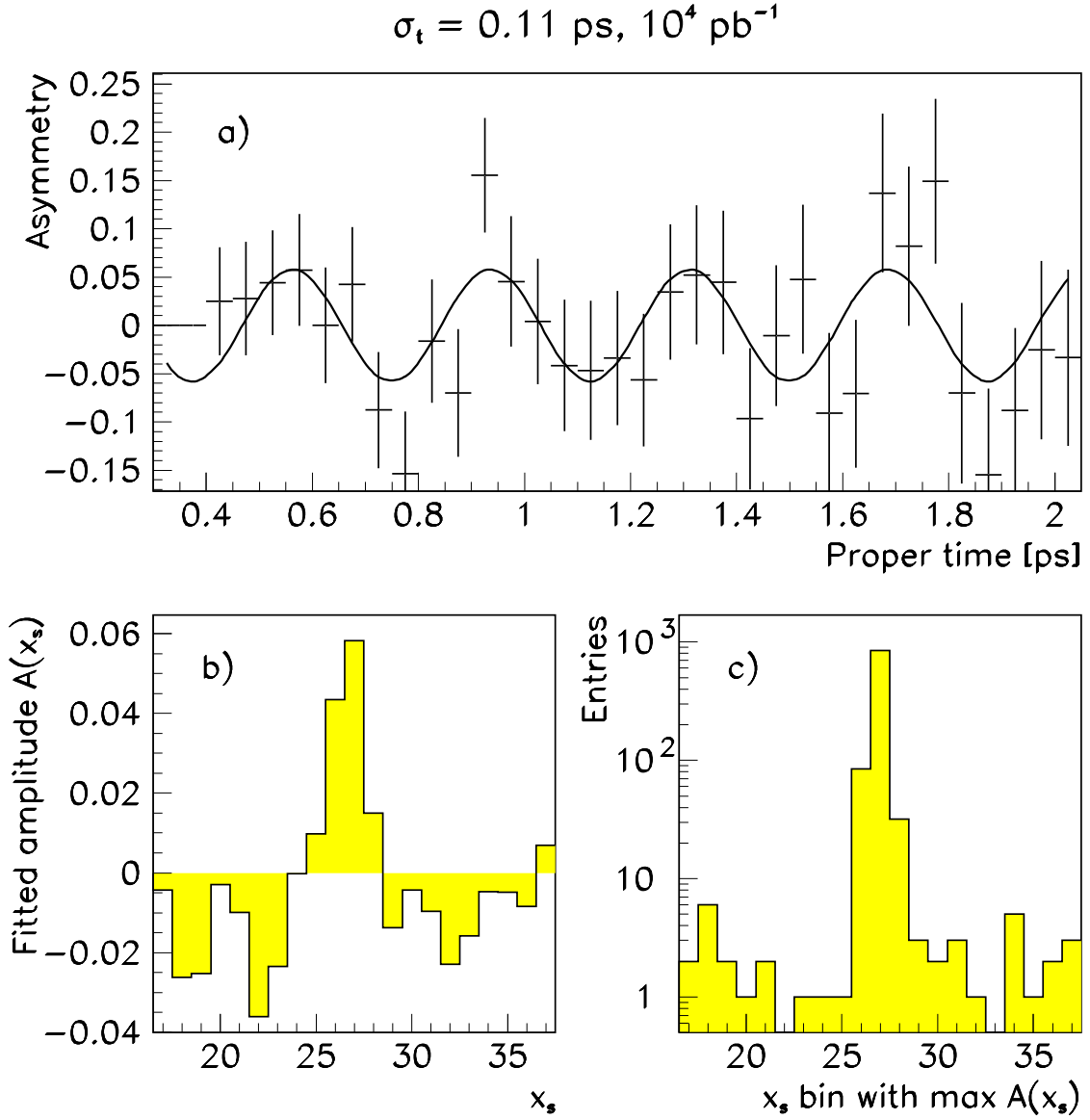
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Figure 6: $x_s = 27$, $\sigma_t = 0.11$ ps and $\int L dt = 10^4$ pb^{-1}; (a) time-dependent asymmetry distribution (single experiment), (b) amplitude $A(x_s)$ fitted to the distribution shown in (a), (c) distribution of x_s giving the highest value of $A(x_s)$ when the experiment was repeated 1000 times.
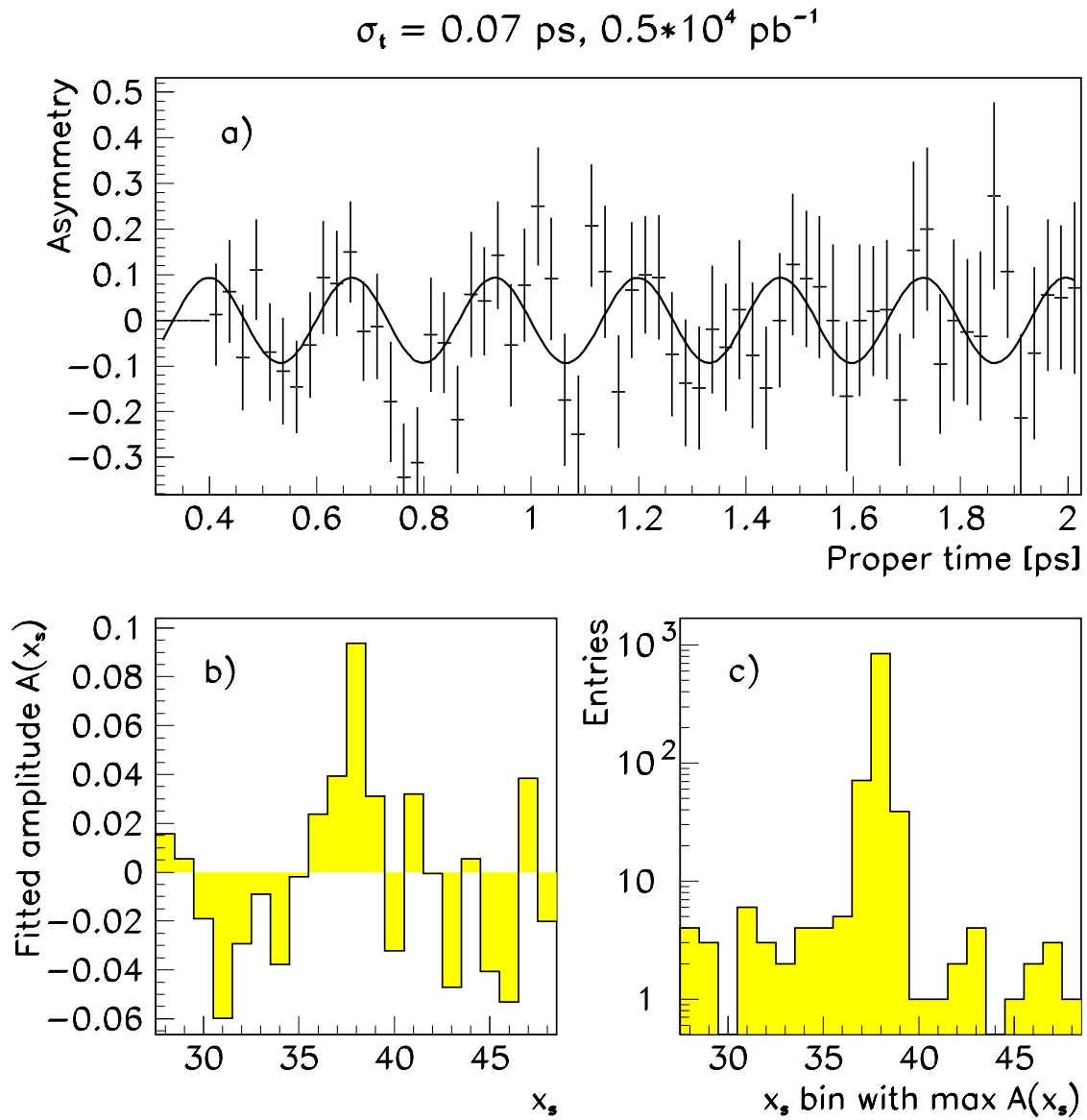
Figure 7: $x_s = 38$, $\sigma_t = 0.07$ ps and $\int L dt = 0.5 \cdot 10^4$ pb^{-1}; (a) time-dependent asymmetry distribution (single experiment), (b) amplitude $A(x_s)$ fitted to the distribution shown in (a), (c) distribution of x_s giving the highest value of $A(x_s)$ when the experiment was repeated 1000 times.
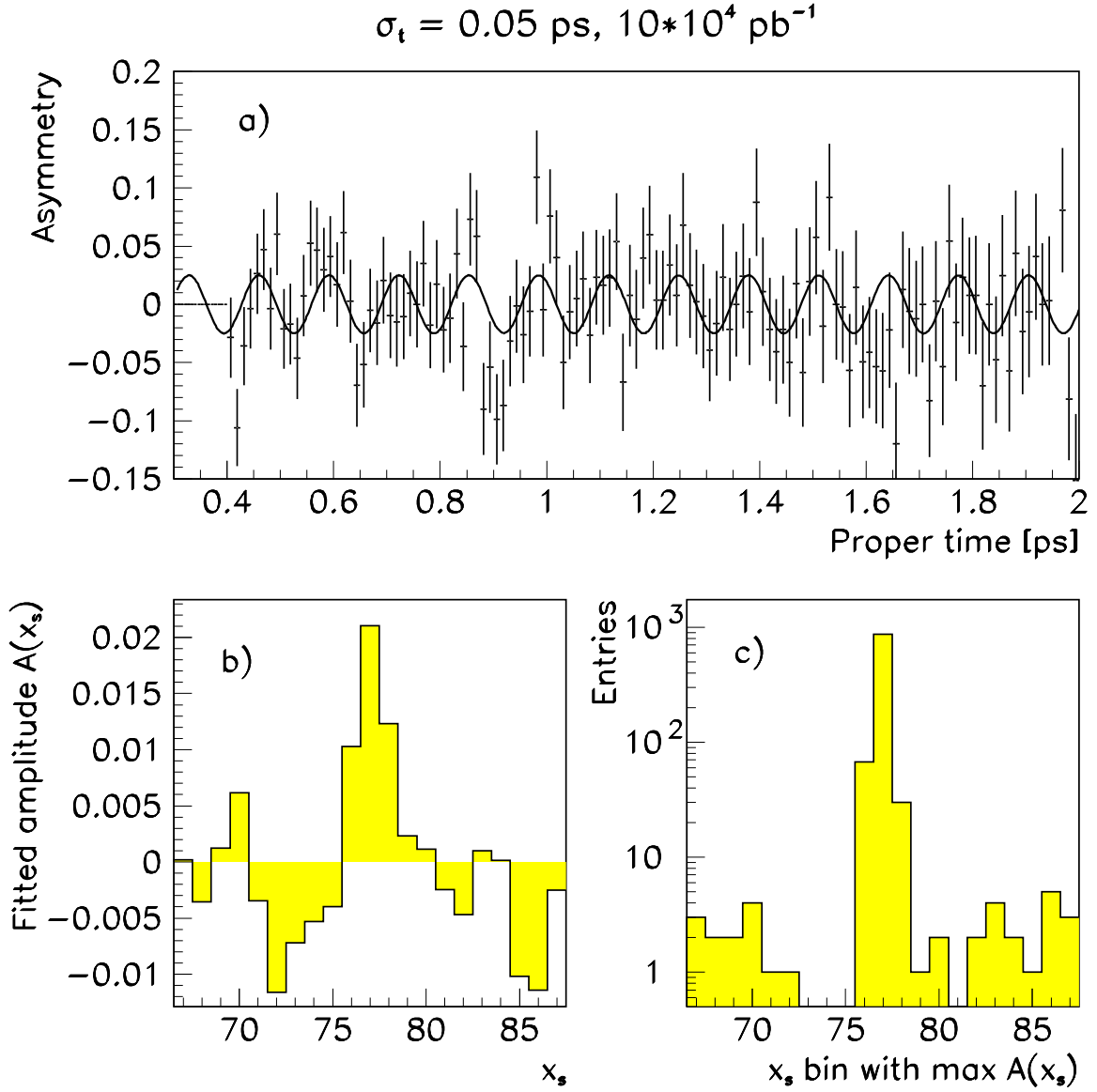
Figure 8: $x_s = 77$, $\sigma_t = 0.05$ ps and $\int L dt = 10 \cdot 10^4$ pb^{-1}; (a) time-dependent asymmetry distribution (single experiment), (b) amplitude $A(x_s)$ fitted to the distribution shown in (a), (c) distribution of x_s giving the highest value of $A(x_s)$ when the experiment was repeated 1000 times.
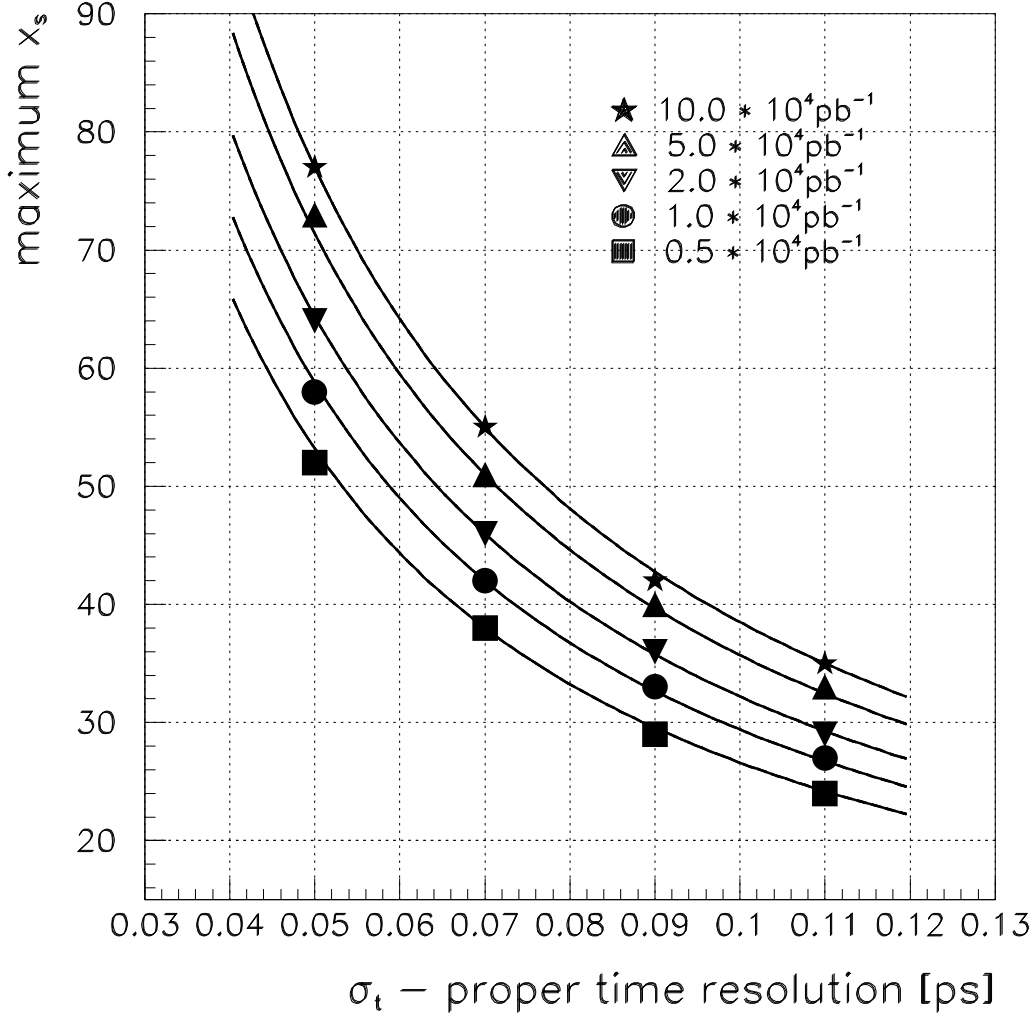
Figure 9: Sensitivity range of the ATLAS experiment for the x_s measurement, as a function of the proper-time resolution σ_t, for various integrated luminosities. The points come from Monte-Carlo calculations, the solid lines show $1/\sigma_t$ dependence.

4 Dependence on signal-to-background ratio

The scan of parameters σ_t and $\int L dt$ described in the previous section was done for the background-to-signal ratio of 1. By increasing the number of background events such that $N_S/(N_B + N_S)$ becomes equal 0.1 we reduce x_s^{max} to 31 for the 'nominal' parameters $\sigma_t = 0.07$ ps and $\int L dt = 10^4$ pb^{-1} (see figure 11).

A summary plot showing the dependence of x_s^{max} on background is shown in figure 12. N_S was kept constant at its predicted value for $\int L dt = 10^4$ pb^{-1} ($N_S = 4480$, see section 2.1), while the number of background event was varied between $N_B = 0$ and $N_B = 9 \times N_S$. It can be observed, that the dependence of x_s^{max} on background
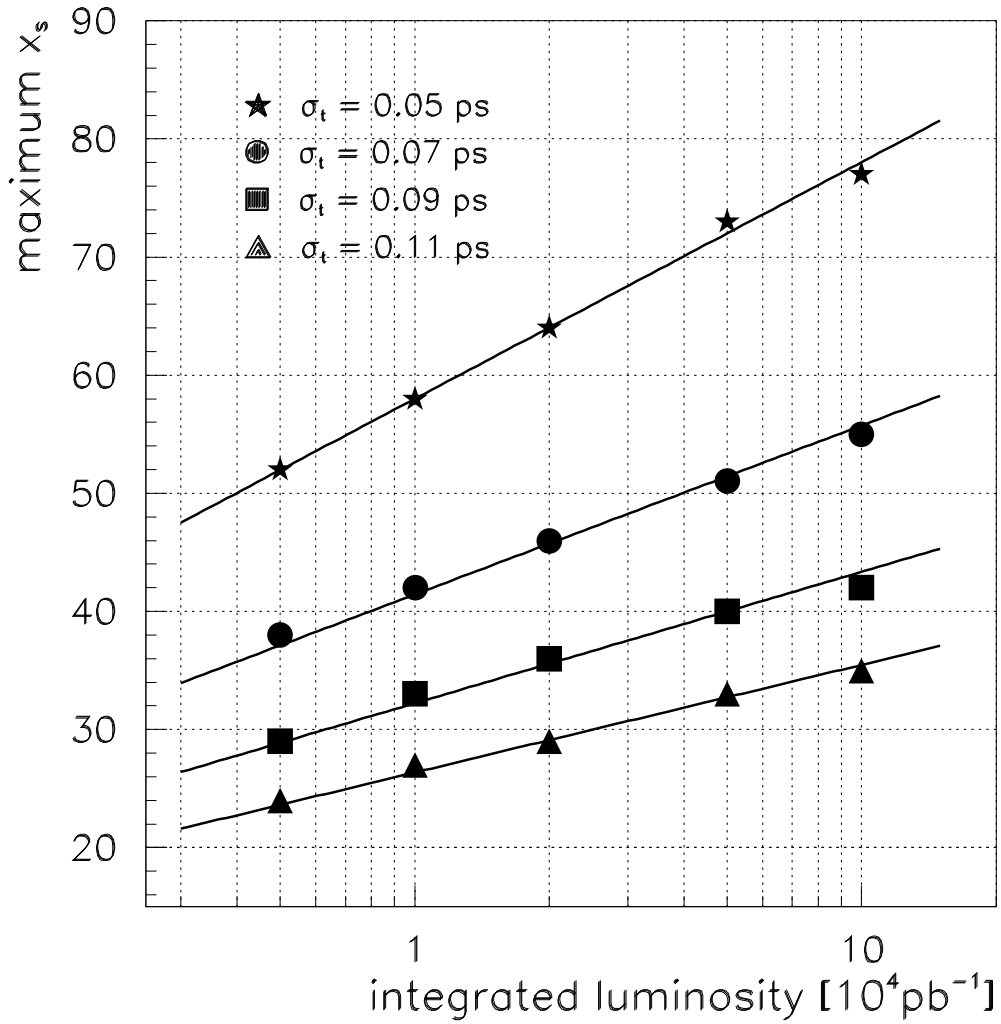
Figure 10: Sensitivity range of the ATLAS experiment for the x_s measurement, as a function of the integrated luminosity, for various proper-time resolutions σ_t. The points come from Monte-Carlo calculations, the solid lines show the $x_s^{max} \simeq (2.90/\sigma_t) \cdot (1 + 0.15 \ln(\int L dt))$ parametrisation, where σ_t is in ps and $\int L dt$ is in units of 10^4 pb^{-1} .

is not strong. It should be noted however, that background had no asymmetry in the simulation and a simple exponential dependence $dN_B/dt \sim \exp(-t/\tau)$ was assumed.
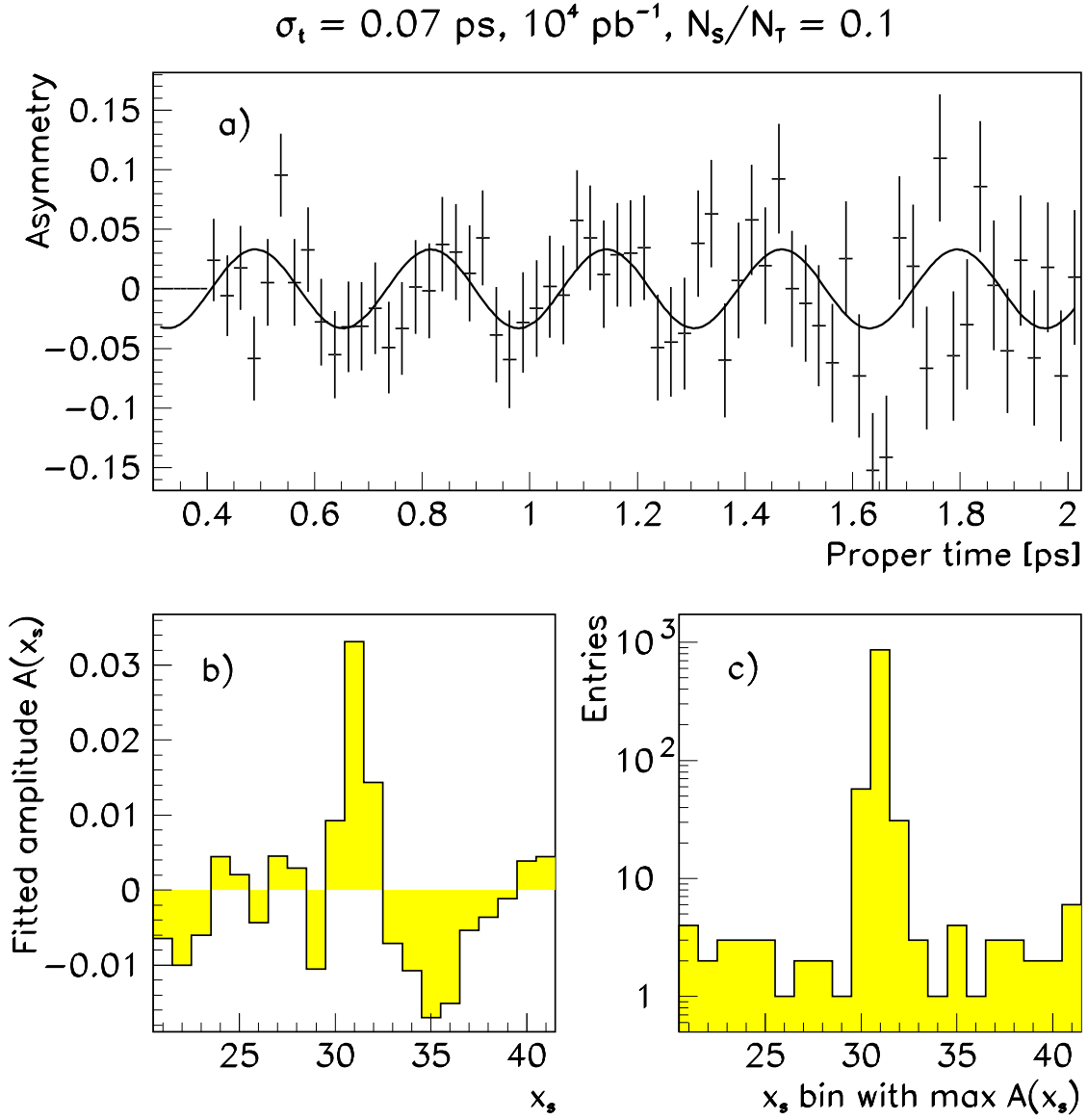
Figure 11: $x_s = 31$, $\sigma_t = 0.07$ ps and $\int L dt = 10^4$ pb^{-1}, $N_B/N_S = 1.0$; (a) time-dependent asymmetry distribution (single experiment), (b) amplitude $A(x_s)$ fitted to the distribution shown in (a), (c) distribution of x_s giving the highest value of $A(x_s)$ when the experiment was repeated 1000 times.
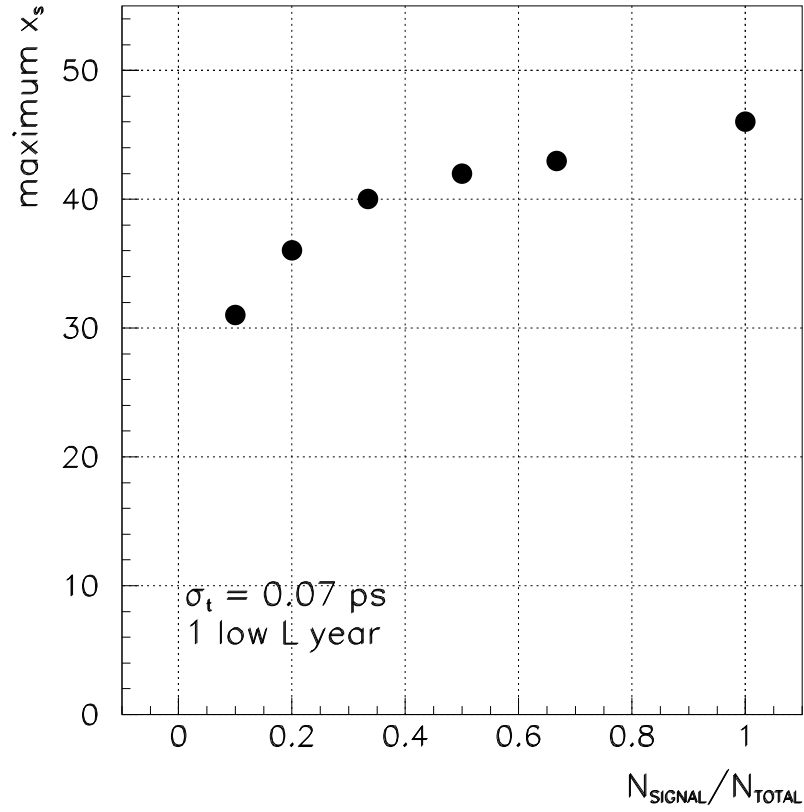
Figure 12: Sensitivity range of the ATLAS experiment for the x_s measurement, as a function of the signal content of the sample, for $\sigma_t = 0.07$ ps and $\int L dt = 10^4$ pb^{-1}.

5   Conclusions

It is justified theoretically, after [4], that the sensitivity range x_s^{max} is proportional to $1/\sigma_t$ (where σ_t is the resolution in the B-decay proper time), for any given integrated luminosity and signal-to-background ratio, independently of the method of defining the sensitivity range.

With the methods described in section 2 it is found that $x_s^{max} = 42$ for nominal ATLAS parameters. The Monte-Carlo study done for various integrated luminosities and for different σ_t values shows, that ATLAS sensitivity range can be parameterized as $x_s^{max} \simeq (2.90/\sigma_t) \cdot (1 + 0.15\ln(\int Ldt))$, where σ_t is in ps and $\int Ldt$ is in units of 10^4 pb^{-1}. It is also found, that the dependence of x_s^{max} on background is relatively weak.

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