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ROXIE - A FEATURE-BASED DESIGN AND OPTIMIZATION PROGRAM FOR SUPERCONDUCTING ACCELERATOR MAGNETS

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Abstract: The paper describes the computer program ROXIE which has been developed for the design of the superconducting magnets for the Large Hadron Collider (LHC) project at CERN. The applied concept of "Features" not only enhances the speed of geometry creation with a minimum input of meaningful engineering data, it also allows design changes to be made with just a few high level commands and thus provides a platform for automated design using numerical field calculation and mathematical optimization techniques.

1 Introduction

The Large Hadron Collider (LHC) project [3] is a superconducting accelerator for protons, heavy ions and electron- proton collisions in the multi-TeV energy range to be installed at CERN in the existing LEP tunnel with a circumference of about 27 km. The new facility will mainly consist of a ring of high field superconducting magnets cooled to 1.9 K with superfluid helium [19]. The LHC requires high field superconducting lattice dipoles and quadrupoles together with about 30 different kinds of magnets for insertion (low-β, cleaning, dump), correction, and dispersion suppression.

The report describes the ROXIE Fortran program (**R**outine for the **O**ptimization of magnet **X**-sections, **I**nverse field computation and coil **E**nd design) which has been developed for the electromagnetic design of the superconducting accelerator magnets for LHC. With its feature-based creation of the complicated 2d and 3d coil configurations and iron cross-sections, design changes can easily be made and propagate automatically through the model, thus providing a platform for automated design using numerical field calculation and mathematical optimization techniques. Together with the interfaces to other CAD-CAM tools, FEM packages, and beam simulation programs, ROXIE represents an approach to an integrated design tool for superconducting magnets.

User manual and examples can be found on the World-Wide Web (WWW): http://roxa33.cern.ch/~russ

2 The feature concept in ROXIE

With the availability of feature concepts in commercial software e.g. EDS Unigraphics [6], and Pro/ENGINEER [21], designers have access to powerful new tools for Computer Aided *Design* rather than Computer Aided *Drawing*. Also developers of computational electromagnetics software have recently concentrated on efficient data exchange between FEM and CAD-CAM based on the new ISO STEP standard [5]. STEP (the Standard for the Exchange of Product data, ISO 10303) with its data specification language EXPRESS fully supports the feature concept. However, there are different attempts to define features and there is no precise definition of what a feature actually is.

Shah [24] defines a feature as a representation of the *engineering meaning or significance of the geometry* of a part or assembly. A feature is a physical constituent of a part, is mappable to a generic shape, has engineering significance and predictable properties. Features can be classified as: Form features, Tolerance features, Assembly features, Functional features, and Material features. Features are functional primitives, which do not only contain the geometrical information (shape, dimensions, position, orientation, tolerances) of a part, but also non-geometric properties such as material name, properties, part number etc..

Feature modelling or "Designing by features" is an extension of parametric modelling (precondition for the use of mathematical optimization methods) to the macroscopic level and makes possible to define with only a few input data the complicated shapes of the device. The Feature Based Design Module (FBDM) (together with the module for the addressing of output data as objectives and the decision making methods) can be seen as the heart of the ROXIE program. After the geometric modelling is done, every feature can be subject to geometric transformations such as translation, rotation, scaling, imaging, while constraints are defined for these operations in order to avoid penetration or physically meaningless structures. Not only the geometric properties of a device can be changed in the optimization process but also material properties, in our case for example number of strands, current density in conductors and strands, filling factors, unit price etc.

The features are composites which can be decomposed into two or more simple features. They inherit common properties from features higher up in the level. The composition for the coils is: Layer - Coil block - Conductor - Strand. The yoke and the collar are composites of simple features i.e. quadrilateral profiles with straight or elliptic edges and possible holes. Whereas the coil features are truly 3d the non coil part is still limited to 2 dimensional cross sections. Table 1 and 2 list the features used in the magnet design together with its most important properties. Basically all predefined and user supplied properties of the features can be used as design variables of the optimization problem.

Feature modelling and mathematical optimization can therefore be combined as an approach towards an integrated design of superconducting magnets.

Coil Features	Properties	
Strands	Dimension	Diameter
	Position	x,y,z
	Orientation	e_x, e_y, e_z
	Material name	Coded strand map number e.g. 1.23
	Number	
	Mat. properties	Cu/SC ratio, Current, Critical current density Jc at B ref. in SC Temperature (T), dJc/dB, dJc/dT
	Location	Conductor no., Block no., Layer no.
Conductors	Shape	Braid, Rutherford type
	Dimension	Height, inner width, outer width, keystone angle Insulation thickness broad and narrow side
	Position	x,y,z
	Orientation	e_x, e_y, e_z
	Name	e.g. 62D6501C (coded)
	Number	
	Mat. properties	No. of strands, Name of strands, Insulation type Current, Current density graded or homogeneous, Temperature, Compression
	Unit price	(per length of conductor and insulation)
	Location	Block no. , Layer no.
Coil blocks	Shape	Rectangular, or cos θ type in cross section, Constant perimeter, Racetrack end with or without inter-turn spacers
	Symmetry	Dipole, Quadrupole .. Dodecapole, asymmetric
	Position	angular position, inclination in xy and yz plane aligned on the winding mandrel (ID) or on the outer radius of the end-spacers,
	Mat. parameters	Conductor type, compaction factor, contraction factor, de-keystoning factor for ends Current, Temperature
	Number	
	Location	Block number, layer number
Spacers	Shape	Elliptic, hyper-elliptic, with or without shelfs
	Mat. properties	Material type (G11), Tolerances, Surface finishing
	Number	
	Location	Layer no.
	Price	
Layers	Symmetry	Multipole symmetry , asymmetric (nested magnets)
	Number	No. of blocks, No. of wedges, No. of end-spacer

Table 1: Features for magnet design (coil part)

Yoke Features	Properties	
Steps	Dimensions	Height and Width
Slots	Dimensions	Positioning angles and inclination angles of the two sides, Depth on the two sides
Holes	Dimensions	Center and radius
Profiles	Shape of edges	elliptical, circular, straight
	Position	x,y,z
	Non geometric	Boundary conditions, FE discretization
	Attributes	No. of neighbouring profiles, Material distribution
Collars	Shape	elliptical, circular, racetrack, combined , separated, with or without insert for field quality reasons,
	Dimensions	Outer radius, Ellipticity
Yoke	Shape	Single aperture, Two-in-one design
	Dimensions	Beam distance, Outer radius
	Mat. prop.	BH curve, Filling factor, Contraction coeff.

Table 2: Features for magnet design (yoke part)

Coil and yoke features can only be treated independently because the FEM solver does not require the meshing of the coils. A reduced vector potential is applied where the excitational field in the iron region is calculated by means of the Biot-Savart's law. In the air region where the coil is situated the excitational field has not be calculated thus avoiding singularities. The method is described in [20].

Fig 1 gives an overview on the program structure.

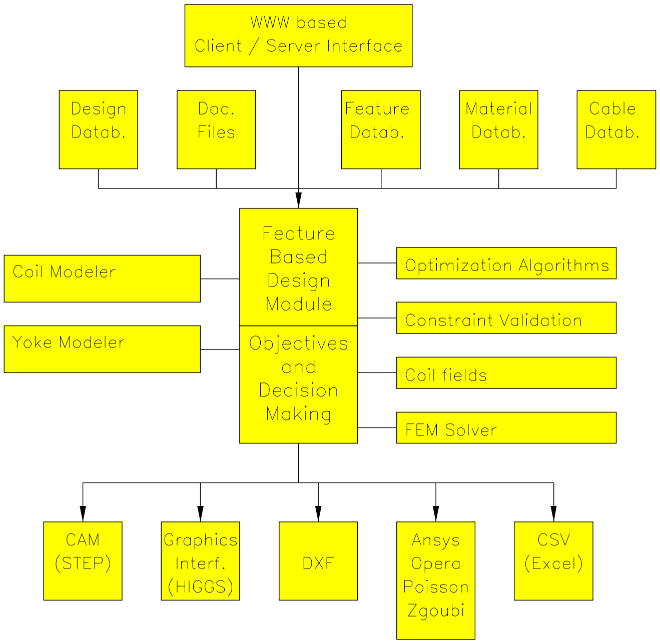

Figure 1: Program structure of the ROXIE program

Special attention was paid to the interfaces to other CAD-CAM tools and FEM packages in view of the integrated design process as described in chapter 6. The heart of the program is the Feature Based Design module together with a module that addresses the relevant output data (objectives for the optimization process) and creates the objective function using various decision making methods.

3 Coil modeller

The program includes routines to define geometrically coil cross sections and coil ends of dipoles, quadrupoles, sextupoles, octupoles and decapoles made of Rutherford type superconducting cables or rectangular shaped braids. The geometric position of coil block arrangements in the cross section of the magnets is calculated from given input data such as the number of conductors per block, conductor type (specified in the cable data base), radius of the winding mandrel and the positioning and inclination angle of the blocks. The fact that the keystoning of the cables is not sufficient to allow their edges to be positioned on the curvature of a circle, is fully respected. This effect increases with the inclination of the coil blocks versus the radial direction. The keystoning of the cable also results in a grading of the current density in the conductor as the cable is more compacted (less voids between the strands) towards the narrow side. Rectangular shaped coil blocks are also possible if the cable is not keystoned. The input parameters for the coil end generation are the z position of the first conductor of each coil block, its inclination angle, the straight section and the size of the inter turn spacers between the conductors. Four options for the coil ends are available:

- Coil end design with and without inter-turn shims and conductors placed on the winding mandrel,
- coil end with grouped conductors aligned at the outer radius of the end-spacers,
- coil end for magnets with rectangular cross sections,
- racetrack coil ends with 2 or 4 straight sections, and solenoids as a special case.

It is assumed that the upper edges of the conductors follow ellipses, hyper-ellipses or circles in the developed sz plane defined by their radial position in the straight subsection and the z position in the yz plane. A de-keystoning factor can be defined for the consideration of a cable shape change in the ends due to the winding process and the fact that a Rutherford type cable made of strands does not have the properties of a solid beam.

4 Yoke modeller

For the iron yoke, the features include iron yokes with single aperture and two-in-one iron yokes both with separated and combined collars. Combined collars can have an iron insert for field quality purpose, separated collars can have elliptical, circular or racetrack shape. The structures are composites of simple features, i.e. quadrilateral profiles with straight and elliptic sections that allow for holes and circular openings. The parameters of these features are size and location, shape of the edges, boundary condition, material distribution, FE discretization and number of neighbouring features.

As the finite element software [20] uses as element type a curvilinear quadrilateral isoparametric finite element with 8 nodes, and because the size and shape of the profiles changes during the optimization process, a fully automatic mesh generator is hardly applicable. All information concerning the neighbouring profiles are collected successively during the build up of the structure. A magnetic material property has to be assigned which is valid over the profile except in holes. The edges of the profiles are marked so that the boundary conditions can be assigned after assembling the cross-section. The profiles are then subdivided into macro elements. The number of the subdivisions can be chosen, but the conformity on the interfaces within neighbouring profiles has to be guaranteed.

5 Optimization techniques

The optimization problems appearing in the magnet design process involve multiple conflicting objectives that must be mutually reconciled. This was first addressed in 1896 by Pareto [18], a social economist who introduced an optimality criterion for vector-optimization problems with conflicting objectives. A Pareto-optimal solution is found if there exists no other solution that will yield an improvement in one objective without the degradation of at least one other objective. Whenever there is a price to be paid for a further improvement of one objective a solution from the Pareto-optimal solution set is found. Methods that guarantee Pareto-optimal solutions were first introduced in the field of economics by Marglin [15], among others, and were only later applied to engineering problems e.g. [23].

Some of the decision-making methods involve additional constraints which are in engineering problems usually nonlinear. The theory of nonlinear optimization with constraints is based on the optimality criterion of Kuhn and Tucker [14], providing the basis for later developments in mathematical programming. Methods for the treatment of nonlinear constraints were developed by Fiacco and McCormick [7], and Rockafellar [22], among others.

The third part in an optimization procedure is the optimization algorithm for the minimization of scalar, unconstrained objective functions. Algorithms using both deterministic, stochastic and genetic elements have been developed in the sixties and covered in various textbooks and articles, e.g., [2, 8, 9, 10].

Table 3 shows a list of the different methods for mathematical optimization implemented in ROXIE. It is important to note that it is the combination of these methods which make an efficient procedure. As there is no general solution to nonlinear optimization problems in the sense that the simplex method is used for the linear optimization problems, it is necessary to provide the user with a set of methods to chose from. In the last chapter some procedures will be described together with the examples.

Decision making methods		
Objective weighting	Zadeh	1963
Distance function	Charnes Cooper	1961
Constrained formulation	Marglin	1967
Pay-off table	Benayoun	1971
Fuzzy set decision making	Bellman Zadeh	1970
Hidden resource evaluation with Lagrange-Multiplier estimation	Kuhn-Tucker	1951
Treatment of nonlinear constraints		
Feasible directions	Zoutendijk	1960
Penalty transformation	Courant	1943
Exact penalty transformation	Pietrzykowski	1969
Augmented Lagrangian technique	Hestenes	1946
Sequential unconstrained minimization (SUMT)	Fiacco, Mc Cormick	1968
Boundary search along active constraints	Appelbaum, Shamash	1977
Optimization Algorithms		
Search methods		
EXTREM	Jacob	1982
Rosenbrock		1960
Powell		1965
Hooke-Jeeves		1962
Gradient methods		
Levenberg-Marquard		1963
Quasi-Newton (DFP)	Davidon-Fletcher-Powell	1963
Neural computing		
Genetic algorithms	Fogel,Holland	1987

Table 3: Elements of optimization procedure available in the ROXIE program.

6 The Integrated Design process

With the feature-based creation of the complicated geometries and the possibility of addressing all significant data for the design and optimization of the device and together with its interfaces to other CAD-CAM packages and field computation packages, the program is increasingly used as an approach towards an integrated design of superconducting magnets. Fig. 2 shows the main steps of an integrated design process with its prime economical and technical aspects. It shows in particular the potential for the application of mathematical optimization routines during the design process.

- **Conceptional design using genetic algorithms.** Genetic algorithms are used for the field synthesis of magnet cross-sections addressing them as current distribution problems. This way, first guesses for the block distribution of the superconducting cables can be found.
- **Geometry layout using the predefined design features.** Once the principle layout is known the geometry is created by means of the design features implemented. This step also defines the design space for the optimization, geometrical constraints and manufacturablity considerations.
- **Electromagnetic Optimization using deterministic algorithms and vector-optimization methods.** The electromagnetic design of the coils

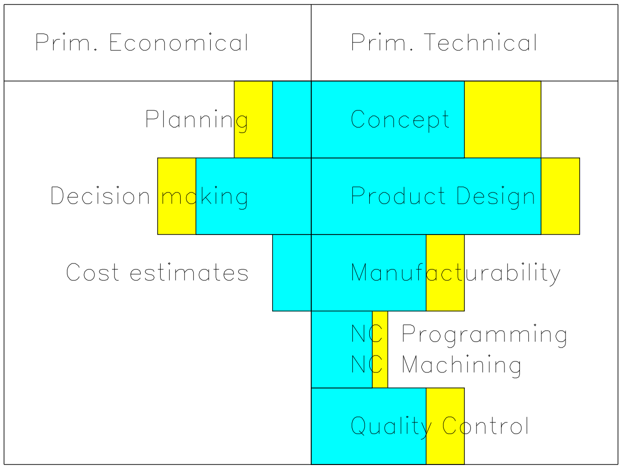

Figure 2: Integrated Design process (dark gray blocks: Application of the ROXIE program; mid gray blocks: Potential for the application of mathematical optimization techniques)

usually starts with the cross section. A high dipole field is required while keeping the higher order multipole content of the main field in the aperture within limits required by beam physics. In a next stage the multipole content for the 3d coil end geometry is optimized by changing the relative position of the coil blocks in the ends. Once an appropriate coil design is found, the optimization focuses on the mimimization of the iron induced effects in the magnets. In two dimensions this can be done by the built in FEM solver for 3d commercial software is applied.

- **Transfer of data to commercial FEM software**. By means of interfaces to the commercially available FEM codes like ANSYS and OPERA the 3d coil geometry is transferred in order to investigate the influence of stray field in the coil end region of the magnet.

- **Tolerance and manufacturablity analysis.** The Lagrange-Multiplier estimation can be used for the evaluation of the hidden-resources in the design as they are a measure for the price which has to be paid when a constraint is increased. From the sensitivity matrix (which can be transferred via an CSV interface into spread-sheet programs e.g. EXCEL) the multipole content can be evaluated as a function of the tolerances on coil block positioning, coil size, asymmetries resulting from the collaring procedure etc.

- **Production of drawings by means of the DXF interface.** The DXF interface creates files for the drawing of the cross-section in the xy and yz planes of the magnet, the developed view in the sz plane and the polygons for the end-spacer manufacture.

- **Production of end-spacers by means of geometrical data transferred into commercial CAD-CAM packages.** The shape of the end-spacers is determined by 9 polygons on the machined surfaces which are then transferred into a CAM system e.g. CATIA, for the calculation and emulation of the cutter movements for machining the piece. The spacers are then machined by means of a 5 axis CNC machines from glass-epoxy tubes (G11).

- **Inverse field calculation for the tracing of manufacturing errors.** The mechanical dimensions of the active parts of the coils are impossible to verify under their operational conditions, after their deformation due to manufacture, warm pre-stressing, cool-down and excitation. The inverse problem solving consists of using optimization routines to find distorted coil geometries which produce exactly the multipole content measured.
- **Display of data in the local conductor coordinate system.** The graphic routines which only use a couple of primitives from the HIGGS (CERN graphics library) programs allow the display of fields and forces in the local conductor coordinate system (parallel and rectangular to the broad side of the cable) as well as in Cartesian coordinates. The routines can also be used in order to transform measured or externally calculated fields from the Cartesian into the local conductor coordinate system.

Below some examples for the applications of the ROXIE program for different magnet types and steps from the design process are given.

7 Acknowledgements

Many thanks to all colleagues who have helped testing the program or who have contributed the information and ROXIE input files of their applications. Their contributions are implicitly acknowledged in the references.

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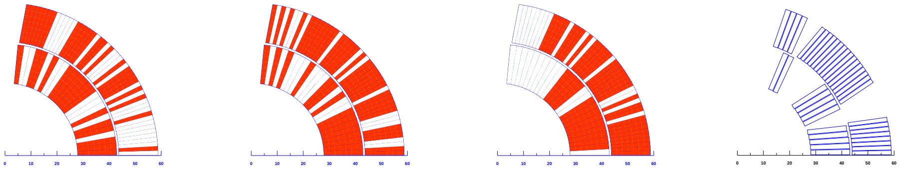

Figure 3: Genetic algorithms are used for the field synthesis of magnet cross-sections addressing them as current distribution problems. The figure shows intermediate steps of the optimization using genetic algorithms after 65, 195 and 4550 function evaluations together with a feasible design obtained by using deterministic methods (from left to right). The number of objective function evaluations necessary shows, that this method can only be used for conceptional design in order to derive new ideas or an initial starting point for the design. The new idea derived here is that adding some conductors to the outer layer coil results in a shielding effect, and the pole angle of the inner layer can be increased in comparison to previously optimized designs. Disadvantages are the higher current density in the cable necessary to produce the same main field (resulting in higher hot spot temperatures at quench) and the reduced margin to quench in the outer layer.

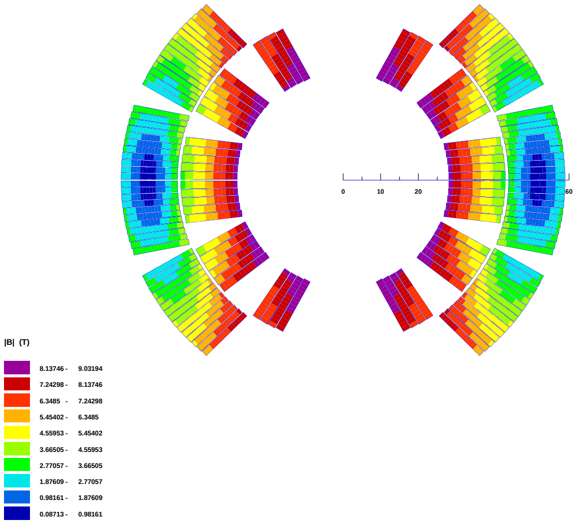

Figure 4: Optimized dipole coil with 5 block structure. Display of the magnetic field modulus. For the optimization an objective weighting function was used together with a deterministic search routine. Usually the algorithm EXTREM by Jacob is used because of its robustness and ease of use. The interesting result is that with only 5 blocks all the higher multipoles up to b9 could be minimized. Older designs always considered 6 blocks.

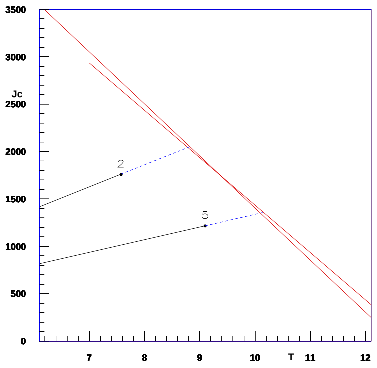

Figure 5: Load line characteristic curves for the 5 block dipole coil, as displayed in fig. 4. Block 2 outer layer, block 5 inner layer. It shows the curves for the critical current density in the superconducting filaments for a given field. The aim is to maximize the main field while keeping the margin to quench (dotted lines) balanced between inner and outer layer

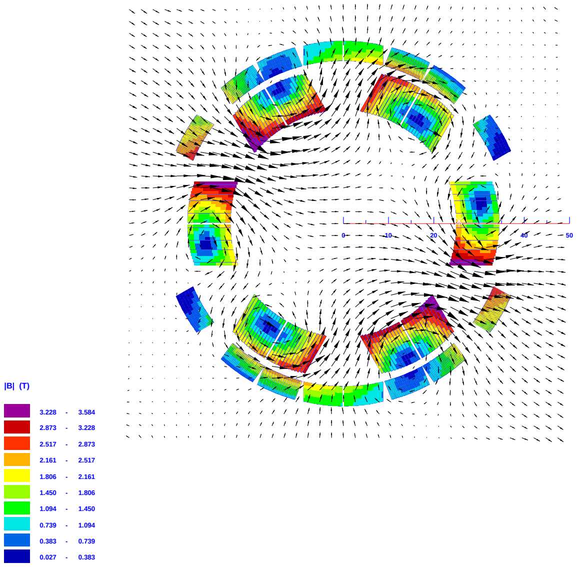

Figure 6: Cross section of combined dipole-sextupole corrector, [11] with field vector display and $\mid B \mid$ in coils. The performance of combined magnets is limited by the peak field enhancement in one coil due to the powering of the other. A very accurate peak field calculation in the coil blocks is therefore precondition for optimization. In all these calculations the self field of the strand is neglected. This corresponds to the measurements from which the critical current density curves (fig. 5) are obtained.

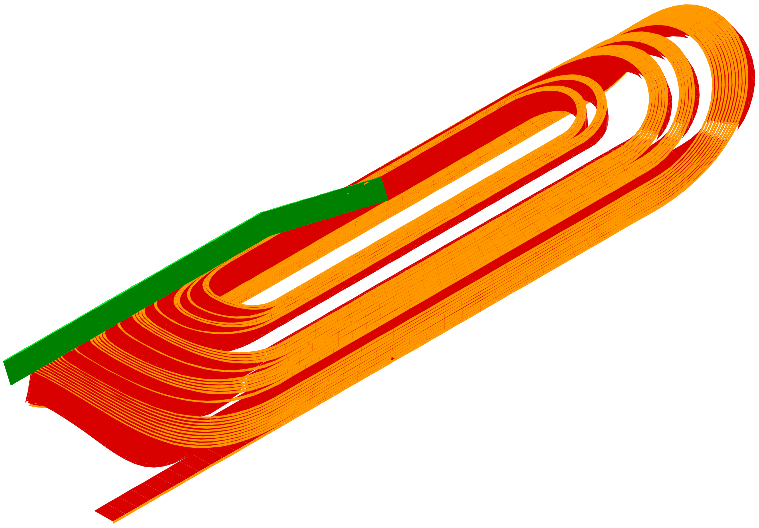

Figure 7: Dipole coil for a separation dipole magnet with its connections. Note the transition of the cable between the coil blocks in the connection side (front left). The coils have so-called constant perimeter ends because the model assumes that upper and lower edges of the cable don't change their length during the winding process around the end-spacers.

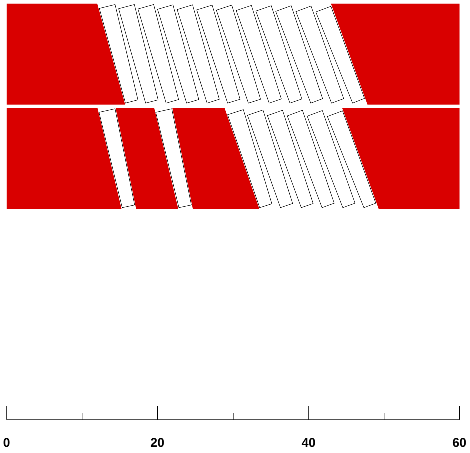

Figure 8: Cut of main quadrupole end (in the yz plane) with conductors placed on the winding mandrel and inter-turn spacers.

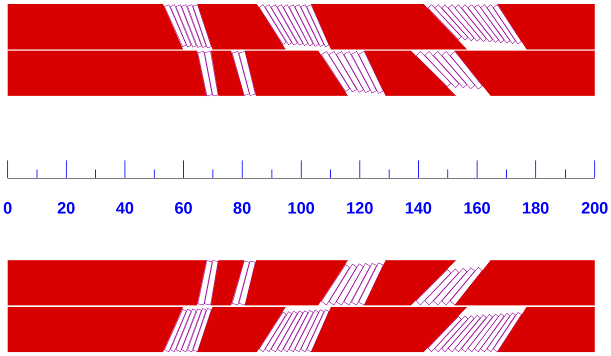

Figure 9: Cut of model dipole end with grouped conductors aligned on the outer radius of the end-spacers. The end-spacers of this model feature shelfs or “shoes” for the support of the turns (c.f. fig. 14). Note that the inter-turn spacers are missing and the inclination angles of the conductors change due to the keystoning of the cable.

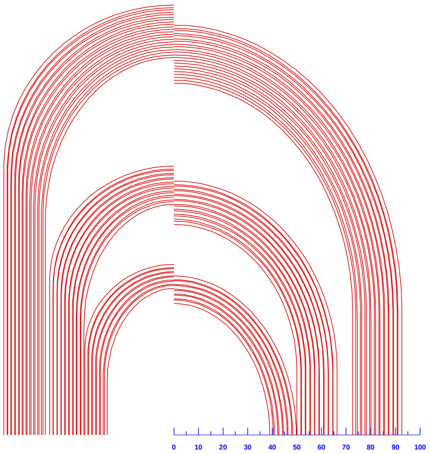

Figure 10: Developed view on outer layer conductors of a model dipole end with grouped conductors. Right: Upper edge of the cable (outer radius) Left: Lower edge. It is assumed that the conductor edges follow ellipses in the developed sz plane. The perimeter of upper and lower edges is the same. Cross section, cuts in yz planes and the developed view can be transferred via DXF files into computer aided drawing packages (c.f. fig. 22)

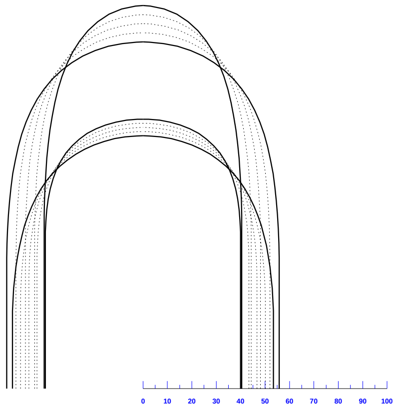

Figure 11: Polygons for end-spacer machining with 5 axis milling machine for dipole model magnet. These polygons are transferred into commercial CAM packages for the calculation of the cutter paths for the milling of the spacer from glass-epoxy tubes.

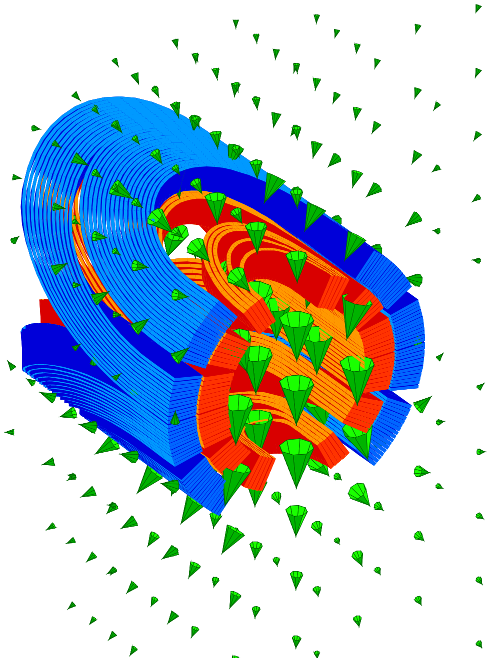

Figure 12: 3D representation of coil end of a dipole model magnet [17] with magnetic field vectors. The optimization problem consists in minimizing the integrated multipole content in the end by shifting the relative position of the blocks in the z direction while keeping the peak field enhancement low.

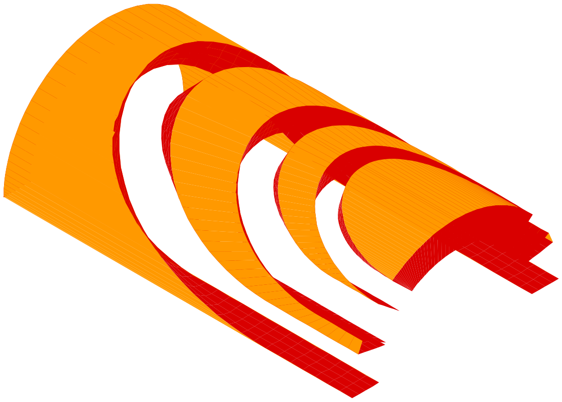

Figure 13: 3D representation of end-spacers for dipole model with conductors aligned on the winding mandrel.

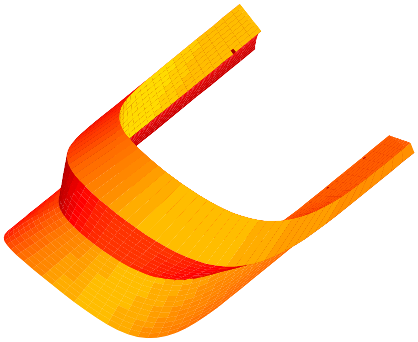

Figure 14: 3D representation of an end-spacer with shelf (in order to align the turns on the outer radius of the spacer).

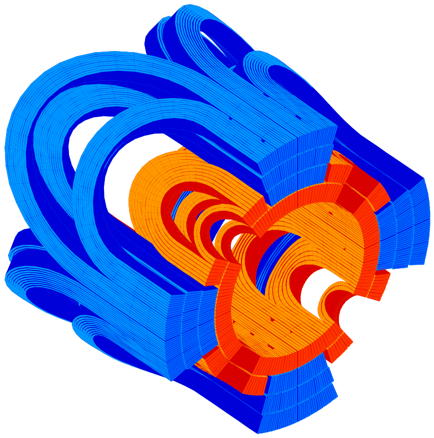

Figure 15: 3D representation of coil end of the insertion quadrupole. 4 layer design with additional grading in the second layer. [26]

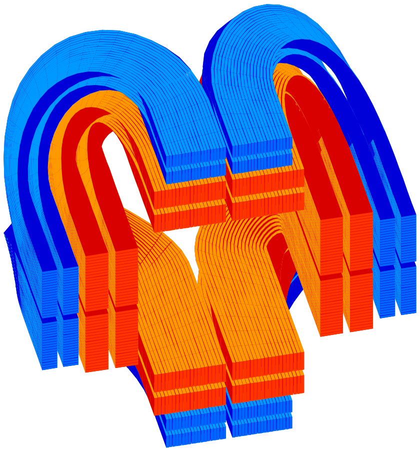

Figure 16: 3D representation of coil end for a insertion quadrupole design with rectangular coil cross section.

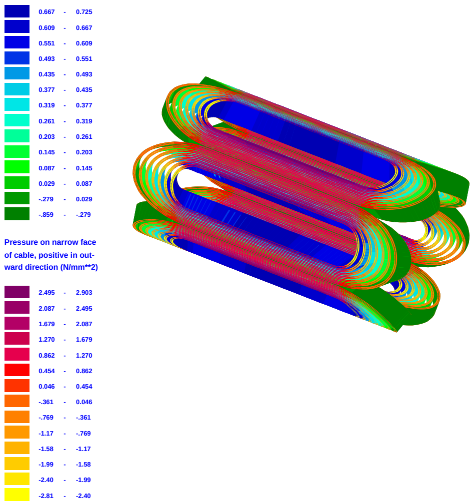

Figure 17: 3D view of sextupole corrector coil, with pressures due to Lorentz-forces on cables displayed in the local conductor coordinate system.

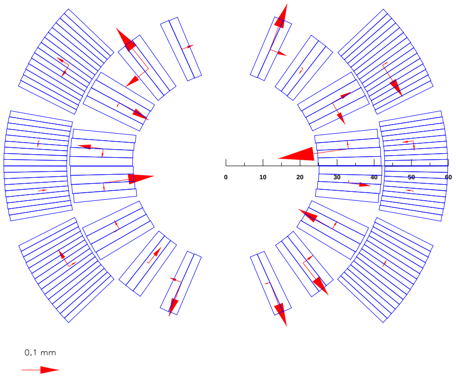

Figure 18: In inverse problem-solving the design variables are a multiple of those for optimization, since the coil positioning errors may well be asymmetric. The objective function has to contain all multipole terms including the skew terms (cos and sin terms of the Fourrier expansion of the field at a given radius). The setup of the objective function is, however, quite simply done in a least-squares objective function. The design variables for the minimization problem are the azimuthal and radial displacements of each coil block and the position of the measurement coil, thus resulting in 50 design variables. The figure shows the displacements of the coil blocks found by the Levenberg-Marquard algorithm after about 1200 function evaluations. As there are far more unknowns than residuals we cannot expect unique solutions to the problem. The problem is ill-conditioned.

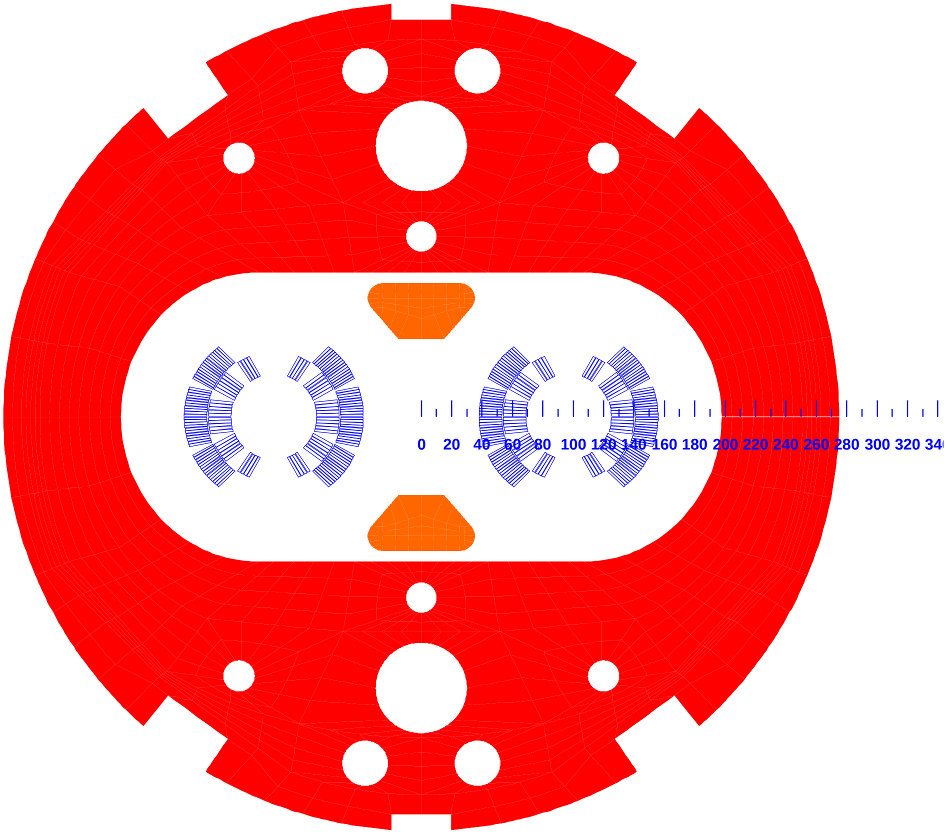

Figure 19: The main dipole consists of separate excitation coils placed around the two beam channels and mounted in one common iron yoke. The objectives for the iron yoke optimization are a high dipole field (max B), low variation of the quadrupole field component versus excitation (min Δb_2), low variation of the sextupole field component versus excitation (min Δb_3), and a small outer yoke radius (min r_Y). The design variables of the optimization problem are the position and radius of the holes in the yoke, the shape of the iron insert, the shape of the collars and the outer yoke radius.

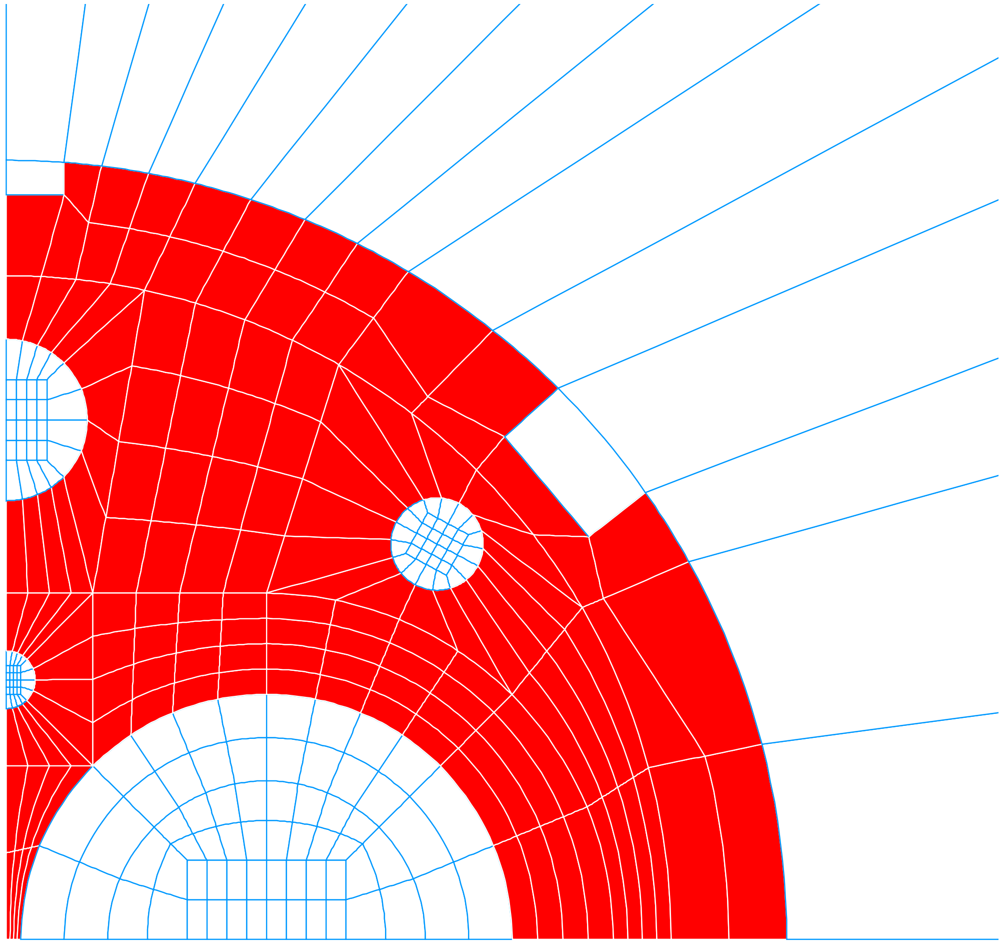

Figure 20: Cross section of a double aperture magnet with separated collars together with the “faceting” used as input for the mesh