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Symmetries of higher-order string gravity actions

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ABSTRACT

In this paper we explicitly prove the invariance of the time-dependent string gravity Lagrangian with up to four derivatives under the global $O(d, d)$ symmetry.

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1 Introduction

Global, continuous symmetries not connected with the diffeomorphism group are very rare in gravitational systems. The first example was discovered by Ehlers for the case of four-dimensional pure gravity with one Killing vector. Later, it was shown by Geroch that in the case of two Killing vectors the symmetry gets enhanced to an infinite Kac-Moody algebra. In string theory, the gravitational multiplet contains not only the graviton, but also a scalar (dilaton) and the antisymmetric tensor (often referred to as torsion). The symmetries of the Ehlers and Geroch type were also shown in this case [4]. Another type of symmetry in such a system was discovered in [1] (without the torsion field the discrete symmetry of the action was discovered in [2] and [3]). It was shown that, for the case of fields depending only on time in an arbitrary number of dimensions (1 time, d space dimensions), the lowest-order Lagrangian exhibits continuous, global $O(d,d)$ symmetry. The symmetry was later extended to the presence of matter [5] or gauge fields [6] and seems to be present in a large number of string-inspired theories containing gravity. In [1] argument was given that the symmetry should be present to all orders in α' in the σ-model expansion (another argument was given in [7]). In [3] it was argued that for the case without the torsion field there should be corrections to fields in the next order in α' to ensure vanishing of the β-functions and in [8] it was demonstrated on one specific example. Since the inclusion of the next-order terms (like curvature squared) can be very important for the stability of the solutions (as was recently discussed for the case with no torsion in [9]), it is the purpose of the present paper to show that the $O(d,d)$ symmetry is explicit in the order α' Lagrangian of gravity coupled to the dilaton and the antisymmetric tensor fields. There is quite a number of authors that have calculated the higher-order effective action coming from string amplitudes or from loop calculations in the σ-models (see for example [10, 11]) that sometimes do not agree with one another. We assumed throughout this paper that the result of [10] is correct, and it turned out that with this assumption the $O(d,d)$ symmetry of the quartic action can be proved.

2 The $O(d,d)$ symmetry in the lowest order

In [1] it was shown that the lowest-order string gravity (gravity coupled to dilaton and the antisymmetric tensor) Lagrangian, for fields depending only on cosmic time, possesses explicit $O(d,d)$ invariance, where d is the number of space dimensions. We will recall here this construction to set the notation. The lowest-order Lagrangian reads (throughout this paper we use the string frame with $e^{-2\phi}$ out front, since the symmetry is most simply realized there)

$$\Gamma^{(0)} = \int d^{d+1}x\sqrt{-g}\; e^{-2\phi}\left\{R + 4(\partial\phi)^2 - \frac{1}{12}H^2\right\}. \tag{2.1}$$

Our metric is $(-,+,\ldots,+)$ and

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho\Gamma^{\mu}{}_{\nu\sigma} - \ldots, \qquad R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}, \qquad H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{cyclic} \tag{2.2}$$

When fields depend only on time, it is possible to bring g and B to the form

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & G(t) \end{pmatrix}, \qquad B_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & B(t) \end{pmatrix}. \tag{2.3}$$

It was shown in [1] that the action (2.1) can then be rewritten as

$$\Gamma^0 = -\int dt e^{-\Phi}\left(\dot{\Phi}^2 + \frac{1}{8}\mathrm{Tr}[\dot{M}_0\eta\dot{M}_0\eta]\right), \tag{2.4}$$

where

$$\Phi = 2\phi - \frac{1}{2}\ln\det G, \tag{2.5}$$

η is the metric for the $O(d,d)$ group in non-diagonal form:

$$\eta = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix} \tag{2.6}$$

and

$$M_0 = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}. \tag{2.7}$$

This M_0 has two important properties [1]: it is symmetric and it belongs to the $O(d,d)$ group:

$$M_0^T = M_0, \qquad M_0\eta M_0 = \eta. \tag{2.8}$$

The action (2.4) is explicitly symmetric under the action of the $O(d,d)$ group:

$$M_0 \to \Omega^T M_0 \Omega, \qquad \Phi \to \Phi, \tag{2.9}$$

where Ω belongs to the $O(d,d)$:

$$\Omega^T\eta\Omega = \eta. \tag{2.10}$$

The general $O(d,d)$ element connected to the identity can be written as:

$$\Omega = \exp\begin{pmatrix} A_1 & A_2 \\ A_3 & -A_1^T \end{pmatrix} \qquad A_2^T = -A_2, \quad A_3^T = -A_3. \tag{2.11}$$

3 The symmetry in the next order without torsion

We start with the following form of fourth order in derivatives action in the string frame (formula 3.24 in [10])

$$\begin{aligned}\Gamma &= \int d^{d+1}x\sqrt{-g}\; e^{-2\phi}\Big\{R+4(\partial\phi)^2-\frac{1}{12}H^2 \\ &-\alpha'\lambda_0\Big[R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}-\frac{1}{2}R^{\mu\nu\sigma\rho}H_{\mu\nu\alpha}H_{\sigma\rho}{}^{\alpha}+ \\ &\frac{1}{24}H_{\mu\nu\lambda}H^{\nu}{}_{\rho\alpha}H^{\rho\sigma\lambda}H_{\sigma}{}^{\mu\alpha}-\frac{1}{8}H_{\mu\rho\lambda}H_{\nu}{}^{\rho\lambda}H^{\mu\sigma\alpha}H^{\nu}{}_{\sigma\alpha}\Big]+O(\alpha'^2)\Big\}\end{aligned} \qquad (3.1)$$

($\lambda_0=-\frac{1}{8}$ for the heterotic string, $-\frac{1}{4}$ for the Bose string and 0 for the superstring).

This is the simplest possible form of the string effective action. If one makes local redefinitions of fields, it does not change the equations of motion (in the redefined fields); however, the symmetry can be easily seen for one choice but impossible to guess for another. Thus we have to try all possible redefinitions to see whether we can bring the action to some suitable form. There are two guidelines for the search. The first one is that the action (when expressed in terms of time derivatives of fields and with all integrations by parts used) contains only first derivatives of fields. It turns out that this can always be done. The second one is that the whole action can be written in terms of Φ and M_0 defined before (but with possible corrections of order α'), since the symmetry is then explicit.

We start to show the techniques involved with the simpler case of vanishing H (then, of course, we do not have the full $O(d,d)$ symmetry but only some discrete subgroup); temporarily, we use the Lagrangian:

$$\Gamma=\int d^{d+1}x\sqrt{-g}\; e^{-2\phi}\left\{R+4(\partial\phi)^2-\alpha'\lambda_0 R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}+O(\alpha'^2)\right\} \qquad (3.2)$$

Requiring only first time derivatives allows for the four structures of order α':

$$\int d^{d+1}x\sqrt{-g}e^{-2\phi}\left[a_1R^2_{GB}+a_2\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)\partial_\mu\phi\partial_\nu\phi+a_3\Box\phi(\partial\phi)^2+a_4(\partial\phi)^4\right] \qquad (3.3)$$

where R^2_{GB} is the Gauss-Bonnet term

$$R^2_{GB}=R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}-4R_{\mu\nu}R^{\mu\nu}+R^2 \qquad (3.4)$$

In order to transform (3.2) to the form (3.3) we use the redefinitions

$$\begin{aligned}\delta g_{\mu\nu} &= \alpha'[b_1R_{\mu\nu}+b_2\partial_\mu\phi\partial_\nu\phi+g_{\mu\nu}(b_3R+b_4(\partial\phi)^2+b_5\Box\phi)] \\ \delta\phi &= \alpha'[c_1R+c_2(\partial\phi)^2+c_3\Box\phi].\end{aligned} \qquad (3.5)$$

Under these redefinitions the action (3.2) is corrected by

$$\begin{aligned}\delta\Gamma &= -\int d^{d+1}x\sqrt{-g}e^{-2\phi}\Big\{\Big(R^{\mu\nu}+2D^\mu\partial^\nu\phi-\frac{1}{2}g^{\mu\nu}(R+4\Box\phi-4(\partial\phi)^2)\Big)\delta g_{\mu\nu}+\\ &\quad +2(R+4\Box\phi-4(\partial\phi)^2)\delta\phi\Big\}\end{aligned} \tag{3.6}$$

Plugging (3.5) into (3.6) we get the form (3.3) when

$$\begin{aligned}\delta g_{\mu\nu} &= -4\alpha'\lambda_0 R_{\mu\nu}\\ \delta\phi &= -\frac{1}{2}\alpha'\lambda_0 R+2\alpha'\lambda_0(\partial\phi)^2;\end{aligned} \tag{3.7}$$

the action then becomes

$$\Gamma^{(1)}=\int\sqrt{-g}e^{-2\phi}\alpha'\lambda_0\left[-R^2_{GB}+16\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)\partial_\mu\phi\partial_\nu\phi-16\Box\phi(\partial\phi)^2+16(\partial\phi)^4\right] \tag{3.8}$$

In order to write the action for fields depending only on cosmic time, we introduce the matrix

$$W:=G^{-1}\dot{G}. \tag{3.9}$$

Then we have

$$\begin{aligned}\Gamma &= \int dte^{-\Phi}\Big\{-\dot{\Phi}^2+\frac{1}{4}\mathrm{Tr}W^2\\ &\quad -\alpha'\lambda_0\left[\frac{1}{8}\mathrm{Tr}W^4-\frac{1}{16}(\mathrm{Tr}W^2)^2+\frac{1}{3}\mathrm{Tr}W^3\dot{\Phi}+\frac{1}{2}(\mathrm{Tr}W^2)\dot{\Phi}^2-\frac{1}{3}\dot{\Phi}^4\right]\Big\}.\end{aligned} \tag{3.10}$$

It is now necessary to list all possible $O(d,d)$ invariants with first time derivatives. There are only four of them:

$$A_1\mathrm{Tr}(\dot{M}_0\eta)^4+A_2(\mathrm{Tr}(\dot{M}_0\eta)^2)^2+A_3\mathrm{Tr}(\dot{M}_0\eta)^2\dot{\Phi}^2+A_4\dot{\Phi}^4. \tag{3.11}$$

Since (we still suppress the B-dependence!)

$$\mathrm{Tr}(\dot{M}_0\eta)^4=2\mathrm{Tr}W^4,\qquad \mathrm{Tr}(\dot{M}_0\eta)^2=-2\mathrm{Tr}W^2, \tag{3.12}$$

we see that there is one term in the action (3.10) that does not belong to this class. In order to make the symmetry explicit we change the definition of M by adding to M_0 a term of order α':

$$M=M_0-\alpha'\lambda_0\begin{pmatrix}-G^{-1}\dot{G}G^{-1}\dot{G}G^{-1} & 0\\ 0 & \dot{G}G^{-1}\dot{G}\end{pmatrix} \tag{3.13}$$

The redefined M satisfies (to order α') the properties (2.8).

With this new definition the total action can be rewritten as (but still without the antisymmetric tensor in M):

$$\begin{aligned}\Gamma &= \int dte^{-\Phi}\left\{-\dot{\Phi}^2 - \frac{1}{8}\mathrm{Tr}(\dot{M}\eta)^2\right. \\ &\left. -\alpha'\lambda_0\left[\frac{1}{16}\mathrm{Tr}(\dot{M}\eta)^4 - \frac{1}{64}(\mathrm{Tr}(\dot{M}\eta)^2)^2 - \frac{1}{4}(\mathrm{Tr}(\dot{M}\eta)^2)\dot{\Phi}^2 - \frac{1}{3}\dot{\Phi}^4\right]\right\}. \quad (3.14)\end{aligned}$$

4 The full $O(d,d)$ symmetry

We now set to prove that (3.14) is actually the proper form of the action after inclusion of the antisymmetric tensor. We have to try all possible redefinitions of the action (3.1) that give only first time derivatives. We may now use, in addition to (3.7), the following redefinitions:

$$\begin{aligned}\delta g_{\mu\nu} &= \alpha'\lambda_0(b_6 H^2_{\mu\nu} + b_7 G_{\mu\nu}H^2) \\ \delta\phi &= \alpha'\lambda_0 c_4 H^2 \\ \delta B_{\mu\nu} &= \alpha'\lambda_0(d_1 D^\lambda H_{\lambda\mu\nu} + d_2 H_{\lambda\mu\nu}\partial^\lambda\phi), \quad (4.1)\end{aligned}$$

where

$$H^2_{\mu\nu} = H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta}, \quad \text{and} \quad H^2 = H_{\mu\alpha\beta}H^{\mu\alpha\beta} \quad (4.2)$$

and the Lagrangian changes as follows:

$$\begin{aligned}\delta\Gamma &= -\int d^{d+1}x\sqrt{-g}e^{-2\phi}\{ \\ &\left(R^{\mu\nu} + 2D^\mu\partial^\nu\phi - \frac{1}{4}(H^2)^{\mu\nu} - \frac{1}{24}g^{\mu\nu}(12R + 48\Box\phi - 48(\partial\phi)^2 - H^2)\right)\delta g_{\mu\nu} + \\ &+ \frac{1}{6}(12R + 48\Box\phi - 48(\partial\phi)^2 - H^2)\delta\phi + \frac{1}{2}(2\partial_\mu\phi H^{\mu\nu\rho} - D_\mu H^{\mu\nu\rho})\delta B_{\nu\rho}\Big\}. \quad (4.3)\end{aligned}$$

The requirement of only first time derivatives allows for, in addition to (3.3), the following structures:

$$\begin{aligned}\Gamma^{(2)} &= \int d^{d+1}x\sqrt{-g}e^{-2\phi}\left[a_1 R^2_{GB} + a_2\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right)\partial_\mu\phi\partial_\nu\phi + a_3\Box\phi(\partial\phi)^2\right. \\ &+ a_4(\partial\phi)^4 + a_5\left(R^{\mu\nu\sigma\rho}H_{\mu\nu\alpha}H_{\sigma\rho}{}^\alpha - 2R^{\mu\nu}H^2_{\mu\nu} + \frac{1}{3}RH^2\right) + a_6 H^2(\partial\phi)^2 \\ &+ a_7\left(D^\mu\partial^\nu\phi H^2_{\mu\nu} - \frac{1}{3}\Box\phi H^2\right) + a_8 H_{\mu\nu\lambda}H^\nu{}_{\rho\alpha}H^{\rho\sigma\lambda}H_\sigma{}^{\mu\alpha} + a_9 H^2_{\mu\nu}H^{2\mu\nu} \\ &\left. + a_{10}(H^2)^2 + a_{11}H^{2\mu\nu}\partial_\mu\phi\partial_\nu\phi + a_{12}\left(D_\mu H^{\mu\nu\rho}H_{\nu\rho\sigma}\partial^\sigma\phi + \frac{1}{6}\Box\phi H^2\right)\right]. \quad (4.4)\end{aligned}$$

Starting from the action (3.1) and trying different redefinitions, we finally arrive at the following form of the action:

$$
\begin{aligned}
\Gamma \;=\; & \int \sqrt{-g} e^{-2\phi} \Big\{ R + 4(\partial\phi)^2 - \frac{1}{12} H^2 \qquad\qquad (4.5)\\
& + \alpha' \lambda_0 \Big[-R^2_{GB} + 16 \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) \partial_\mu \phi \partial_\nu \phi - 16 \Box\phi (\partial\phi)^2 + 16 (\partial\phi)^4 \\
& + \frac{1}{2} \left(R^{\mu\nu\sigma\rho} H_{\mu\nu\alpha} H_{\sigma\rho}{}^{\alpha} - 2 R^{\mu\nu} H^2_{\mu\nu} + \frac{1}{3} R H^2 \right) - 2 \left(D^\mu \partial^\nu \phi H^2_{\mu\nu} - \frac{1}{3} \Box\phi H^2 \right) \\
& - \frac{2}{3} H^2 (\partial\phi)^2 - \frac{1}{24} H_{\mu\nu\lambda} H^{\nu}{}_{\rho\alpha} H^{\rho\sigma\lambda} H_{\sigma}{}^{\mu\alpha} + \frac{1}{8} H^2_{\mu\nu} H^{2\mu\nu} - \frac{1}{144} (H^2)^2 \Big] \Big\} .
\end{aligned}
$$

Making the field redefinitions of order α' is equivalent to all possible substitutions of the lowest-order equations of motion in the higher-order Lagrangian. The lowest-order equations of motion are:

$$
\begin{aligned}
R_{\mu\nu} + 2 D_\mu \partial_\nu \phi - \frac{1}{4} H^2_{\mu\nu} = 0, \qquad & \Box\phi - 2(\partial\phi)^2 + \frac{1}{12} H^2 = 0 \\
R + 4\Box\phi - 4(\partial\phi)^2 - \frac{1}{12} H^2 = 0, \qquad & D^\lambda H_{\lambda\mu\nu} - 2 H_{\mu\nu}{}^{\lambda} \partial_\lambda \phi = 0 \qquad (4.6)
\end{aligned}
$$

Using them and the Bianchi identities for curvature and torsion it is relatively straightforward to show the equivalence "on-shell" of (4.5) with (3.1). The action (4.5) corresponds to the choice in (4.1):

$$
b_6 = 0, \quad b_7 = 0, \quad c_4 = -\frac{1}{24}, \quad d_1 = 0, \quad d_2 = 4 \qquad (4.7)
$$

To write the result (4.5) for the case of fields depending only on time, we introduce (in addition to W defined before) the matrix Y:

$$
Y := G^{-1} \dot{B}. \qquad (4.8)
$$

We have

$$
\begin{aligned}
\Gamma \;=\; & \int dt e^{-\Phi} \Big\{ -\dot{\Phi}^2 + \frac{1}{4} \mathrm{Tr} W^2 - \frac{1}{4} \mathrm{Tr} Y^2 \\
& + \alpha' \lambda_0 \Big[-\frac{1}{8} \mathrm{Tr} W^4 + \frac{1}{16} (\mathrm{Tr} W^2)^2 - \frac{1}{3} \mathrm{Tr} W^3 \dot{\Phi} - \frac{1}{2} (\mathrm{Tr} W^2) \dot{\Phi}^2 + \frac{1}{3} \dot{\Phi}^4 \\
& + \frac{1}{2} \mathrm{Tr}(W^2 Y^2) + \frac{1}{4} \mathrm{Tr}(WYWY) - \frac{1}{8} \mathrm{Tr} W^2 \mathrm{Tr} Y^2 + \dot{\Phi} \mathrm{Tr}(W Y^2) + \frac{1}{2} \dot{\Phi}^2 \mathrm{Tr} Y^2 \\
& + \frac{3}{8} \mathrm{Tr} Y^4 + \frac{1}{16} (\mathrm{Tr} Y^2)^2 \Big] \Big\} . \qquad (4.9)
\end{aligned}
$$

In order to compare it to the $O(d,d)$ symmetric form, we need the expressions

$$
\begin{aligned}
\mathrm{Tr}(\dot{M}_0 \eta)^4 &= 2 \mathrm{Tr} W^4 + 2 \mathrm{Tr} Y^4 - 8 \mathrm{Tr}(W^2 Y^2) + 4 \mathrm{Tr}(WYWY) \\
\mathrm{Tr}(\dot{M}_0 \eta)^2 \dot{\Phi}^2 &= (-2 \mathrm{Tr} W^2 + 2 \mathrm{Tr} Y^2) \dot{\Phi}^2 \\
(\mathrm{Tr}(\dot{M}_0 \eta)^2)^2 &= (-2 \mathrm{Tr} W^2 + 2 \mathrm{Tr} Y^2)^2. \qquad (4.10)
\end{aligned}
$$

We see that our result (4.9) contains a number of terms that are not of this form, so we redefine M:

$$M = M_0 - \alpha' \lambda_0 \begin{pmatrix} \alpha & \beta \\ \beta^T & \gamma \end{pmatrix}, \tag{4.11}$$

where

$$\begin{aligned} \alpha &= -G^{-1}\dot{G}G^{-1}\dot{G}G^{-1} + G^{-1}\dot{B}G^{-1}\dot{B}G^{-1} \\ \beta &= G^{-1}(\dot{G}G^{-1}\dot{G} - \dot{B}G^{-1}\dot{B})G^{-1}B - G^{-1}(\dot{G}G^{-1}\dot{B} + \dot{B}G^{-1}\dot{G}) \\ \gamma &= \dot{G}G^{-1}\dot{G} - \dot{B}G^{-1}\dot{B} - (\dot{G}G^{-1}\dot{B} + \dot{B}G^{-1}\dot{G})G^{-1}B \\ &\quad -B(-G^{-1}\dot{G}G^{-1}\dot{G}G^{-1} + G^{-1}\dot{B}G^{-1}\dot{B}G^{-1})B - BG^{-1}(\dot{G}G^{-1}\dot{B} + \dot{B}G^{-1}\dot{G}). \end{aligned} \tag{4.12}$$

To order α' the redefined M satisfies (2.8), so that the redefinition is itself (time- and field-dependent) an $O(d,d)$ rotation. To make the properties (2.8) manifest, we write the redefinition (4.11) as

$$M = \omega^T M_0\, \omega, \tag{4.13}$$

where ω is in the form (2.11), with:

$$\begin{aligned} A_1 &= -\alpha' \lambda_0 \left[-\frac{1}{2}\dot{G}G^{-1}\dot{G}G^{-1} + \frac{1}{2}\dot{B}G^{-1}\dot{B}G^{-1} \right] \\ A_2 &= -\alpha' \lambda_0 \left[-\dot{G}G^{-1}\dot{B} - \dot{B}G^{-1}\dot{G} + \frac{1}{2}(\dot{G}G^{-1}\dot{G} - \dot{B}G^{-1}\dot{B})G^{-1}B \right. \\ &\quad \left. + \frac{1}{2}BG^{-1}(\dot{G}G^{-1}\dot{G} - \dot{B}G^{-1}\dot{B}) \right] \\ A_3 &= 0. \end{aligned} \tag{4.14}$$

With this new M, the action (4.9) is exactly in the form anticipated before in eq. (3.14):

$$\begin{aligned} \Gamma &= \int \frac{dt}{N} e^{-\Phi} \left\{ -\dot{\Phi}^2 - \frac{1}{8}\mathrm{Tr}(\dot{M}\eta)^2 \right. \\ &\quad \left. - \frac{\alpha' \lambda_0}{N^2} \left[\frac{1}{16}\mathrm{Tr}(\dot{M}\eta)^4 - \frac{1}{64}(\mathrm{Tr}(\dot{M}\eta)^2)^2 - \frac{1}{4}(\mathrm{Tr}(\dot{M}\eta)^2)\dot{\Phi}^2 - \frac{1}{3}\dot{\Phi}^4 \right] \right\}. \end{aligned} \tag{4.15}$$

We have introduced the lapse function N (in the first order in derivatives action, it is a trivial replacement $dt \to N dt$), since it gives one more equation of motion (called the "g_{00}" equation in [1]) and only afterwards we put N to 1.

This action is explicitly $O(d,d)$-invariant under (2.9). It looks, however, like a little miracle that the coefficients in (3.1) coming from the string amplitudes are exactly such that they give the explicit $O(d,d)$ symmetry of (4.15). In comparison with the lowest-order case, now the $O(d,d)$ symmetry acts in a more complicated way, as a rotation of not only fields but fields with their derivatives.

The form of the action (3.8) needed to exhibit the symmetry is remarkably the same as the unique ("off-shell") form of the action found in [13] (eq. (20) there). The comparison of the full action (4.5) with the result of [14] is more difficult since there are apparent contradictions between this reference and [10, 13]. However, our redefinition (4.7) is exactly the same as the redefinition used in [14] and we suspect that the "off-shell" conformal invariance also leads to the unique action (4.5) which is a remarkable feature pointing out to a deeper structure behind the $O(d,d)$ symmetry.

Since the $O(d,d)$ symmetry is continuous and global, it has an associated conserved current, which means, for a theory depending only on time, that the current should be constant (it is an "integrated once" equation of motion for M). In analogy to [1] we call this constant A:

$$A = \text{const} = e^{-\Phi}\left\{M\eta\dot{M} + 2\alpha'\lambda_0\left[\frac{1}{2}M(\eta\dot{M})^3 - \frac{1}{8}M\eta\dot{M}\,\text{Tr}(\dot{M}\eta)^2 - M\eta\dot{M}\dot{\Phi}^2\right]\right\} \tag{4.16}$$

where $A^T = -A$ and $A\eta M = -M\eta A$ ([1]).

The N equation reads:

$$\begin{aligned} 0 \;=\; & -\dot{\Phi}^2 - \frac{1}{8}\text{Tr}(\dot{M}\eta)^2 \\ & -3\alpha'\lambda_0\left[\frac{1}{16}\text{Tr}(\dot{M}\eta)^4 - \frac{1}{64}(\text{Tr}(\dot{M}\eta)^2)^2 - \frac{1}{4}(\text{Tr}(\dot{M}\eta)^2)\dot{\Phi}^2 - \frac{1}{3}\dot{\Phi}^4\right] \end{aligned} \tag{4.17}$$

Equations (4.16) and (4.17) are non-linear in fields but (as a result of the existence of symmetry) first order in derivatives. The analysis of these equations and their solutions will appear in a subsequent publication [15].

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