CERN/PS 96-41 (DI)

OPTICAL DESIGN OF A BEAM DELIVERY SYSTEM USING A ROTATOR

Michael Benedikt and Christian Carli

Abstract

Hadron therapy for cancer is greatly improved by the use of a gantry. A gantry makes it possible to deliver the radiation dose to the tumour from different directions, so that the entry dose is spread out. The problem for the machine designer is to make the gantry optics independent of the gantry rotation. This results in complicated matching conditions; especially if the emittances in the two transverse phase spaces are unequal. We recapitulate the theory of a rotator to feed a single gantry and we generalize to beam delivery systems using one rotator to feed several gantries. We give some design examples for rotators and outline some criteria that are important for keeping reasonable beam sizes in the rotators. We give a modular design for a beam delivery system; these modules can be combined in different ways, to give systems consisting of one rotator and an arbitrary number of gantries.

Contents

1 INTRODUCTION

In an accelerator facility for radiation therapy, using protons and light ions, the beam delivery system (BDS) is particularly sophisticated. For conformal tumour treatment it is an advantage to have variable incidence angles, which is achieved by the use of a gantry. A *gantry* is a section of beam line, which can be rotated around the patient with respect to the horizontal axis. This raises the optical problem of matching the beam coming from the accelerator to the rotating gantry, so that the patient always "*sees*" the same beam.

Two ways of matching the rotating optics of the gantry to the fixed optics of the machine, while keeping the beam behaviour in the gantry itself independent of the rotation angle, have been proposed. The first, which we shall call the "*symmetric beam method*", severely restricts flexibility by imposing equal emittances, equal Twiss functions and zero dispersion at the entry to the gantry. The second, which we shall call the "*rotator method*", avoids these restrictions. A *rotator* is a section of bending-free transfer line that is rotated in proportion to the gantry angle. The design of a rotator has to be done with great care to avoid problems with beam size, alignment and chromaticity. The rotator method is less well known and to the authors' best knowledge it was first proposed by Lee Teng [1].

For proton therapy one has a choice between a cyclotron and a synchrotron but for ions (e.g. carbon or oxygen), the only practical solution is the latter. The cyclotron can deliver beams of quasi-equal emittance in the two planes, suitable for the symmetric beam or the rotator method, whereas a synchrotron, employing resonant extraction, delivers asymmetric beams (small emittance in the plane of extraction) [2], which imposes the use of a rotator. Since there is no restriction on the dispersion function when using a rotator, this has the fortuitous advantage of simplifying the gantry design.

In this paper, we describe the principles of the rotator method, some design examples of rotators are given (FODO, triplet, doublet/FODO) and the advantages of the different structures are discussed. Some draft-designs for the transfer lines and the gantry are presented and some modular layouts of the transfer lines, that make it possible to run a complete complex with just one rotator, are proposed. A formalism to describe the beam size in rotated structures is derived. For completeness, the symmetric beam method is explained in Appendix A.

2 PRINCIPLES OF THE ROTATOR METHOD

2.1 Beam Rotator feeding one Gantry

In the face of the complex problem of matching to the gantry, the rotator provides an efficient solution, comprising a rotatable, bending-free section of transfer line with phase advances of 2π and π in the two transverse planes. Consider the transfer matrix of a rotator (we will call this matrix $M_{I,-I}$ to denote that the transfer matrix for the horizontal and the vertical planes are the identity and minus-identity matrices, respectively):

$$M_{I,-I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1}$$

If this section is rotated physically by an angle β with respect to the fixed beamline coming from the accelerator, the positions and angles of each particle at the junction can be transfered into the local coordinate system, fixed to the rotator, by multiplication with the 4x4 rotation matrix $M_R(\beta)$:

$$M_R(\beta) = \begin{pmatrix} \cos(\beta) & 0 & \sin(\beta) & 0 \\ 0 & \cos(\beta) & 0 & \sin(\beta) \\ -\sin(\beta) & 0 & \cos(\beta) & 0 \\ 0 & -\sin(\beta) & 0 & \cos(\beta) \end{pmatrix}. \tag{2}$$

We now add a gantry directly after the rotator and the rotator is set to half the gantry angle α. In order to describe the positions and angles of all particles in the local coordinate system of the gantry, one needs the overall transfer matrix M_O from the end of the fixed beam line to the entry of the gantry, which is obtained by multiplying the component matrices:

$$M_O = \begin{pmatrix} \cos(\frac{\alpha}{2}) & 0 & \sin(\frac{\alpha}{2}) & 0 \\ 0 & \cos(\frac{\alpha}{2}) & 0 & \sin(\frac{\alpha}{2}) \\ -\sin(\frac{\alpha}{2}) & 0 & \cos(\frac{\alpha}{2}) & 0 \\ 0 & -\sin(\frac{\alpha}{2}) & 0 & \cos(\frac{\alpha}{2}) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \cos(\frac{\alpha}{2}) & 0 & \sin(\frac{\alpha}{2}) & 0 \\ 0 & \cos(\frac{\alpha}{2}) & 0 & \sin(\frac{\alpha}{2}) \\ -\sin(\frac{\alpha}{2}) & 0 & \cos(\frac{\alpha}{2}) & 0 \\ 0 & -\sin(\frac{\alpha}{2}) & 0 & \cos(\frac{\alpha}{2}) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} .$$

The righthand side shows the gratifying result that the normal modes [3] of the fixed, incoming beamline map directly into the normal modes of the gantry, independent of the gantry angle.

$$M_O = M_R\left(\frac{\alpha}{2}\right) \cdot M_{I,-I} \cdot M_R\left(\frac{\alpha}{2}\right) = M_{I,-I} \quad . \tag{3}$$

It is worth stressing that this result is mathematically exact under all conditions when considering first-order transfer matrices through perfectly aligned structures. In fact, the above derivation imposes no conditions on the Twiss parameters, dispersion functions or emittances at the junctions between the different parts of the line (incoming line, rotator, gantry). However, the above does not always ensure a practical solution from the point of view of alignment tolerances, aperture requirements and chromatic aberrations. For these reasons it is prudent to design the rotator junctions with equal, or approximately equal Twiss functions, $\beta_x \simeq \beta_y$ and $\alpha_x \simeq \alpha_y$, to avoid any exotic beam behaviour. This will be discussed further in the following sections for different rotator designs.

2.2 Generalization to a BDS with several Gantries

The aim of this section is to show that, with appropriate generalizations, the above idea can be extended in order to construct BDSs consisting of one rotator, feeding several gantries. For such systems, *extension sections*, with optical properties similar to the rotator, are inserted between rotator and gantries.

First, we note that it is sufficient, but not necessary, that the transfer matrix from the entrance of the rotator to the entrance of the gantry is $M_{I,-I}$. One of our main interests is to simplify the gantry design by allowing finite dispersion at the gantry entry and this requires the same horizontal transfer matrix of I or $-I$ from the rotator to all the gantries. However, the choice in the vertical plane between I or $-I$ is free for each gantry. We loose no generality by choosing the horizontal transfer matrix to be unity. Thus we ask the overall transfer matrix to be either $M_{I,-I}$ or $M_{I,I}$. If extension modules, with horizontal and vertical transfer matrices being any combination of I and $-I$, are inserted, it is always possible to find an appropriate rotation angle β for the rotator that gives the required overall transfer matrix of $M_{I,-I}$ or $M_{I,I}$. The four possible extension modules are tabulated in Table 1 with the corresponding overall transfer matrices to the gantry and the relevant rotator angles, expressed in terms of the gantry angle α.

Inserting an extension module with the transfer matrix $M_{I,I}$ is trivial and the particle distribution remains unchanged at the entrance to the gantry. We will consider in more detail the second case in Table 1. To get the overall transfer matrix from the entrance of the rotator to the entrance of the gantry, the following matrices have to be multiplied in beam order:

- Matrix, describing the rotation at the entry to the rotator (β).
- Rotator transfer matrix ($M_{I,-I}$).

Extension module	Rotator angle β	Overall transfer M_O
$M_{I,I}$	$\alpha/2$	$M_{I,-I}$
$M_{I,-I}$	$-\alpha/2$	$M_{I,I}$
$M_{-I,I}$	$\pi-\alpha/2$	$M_{I,I}$
$M_{-I,-I}$	$\pi+\alpha/2$	$M_{I,-I}$

Table 1: Characteristics of single-rotator, multi-gantry delivery systems

- Matrix, describing the reverse rotation at the exit of the rotator and entry to the extension module $(-\beta)$.
- Transfer matrix through the extension module (in this case $M_{I,-I}$).
- Matrix, describing the rotation (α) of the gantry with respect to the fixed extension module.

Now the rotation angle β of the rotator has to be chosen in such a way that the overall transfer matrix is either $M_{I,I}$ or $M_{I,-I}$. It follows directly that for the chosen extension module $\beta = -\frac{\alpha}{2}$ and one obtains:

$$M_O = M_R(-\frac{\alpha}{2}) \cdot M_{I,-I} \cdot M_R(\frac{\alpha}{2}) \cdot M_{I,-I} M_R(\alpha) = M_{I,-I} \quad .$$

The overall transfer matrix is independent of the rotation angle α of the gantry and indeed of the required form ($M_{I,I}$ or $M_{I,-I}$).

3 BEAM SIZES AND σ-MATRIX FORMALISM

In this section we recapitulate briefly the σ-matrix formalism as given in References [3, 4]. The method is then applied to rotating sections of transfer line and demonstrated by computing the beam size inside the rotator (Section 4.2). The same technique can be applied to the extension modules in which a very similar situation exists. Although the extension modules are fixed, there is a changing correlation between the two transverse phase spaces according to the gantry angle.

Let $\vec{\xi} = (x, x', y, y')$ be a position vector, containing the transverse phase space coordinates of a particle. The statistical average of any distribution of particles in phase space is then given by the σ-matrix, defined as: $\sigma_{ij} :=< \xi_i \xi_j >$ i.e.

$$\sigma = \begin{pmatrix} < x^2 > & < xx' > & < xy > & < xy' > \\ < xx' > & < x'^2 > & < x'y > & < x'y' > \\ < xy > & < x'y > & < y^2 > & < yy' > \\ < xy' > & < x'y' > & < yy' > & < y'^2 > \end{pmatrix}$$

where the brackets $<>$ mean the expectation values. It can be shown that the σ-matrix propagates under the action of any linear transformation matrix S according to:

$$\sigma_{ij} \Rightarrow S_{il} \xi_l S_{jm} \xi_m = S_{il} \sigma_{lm} S^T_{mj}. \tag{4}$$

For the present purpose of calculating beam sizes along the rotator, S will take the forms of a transfer matrix M, a rotation matrix M_R, or any multiplication of these matrices.

To make the liaison between the Twiss parameters and emittances of the incoming beamline and the σ-matrix formulation, we make the following equivalences for both transverse phase spaces:

$$\varepsilon_x = \sqrt{< x^2 >< x'^2 > - < xx' >^2}$$

$$\beta_x = \frac{<x^2>}{\varepsilon_x} \qquad \alpha_x = -\frac{<xx'>}{\varepsilon_x} \qquad \gamma_x = \frac{<x'^2>}{\varepsilon_x}$$

For an uncoupled beam, i.e. no correlations between the two transverse phase spaces, all elements of the σ-matrix, coupling the horizontal and the vertical phase space, vanish and thus, the σ-matrix reduces to two 2×2 matrices which can be written in terms of the Twiss parameters and the 1σ-emittances, using the equivalences above.

$$\sigma_{n.c.} = \begin{pmatrix} \varepsilon_x\beta_x & -\varepsilon_x\alpha_x & 0 & 0 \\ -\varepsilon_x\alpha_x & \varepsilon_x\gamma_x & 0 & 0 \\ 0 & 0 & \varepsilon_y\beta_y & -\varepsilon_y\alpha_y \\ 0 & 0 & -\varepsilon_y\alpha_y & \varepsilon_y\gamma_y \end{pmatrix}. \tag{5}$$

In our case we assume the beam to be uncoupled when extracted from the accelerator. Inside the rotator and the extension modules, the beam is coupled, but because of the special optical properties of these sections, the beam is again uncoupled inside the gantry.

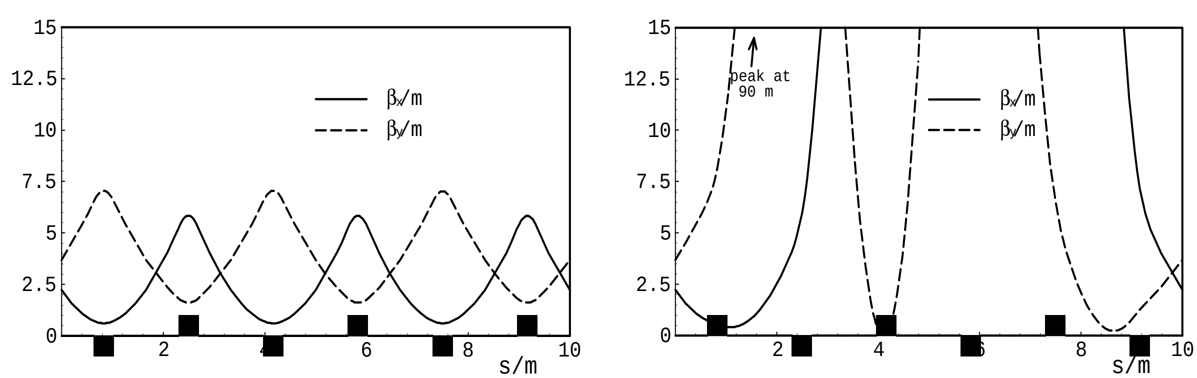
Following this formalism the σ-matrix at any point in the beam line can be computed from the σ-matrix at the beginning by using the appropriate matrices M and M_R. The horizontal and vertical beam sizes are simply given as the square roots of the elements σ_{11} and σ_{33}.

4 ROTATORS

4.1 Optical Designs

In the matrix proof given in Section 2.1, the rotator is represented by its 4x4 transfer matrix and 4x4 rotation matrices at the entry and exit. In the final result, the overall transfer matrix maps the incoming normal modes to those of the gantry without cross terms. The only constraint for rotator design derived by this formalism is that the normal modes in the rotated structure must advance by 2π in one plane and by π in the other. This may give the impression that designing a rotator is a simple task. But one has to be aware of the fact that the overall transfer matrix cannot give any information about beamsizes, chromatic effects, etc. inside the structures.

The following Figs. 1-3 show the geometry and β-functions of three different rotators with the prerequisite phase advances (for simplicity we consider $\mu_x = 2\pi$ and $\mu_y = \pi$).



(a) (b)

Figure 1: β-functions in a three-cell regular FODO rotator (a) not rotated (b) rotated by $\frac{\pi}{2}$

Fig. 1 shows a regular FODO-structure consisting of three cells. The quadrupoles are in two families. The horizontal and vertical phase advances per cell are $(2/3)\,\pi$ and $(1/3)\,\pi$. This section was matched to $\beta_x = 2.2m$, $\alpha_x = 1.8$, $\beta_y = 3.7m$ and $\alpha_y = -1.8$. Fig. 1a shows the β-functions for the unrotated structure (the design case of the line) whereas Fig. 1b shows the β-functions for the same structure, rotated by $\frac{\pi}{2}$. Comparison of (a) and (b) dramatically illustrates the problem, inherent in this type of structure. The rotation is equivalent to a change from focusing to defocusing in the unrotated structure and leads to exceedingly large fluctuations of the β-functions compared to the design case. This behaviour arises due to the opposite signs of the α-functions at the input. Note, this is not a mismatch. Both cases map the input to the output correctly, confirming the mathematics.

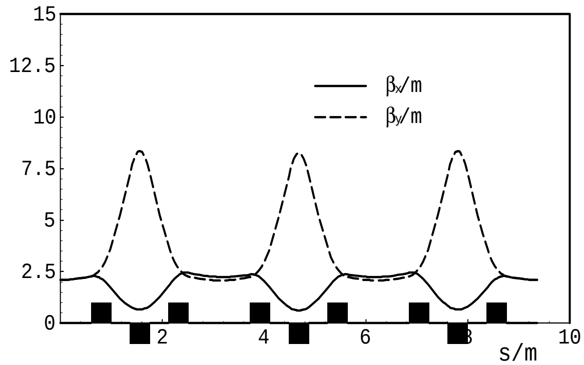


Figure 2: β-functions in a three-cell regular triplet rotator

A regular triplet structure avoids this problem. Fig. 2 shows a structure with three triplets. As for the previous FODO structure only two power supplies are needed. The phase advances per triplet are the same as above but because of the symmetry α_x and α_y are always zero at the input and inter-cell positions. The structure was matched to $\beta_x = \beta_y = 2.1m$. This type of rotator, matched with equal Twiss parameters at the entry, ensures well controlled Twiss functions at any rotation angle.

The third rotator, shown in Fig. 3, uses a doublet to match symmetric Twiss parameters in the two transverse planes into a short FODO channel of only three quadrupoles. The two doublets succeed in matching the beam into this channel in such a way, that the exotic beam behaviour seen in the FODO rotator (Fig. 1) is completely subdued. The structure was matched to $\alpha_x = \alpha_y = 0$ and $\beta_x = \beta_y = 4m$. The central FODO channel gives the required phase advances of 2π horizontal and π vertical. One advantage of this rotator, compared to the triplet structure, is that only seven quadrupoles are needed. On the other hand, the triplet is working with only two power supplies, whereas this structure requires four.

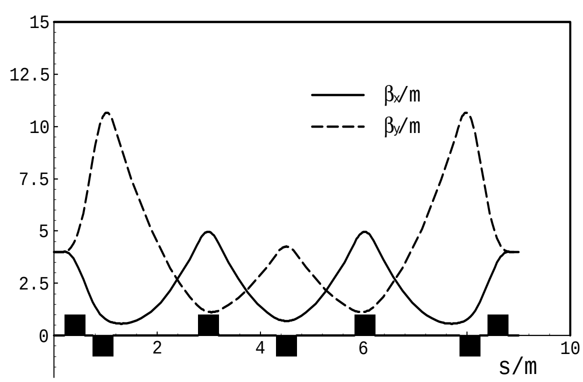


Figure 3: β-functions in a composite doublet/FODO rotator

4.2 Beam Sizes

Inside the rotator, we may be tempted to focus strongly, in order to get a large phase advance in a short distance, but small beam sizes usually lead to large sizes elsewhere and in a transfer line we would like to maintain moderate beam sizes for several reasons :

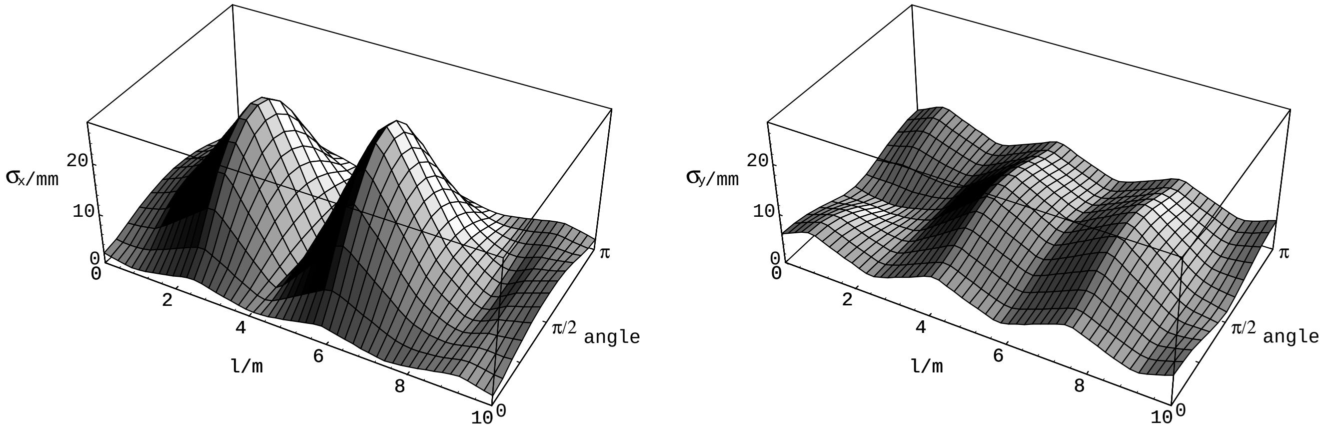
- Apertures of magnets and the vacuum chamber
- Chromatic aberrations
- Alignment tolerances and power supply specifications

The following Figs. 4, 5 and 6 show the beam sizes in the three rotator examples. The calculations have been made using the theory given in Section 3. In each case there are two plots showing the 1σ-beam size seen in the local coordinates, fixed to the rotator, as a function of the rotation angle. As already mentioned in the introduction, a slow extracted beam from a synchrotron usually has a significantly smaller transverse emittance in the extraction plane and to take this effect into account a horizontal emittance of $2 \cdot 10^{-6}\pi\ m$ and a vertical emittance of $10 \cdot 10^{-6}\pi\ m$ were assumed.

The FODO structure of Fig. 4 exhibits the extreme ratio of the maximum to minimum beam size of 43, whereas the other two structures in Figs. 5 and 6 control this ratio to less than 3.3. This smoother behaviour gives a more regular phase advance and provides structures which are less sensitive to alignment tolerances and chromatic errors. It is indeed quite remarkable that these structures can provide a behaviour which is comparable to a static line. In fact in a practical design, the aperture requirements of the rotator would probably be no more demanding than the requirements in the various matching quadrupoles after extraction. Table 2 summarizes the maximal and minimal beam sizes at 0^o, 45^o and 90^o for the three rotators.

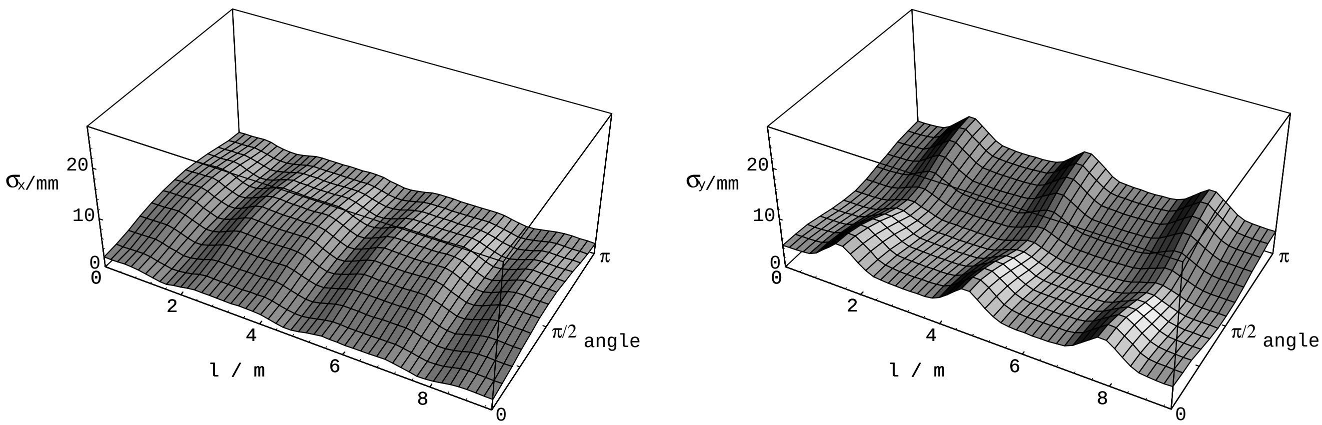
	FODO			Triplet			Doublet/FODO		
	0^o	45^o	90^o	0^o	45^o	90^o	0^o	45^o	90^o
$\sigma_{h,max}[mm]$	3.4	14	28	2.2	3.9	5	3.3	5.4	7
$\sigma_{h,min}[mm]$	1.1	1.3	0.65	1.1	2	2.5	1	1.9	2.4
$\sigma_{h,max}/\sigma_{h,min}$	3	11	43	2	2	2	3.3	2.8	2.9
$\sigma_{v,max}[mm]$	8.4	15	13.3	9.1	7	4.1	10	8	4.6
$\sigma_{v,min}[mm]$	4	3.5	0.9	4.5	3.5	2	3.3	2.6	1.5
$\sigma_{v,max}/\sigma_{v,min}$	2.1	4	15	2	2	2	3	3.1	3

Table 2: Maximal and minimal beam sizes in rotators



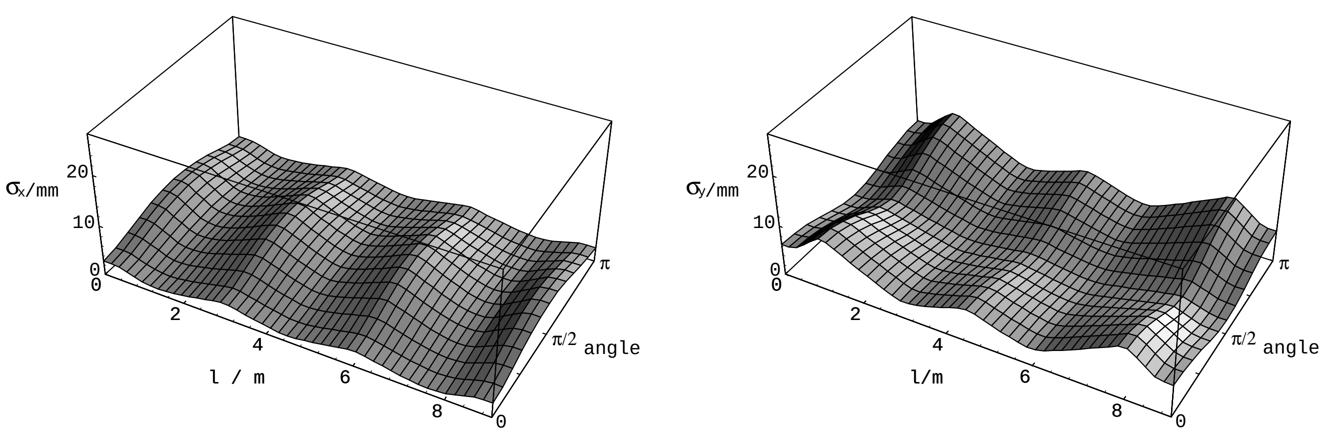
(a) (b)

Figure 4: Horizontal (a) and vertical (b) beam sizes in a FODO rotator



(a) (b)

Figure 5: Horizontal (a) and vertical (b) beam sizes in a triplet rotator



(a) (b)

Figure 6: Horizontal (a) and vertical (b) beam sizes in a doublet/FODO rotator

5 DESIGN OF A MODULAR BEAM DELIVERY SYSTEM WITH ROTATORS

In this section we are presenting a design of a BDS which incorporates the ideas developed above. In the design of the BDS we assumed, that the width $\delta := \Delta p/p$ of the momentum distribution is small enough that it does not have an excessive impact on the beam size. More quantitatively we would like,

$$\gamma D^2 + 2\alpha DD' + \beta D'^2 \quad \lesssim \quad \frac{\varepsilon}{\delta^2} \quad . \tag{6}$$

Note, that the above expression is invariant in bending-free regions and it varies only slightly inside the extension modules with bending. If there is a large momentum spread (this might occur, when a fixed-energy cyclotron is used, since this implies the use of absorbers for energy adjustment), the dispersion may contribute dominantly to the beam size. In this case, the considerations about the optics must be reviewed and the contributions of the dispersion and the betatron oscillations merged. The earlier conclusion, that a FODO rotator results in exotic beam behaviour still applies. Here we concentrate on a BDS accepting a small momentum spread and thus we can apply directly the principles from Section 4.

The minimum possible bending radius of the entire BDS is $1.6m$ (in the gantry), this is compatible with the use of conventional magnets for delivering protons (say with a kinetic energy $T = 250MeV \Longrightarrow$ beam rigidity $p/q = 2.43Tm \Longrightarrow$ maximal field $1.5T$). We assumed that the emittances to be transfered are $\varepsilon_x = 2 \cdot 10^{-6}\pi\ m$, $\varepsilon_y = 5 \cdot 10^{-6}\pi\ m$. At the treatment volume, the following constraints are applied to the optical parameters:

- Twiss β-functions of $\beta_x = 2m$, $\beta_y = 0.80m$ to have a round beam with (1σ) equal $2mm$.
- Dispersion $D = 0$, so that the beam size is not increased and to prevent a strange shape of the voxel, i.e. a correlation of the position x and the longitudinal coordinate. This constraint linked to a requirement for zero disperion at the entry to the gantry is mainly responsible for the considerable length of some other designs [5, 6].
- Derivative of the dispersion $D' = 0$. This constraint is not very stringent and is one that has been removed by other authors e.g. Ref. [6]. However, no significant improvements for the present design were found and the constraint has been maintained.
- Parallel scanning of the tumour by the beam is preferred (to reduce the entry dose) although it is not absolutely necessary. In the specified gantry, parallel scanning is implemented in the horizontal plane. In the vertical plane the effective source to isocentre distance is $8m$.

The following design features were implemented:

- Use of a rotator to allow unequal emittances in the two transverse planes and finite dispersion at the entry to the gantry. Thus, it is no longer necessary to close the dispersion bump inside the gantry, so facilitating the optics design.
- Addition of extension modules, whose phase advances are multiples of π, to serve several gantries with a single rotator.
- Equal Twiss parameters in both transverse planes at the entry to the rotator (the Twiss functions are then automatically equal at all junctions between rotator, extension modules and gantries).

We foresee an overall modular layout of the BDS. The different modules are:

- Matching from the accelerator to the BDS : Note that this section will depend on the output of the accelerator and will have to be redesigned for a specific machine. Twiss functions and the dispersion along the line are shown in Fig. 7.
- Matching to the rotator and launching the dispersion for the gantry: This module matches the Twiss parameters needed for the rotator (which the gantry must accept) and provides

the correct dispersion for the gantry (which the rotator must be able to transmit). The 1:1 or 1:-1 mappings of the subsequent modules mean that the dispersion and also the Twiss functions are transmitted directly to the gantry. Twiss functions and the dispersion along the line are shown in Fig. 8.

- Rotator consisting of a doublet matching into a FODO channel and a doublet: phase advance 2π and π in the horizontal and vertical planes; at entrance/exit the Twiss parameters are symmetric in both transverse phase spaces with $\beta = 4m$ and $\alpha = 0$. Twiss functions and dispersion along the line are shown in Fig. 9.
- Extension modules with phase advances 2π and π. An extension module comprises a straight-through channel or a deflected channel, which is chosen by powering a pair of dipoles.
  - Extension without deflection : the same optics as for the rotator is used (Fig. 9).
  - Extension with deflection : an optics similar to the rotator is used with bending magnets, inserted at appropriate positions between the doublets and the FODO channel, in order to make a closed dispersion bump. Twiss functions and the dispersion along the line are shown in Fig. 10.
- Isocentric gantry:
  - No optical elements or transverse scanning system after the last deflection towards the patient; otherwise the radius of the gantry is enlarged.
  - There are two transverse scanning magnets; one for each plane (see Fig. 12).
  - For the moment, we imagine conventional magnets. If superconducting dipoles are used for the last bending, the edge focusing has to be replaced by a quadrupole between the two dipoles.
  - This particular gantry design is intended for fast scanning in both transverse planes and slow energy scanning. This system is adapted for a synchrotron.

  The dispersion at the entrance to the gantry is not zero, because we profit from the fact, that the dispersion is rotated by the rotator. Twiss parameters and the dispersion along the gantry are shown in Fig. 11.

The modules listed above can be combined in different ways, resulting in more or less complicated BDSs. Two possible combinations are illustrated. The simplest configuration is a single gantry directly following a rotator. This is shown in Figs. 13 and 14. A more complicated BDS, that comprises 3 gantries in a row is shown in Figs. 15 and 16. When the beam is delivered, for example to the second gantry, it passes the rotator, one extension module without bending and one with bending, to finally reach the gantry (powered elements are shown with dark shading).

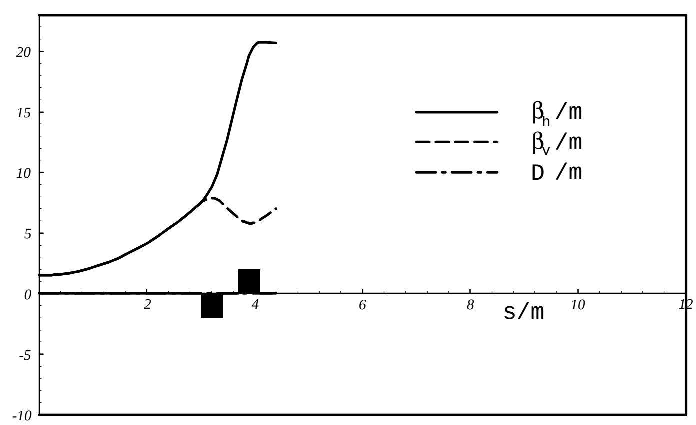


Figure 7: Twiss functions and dispersion in the module matching the extraction to the BDS;

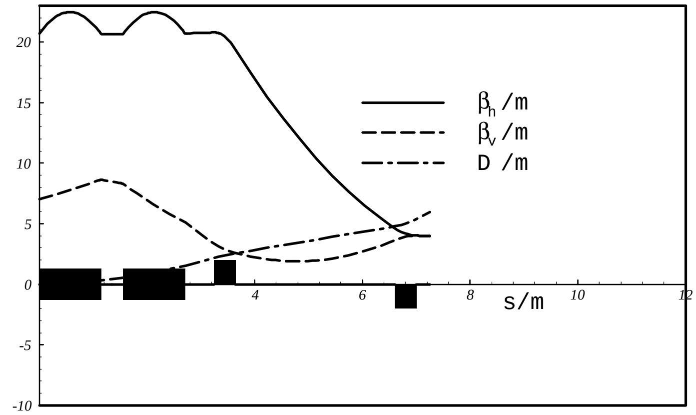


Figure 8: Twiss functions and dispersion in the module generating the dispersion and matching to the rotator

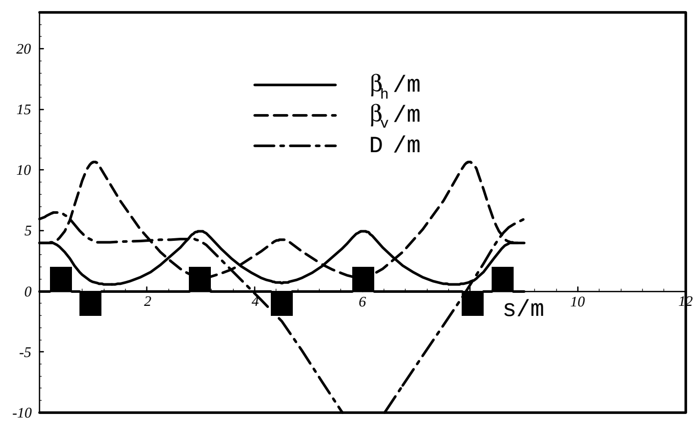


Figure 9: Twiss functions and dispersion in the rotator

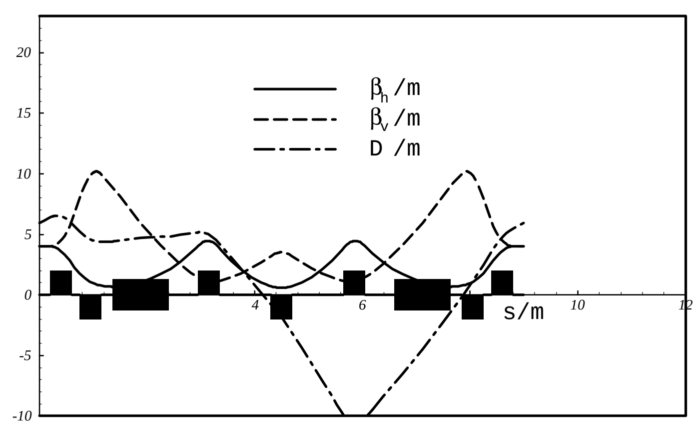


Figure 10: Twiss functions and dispersion in the extension module with deflection.

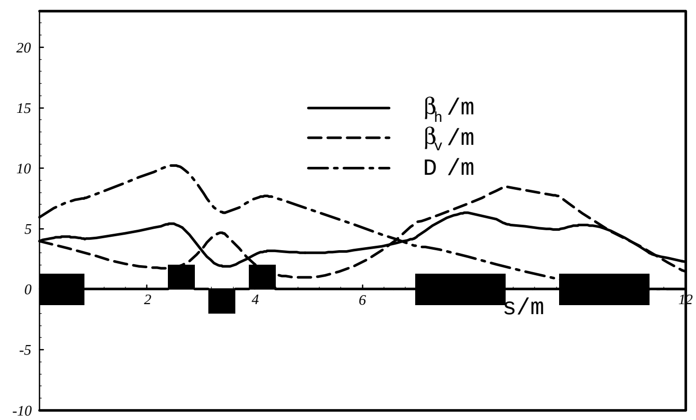


Figure 11: Twiss functions and dispersion in the gantry

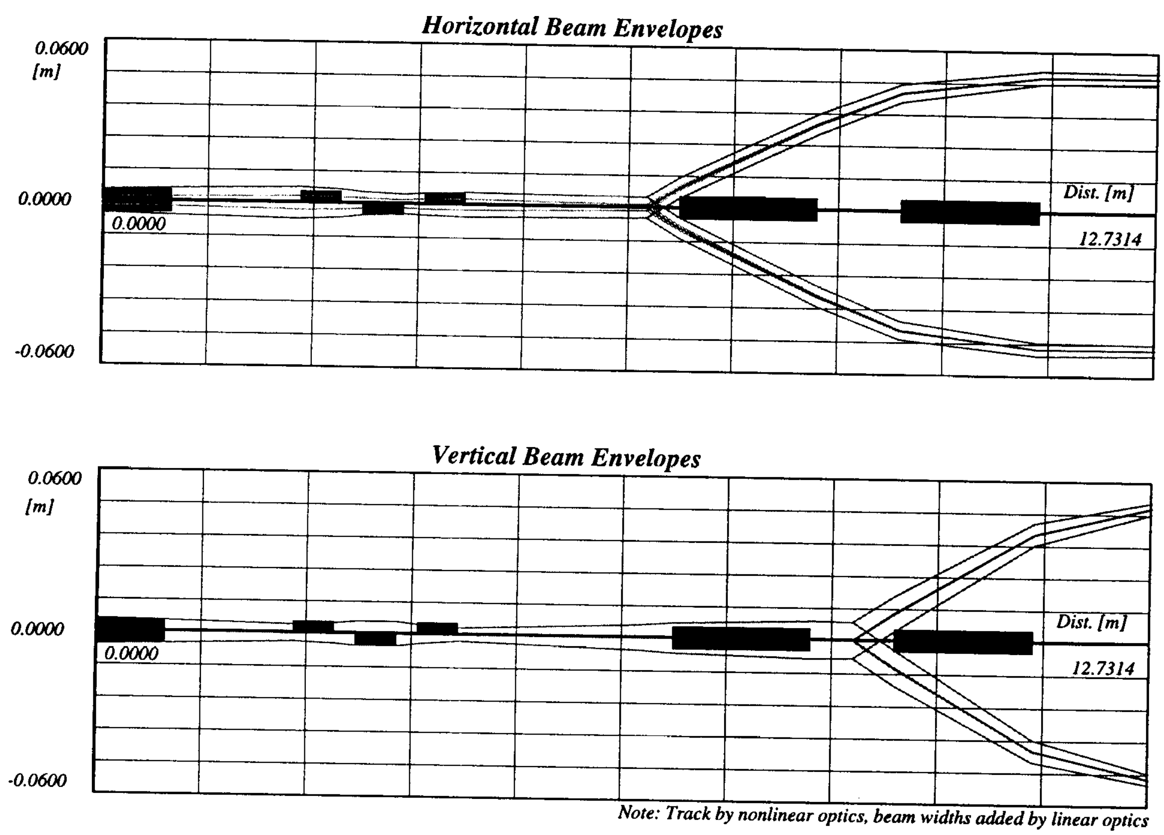


Figure 12: Envelopes inside the gantry for horizontally and vertically scanned beam

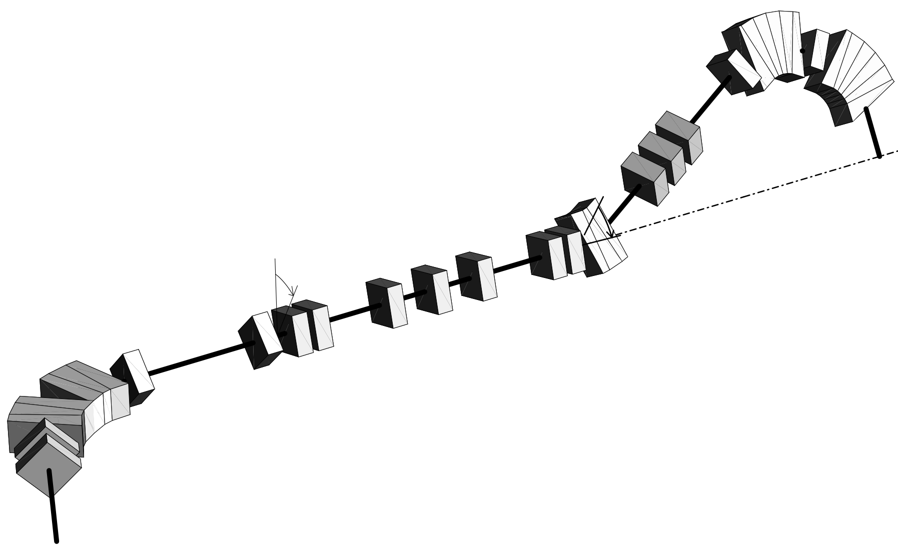


Figure 13: 3D view of the minimal BDS with only one gantry connected directly to the rotator; the gantry is rotated by 60 degrees

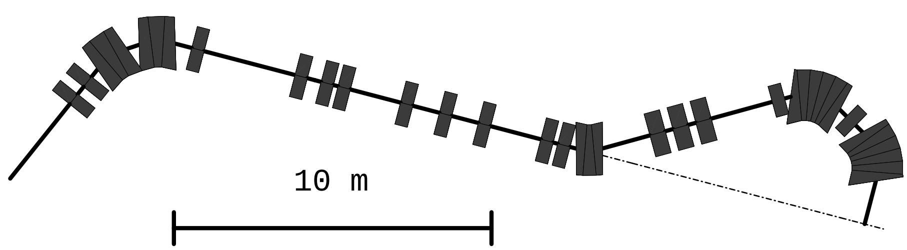


Figure 14: Top view of the minimal BDS with only one gantry connected directly to the rotator; the gantry is rotated by 60 degrees

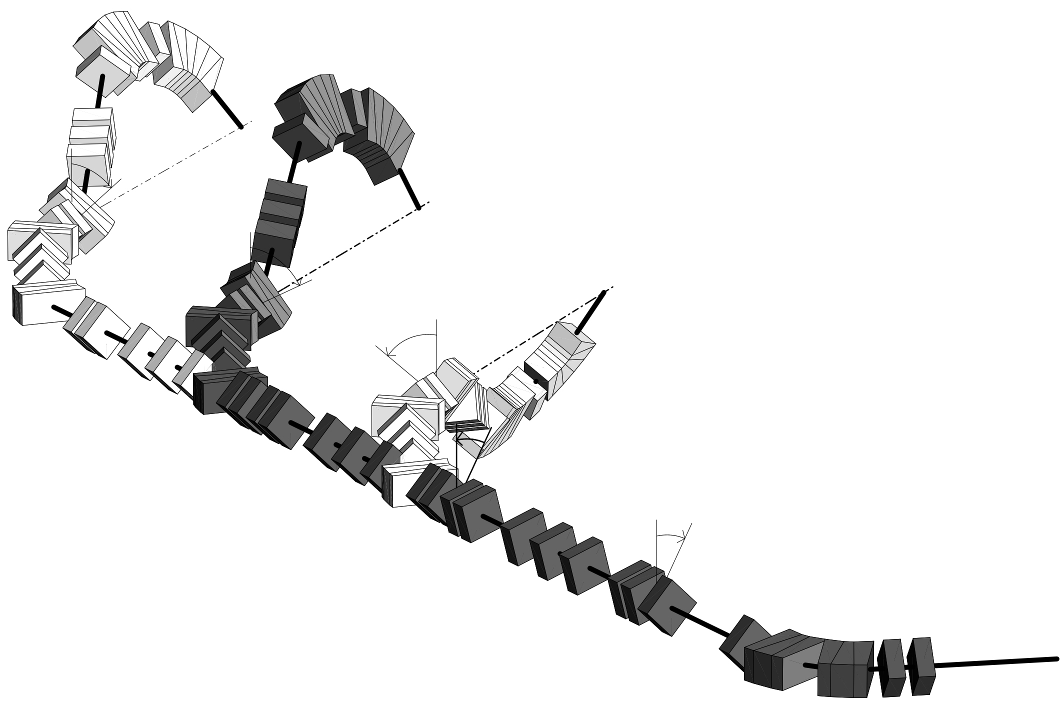


Figure 15: 3D view of a modular BDS with 3 gantries in a row

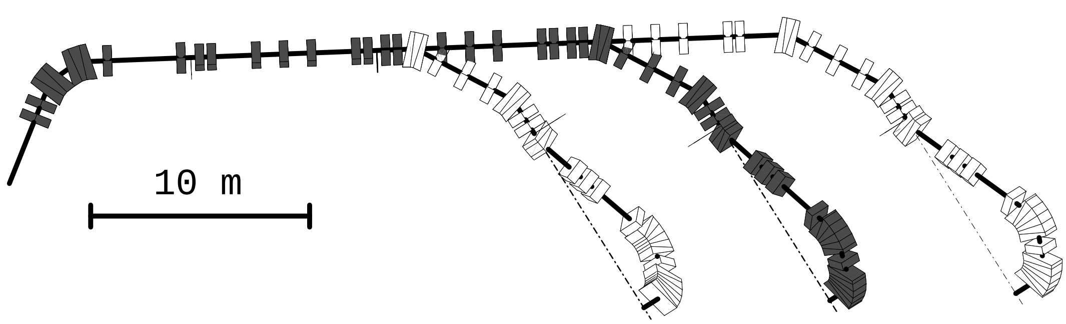


Figure 16: Top view of a modular BDS with 3 gantries in a row

6 CONCLUSIONS AND OUTLOOK

The theory for using a rotator to match a beam into a gantry has been reviewed. This scheme is generalized to one rotator feeding several gantries. Some design principles about rotators and a formalism to calculate the beam size, when this device is rotated, are presented. In particular, it was pointed out, that a regular FODO structure is not well suited for a rotator; but structures with symmetry of the Twiss functions between the two transverse planes (at the entry and the exit of the rotator) are proposed.

A modular design for a BDS is shown. The different modules can be combined, giving systems with one or several gantries and various geometries. In this report we concentrate on a BDS for beams with relatively small momentum spreads. We have tried to keep reasonable β-functions throughout the designs and expect, therefore, that chromatic aberrations and alignment tolerances will be acceptable. However, as one of the next steps, the chromatic effects and sensitivity to alignment errors, especially in the rotating structures, need to be evaluated.

Another possible generalization concerns the design of a beam delivery system for a larger momentum spread. In this case, zero dispersion at the treatment volume becomes more important and consequently the advantages of the rotator more obvious. The beam size in the rotator, the subsequent lines and the gantry, may be dominated by dispersive effects. Considerations about the beam sizes have to be revised in the sense that the betatron oscillations and the dispersive effects have to be merged, leading to some "effective" emittance and β-functions.

ACKNOWLEDGEMENTS

This work was performed at CERN as a contribution to the Proton Ion Medical Machine Study (PIMMS) that is being hosted by the PS Division. The authors would like to thank Phil Bryant for his help.

References

[1] L. C. Teng, *private communications*, laboratory notebook (Jan. 1970) and int. rep. LL-134 (Oct. 1986)

[2] P. Bryant, private communication *Notes of an Informal Meeting of the Proton-Ion Medical Machine Study Group*, CERN/PS (Feb. 1996)

[3] J. Buon, *A Statistical Description of a Particle Beam Subjected to a Linear and Coupled Betatron Motion*, LAL/RT 96-03 (April 1996)

[4] D. C. Carey, *The Optics of Charged Particle Beams*, Harwood Academic Publishers (1987)

[5] E. Pedroni, *Beam Delivery*, Proceedings of the First International Symposium on Hadrontherapy, (Elsevier, 1994) ISBN 0-444-81918-5

[6] M. Pavlovic, *Beam-Optics Study of the Gantry Beam Delivery System for Light-Ion Cancer Therapy*, GSI-Preprint, 95-83 (1995)

[7] H. Grote and C. Iselin, *The MAD Program (Methodic Accelerator Design)*, CERN/SL/90-13 (AP) (1990)

Appendix

A SYMMETRIC BEAM METHOD

In the symmetric beam method, the gantry is matched directly to the fixed beam line coming from the accelerator. In order to obtain always the same optics inside the gantry, independent of its angle, the beam has to be symmetric with respect to rotations at the entry to the gantry. To derive the necessary conditions we will use the σ-matrix formalism given in Section 3. We compute the σ-matrix at the entrance to the gantry as a function of the rotation angle and require, that the result is independent of this angle. The σ-matrix for the uncoupled beam before the rotation is of the form given in Equation 5 and the matrix, describing the rotation of the gantry is given in Equation 2. For the σ-matrix at the gantry entrance, i.e. immediately after the rotation one obtains :

$$M_R(\alpha) \cdot \sigma_{n.c.} \cdot M_R^T(\alpha) =$$

$$= \begin{pmatrix} \cos^2(\alpha)\varepsilon_x\beta_x + \sin^2(\alpha)\varepsilon_y\beta_y & -\cos^2(\alpha)\varepsilon_x\alpha_x - \sin^2(\alpha)\varepsilon_y\alpha_y & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\beta_x + \varepsilon_y\beta_y) & \cos(\alpha)\sin(\alpha)(\varepsilon_x\alpha_x - \varepsilon_y\alpha_y) \\ -\cos^2(\alpha)\varepsilon_x\alpha_x - \sin^2(\alpha)\varepsilon_y\alpha_y & \cos^2(\alpha)\varepsilon_x\gamma_x + \sin^2(\alpha)\varepsilon_y\gamma_y & \cos(\alpha)\sin(\alpha)(\varepsilon_x\alpha_x - \varepsilon_y\alpha_y) & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\gamma_x + \varepsilon_y\gamma_y) \\ \cos(\alpha)\sin(\alpha)(-\varepsilon_x\beta_x + \varepsilon_y\beta_y) & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\alpha_x + \varepsilon_y\alpha_y) & \sin^2(\alpha)\varepsilon_x\beta_x + \cos^2(\alpha)\varepsilon_y\beta_y & -\sin^2(\alpha)\varepsilon_x\alpha_x - \cos^2(\alpha)\varepsilon_y\alpha_y \\ \cos(\alpha)\sin(\alpha)(-\varepsilon_x\alpha_x + \varepsilon_y\alpha_y) & \cos(\alpha)\sin(\alpha)(-\varepsilon_x\gamma_x + \varepsilon_y\gamma_y) & -\sin^2(\alpha)\varepsilon_x\alpha_x - \cos^2(\alpha)\varepsilon_y\alpha_y & \sin^2(\alpha)\varepsilon_x\gamma_x + \cos^2(\alpha)\varepsilon_y\gamma_y \end{pmatrix}$$

From the above σ-matrix one can deduce, that a beam distribution independent of the gantry rotation angle α can only be obtained, if the following constraints are fulfilled at the entry to the gantry:

$$\begin{array}{lrl} \text{Equal emittances} & & \varepsilon_x = \varepsilon_y \\ \text{Equal Twiss parameters} & & \beta_x = \beta_y \\ & & \alpha_x = \alpha_y \\ & \Rightarrow & \gamma_x = \gamma_y \end{array}$$

The optical constraint, to have the same Twiss parameters for both, the horizontal and the vertical phase space, turned out to be not very limiting in practice, i.e. it is relatively easy to fulfill. The constraint, that the emittances in the two transverse planes have to be identical is a severe problem, if the gantry is to be used with beam extracted resonantly from a synchrotron. Although a linac and a cyclotron can in principle deliver equal emittances, it is not easy to prove that this is the case. On-line emittance measurements at low intensity are not trivial and even under ideal conditions emittance is a more difficult quantity to measure than say beam position. It may be that a rotator would be an advisable addition to all systems.

B MAD INPUT FILE FOR ALL MODULES OF THE BDS

In order to give the exact geometry and the beam optics parameters of the modular BDS we give the Input-File for a MAD [7] run. The following is the Input File feeding the second gantry in the BDS comprising 3 gantries in a row; all modules are contained and thus the generalization to other combinations of the modules is straightforward.

```
TITLE, S="Modular Beam Delivery to three Gantrys"

 bet = 30*RADDEG  ! Gantry rotation angle : 30 degrees
 G_rot   : SROT, ANGLE= bet        ! rotation angle of the gantry
 R_roti  : SROT, ANGLE= 0.5 * bet  ! rotation of rotator
 R_roto  : SROT, ANGLE=-0.5 * bet  ! rotation back to line fixed in space

 M_D3   : DRIFT, L=0.30
 M_QF   : QUADRUPOLE, L=0.40, K1= 1.3761
 M_D2   : DRIFT, L=0.30
 M_QD   : QUADRUPOLE, L=0.40, K1=-1.3753
 M_D1   : DRIFT, L=3.0
 ! Matching Accelerator - section to 'produce' the dispersion
 ADAP   : LINE = ( M_D1, M_QD, M_D2, M_QF, M_D3 )

 D_D3   : DRIFT, L=0.25
 D_QD   : QUADRUPOLE, L=0.40, K1=-0.7421
 D_D2   : DRIFT, L=2.9610
 D_QF   : QUADRUPOLE, L=0.40, K1= 0.4560
 D_D1   : DRIFT, L=0.5385
 D_BEN  : SBEND, L=66*RADDEG, ANGLE=33*RADDEG, E1=16.5*RADDEG, E2=16.5*RADDEG
 D_DB   : DRIFT, L=0.20
 ! Matching to the rotator - 'produce' the dispersion needed at gantry entrance
 DISP   : LINE = ( D_BEN, D_DB, D_DB, D_BEN, D_D1, D_QF, D_D2, D_QD, D_D3 )

 R_rot  : SROT, ANGLE=0.3446
 R_Dr   : DRIFT, L=0.20
 R_QFD  : QUADRUPOLE, L=0.40, K1= 2.9332
 R_DD   : DRIFT, L=0.15
 R_QDD  : QUADRUPOLE, L=0.40, K1=-3.0883
 R_Di1  : DRIFT, L=1.6344
 R_QF   : QUADRUPOLE, L=0.40, K1= 2.8577
 R_Di2  : DRIFT, L=2.75-R_Di1[L]
 R_QD   : QUADRUPOLE, L=0.20, K1=-2.3382
 ! Rotator (and (1:1)/(1:-1) prolongation section)
 ROTA  : LINE = ( R_Dr, R_QFD, R_DD, R_QDD, R_Di1, R_QF, R_Di2, R_QD, &
                 R_QD, R_Di2, R_QF, R_Di1, R_QDD, R_DD, R_QFD, R_Dr )

 B_Dr   : DRIFT, L=0.20
 B_QFD  : QUADRUPOLE, L=0.40, K1= 2.6610
 B_DD   : DRIFT, L=0.15
 B_QDD  : QUADRUPOLE, L=0.40, K1=-2.5152
 B_Di11 : DRIFT, L=0.2112
 B_Ben  : SBEND, L=2.0*PI/6., ANGLE=PI/6., E1=PI/12., E2=PI/12.
 B_Di12 : DRIFT, L=0.75-B_Di11[L]
 B_QF   : QUADRUPOLE, L=0.40, K1= 3.0182
 B_Di2  : DRIFT, L=0.95
 B_QD   : QUADRUPOLE, L=0.20, K1=-2.5073
 ! prolongation section with deflection
 B_1TO1 : LINE = ( B_Dr, B_QFD, B_DD, B_QDD, B_Di11, B_Ben, B_Di12,   &
                  B_QF, B_Di2, B_QD,                                  &
                  B_QD, B_Di2, B_QF, B_Di12, B_Ben, B_Di11, B_QDD,    &
                  B_DD, B_QFD, B_Dr )
```

```
EC   = 0.2924
EE   = 0.4078
G_DF    : DRIFT, L=1.40
G_DH    : DRIFT, L=0.50
G_B2    : SBEND, L=1.60*PI/3., ANGLE=PI/3., E1=EC, E2=EE
G_B1    : SBEND, L=1.60*PI/3., ANGLE=PI/3., E1=EE, E2=EC
G_DSC1 : DRIFT, L=2.1935
G_DSC2 : DRIFT, L=0.40
G_QF2  : QUADRUPOLE, L=0.50, K1= 1.1507
G_DD2  : DRIFT, L=0.25
G_QD   : QUADRUPOLE, L=0.50, K1=-2.8055
G_DD1  : DRIFT, L=0.25
G_QF1  : QUADRUPOLE, L=0.50, K1= 1.6336
G_DUP  : DRIFT, L=4.242562-G_DD1[L]-G_DD2[L]-G_DSC1[L]
G_BUP  : SBEND, L=1.60*PI/6., ANGLE=-PI/6., E1=-PI/12., E2=-PI/12.
! the gantry itself
GANTRY : LINE = ( G_BUP, G_DUP, G_QF1, G_DD1,     &
                  G_QD, G_DD2, G_QF2,  &
                  G_DSC1, G_DSC2, G_B1, G_DH, G_DH, G_B2, G_DF )

ALL2   : LINE = ( ADAP, DISP, R_roti, ROTA, R_roto, ROTA, B_1TO1,   &
                  G_rot, GANTRY )

 USE, ALL2
 SELECT, TWISS, FULL
 SURVEY
 TWISS, BETX=1.50, BETY=1.50, COUPLE
```