Improving the equivalent-photon approximation in electron–positron collisions

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Abstract

The validity of the equivalent-photon approximation for two-photon processes in electron–positron collisions is critically examined. Commonly used forms to describe hadronic two-photon production are shown to lead to sizeable errors. An improved two-photon luminosity function is presented, which includes beyond-leading-logarithmic effects and scalar-photon contributions. Comparisons of various approximate expressions with the exact calculation in the case of the total hadronic cross section are given. Furthermore, effects of the poorly known low-Q^2 behaviour of the virtual hadronic cross sections are discussed.

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With the advent of LEP2, measurements of two-photon processes in a new domain of $\gamma\gamma$ cm energies W will soon become feasible [1]. Together with HERA measurements [2] of γp collisions, detailed insights into the photon structure at the highest energies are ahead of us. At HERA, measurements of the scattered lepton in the luminosity system restricts the photon virtuality Q^2 in tagged γp collisions to values below about $0.02\,\mathrm{GeV}^2$. This number is well below the typical hadronic scale $Q_0 \sim m_\rho$ and, hence, the extrapolation to the real photon–proton cross section is under good control. In contrast, two-photon processes at LEP cannot be measured at such low virtualities Q_i^2 of the two photons. Either antitagging conditions are imposed or the scattered leptons are measured at rather large angles (single- or double-tag events). In the first case, photon virtualities up to several GeV^2 are included in the data sample, while Q^2 values well above $1\,\mathrm{GeV}^2$ are selected in the second case. Even the advent of the so-called very small angle tagger will select events with an average Q^2 of still about $0.5\,\mathrm{GeV}^2$ [1] (see also table 1).

In any case, the extrapolation of hadronic two-photon processes to zero Q^2 is highly non-trivial, in particular in view of the recent very low-Q^2 data from HERA [3], which show a significant change in the W-dependence of the virtual γp cross section as Q^2 exceeds m_ρ^2. We hasten to add that measurements of hadronic two-photon cross sections at non-zero photon virtualities might give a glimpse on the elusive QCD Pomeron [1, 4]. In this letter we estimate the uncertainties associated with the extraction of the theoretically interesting hadronic two-photon cross sections from the measured $\mathrm{e}^+\mathrm{e}^-$ ones and propose an improved equivalent-photon approximation (EPA). Let us recall that the EPA is implemented, in one way or another, in practically all programmes [5–10] to generate hadronic two-photon interactions.

The concept of the two-photon luminosity function $\mathcal{L}(\tau)$ arises when one relates the cross section for the scattering of two *real* photons $\sigma_{\gamma\gamma}(W^2)$ to the measured $\mathrm{e}^+\mathrm{e}^-$ cross section $\sigma(s)$

$$\sigma(s) = \int_{\tau_{\mathrm{th}}}^{1} \mathrm{d}\tau\, \sigma_{\gamma\gamma}(W^2 = \tau s)\, \mathcal{L}(\tau)\ . \tag{1}$$

Here $\sqrt{s} = 2E$ denotes the $\mathrm{e}^+\mathrm{e}^-$ c.m. energy and W the $\gamma\gamma$ c.m. ("hadronic") energy. Usually [11], $\mathcal{L}(\tau)$ is calculated as the product (or, more precisely, the convolution) of two equivalent-photon approximations (EPA):

$$\begin{aligned}
\mathcal{L}_{\mathrm{EPA}}(\tau) &= \frac{1}{\tau} \int N(x_1)\, N(x_2 = \tau/x_1)\, \frac{\mathrm{d}x_1}{x_1} \\
N(x_i) &= \frac{\alpha}{2\pi} \left\{ \left[1 + (1-x_i)^2\right] \ln \frac{Q^2_{i\,\mathrm{max}}}{Q_{i\,\mathrm{min}}} - 2m_{\mathrm{e}}^2 x_i^2 \left[\frac{1}{Q^2_{i\,\mathrm{min}}} - \frac{1}{Q^2_{i\,\mathrm{max}}}\right]\right\} \\
Q^2_{i\,\mathrm{min}} &= Q^2_{i0} \equiv \frac{m_{\mathrm{e}}^2\, x_i^2}{1-x_i} \qquad (\theta_{i\,\mathrm{min}} = 0) \\
&= (1-x_i)\ E^2\, \theta^2_{i\,\mathrm{min}} \qquad (\theta_{i\,\mathrm{min}} \neq 0) \\
Q^2_{i\,\mathrm{max}} &= \min\left\{m_\rho^2, (1-x_i)\ E^2\, \theta^2_{i\,\mathrm{max}}\right\} \\
\max\{x_{1\,\mathrm{min}}, \tau/x_{2\,\mathrm{max}}\} &\leq x_1 \leq \min\{x_{1\,\mathrm{max}}, \tau/x_{2\,\mathrm{min}}, 1\}\ .
\end{aligned} \tag{2}$$

The limits on the photon virtualities Q_i^2 and the scaled photon energies $x_i = \omega_i/E$ are determined by the experimental (anti-)tagging cuts on the angles θ_i and energies E_i of the scattered electrons $x_{i\,\mathrm{min}} = 1 - E_i^{\mathrm{max}}/E$, $x_{i\,\mathrm{max}} = 1 - E_i^{\mathrm{min}}/E$.

It is important to realize the three approximations that lead to (2) [11]. First, the Q_i^2 dependence of σ_{TT}, the cross section for transverse photons, is neglected. Rather, for a hadronic cross section the Q_i^2 integrations are cut off at $Q_i^2 = m_\rho^2$ in (2). For $Q_i^2 \lesssim m_\rho^2$ the uncertainty is of the order of Q_i^2/m_ρ^2. Clearly, the approximation will break down for tagged events where $Q^2 \gtrsim m_\rho^2$.

Second, scalar-photon contributions are neglected. Again, for $Q_i^2 \lesssim m_\rho^2$ the uncertainty is bound by Q_i^2/m_ρ^2, but scalar-photon contributions can potentially be large in tagged events.

Third, the kinematics is not treated exactly. Rather it is based on the approximation $Q_i^2 \ll W^2$. For $Q_i^2 \lesssim W^2$, the uncertainty is of order Q_i^2/W^2 and, hence, presumably small for the measurement of cross sections at large W ($W > 5\,\mathrm{GeV}$, say,) since the dynamics will strongly suppress Q^2 values larger than a few GeV^2.

We now investigate the importance of the various contributions and propose an improved luminosity function. Consider the reaction $\mathrm{e}^+(p_1) + \mathrm{e}^-(p_2) \to \mathrm{e}^+(k_1) + \mathrm{e}^-(k_2) + X$ proceeding through the two-photon process $\gamma(q_1) + \gamma(q_2) \to X$, $q_i = p_i - k_i$, $Q_i^2 = -q_i^2$, $W^2 = (q_1 + q_2)^2$. In general, any two-photon process is described by five non-trivial structure functions (two more for polarized initial electrons). Three of these can be expressed through the cross sections σ_{ab} for scalar ($a, b = S$) and transverse photons ($a, b = T$) ($\sigma_{ST} = \sigma_{TS}(q_1 \leftrightarrow q_2)$). The other two structure functions τ_{TT} and τ_{TS} correspond to transitions with spin-flip for each of the photons (with total helicity conservation, of course). We emphasize that the hadronic physics is fully encoded in these structure functions while the connection with the measured $\mathrm{e}^+\mathrm{e}^-$ cross section is a pure matter of QED. This connection is most transparent [1] if we introduce $\tilde{\phi}$, the angle between the scattering planes of the colliding e^+ and e^- in the *photon* c.m.s.:

$$\begin{aligned} \mathrm{d}\sigma = {} & \frac{\alpha^2}{16\,\pi^4\,Q_1^2\,Q_2^2} \sqrt{\frac{(W^2 + Q_1^2 + Q_2^2)^2 - 4\,Q_1^2 Q_2^2}{s\,(s - 4\,m_\mathrm{e}^2)}}\ \frac{\mathrm{d}^3 k_1}{E_1}\,\frac{\mathrm{d}^3 k_2}{E_2} \Big\{ 4\,\rho_1^{++}\rho_2^{++}\,\sigma_{TT} \\ & + 2\,\rho_1^{++}\rho_2^{00}\,\sigma_{TS} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{ST} + \rho_1^{00}\rho_2^{00}\,\sigma_{SS} \\ & + 2\,|\rho_1^{+-}\,\rho_2^{+-}|\,\tau_{TT}\,\cos 2\tilde{\phi} - 8\,|\rho_1^{+0}\rho_2^{+0}|\,\tau_{TS}\,\cos\tilde{\phi} \Big\}\ . \end{aligned} \tag{3}$$

In exact treatments of the phase space, the latter is most often [11–14] expressed in terms of the virtualities Q_i (or the polar angles θ_i) and energies $\omega_i = q_i \cdot (p_1 + p_2)/\sqrt{s}$ of the photons, and of the angle ϕ between the planes of the two scattered electrons defined in the *laboratory* c.m.s.:

$$\frac{\mathrm{d}^3 k_1}{E_1}\,\frac{\mathrm{d}^3 k_2}{E_2} = \frac{2\,\pi}{s - 4\,m_\mathrm{e}^2}\,\mathrm{d}Q_1^2\,\mathrm{d}Q_2^2\,\mathrm{d}\omega_1\,\mathrm{d}\omega_2\,\mathrm{d}\phi\ . \tag{4}$$

Note that, in general, $\tilde{\phi} \neq \phi$ and the hadronic energy W depends non-trivially on the integration variables

$$W^2 = W_A^2 + \sqrt{\frac{W_B^4}{4\,(E^2 - m_\mathrm{e}^2)^2}}\ \cos\phi\ , \tag{5}$$

where

$$W_A^2 = 4\,\omega_1\omega_2 - \frac{{Q_2}^2\,(E - \omega_1)}{E} - \frac{{Q_1}^2\,(E - \omega_2)}{E} + \frac{{Q_2}^2{Q_1}^2}{2\,E^2}$$

[1]See, for example, [11] where also explicit expressions for the density matrices ρ_i^{ab} of virtual photons can be found.

$$+\frac{2\,{m_\mathrm{e}}^2\omega_1\omega_2}{E^2-{m_\mathrm{e}}^2}+\frac{{m_\mathrm{e}}^2\left(2\,E\,{Q_2}^2\omega_1+2\,E\,{Q_1}^2\omega_2+{Q_2}^2{Q_1}^2\right)}{2\,E^2\left(E^2-{m_\mathrm{e}}^2\right)} \tag{6}$$

$$\begin{aligned}W_B^4 = &-{Q_2}^2{Q_1}^2\left(4\,E^2-4\,E\,\omega_2-{Q_2}^2\right)\left(-4\,E^2+4\,E\,\omega_1+{Q_1}^2\right)\\ &-4\,{Q_2}^2{m_\mathrm{e}}^2{\omega_1}^2\left(4\,E^2-4\,E\,\omega_2-{Q_2}^2\right)-4\,{Q_1}^2{\omega_2}^2{m_\mathrm{e}}^2\left(4\,E^2-4\,E\,\omega_1-{Q_1}^2\right)\\ &+4\,{Q_2}^2{Q_1}^2{m_\mathrm{e}}^2\left({Q_1}^2+{Q_2}^2-8\,E^2+4\,E\,\omega_2+4\,E\,\omega_1\right)\\ &+16\,{\omega_2}^2{\omega_1}^2{m_\mathrm{e}}^4+16\,{m_\mathrm{e}}^4\left({Q_1}^2{\omega_2}^2+{Q_2}^2{Q_1}^2+{Q_2}^2{\omega_1}^2\right)\ .\end{aligned} \tag{7}$$

It is only for small Q_1 (and/or small Q_2) and small m_e that $\tilde{\phi}$ coincides with ϕ and W becomes independent of ϕ and, hence, the azimuthal integration becomes trivial.

In order to obtain the most general $\mathrm{e^+e^-}$ cross section at *fixed* W, equivalently at fixed τ, one needs to transform from one of the integration variables of (4) to W [15–18], for example from ϕ to W [15]

$$\frac{\mathrm{d}^3k_1}{E_1}\frac{\mathrm{d}^3k_2}{E_2}=\frac{4\,\pi\,\sqrt{s}}{\sqrt{s-4\,m_\mathrm{e}^2}}\,\mathrm{d}Q_1^2\,\mathrm{d}Q_2^2\,\mathrm{d}W^2\,\frac{\mathrm{d}\omega_1\,\mathrm{d}\omega_2}{\sqrt{-16\,\Delta_4}}\ , \tag{8}$$

where the Gram determinant Δ_4 is a quadratic function in each ω_i. Since the virtual-photon cross sections σ_{ab} do not depend on x_i, the $\mathrm{e^+e^-}$ cross section at fixed $\tau = W^2/s$ can be written as

$$\begin{aligned}\frac{\mathrm{d}\sigma(s,\tau)}{\mathrm{d}\tau} &\equiv \mathcal{L}(\tau,s,m_\mathrm{e}^2,\mathrm{cuts})\,\sigma_{\gamma\gamma}(W^2=\tau s)\\ &= \sum_{a,b=T,S}\int \mathrm{d}Q_1^2\,\mathrm{d}Q_2^2\,J_{ab}(\tau,Q_1^2,Q_2^2;s,m_\mathrm{e}^2;\mathrm{cuts})\,\sigma_{ab}(W^2=\tau s,Q_1^2,Q_2^2)\ ,\end{aligned} \tag{9}$$

where

$$J_{ab}=\frac{1}{\pi}\,\frac{\mathrm{d}\omega_1\,\mathrm{d}\omega_2}{\sqrt{-16\,\Delta_4}}\,\frac{\alpha^2}{2\,\pi^2\,Q_1^2\,Q_2^2}\,\frac{s\sqrt{X}}{s-4\,m^2}\,\rho_1^{ab}\,\rho_2^{ab}\ . \tag{10}$$

Here we have neglected the $\tilde{\phi}$-dependent terms, which were significant only when experimental situations with asymmetric cuts on the two scattered leptons would not be averaged. If one is interested in only the total cross section at fixed W, analytic results can be obtained for the functions J_{ab}. Indeed, an explicit expression for J_{TT} has been derived in [15]. However, in experimental measurements one always applies cuts on the angles and energies of scattered electrons, in which case exact results for J_{ab} cannot be obtained analytically. On the other hand, accurate numerical results (better than 1%, say) are not easily obtained since (9) involves four non-trivial integrations.

For applications at LEP, where the experimental interest is focused on two-photon physics at high W, one can proceed further analytically by expanding in Q_i^2/W^2. We start from the observation that, for $m_\mathrm{e}^2/s \ll 1$, $m_\mathrm{e}^2/W^2 \ll 1$, and when at least one Q_i is small compared with W, the ω integration measure can be replaced by the approximate expression with a Dirac delta distribution:

$$\frac{1}{\pi}\,\frac{\mathrm{d}\omega_1\,\mathrm{d}\omega_2}{\sqrt{-16\,\Delta_4}}=\frac{1}{s}\left\{\frac{\mathrm{d}\omega_1\,\mathrm{d}\omega_2}{4}\delta\left(\omega_1\omega_2-W^2/4\right)+\mathcal{O}(m_\mathrm{e}^2/s,m_\mathrm{e}^2/W^2,Q_i^2/W^2)\right\}\ . \tag{11}$$

The approximation (11) is justified for $W \gg m_\rho$, since hadronic two-photon cross sections σ_{ab} vanish quickly for $Q_i^2 \gtrsim m_\rho^2$. This yields

$$x_1\,\tau\,\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}x_1}=\int \mathrm{d}Q_1^2\int \mathrm{d}Q_2^2\sum_{a,b=T,S}f_a(x_1,Q_1^2)\,f_b(x_2,Q_2^2)\,\frac{\sigma_{ab}(W^2,Q_1^2,Q_2^2)}{\sigma_{\gamma\gamma}(W^2)}\ , \tag{12}$$

where

$$f_T(x,Q^2) = \frac{\alpha}{2\,\pi}\left\{\frac{1+(1-x)^2}{Q^2} - \frac{2\,m_{\rm e}^2\,x^2}{Q^4}\right\} \equiv f_{\rm LLA} + f_{\rm NL}$$
$$f_S(x,Q^2) = \frac{\alpha}{\pi}\,\frac{1-x}{Q^2}\ . \tag{13}$$

The Q_i^2-integration limits are determined by the experimental (anti-)tagging cuts on the angles of the scattered electrons $\theta_{i\,\rm min,max}$. For $m_{\rm e} \ll \omega_i$ we find

$$Q^2_{i\,\rm min} = Q^2_{i0} + 4E^2(1-x_i)\sin^2\frac{1}{2}\theta_{i\,\rm min} \approx Q^2_{i0} + E^2(1-x_i)\theta^2_{i\,\rm min}$$
$$Q^2_{i\,\rm max} = Q^2_{i0} + 4E^2(1-x_i)\sin^2\frac{1}{2}\theta_{i\,\rm max} \approx Q^2_{i0} + E^2(1-x_i)\theta^2_{i\,\rm max}\ . \tag{14}$$

Note the inclusion of the usually neglected term $Q^2_{i0} = m_{\rm e}^2\,x_i^2/(1-x_i)$ in the upper limit of Q^2. Its presence improves the behaviour for $x_i \to 1$. In the limit of small maximum scattering angle $\theta_{1\,\rm max}$ and $\theta_{1\,\rm min} = 0$, the Q_1^2-limits (14) coincide with the results quoted in [19] found from an analysis of ep collisions.

In order to further proceed analytically we make use of a second observation. At $Q_i^2 \ll W^2$ one can, to a very good approximation, assume factorization of the W and Q_i dependences:

$$\sigma_{ab}(W^2,Q_i^2) = h_a(Q_1^2)\,h_b(Q_2^2)\,\sigma_{\gamma\gamma}(W^2)\ . \tag{15}$$

Then (9) can be reduced to the single product of a "true" two-photon luminosity function and the real $\gamma\gamma$ cross section, the former given by

$$\mathcal{L} = \frac{1}{\tau}\int\frac{{\rm d}x_1}{x_1}\prod_{i=1}^{2}\left\{\int {\rm d}Q_i^2\,\left[f_T(x_i,Q_i^2)\,h_T(Q_i^2) + f_S(x_i,Q_i^2)\,h_S(Q_i^2)\right]\right\}\ . \tag{16}$$

Equation (16) defines the improved EPA. We emphasize that, in general, the luminosity function does not factorize into the product of two equivalent-photon approximations; the approximation $Q_i^2 \ll W^2$ is essential.

We now present our results for four typical experimental situations, antitag events (both electrons in the antitag region), low-Q^2 and high-Q^2 single-tag events (one electron in the antitag region and the other in the tag region), and low-Q^2 double-tag events. We choose $\sqrt{s} = 130\,$GeV, $W = 10\,$GeV and define the regions in table 1.

Tag region	$E_{1\,\rm min}$	$E_{1\,\rm max}$	$\theta_{1\,\rm min}$	$\theta_{1\,\rm max}$	$Q^2_{1\,\rm min}$	$Q^2_{1\,\rm max}$
Antitag	0	65	0	1.43	0.572×10^{-12}	5.26
High-Q^2 tag	30	65	1.55	3.67	2.85	34.7
Low-Q^2 tag	30	65	0.28	0.68	0.0931	1.19

Table 1: The cuts on the scattered electrons used in this paper. For convenience, the corresponding Q^2-ranges are also given ($\sqrt{s} = 130\,$GeV, $W = 10\,$GeV; energies in GeV, angles in degrees, and squared momentum transfers in GeV^2).

In order to appreciate the uncertainty associated with the poorly known Q^2 fall off of the hadronic cross sections, we consider three models. The first follows from a parametrization [20] of the γ^*p cross section calculated in a model of generalized vector-meson dominance (GVMD):

$$
\begin{aligned}
h_T(Q^2) &= r\, P_1^{-2}(Q^2) + (1-r)\, P_2^{-1}(Q^2) \\
h_S(Q^2) &= \xi \left\{ r\, \frac{Q^2}{m_1^2}\, P_1^{-2}(Q^2) + (1-r) \left[\frac{m_2^2}{Q^2} \ln P_2(Q^2) - P_2^{-1}(Q^2) \right] \right\} \\
P_i(Q^2) &= 1 + \frac{Q^2}{m_i^2} \, ,
\end{aligned}
\tag{17}
$$

where we take $\xi = 1/4$, $r = 3/4$, $m_1^2 = 0.54\,\mathrm{GeV}^2$ and $m_2^2 = 1.8\,\mathrm{GeV}^2$.

The second model [21] adds a continuum contribution to simple (diagonal, three-mesons only) vector–meson dominance (VMDc):

$$
\begin{aligned}
h_T(Q^2) &= \sum_{V=\rho,\omega,\rho} r_V \left(\frac{m_V^2}{m_V^2 + Q^2} \right)^2 + r_c \frac{m_0^2}{m_0^2 + Q^2} \\
h_S(Q^2) &= \sum_{V=\rho,\omega,\rho} \frac{\xi\, Q^2}{m_V^2}\, r_V \left(\frac{m_V^2}{m_V^2 + Q^2} \right)^2 \, ,
\end{aligned}
\tag{18}
$$

where $r_\rho = 0.65$, $r_\omega = 0.08$, $r_\phi = 0.05$, and $r_c = 1 - \sum_V r_V$. Since photon-virtuality effects are often estimated by using a simple ρ-pole only, we consider also the model defined by (ρ-pole):

$$
\begin{aligned}
h_T(Q^2) &= \left(\frac{m_\rho^2}{m_\rho^2 + Q^2} \right)^2 \\
h_S(Q^2) &= \frac{\xi\, Q^2}{m_\rho^2} \left(\frac{m_\rho^2}{m_\rho^2 + Q^2} \right)^2 \, .
\end{aligned}
\tag{19}
$$

In all cases, the Q_i^2 integrations can be performed analytically so that one is left with only a single one-dimensional numerical integration. A Fortran program is available on request from the author.

Table 2 gives the results for the three models obtained from the improved EPA (16) in comparison with the exact results obtained[2] from (9). Table 2 contains also the results of four approximations, two of which are frequently used EPAs: (i) the ρ-pole model without scalar-photon contributions, i.e. $h_S(Q_i^2) = 0$; (ii) the ρ-pole model in the leading-logarithmic approximation, i.e. neglecting, besides scalar-photon contributions, also the m_e^2/Q^4 term in f_T ($f_\mathrm{NL} = 0$ in (13)); (iii) the EPA defined in (2) [11]; and (iv) the EPA obtained by integrating (2) with logarithmic accuracy [11]:

$$
\begin{aligned}
\mathcal{L}_{PDG} &= \frac{1}{\tau} \left(\frac{\alpha}{\pi} \right)^2 \left[\left(\ln \frac{Q_m^2}{\tau\, m_\mathrm{e}^2} - 1 \right)^2 f(\tau, \tau_m) - \frac{1}{3} \ln^3 \frac{1}{\tau_m} \right] \\
f(x,y) &= \left(1 + \frac{x}{2} \right)^2 \ln \frac{1}{y} - 2 \sqrt{\frac{x}{y}} \left(1 + \frac{x}{2} \right) (1-y) + \frac{x}{2y} \left(1 - y^2 \right) \\
Q_m &= \min \{ m_\rho, \theta_\mathrm{max} E \} \qquad ; \qquad \tau_m = \max \left\{ \frac{4\, \omega_\mathrm{min}^2}{W^2}, \tau \right\} \, .
\end{aligned}
\tag{20}
$$

[2]Details of the exact integration (9) will be described elsewhere [23].

Tag region	Antitag [$\times 10^{-2}$]		Single low-Q^2 tag [$\times 10^{-4}$]	
Model	exact	approx.	exact	approx.
GVMD	105	105	608	610
VMDc	105	105	622	624
ρ-pole	102	102	541	543
ρ-pole, $h_S = 0$	99.5	99.5	493	495
ρ-pole, $h_S = f_{\rm NL} = 0$		110		522
EPA (2)		112		974
PDG (20)		114		—
Tag region	**Single high-Q^2 tag [$\times 10^{-5}$]**		**Double low-Q^2 tag [$\times 10^{-5}$]**	
Model	exact	approx.	exact	approx.
GVMD	789	851	355	357
VMDc	696	749	370	373
ρ-pole	267	286	290	292
ρ-pole, $h_S = 0$	94.9	101	246	247
ρ-pole, $h_S = f_{\rm NL} = 0$		106		247
EPA (2)		0		862

Table 2: The two-photon luminosity $\mathcal{L}$ at $\sqrt{s} = 130\,\mathrm{GeV}$ and $W = 10\,\mathrm{GeV}$ for various models of the two-photon cross sections calculated exactly by integrating (9), in the improved EPA (16), and in the two commonly used EPA (2) and (20).

For $W^2 \geq 4\,E\omega_{\min}$, (20) reduces to the expression quoted by the PDG [22].

The fastest (in terms of CPU time) estimate is, of course, obtained with the closed expression (20) and is accurate to about 10% for the antitag case. Clearly, (20) cannot be used for single-tag cases, but also the double-tag case is beyond its validity. The evaluation of the improved EPA (16) or the standard EPA (2) is still very fast and, definitely, much shorter than the evaluation of the exact result (9). The result of the standard EPA (2) is slightly better than that of (20) for the antitag case. More importantly, (2) can also be applied to the low-Q^2-tag cases, at least for an order-of-magnitude estimate: the results for the single and double low-Q^2-tag cases are overestimated by factors of 1.6 and 2.4, respectively. Results for high-Q^2 tags cannot be obtained since the lower-Q^2 limit exceeds m_ρ^2. An additional drawback of the standard EPA is the fact that one cannot investigate uncertainties in the two-photon luminosity arising from the poorly known low-Q^2 behaviour of the virtual hadronic cross sections.

As can be seen from table 2, the improved two-photon luminosity function (16) works extremely well in the single-tag and antitag cases. The accuracy is better than 1% and the differences compared to the exact results are smaller than uncertainties associated with the low-Q^2 extrapolation of the hadronic cross sections. Even for the high-Q^2 case, the accuracy stays better than 10%.

The effect of neglecting the non-logarithmic term proportional to m_{e}^2/Q_i^2 in f_T is as large as about 10% for the antitag case. The importance of this term clearly diminishes with increasing average Q_i^2. In the case of double low-Q^2 tag it becomes essentially negligible.

The differences between the two models GVMD and VMDc is rather small: in the low-Q^2-tag cases less than 2% and in the high-Q^2-tag case about 6%. This result is not surprising as the two models are quite similar. Yet, the uncertainty in the extraction of real $\gamma\gamma$ cross section, particularly in tagged events, can well be much bigger. Differences of about 7% (12%) are found between the VMDc and ρ-pole models for the single (double) low-Q^2-tag cases. The high-Q^2-tag results of these two models even differ by more than a factor of 2! These differences can be traced back to the extra monopole factor (continuum term) in the transverse-photon part of the VMDc model. On top of that, there is a similarly large effect of scalar photons.

A correct understanding of the hadronic cross sections at low photon virtualities is hence indispensable for a precision measurement of the total hadronic $\gamma\gamma$ cross section. The argument can, of course, be turned around: given the large sensitivity to the low-Q^2 behaviour, and provided the total cross section can be measured in various modes (antitag, low- and high-Q^2 single- and double-tag events) the transition region towards zero Q^2 can be investigated at LEP. It goes without saying that with only minor modifications, the above-proposed improved luminosity function can also be used for other hadronic reactions, for example high-p_T jet production.

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