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CLASSICAL AND QUANTUM $N = 2$ SUPERSYMMETRIC BLACK HOLES

Klaus Behrndt[a], **Gabriel Lopes Cardoso**[b], **Bernard de Wit**[c], **Renata Kallosh**[d], **Dieter Lüst**[a], **Thomas Mohaupt**[a 1]

[a] *Humboldt-Universität zu Berlin, Institut für Physik, D-10115 Berlin, Germany*
[b] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*
[c] *Institute for Theoretical Physics, Utrecht University, 3508 TA Utrecht, Netherlands*
[d] *Physics Department, Stanford University, Stanford, CA 94305-4060, USA*

ABSTRACT

We use heterotic/type-II prepotentials to study quantum/classical black holes with half the $N = 2, D = 4$ supersymmetries unbroken. We show that, in the case of heterotic string compactifications, the perturbatively corrected entropy formula is given by the tree-level entropy formula with the tree-level coupling constant replaced by the perturbative coupling constant. In the case of type-II compactifications, we display a new entropy/area formula associated with axion-free black-hole solutions, which depends on the electric and magnetic charges as well as on certain topological data of Calabi–Yau three-folds, namely the intersection numbers, the second Chern class and the Euler number of the three-fold. We show that, for both heterotic and type-II theories, there is the possibility to relax the usual requirement of the non-vanishing of some of the charges and still have a finite entropy.

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[1] email: behrndt@qft2.physik.hu-berlin.de, cardoso@surya11.cern.ch, bdewit@fys.ruu.nl, kallosh@physics.stanford.edu, luest@qft1.physik.hu-berlin.de, mohaupt@qft2.physik.hu-berlin.de

1 Introduction

Recently there has been considerable progress in the understanding of microscopic and macroscopic properties of supersymmetric black holes in string theory. Using the Dirichlet-brane interpretation of type-II solitons, the microscopic entropy of certain stringy black holes could be explicitly calculated [1] in agreement with the macroscopic Bekenstein-Hawking entropy formula. In [2] it was shown that, while the values of the moduli at spatial infinity are more or less arbitrary parameters, their values at the horizon are entirely fixed in terms of the (quantized) magnetic and electric charges of the black hole. In contradistinction with the black-hole mass, which is governed by the value of the central charge at infinity and thus depends on the charges and the moduli values at infinity, the entropy-area formula is given in terms of the central charge at the horizon [3]. Because the moduli are fixed at the horizon in terms of the charges, irrespective of their possible values at spatial infinity, the entropy-area is thus expressible in terms of the charges. This result is natural from the point of view that the entropy should follow from a counting of independent quantum-mechanical states, which seems to preclude any dependence on continuous parameters such as the moduli values at spatial infinity. In [3] it was also shown that the central charge acquires a minimal value at the horizon and that the extremization of the central charge provides the specific moduli values at the horizon.

The restricted behaviour at the horizon is related to the enhancement to full $N = 2$ supersymmetry near the horizon, while globally the field configurations leave only half the supersymmetries unbroken (so that we are dealing with true BPS states). Thus the black holes can be regarded as solitonic solutions that interpolate between the maximally supersymmetric field configurations at spatial infinity and at the horizon. Particularly simple solutions, called double extreme black holes [4], are given by those configurations where the moduli take constant values from the horizon up to spatial infinity.

Using the extremization procedure of [3], the macroscopic entropy formulae for $N = 4$ and $N = 8$ extreme black hole solutions [5] were obtained in perfect agreement with the construction of explicit black-hole solutions. The $N = 4, 8$ entropies are completely unique and they depend only on the quantized magnetic/electric charges and they are invariant under the perturbative and non-perturbative duality symmetries, such as T-duality, S-duality [6] and string/string duality [7, 8, 9].

In four-dimensional $N = 2$ string theories new features of black-hole physics arise which destroy the uniqueness of the $N = 2$ entropy formula. In particular there exists a large number of different $N = 2$ string vacua so that the extreme black-hole solutions depend on the specific details of the particular $N = 2$ string model. Consequently the same features are present for the $N = 2$ entropy formula. Nevertheless, the $N = 2$ entropy, being proportional to the extremized $N = 2$ central charge Z, still depends on the quantized magnetic/electric charges, although the nature of the dependence is governed by the particular string model. The $N = 2$ central charge Z and the $N = 2$ BPS spectrum can be directly calculated from the $N = 2$ holomorphic prepotential which describes the two-derivative couplings of the $N = 2$ vector multiplets in the effective $N = 2$ string action [10] (or, in a symplectic basis where the prepotential does not exist [11], from the symplectic sections). Therefore the parameters of the prepotential of a given $N = 2$ string model determine the black-hole entropy as well as the values of the scalar fields at the horizon.

Depending on whether one is discussing heterotic or type-II $N = 2$ string vacua, the parameters of the prepotential have a rather different interpretation. To be more specific, let us first consider four-dimensional $N = 2$ heterotic string compactifications on $K3 \times T_2$, where the number of vector multiplets N_V (not counting the graviphoton), the number of hypermultiplets N_H and the couplings are specified by a particular choice of the $SU(2)$ instanton gauge bundle. The classical prepotential is completely universal and corresponds to a scalar non-linear σ-model based on the coset space $\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2,N_V-1)}{SO(2)\times SO(N_V-1)}$. Extremizing the corresponding central charge Z the classical $N = 2$ black hole entropy and the moduli on the horizon have been computed explicitly [4, 12, 13], and the result agrees with the truncated $N = 4$ formulae.

Since in heterotic $N = 2$ string compactifications the dilaton field S can be described by a vector multiplet, the heterotic prepotential receives perturbative corrections only at the one-loop level [15, 16]; in addition there are non-perturbative contributions. The heterotic one-loop corrections to the prepotential, being independent of the dilaton S, split into a cubic polynomial, a constant term and an infinite series of terms which are exponentially suppressed in the decompactification limit of large moduli fields. It is an interesting observation that the coefficients of the exponential terms are given in terms of q-expansion coefficients of certain modular forms as explicitly shown for models with $N_V = 3, 4$ in [17, 18, 19]. Thus, the one-loop black hole solutions are determined by an infinite set of integer numbers; hence the extremization problem of the corresponding one-loop central charges is very involved and difficult. Nevertheless, we are able to derive a simple formula for the black-hole entropy in terms of the heterotic string-coupling and the target-space duality-invariant inner product of the charges, which holds to all orders in perturbation theory. This formula does not depend explicitly on the values of the moduli fields. At the horizon the values of the moduli can be determined explicitly in certain cases when neglecting all exponential terms in the large moduli limit. Hence new quantum features of black holes already become important when considering only cubic corrections to the classical prepotential.

It is well established that the $N = 2$ heterotic string on $K3 \times T_2$ is dual to type-IIA (IIB) compactification on a suitably chosen Calabi–Yau three-fold [20, 21, 22]. In fact, it was shown [20, 23, 24, 25, 18, 19] for models with $N_V = 3, 4$ that the type-IIA and heterotic prepotential agree in heterotic weak-coupling limit. On the type-IIA side the $N = 2$ prepotential of the Kähler class moduli is completely classical because the type-II dilaton corresponds to a hypermultiplet and has no couplings to the vector fields. More specifically, the cubic couplings of the type-IIA prepotential are determined by the topological intersection numbers of the corresponding Calabi–Yau space; the coefficients of the exponential terms are given in terms of the rational Calabi–Yau instanton numbers. In this paper we focus on the limit of large Kähler-class moduli, i.e. we will discuss the influence of the classical intersection numbers C_{ABC}, as well as terms constant and linear in the moduli, on the Calabi–Yau black-hole solutions. The linear ones are related to the second Chern class of the three-fold and the constant one is related to the Euler characteristic [26]. Hence we find new entropy formulae which depend only on the magnetic/electric charges and topological data on the Calabi–Yau manifold.

Our paper is organized as follows. In the next section we will briefly introduce the $N = 2$ vector couplings and the $N = 2$ central charge in terms of the $N = 2$ prepotential. We will recall the structure of the prepotentials in four-dimensional $N = 2$ heterotic and type-IIA string vacua, and also their relations via heterotic/typeII string-string duality. In section 3 we show that there is a rather elegant and simple way to find the solutions of the extremization problem of the $N = 2$ central charge, which can be used to compute the values of the moduli on the black-hole horizon and the black-hole entropy as a function of the quantized electric/magnetic charges. While these solutions cannot be determined in full generality, we can generally prove a simple formula for the entropy for perturbative heterotic vacua, as a product of the inverse square of the perturbative string-coupling constant (which itself depends on the electric/magnetic charges) and the target-space duality-invariant inner product of the charges. A particular class of solutions that can generally be evaluated for cubic prepotentials, is the class of non-axionic black-holes. This result covers the type-IIA Calabi–Yau black-hole entropy in case of small contributions of the rational instanton configurations, i.e., in the limit of large Kähler-class moduli. We will also discuss the influence of linear terms in the prepotential on the black hole entropy. In the Calabi–Yau case these linear terms are related to the second Chern class of the three-fold [26]. In section four we discuss the relation of our solution to intersecting branes in higher dimensions and suggest their M-theory interpretation. In the last section we summarize our results.

2 The $N = 2$ prepotential in heterotic and type-IIA string vacua

2.1 General formulae

The vector couplings of $N = 2$ supersymmetric Yang-Mills theory are encoded in a holomorphic function $F(X)$, where the X denote the complex scalar fields of the vector supermultiplets. With local supersymmetry this function depends on one extra field, in order to incorporate the graviphoton. The theory can then be encoded in terms of a holomorphic function $F(X)$ which is homogeneous of second degree and depends on complex fields X^I with $I = 0, 1, \ldots N_V$. Here N_V counts the number of physical vector multiplets.

The resulting *special geometry* [10, 27] can be defined more abstractly in terms of a symplectic section V, also referred to as period vector: a $(2N_V + 2)$-dimensional complex symplectic vector, expressed in terms of the holomorphic prepotential F according to

$$V = \begin{pmatrix} X^I \\ F_J \end{pmatrix} \quad , \tag{2.1}$$

where $F_I = \partial F/\partial X^I$. The N_V physical scalar fields of this system parametrize an N_V-dimensional complex hypersurface, defined by the condition that the section satisfies a constraint

$$\langle \bar{V}, V \rangle \equiv \bar{V}^{\mathrm{T}} \Omega V = -i, \tag{2.2}$$

with Ω the antisymmetric matrix

$$\Omega = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} . \tag{2.3}$$

The embedding of this hypersurface can be described in terms of N_V complex coordinates z^A $(A = 1, \ldots, N_V)$ by letting the X^I be proportional to some holomorphic sections $X^I(z)$ of the complex projective space. In terms of these sections the X^I read

$$X^I = e^{\frac{1}{2}K(z,\bar{z})}\, X^I(z)\,, \tag{2.4}$$

where $K(z, \bar{z})$ is the Kähler potential, to be introduced below. In order to distinguish the sections $X^I(z)$ from the original quantities X^I, we will always explicitly indicate their z-dependence. The overall factor $\exp[\frac{1}{2}K]$ is chosen such that the constraint (2.2) is satisfied. Furthermore, by virtue of the homogeneity property of $F(X)$, we can extract an overall factor $\exp[\frac{1}{2}K]$ from the symplectic sections (2.1), so that we are left with a holomorphic symplectic section. Clearly this holomorphic section is only defined projectively, i.e., modulo multiplication by an arbitrary holomorphic function. On the Kähler potential these projective transformations act as Kähler transformations, while on the sections V they act as phase transformations.

The resulting geometry for the space of physical scalar fields belonging to vector multiplets of an $N = 2$ supergravity theory is a special Kähler geometry, with a Kähler metric $g_{A\bar{B}} = \partial_A \partial_{\bar{B}} K(z,\bar{z})$ following from a Kähler potential of the special form

$$K(z,\bar{z}) = -\log\Big(i\bar{X}^I(\bar{z})F_I(X^I(z)) - iX^I(z)\bar{F}_I(\bar{X}^I(\bar{z}))\Big). \tag{2.5}$$

A convenient choice of inhomogeneous coordinates z^A are the *special* coordinates, defined by

$$X^0(z) = 1\,, \quad X^A(z) = z^A\,, \qquad A = 1,\ldots,N_V. \tag{2.6}$$

In this parameterization the Kähler potential can be written as [28]

$$K(z,\bar{z}) = -\log\Big(2(\mathcal{F}+\bar{\mathcal{F}}) - (z^A - \bar{z}^A)(\mathcal{F}_A - \bar{\mathcal{F}}_A)\Big)\,, \tag{2.7}$$

where $\mathcal{F}(z) = i(X^0)^{-2}F(X)$.

We should point out that it is possible to rotate the basis specified by (2.1) by an $Sp(2N_V+2,\mathbf{Z})$ transformation in such a way that it is no longer possible to associate them to a holomorphic function [11]. As long as all fundamental fields are electrically neutral (which is the case in the context of this paper), this is merely a technical problem, as one can always rotate back to the basis where a prepotential exists [29]. As shown in [11] the supergravity Lagrangian can be expressed entirely in terms of the symplectic section V, without restricting its parameterization so as to correspond to a prepotential $F(X)$.

The Lagrangian terms containing the kinetic energies of the gauge fields are

$$4\pi\,\mathcal{L}^{\rm gauge} \;=\; -\tfrac{i}{8}\Big(\mathcal{N}_{IJ}\,F^{+I}_{\mu\nu}F^{+\mu\nu J} \;-\; \bar{\mathcal{N}}_{IJ}\,F^{-I}_{\mu\nu}F^{-\mu\nu J}\Big)\,, \tag{2.8}$$

where $F^{\pm I}_{\mu\nu}$ denote the selfdual and anti-selfdual field-strength components and $\mathcal{N}_{IJ}(z,\bar{z})$ is the field-dependent tensor that comprises the inverse gauge couplings $g^{-2}_{IJ} = \frac{i}{16\pi}(\mathcal{N}_{IJ} - \bar{\mathcal{N}}_{IJ})$ and the generalized θ angles $\theta_{IJ} = \frac{\pi}{2}(\mathcal{N}_{IJ} + \bar{\mathcal{N}}_{IJ})$.

Now we define the tensors $G^{\pm}_{\mu\nu I}$ as

$$G^{+}_{\mu\nu I} = \mathcal{N}_{IJ}F^{+J}_{\mu\nu}\,, \quad G^{-}_{\mu\nu I} = \bar{\mathcal{N}}_{IJ}F^{-J}_{\mu\nu}\,, \tag{2.9}$$

which describe the (generalized) electric displacement and magnetic fields. The set of Bianchi identities and equation of motion for the Abelian gauge fields are invariant under the transformations

$$
\begin{aligned}
F^{+I}_{\mu\nu} \longrightarrow \tilde{F}^{+I}_{\mu\nu} &= U^I{}_J \, F^{+J}_{\mu\nu} + Z^{IJ} \, G^+_{\mu\nu J} \, . \\
G^+_{\mu\nu I} \longrightarrow \tilde{G}^+_{\mu\nu I} &= V_I{}^J \, G^+_{\mu\nu J} + W_{IJ} \, F^{+J}_{\mu\nu} \, ,
\end{aligned}
\tag{2.10}
$$

where U, V, W and Z are constant, real, $(N_V+1)\times(N_V+1)$ matrices, which have to satisfy the symplectic constraint

$$
\mathcal{O}^{-1} = \Omega \, \mathcal{O}^{\mathrm{T}} \, \Omega^{-1} \qquad \text{where} \quad \mathcal{O} = \begin{pmatrix} U & Z \\ W & V \end{pmatrix} \, . \tag{2.11}
$$

The target-space duality group Γ is a certain subgroup of $Sp(2N_V+2, \mathbf{Z})$. It follows that the magnetic/electric charge vector $Q = (p^I, q_J)$, defined by $(\oint F^I, \oint G_J) = (2\pi p^I, 2\pi q_J)$, transforms as a symplectic vector, where we stress that the identification of magnetic and electric charges is linked to the symplectic basis. Since $N=2$ supersymmetry relates the X^I to the field strengths $F^{+I}_{\mu\nu}$, while the F_I are related to the $G_I^{+\mu\nu}$, the period vector V also transforms as a symplectic vector:

$$
\begin{aligned}
\tilde{X}^I &= U^I{}_J \, X^J + Z^{IJ} \, F_J \, , \\
\tilde{F}_I &= V_I{}^J \, F_J + W_{IJ} \, X^J \, .
\end{aligned}
\tag{2.12}
$$

Finally consider $N=2$ BPS states, whose masses are equal to the central charge Z of the $N=2$ supersymmetry algebra. In terms of the magnetic/electric charges Q and the period vector V the BPS masses take the following form [11]:

$$
M^2_{BPS} = |Z|^2 = |\langle Q, V\rangle|^2 = e^K |q_I X^I(z) - p^I F_I(z)|^2 = e^{K(z,\bar{z})} \, |\mathcal{M}(z)|^2. \tag{2.13}
$$

It follows that M^2_{BPS} is invariant under symplectic transformations (2.12).

An example of a prepotential arising in string compactifications is given by the cubic prepotential

$$
F(X) = d_{ABC} \frac{X^A X^B X^C}{X^0} \, , \tag{2.14}
$$

where d_{ABC} are some real constants. The corresponding Kähler potential is given by

$$
K(z,\bar{z}) = -\log \Big(-i d_{ABC} \, (z-\bar{z})^A (z-\bar{z})^B (z-\bar{z})^C \Big) \, . \tag{2.15}
$$

In the case of heterotic string compactifications, both the classical prepotential as well as certain perturbative corrections to it are described by a cubic prepotential of the type (2.14). In the case of type IIA compactifications, the d_{ABC} are just proportional to the classical intersection numbers: $d_{ABC} = -\frac{1}{6} C_{ABC}$.

2.2 The heterotic prepotential

In the following, we will discuss a class of heterotic $N = 2$ models, obtained by compactifying the $E_8 \times E_8$ string on $K3 \times T_2$. The moduli z^A ($A = 1, \ldots, N_V$) comprise the dilaton S, the two toroidal moduli T and U as well as Wilson lines V^i ($i = 1, \ldots, N_V - 3$):

$$S = -iz^1 \, , \quad T = -iz^2 \, , \quad U = -iz^3 \, , \quad V^i = -iz^{i+3} \, . \tag{2.16}$$

We will, in the following, collectively denote the moduli T, U and V^i by T^a, so that $a = 2, \ldots, N_V$. The generic unbroken Abelian gauge group $U(1)^{N_V+1}$ depends on the specific choice of $SU(2)$ bundles with instanton numbers $(d_1, d_2) = (12 - n, 12 + n)$ when compactifying to six dimensions on $K3$ (see [20, 30, 31] for details). For example, for $n = 0, 1, 2$, a complete Higgsing is possible which leads to the three-parameter S-T-U models with no Wilson-line moduli ($N_V = 3$). It is, however, also possible to not completely Higgs away the six-dimensional gauge group $E_7 \times E_7$, and for $n = 0, 1, 2$ one obtains in this way heterotic models with one Wilson-line modulus V. Here we have four vector multiplets, so that we are dealing with a four-parameter S-T-U-V model ($N_V = 4$).

For this class of models, the heterotic prepotential has the form

$$\mathcal{F}^{\text{het}} = -ST^a \eta_{ab} T^b + h(T^a) + f^{\text{NP}}(e^{-2\pi S}, T^a) \, , \tag{2.17}$$

where

$$T^a \eta_{ab} T^b = T^2 T^3 - \sum_{I=4}^{N_V} (T^I)^2, \qquad a, b = 2, \ldots, N_V \, . \tag{2.18}$$

The dilaton S is related to the tree-level coupling constant and to the theta angle by $S = 4\pi/g^2 - i\theta/2\pi$. The first term in (2.17) is the classical part of the heterotic prepotential, $h(T^a)$ denotes the one-loop contribution and f^{NP} is the non-perturbative part, which is exponentially suppressed for small coupling. Note that the perturbative corrections are entirely due to one-loop effects, owing to nonrenormalization theorems. In the following we focus on the perturbative contributions.

The classical prepotential leads to the metric of the special Kähler manifold $\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2,N_V-1)}{SO(2) \times SO(N_V-1)}$ with corresponding tree-level Kähler potential

$$K \;=\; -\log\Big[(S + \bar{S})\Big] - \log\Big[(T^a + \bar{T}^a)\eta_{ab}(T^b + \bar{T}^b)\Big]. \tag{2.19}$$

Due to the required embedding of the T-duality group into the $N = 2$ symplectic transformations, it follows [15, 16] that the heterotic one-loop prepotential $h(T^a)$ must obey well-defined transformation rules under this group. The function $h(T^a)$ leads to the following modified Kähler potential [15], which represents the full perturbative contribution,

$$K = -\log[(S+\bar{S}) + V_{GS}(T^a, \bar{T}^a)] - \log\Big[(T^a + \bar{T}^a)\eta_{ab}(T^b + \bar{T}^b)\Big], \tag{2.20}$$

where

$$V_{GS}(T^a, \bar{T}^a) = \frac{2(h+\bar{h}) - (T^a + \bar{T}^a)(\partial_{T^a} h + \partial_{\bar{T}^a}\bar{h})}{(T^a + \bar{T}^a)\eta_{ab}(T^b + \bar{T}^b)} \tag{2.21}$$

is the Green-Schwarz term [32] describing the mixing of the dilaton with the moduli T^a. Note that the true perturbative coupling constant is given by

$$\frac{4\pi}{g^2_{\rm pert}} = \tfrac{1}{2}\Big(S + \bar{S} + V_{GS}(T^a, \bar{T}^a)\Big)\ . \tag{2.22}$$

To be more specific, let us recall the precise form of the one-loop prepotential $h(T^a)$ [17, 18, 19]. For simplicity we limit the discussion here to the models with $N_V = 4$. Any of the S-T-U-V models considered can be simply truncated to the three-parameter S-T-U model upon setting $V \to 0$. For the class of S-T-U-V models considered here, the one-loop prepotential is given by

$$h(T,U,V) = p_n(T,U,V) - c - \frac{1}{4\pi^3}\sum_{\substack{k,l,b\in\mathbf{Z}\\(k,l,b)>0}} c_n(4kl-b^2)Li_3(\mathbf{e}[ikT+ilU+ibV]), \tag{2.23}$$

where $c = \frac{c_n(0)\zeta(3)}{8\pi^3}$ and $\mathbf{e}[x] = \exp 2\pi i x$. The coefficients $c_n(4kl-b^2)$ are the expansion coefficients of particular Jacobi modular forms [19]. p_n is a cubic polynomial of the form [33, 34, 19]

$$p_n(T,U,V) = -\tfrac{1}{3}U^3 - (\tfrac{4}{3}+n)V^3 + (1+\tfrac{1}{2}n)UV^2 + \tfrac{1}{2}nTV^2\ . \tag{2.24}$$

It is important to note that the expression (2.23) is valid in the specific Weyl chamber $\mathrm{Re}\,T > \mathrm{Re}\,U > 2\mathrm{Re}\,V$.

Now consider taking the limit $S,T,U,V \to \infty$ subject to $\mathrm{Re}\,S > \mathrm{Re}\,T > \mathrm{Re}\,U > 2\mathrm{Re}\,V$, in which all non-perturbative as well as perturbative exponential terms are suppressed. Then, the heterotic prepotential is simply given by the cubic polynomial $\mathcal{F}^{\rm het} = -S(TU - V^2) + p_n(T,U,V)$. In the limit $V \to 0$, the perturbative prepotential is completely universal. In the large-moduli limit $S,T,U \to \infty$ ($\mathrm{Re}\ S > \mathrm{Re}\ T > \mathrm{Re}\ U$), which is the decompactification limit to 5 dimensions, the prepotential of these three-parameter models takes the form

$$\mathcal{F}^{\rm het} = -STU - \tfrac{1}{3}U^3 - c, \tag{2.25}$$

where $c = \bar{c} = \frac{c_{STU}(0)\zeta(3)}{8\pi^3}$ and $c_{STU}(kl) = \sum_b c_n(4kl - b^2)$ (for any n). Using (2.21) it is straightforward to compute the one-loop term V_{GS} which follows from the prepotential (2.25):

$$V_{GS}(T, \bar{T}, U, \bar{U}) = \frac{(U + \bar{U})^2}{3(T + \bar{T})} - \frac{4c}{(T + \overline{T})(U + \overline{U})} \quad . \tag{2.26}$$

2.3 The type-IIA prepotential

As already mentioned, the prepotential in type-IIA Calabi–Yau compactifications, which depends on the Kähler-class moduli t^A ($A = 1, \ldots, N_V = h_{1,1}$), is of purely classical origin. Nevertheless it has the same structure as the heterotic prepotential. In fact, for dual heterotic/type-IIA pairs the prepotentials are identical upon a suitable identification of the Kähler-class moduli t^A in terms of the heterotic fields S and T^a.

The type-IIA prepotential has the following general structure [35]:

$$\mathcal{F}^{\rm II} = -\frac{1}{6} C_{ABC} t^A t^B t^C - \frac{\chi\zeta(3)}{2(2\pi)^3} + \frac{1}{(2\pi)^3} \sum_{d_1,\ldots,d_h} n^r_{d_1,\ldots,d_h} Li_3(\mathbf{e}[i \sum_A d_A t^A]), \tag{2.27}$$

where we work inside the Kähler cone $\sigma(K) = \{\sum_A t^A J_A | t^A > 0\}$. (The J_A denote the (1,1)-forms of the Calabi–Yau three-fold M, which generate the cohomology group $H^2(M, \mathbf{R})$). The cubic part of the type-IIA prepotential is given in terms of the classical intersection numbers C_{ABC}, whereas the coefficients $n^r_{d_1,\ldots,d_h}$ of the exponential terms denote the rational instanton numbers of genus 0. Hence, in the limit of large Kähler class moduli, $t^A \to \infty$, only the classical part, related to the intersection numbers, survives. Consider, for example, the four-parameter model based on the compactification on the Calabi–Yau three-fold $P_{1,1,2,6,10}(20)$ with $h_{1,1} = 4$ and Euler number $\chi = -372$ [33]. The cubic intersection-number part of the type-IIA prepotential for this model is given as

$$\begin{aligned} -\mathcal{F}^{\rm II}_{\rm cubic} &= t^2((t^1)^2 + t^1 t^3 + 4t^1 t^4 + 2t^3 t^4 + 3(t^4)^2) + \tfrac{4}{3}(t^1)^3 + 8(t^1)^2 t^4 \\ &\quad + t^1 (t^3)^2 + 2(t^1)^2 t^3 + 8t^1 t^3 t^4 + 2(t^3)^2 t^4 + 12 t^1 (t^4)^2 + 6t^3 (t^4)^2 + 6(t^4)^3. \end{aligned} \tag{2.28}$$

Some of the rational instanton numbers $n^r_{d_1,d_2,d_3,d_4}$ for this model are displayed in [33, 19]. This model is dual to the previously discussed heterotic string compactification with $N_V = 4$ and $n = 2$ [19]. The necessary identification of heterotic and type-IIA moduli is given as

$$t^1 = U - 2V, \qquad t^2 = S - T, \qquad t^3 = T - U, \qquad t^4 = V \ , \tag{2.29}$$

and one can explicitly check that for some instanton numbers and for the Euler number the relations

$$n^r_{l+k,0,k,2l+2k+b} = -2c_2(4kl - b^2), \quad \chi = 2c_2(0) \tag{2.30}$$

are indeed satisfied. In addition, the cubic heterotic prepotential p_2 (cf. (2.24)) and the Calabi–Yau prepotential (cf. (2.28)) agree.

Finally, let us mention that, in a particular symplectic basis, it is very convenient to add to the Calabi–Yau prepotential (2.27) a topological term which is determined by the second Chern class c_2 of the three-fold M, which gives rise to terms linear in the Kähler-class moduli fields:

$$\mathcal{F}^{\mathrm{II}} = -\frac{1}{6} C_{ABC} t^A t^B t^C + \sum_A^h \frac{c_2 \cdot J_A}{24} t^A + \cdots . \tag{2.31}$$

The real numbers $c_2 \cdot J_A = \int_M c_2 \wedge J_A$ are the expansion coefficents of c_2 with respect to the basis J_A^* of the cohomology group $H^4(M, \mathbf{R})$ which is dual to the basis J_A of $H^2(M, \mathbf{R})$ (i.e. $\int_M J_A^* \wedge J_B = \delta_{AB}$). It is clear from eq.(2.12) that adding such a linear term to the prepotential is equivalent to performing a symplectic transformation with $U = V = 1$, $Z = 0$ and $W_{0A} = \frac{c_2 \cdot J_A}{24}$; hence it has just the effect of a constant shift in the theta angles [36]. In the next section we will see that adding such a topological linear term to the prepotential may have interesting effects on the $N = 2$ black hole entropy as a function of the magnetic/electric charges.

As an example we consider the three parameter model based on the Calabi-Yau $P_{1,1,2,8,12}(24)$ with $h_{1,1} = 3$ and $\chi = -480$, which is dual to the heterotic string compactification with $N_V = 3$ and $n = 2$. The corresponding prepotential can be simply obtained from (2.28) by setting $V = 0$. Here the linear topological term takes the form

$$\sum_A^3 \frac{c_2 \cdot J_A}{24} t^A = \tfrac{23}{6} t^1 + t^2 + 2t^3 = S + T + \tfrac{11}{6} U . \tag{2.32}$$

3 $N = 2$ Supersymmetric black holes

In this section we consider extreme dyonic black holes in the context of $N = 2$ supergravity. The fields corresponding to these black holes spatially interpolate between two maximally supersymmetric field configurations. One is the trivial flat space at spatial infinity, which allows constant values for the moduli fields. The other is the Bertotti-Robinson metric near the horizon, where the fields are restricted to (covariantly) constant moduli and graviphoton field strength (the latter is directly related to the value of the central charge at the horizon). The interpolating fields leave only half the supersymmetries invariant, so that we are dealing with true BPS states. For these black holes, the mass is equal to the central charge taken at spatial infinity, so that

$$M_{ADM}^2 = |Z_\infty|^2 = e^{K(z,\bar{z})} \left. |\mathcal{M}(z)|^2 \right|_\infty , \tag{3.1}$$

where the moduli fields z are taken at spatial infinity. Hence the mass depends generically on the magnetic/electric charges and the asymptotic values of the moduli fields.

Near the horizon the values of the moduli fields, and thus the value of the central charge, are strongly restricted by the presence of full $N = 2$ supersymmetry. In [3] it was proved that this implies that the central charge becomes extremal on the horizon. The result of this is that one can express the values of the moduli at the horizon in terms of the magnetic/electric charges p^I and q_I. The value of the central charge at the horizon is related to the Hawking-Bekenstein entropy,

$$\frac{\mathcal{S}}{\pi} = |Z_{\rm hor}|^2 \,, \tag{3.2}$$

where we have conveniently adjusted the value of Newton's constant. The area of the black hole, which equals four times the entropy, has an interpretation as the mass of the Bertotti-Robinson universe. The crucial observation here is that the entropy and related quantities depend only on the quantized magnetic/electric charges (with $N = 2$ supersymmetry the nature of this dependence is governed by the particular string vacuum), while the mass of the black hole depends on the charges as well as on the asymptotic values of the moduli. The latter are, in principle, arbitrary parameters that do not depend on the charges and, when approaching the black hole, evolve according to a damped geodesic equation towards the fixed-point values at the horizon, which are given in terms of the charges.

There exist so-called double extreme black holes, introduced in [4], for which the moduli remain constant away from the horizon. In that case the central charge remains constant and thus the black hole mass is equal to the Bertotti-Robinson mass. The moduli at spatial infinity take the same values as near the horizon, so that the black-hole mass itself is now also a function of the magnetic/electric charges. Consequently, for double extreme black holes we find that M_{ADM} is a function of the p^I and q_I.

In this section we study the extremization problem at the black-hole horizon to obtain the value of the moduli and the black-hole entropy as a function of the charges p^I and q_I. We cast this problem in a convenient form, which can be formulated in terms of a variational principle (cf. (3.12)). This allows us to construct a variety of explicit solutions. Then, in the second subsection, we consider the black-hole entropy for heterotic $N = 2$ supersymmetric string compactifications to all orders in string perturbation theory and derive a general formula for the entropy. An important feature of this formula is its invariance under target-space duality. In the next subsection we consider so-called non-axionic black holes, where one can conveniently obtain explicit solutions. Finally in subsection 3.4 we consider the entropy for type-II compactifications.

3.1 Extremization of the $N = 2$ central charge

Let us start and exhibit some features of the double extreme black holes, for which the moduli remain constant. The metric of these black holes is of the extreme Reissner-Nordstrom form with the mass equal to the square root of the area divided by 4π. In isotropic coordinates the metric is

$$ds^2 = -\left(1 + \frac{\sqrt{A/4\pi}}{r}\right)^{-2} dt^2 + \left(1 + \frac{\sqrt{A/4\pi}}{r}\right)^{2} d\vec{x}^2 \quad . \tag{3.3}$$

One can also present the metric as ($\tilde{r} = r + \sqrt{A/4\pi}$)

$$ds^2 = -\left(1 - \frac{\sqrt{A/4\pi}}{\tilde{r}}\right)^{2} dt^2 + \left(1 - \frac{\sqrt{A/4\pi}}{\tilde{r}}\right)^{-2} d\tilde{r}^2 + \tilde{r}^2 d^2\Omega \quad . \tag{3.4}$$

The mass is defined via the large-$\tilde{r}$ expansion

$$g_{tt} = \Big(1 - \frac{2M_{ADM}}{\tilde{r}} + \cdots\Big) \tag{3.5}$$

and the metric shows that

$$M_{ADM} = \sqrt{\frac{A}{4\pi}} \quad . \tag{3.6}$$

In this form it is clear that the horizon is at $g_{tt} = 0 \Longrightarrow \tilde{r} = \sqrt{A/4\pi}$. Therefore the area of the horizon is indeed given by

$$4\pi(\tilde{r}^2)_{\rm hor} = A \quad . \tag{3.7}$$

As discussed above, to obtain the value of the moduli at the horizon for extreme $N = 2$ black holes, one can determine the extremal value of the central charge in moduli space. This implies that

$$\partial_A |Z| = 0 \ . \tag{3.8}$$

These equations are difficult to solve in general. They are, however, equivalent to the following set of equations [3]

$$\bar{Z}\, V - Z\, \bar{V} = iQ \,, \tag{3.9}$$

where Q is the magnetic/electric charge vector $Q = (p^I, q_J)$. The above relation is closely related to the fact that the field configurations are fully supersymmetric at the horizon. Here we note that these equations can be independently justified on the basis of symplectic covariance. Assuming that the moduli near the horizon depend exclusively on the magnetic/electric charges and satisfy equations of motions that transform in a well-defined way under symplectic (duality) reparametrizations, the symplectic period vector must be proportional to the symplectic charge vector. As the period vector is complex and the charge vector is real, there is a complex proportionality factor which must be a symplectic invariant. Using (2.2) we derive that this factor is precisely the central charge Z and find the above result (3.9).

From (3.9) one can determine the period vector, which is defined in terms of N_V complex moduli. We do this by reformulating the equation and the corresponding expression of the black-hole entropy in terms of a variational principle. To do this, we first introduce a new symplectic vector Π by

$$\Pi = \begin{pmatrix} Y^I \\ F_J(Y) \end{pmatrix} \qquad \text{where } \; Y^I \equiv \bar{Z}\, X^I \,. \tag{3.10}$$

Observe that Y^I and thus the vector Π is $U(1)$ invariant, so that it is not subject to Kähler transformations. In terms of Π, (3.9) and (3.2) turn into

$$\Pi - \bar{\Pi} = iQ \,, \qquad \frac{\mathcal{S}}{\pi} = |Z_{\rm hor}|^2 = i\langle \bar{\Pi}, \Pi \rangle \,. \tag{3.11}$$

The equations (3.11) are governed by a variational principle associated with a 'potential'

$$\mathcal{V}_Q(Y,\bar{Y}) \equiv -i\langle \bar{\Pi}, \Pi\rangle - \langle \bar{\Pi} + \Pi, Q\rangle \,. \tag{3.12}$$

$\mathcal{V}_Q$ takes an extremal value whenever Y and $\bar{Y}$ satisfy the first equation (3.11). This extremal value is given by the second expression (3.11) for the entropy. Using (2.5) the entropy can be also written as

$$\frac{\mathcal{S}}{\pi} = |Y^0|^2 \exp\Big[- K(z,\bar{z})\Big]\Big|_{\rm hor} \,. \tag{3.13}$$

where Y^0 and the special coordinates z^A are evaluated at the horizon.

Let us now consider the construction of solutions to the equations (3.11). Written in components they read

$$Y^I - \bar{Y}^I = ip^I \,, \qquad F_I(Y) - \bar{F}_I(\bar{Y}) = iq_I \,. \tag{3.14}$$

To solve these equations it does not help to go to a special symplectic basis (although the equations may take a more 'suggestive' form), as this only corresponds to taking linear combinations. Although we assumed the existence of the holomorphic prepotential, the above variational principle can also be formulated in a basis where such a prepotential does not exist, but for the purpose of this paper this feature is not important. The components of Π comprise $2N_V + 2$ complex quantities, but only $N_V + 1$ of them are independent (as the others are determined in terms of the prepotential). So generically, the above equation fixes Π in terms of p^I and q_J. Before considering an explicit example, we note the following convenient relations, which follow from (3.14) for p^0 and p^A,

$$(z^A - \bar{z}^A)Y^0 = i(p^A - p^0\bar{z}^A)\,. \tag{3.15}$$

As an example, consider the following cubic prepotential

$$F(Y) = -b\,\frac{Y^1Y^2Y^3}{Y^0} + a\,\frac{(Y^3)^3}{Y^0}\quad . \tag{3.16}$$

The solution to (3.14) for a general magnetic/electric charge vector (p^I, q_I) where $I = 0, 1, 2, 3$, reads

$$\begin{aligned} Y^0 &= \frac{p^3 + ip^0\bar{U}}{U+\bar{U}}\;, \qquad Y^1 = -\frac{Y^0}{p^3 + ip^0\bar{U}}\Big(- ip^1\bar{U} + \frac{q_2}{b}\Big)\;, \\ Y^2 &= -\frac{Y^0}{p^3 + ip^0\bar{U}}\Big(- ip^2\bar{U} + \frac{q_1}{b}\Big)\;, \qquad Y^3 = iU\,Y^0\;, \end{aligned} \tag{3.17}$$

where U is determined by the following equation

$$\begin{aligned} q_0 - iq_3\bar{U} &= \frac{b}{p^3 + ip^0\bar{U}}\Big(- ip^1\bar{U} + \frac{q_2}{b}\Big)\Big(- ip^2\bar{U} + \frac{q_1}{b}\Big) \\ &\quad + a\,p^3(U^2 + 2U\bar{U} - 2\bar{U}^2) + ia\,p^0U\bar{U}(U + 2\bar{U})\quad . \end{aligned} \tag{3.18}$$

The entropy can be determined as a function of U, by making use of (3.13),

$$\frac{\mathcal{S}}{\pi} = \left|\frac{U+\bar{U}}{p^3 - ip^0U}\right|^2\left\{\frac{(b\,p^1p^3 + q_2p^0)(b\,p^2p^3 + q_1p^0)}{b} - a\right\}\,. \tag{3.19}$$

What remains is to solve (3.18). For the case $a = 0$, $b = 1$, this is a quadratic equation for U with solution

$$U = i\frac{q_0p^0 + q_1p^1 + q_2p^2 - q_3p^3}{2(q_3p^0 + p^1p^2)} \pm \sqrt{\frac{q_1q_2 - q_0p^3}{q_3p^0 + p^1p^2} - \frac{(q_0p^0 + q_1p^1 + q_2p^2 - q_3p^3)^2}{4(q_3p^0 + p^1p^2)^2}}\;. \tag{3.20}$$

These solutions with the corresponding value for the entropy can be compared to previous results [12, 4, 13] (for the results of the second and third work this comparison requires a conversion to the appropriate symplectic basis).

3.2 Perturbative entropy formula for heterotic string compactifications

The classical entropy formula for $N = 2$ supersymmetric heterotic string compactifications has been derived in the perturbative string basis [4, 13]. It was shown to be invariant (as should be expected) under the target-space duality group, which, at the classical level, is just equal to $SO(2, N_V - 1)$. The entropy was also constructed in the symplectic basis corresponding to the first term in (2.17) in [12].

In this section we derive the entropy formula, but now to all orders of string perturbation theory. It reads

$$\frac{\mathcal{S}}{\pi} = \frac{8\pi}{g^2_{\rm pert}}\Big|_{\rm hor} \left(p^0 q_1 + p^a \eta_{ab} p^b\right) \ . \tag{3.21}$$

The perturbative string coupling depends on the values of the dilaton field and the moduli at the horizon. The charges p and q refer to the magnetic/electric charges as defined in the symplectic basis associated with (2.17). This is, however, not the basis defined by perturbative string theory, where the magnetic charges are equal to $N^I = (p^0, q_1, p^2, \ldots)$. These magnetic charges transform linearly under target-space duality transformations,

$$N^I \to \hat{U}^I{}_J \, N^J \ , \tag{3.22}$$

where the matrix $\hat{U}$ belongs to a subgroup of $SO(2, N_V - 1, \mathbf{Z})$. In terms of the string basis we find that we are dealing with an invariant under these transformations [11, 15, 13]

$$p^0 q_1 + p^a \eta_{ab} p^b = N^0 N^1 + N^a \eta_{ab} N^b \equiv \langle N, N \rangle \ . \tag{3.23}$$

Thus, in the perturbative string basis, eq. (3.21) reads

$$\frac{\mathcal{S}}{\pi} = \frac{8\pi}{g^2_{\rm pert}}\Big|_{\rm hor} \langle N, N \rangle \ . \tag{3.24}$$

Due to nonrenormalization theorems, this result is true to all orders in perturbation theory and takes precisely the same form as the classical entropy formula [13], with the tree-level coupling constant replaced by its full perturbative value. As the latter is invariant under target-space duality [15], the perturbative entropy formula is invariant under target-space duality. In fact, since $\langle N, N \rangle$ is invariant under target-space duality transformations, whereas the dilaton is not, at least not beyond the classical level, it was natural to expect that the corrected entropy formula should be given by the tree-level formula with the dilaton replaced by some one-loop target-space duality invariant object. It is gratifying to see that this object is precisely the true loop-counting parameter of heterotic string theory.

Let us now show that (3.21) indeed holds. Inserting eqs. (2.20) into eq. (3.13) yields

$$\frac{\mathcal{S}}{\pi} = (S + \bar{S} + V_{GS}) \, |Y^0|^2 \, (T^a + \bar{T}^a) \eta_{ab} (T^b + \bar{T}^b) \ . \tag{3.25}$$

Using that $T^a = -iz^a$, and inserting (3.15) and its complex conjugate into (3.25) yields

$$|Y^0|^2 \, (T^a + \bar{T}^a) \eta_{ab} (T^b + \bar{T}^b) = (p^a + i p^0 \bar{T}^a) \eta_{ab} (p^b - i p^0 T^b) \ . \tag{3.26}$$

On the other hand, it follows from (3.14) and (2.17) that

$$F_1(Y) - \bar{F}_1(\bar{Y}) = -\frac{Y^a \eta_{ab} Y^b}{Y^0} + \frac{\bar{Y}^a \eta_{ab} \bar{Y}^b}{\bar{Y}^0} = i q_1 \quad . \tag{3.27}$$

Using that $\bar{Y}^a = Y^a - i p^a$, we obtain

$$\begin{aligned} F_1(Y) - \bar{F}_1(\bar{Y}) &= -\frac{Y^a \eta_{ab} Y^b}{Y^0} + \frac{\bar{Y}^a \eta_{ab} (Y^b - i p^b)}{\bar{Y}^0} \\ &= -i T^a \eta_{ab} Y^b - i \bar{T}^a \eta_{ab} (Y^b - i p^b) \\ &= Y^0 (T^a + \bar{T}^a) \eta_{ab} T^b - \bar{T}^a \eta_{ab} p^b = i q_1 \quad . \end{aligned} \tag{3.28}$$

Using once more (3.15), we establish

$$p^0 q_1 + p^a \eta_{ab} p^b = (p^a + i p^0 \bar{T}^a) \eta_{ab} (p^b - i p^0 T^b) \quad . \tag{3.29}$$

Combining (3.25), (3.26) and (3.29) and using the expression for the perturbative string-coupling constant (2.22) yields the desired result (3.21).

3.3 The axion-free case

Axion-free solutions are solutions with Re $z^a = 0$. For these solutions (3.15) takes the form

$$z^A (2Y^0 - i p^0) = i p^A \, . \tag{3.30}$$

First let us assume that $2Y^0 - i p^0 = Y^0 + \bar{Y}^0 = \lambda \neq 0$. In that case we easily derive the following result for the Y^I,

$$Y^0 = \tfrac{1}{2}(\lambda + i p^0) \, , \qquad Y^A = i p^A \, \frac{\lambda + i p^0}{2\lambda} \, . \tag{3.31}$$

Consider the second set of equations (3.14) applied to an arbitrary prepotential of the heterotic/type-II form,

$$F(Y) = \frac{d_{ABC} \, Y^A Y^B Y^C}{Y^0} + i c (Y^0)^2 \, , \tag{3.32}$$

where c is a real constant. In principle we could allow additional quadratic terms, which would still be explicitly solvable, at least for axion-free solutions. Arbitrary quadratic terms with *real* coefficients can be easily incorporated by making use of suitable symplectic reparametrization. This will be discussed in the next subsection. For the case above (3.14) now yields the following equations,

$$q_0 = \frac{d_{ABC} \, p^A p^B p^C}{\lambda^2} + 2 c \lambda \, , \qquad q_A = -\frac{3 p^0}{\lambda^2} \, d_{ABC} \, p^B p^C \, , \tag{3.33}$$

leading to the condition

$$3p^0 q_0 + p^A q_A = 6c\lambda\, p^0 \,. \tag{3.34}$$

Observe that the first condition (3.33) can only be satisfied for $(q_0 - 2\lambda)\, d_{ABC} p^A p^B p^C > 0$. The entropy can be computed from (3.13) and reads

$$\frac{\mathcal{S}}{\pi} = -2(q_0 - 2c\lambda)\Big[\lambda + \frac{(p^0)^2}{\lambda}\Big] \,. \tag{3.35}$$

For $cp^0 \neq 0$ we can express λ in terms of the charges,

$$\lambda = \frac{3p^0 q_0 + p^A q_A}{6c\, p^0} \,, \tag{3.36}$$

On the other hand, when $cp^0 = 0$ we have a constraint on the charges,

$$3p^0 q_0 + p^A q_A = 0 \,. \tag{3.37}$$

For $c = 0$ and $q_0 \neq 0$ we can express λ as

$$\lambda = \pm\sqrt{\frac{d_{ABC}\, p^A p^B p^C}{q_0}} \,. \tag{3.38}$$

Plugging this into (3.33) one can express the charges q_A in terms of the remaining ones, q_0, p^0, p^A. Positivity of the entropy requires $q_0 \lambda < 0$. In the following we choose the moduli z^A to live on the upper-half plane Im $z^A > 0$ and for convenience we restrict ourselves to charges with $q_0 < 0$ and $p^A > 0$. Then the moduli z^A take the form

$$z^A = i\, p^A \sqrt{\frac{q_0}{d_{ABC}\, p^A p^B p^C}} \,. \tag{3.39}$$

As a special case, consider the non-axionic solution (3.33) with $c = p^0 = 0$ and, consequently, $q_A = 0$. This constitutes a solution with only $N_V + 1$ independent, non-vanishing charges, which we take to satisfy $p^A > 0$, $q_0 < 0$, for definiteness. The entropy is given by

$$\frac{\mathcal{S}}{\pi} = 2\sqrt{q_0 d_{ABC} p^A p^B p^C} \,, \tag{3.40}$$

and the moduli are given in (3.39). In particular, for the cubic prepotential (3.16) we find that

$$z^1 = p^1\,\frac{z^3}{p^3} \,, \qquad z^2 = p^2\,\frac{z^3}{p^3} \,, \qquad z^3 = i\sqrt{\frac{q_0 p^3}{-b\, p^1 p^2 + a(p^3)^2}} \,, \tag{3.41}$$

as well as

$$\frac{\mathcal{S}}{\pi} = 2\sqrt{-q_0 (b\, p^1 p^2 p^3 - a(p^3)^3)} \,. \tag{3.42}$$

For the values $b = -3a = 1$, the cubic prepotential (3.16) describes the one-loop corrected heterotic prepotential (2.25) of the S-T-U model in the decompactification limit Re $S >$

Re $T >$ Re $U \to \infty$. Consistency of this limit requires the following ordering of the absolute values of the charges: $-q_0 \gg p^1 > p^2 > p^3 \gg 0$. It will be shown in section 4 that this hierarchy of charges also guarantees the suppression of α' corrections.

Also note that the solution (3.38) we found for the case $cp^0 = 0$ is a good approximate solution for the general case $c \neq 0$ (with general p^0). Recalling that $c = \frac{\chi\zeta(3)}{16\pi^3}$, which is of order 1 for typical Calabi-Yau Euler numbers χ with $|\chi| \leq 1000$, we expect that the constant term in the prepotential will only give a small contribution when the moduli are large. Comparing the exact solution for λ in the case $c \neq 0$ to the solution (3.38) one can show that both differ by terms of order $\sqrt{\frac{c^2 d_{ABC} p^A p^B p^C}{q_0^3}}$, which is small for $|q_0| \gg |p^A|$, i. e. for large moduli.

The second class of solutions corresponds to $Y^0 = \frac{1}{2} i p^0$, which implies (for finite z^A) that all the p^A must vanish. Now the stabilization equations (3.14) imply that

$$q_A = 3p^0\, d_{ABC}\, z^B z^C\,, \qquad q_0 = 0\,. \tag{3.43}$$

Hence the only nonzero charges are p^0 and (some of) the q_A. The above N_V quadratic equations for the N_V purely imaginary parameters z^A can usually be solved straightforwardly. Note that there is no dependence on the constant term c in this case, because Y^0 is purely imaginary.

To demonstrate this second solution we reconsider the prepotential corresponding to (3.16). The equations for q_A take the form

$$q_1 = b\, p^0\, TU\,, \qquad q_2 = b\, p^0\, SU\,, \qquad q_3 = b\, p^0\, ST - 3a\, p^0\, U^2\,, \tag{3.44}$$

with S, T, U real. These solutions can be solved for S, T and U,

$$\begin{aligned} 2\sqrt{\frac{b\, p^0 q_1}{q_2}}\, S = 2\sqrt{\frac{b\, p^0 q_2}{q_1}}\, T &= \sqrt{q_3 + 2\sqrt{\frac{-3a\, q_1 q_2}{b}}} + \sqrt{q_3 - 2\sqrt{\frac{-3a\, q_1 q_2}{b}}}\,, \\ 2\sqrt{-3a\, p^0}\, U &= \sqrt{q_3 + 2\sqrt{\frac{-3a\, q_1 q_2}{b}}} - \sqrt{q_3 - 2\sqrt{\frac{-3a\, q_1 q_2}{b}}}\,. \end{aligned} \tag{3.45}$$

The charges and the coefficients a and b must be chosen such that S, T and U are positive.

3.4 The entropy formula in type-II compactifications

The entropy formula for extreme black holes in type-II compactifications will depend on electric and magnetic charges as well as on topological data of the Calabi–Yau manifold, on which one has compactified the type-II string theory. The topological data appearing in the prepotential are the classical intersection numbers C_{ABC} as well as the expansion coefficients $c_2 \cdot J_A$ of the second Chern class c_2 of the three-fold, which were defined in section 2.3. These data are related to the (real) coefficients d_{ABC}, c and W_{0A} of the associated prepotential,

$$F(Y) = \frac{d_{ABC}\, Y^A Y^B Y^C}{Y^0} + W_{0A}\, Y^0 Y^A + ic(Y^0)^2\,, \tag{3.46}$$

by $d_{ABC} = -\frac{1}{6} C_{ABC}$ and $W_{0A} = \frac{c_2 \cdot J_A}{24}$.

For extreme black holes based on the prepotential (3.46), the entropy formula will generically be given by

$$\frac{\mathcal{S}}{\pi} = |Z_{\rm hor}|^2 = \tfrac{1}{4}\, A\left((p^I, q_I), C_{ABC}, c_2 \cdot J_A, \chi\right) \quad . \tag{3.47}$$

As is well known, quadratic polynomials with real coefficients can be introduced into any $N = 2$ prepotential by a suitable symplectic reparametrization. So the above case (3.46) is covered by our previous analysis, provided we perform the corresponding symplectic rotation on the associated charges,

$$\begin{pmatrix} \tilde{p}^I \\ \tilde{q}_I \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ W & 1 \end{pmatrix} \begin{pmatrix} p^I \\ q_I \end{pmatrix} \quad , \tag{3.48}$$

where $W_{AB} = W_{00} = 0$. Note that a non–vanishing W_{00} would not allow us to eliminate the term $ic(Y^0)^2$ in the prepotential, because W_{00} must be real, wheras ic is imaginary. More general theta shifts with $W_{AB} \neq 0$, $W_{00} \neq 0$ would generate quadratic and constant terms in Y^0 with real coefficents. We will discard these terms, because they don't have a topological interpretation.

In the following the electric and magnetic charges of the former solution are denoted by $\tilde{q}_I$ and $\tilde{p}^I$, respectively, whereas the electric and magnetic charges of the latter are denoted by q_I and p^I. Note that this symplectic transformation induces a shift to the theta angles and thus a corresponding shift of the electric charges [36]. Thus, it follows that the entropy for the former solution can be computed from the entropy for the latter by performing the above substitution of the electric charges.

Consider, for instance, the axion-free solution (3.40) discussed in the previous subsection, based on the cubic prepotential (2.14), with $p^0 = q_A = 0$ and setting $c = 0$. Then we have for the symplectically transformed solution that

$$q_0 = \tilde{q}_0 - W_{0A} \tilde{p}^A \quad , \tag{3.49}$$

and for its entropy that

$$\frac{\mathcal{S}}{\pi} = 2\sqrt{(\tilde{q}_0 - W_{0A}\tilde{p}^A) d_{BCD} \tilde{p}^B \tilde{p}^C \tilde{p}^D} \quad . \tag{3.50}$$

Thus, we can in particular set $\tilde{q}_0 = 0$, that is, we have a solution that is determined by magnetic charges $\tilde{p}^A$ only, which is non-singular and has non-vanishing entropy

$$\frac{\mathcal{S}}{\pi} = 2\sqrt{-(W_{0A}\tilde{p}^A)d_{BCD}\tilde{p}^B\tilde{p}^C\tilde{p}^D} \ . \tag{3.51}$$

In the effective action, the term proportional to W in (3.46) manifests itself in the presence of the additional term in the action

$$\delta S \sim \int W_{A0}\, F^A \wedge F^0 \ . \tag{3.52}$$

Since F^0 is an electric gauge field and F^A is a magnetic monopole field, this integral is non-vanishing.

4 Relation to higher-dimensional geometries

The black-hole solutions discussed so far appeared in the context of either a compactification of the heterotic string on $K3 \times T_2$ or of the type-II string on a Calabi–Yau three-fold. Type-II string theory, on the other hand, is dual [9] to M-theory compactified on $CY \times S_1$ [39]. In this section we discuss how the black-hole geometries associated with (3.40) arise from a compactification of the higher-dimensional spacetime, that is, by a compactification of M-theory. We focus on those black-hole geometries that can either be obtained by a type-II string compactification on a Calabi–Yau three-fold with $h_{1,1} = 3$, or that are associated with the S-T-U models on the heterotic side.

On the M-theory side, we can regard these black-hole solutions as arising from compactifications of certain 11-dimensional solutions describing three intersecting M-5-branes with a boost along the common string. Let us first consider the simplest such 11-dimensional solution, which can be compactified on a 6-dimensional torus, [37]:

$$\begin{aligned} ds^2_{11} = \tfrac{1}{(H^1H^2H^3)^{\frac{1}{3}}}\Big[&du\, dv + H_0\, du^2 + H^1H^2H^3\, d\vec{x}^2 + \\ &+ H^1(dy_1^2 + dy_2^2) + H^2(dy_3^2 + dy_4^2) + H^3(dy_5^2 + dy_6^2)\Big] \ . \end{aligned} \tag{4.1}$$

Here, the H^1, H^2 and H^3 parametrize the three 5-branes and they are harmonic functions with respect to $\vec{x}$. The internal space is spanned by the coordinates y. Each 5-brane wraps around a 4-cycle; e.g. the H^1-5-brane around (y_3, y_4, y_5, y_6), and any two 4-cycles intersect each other in a 2-cycle.

Next, let us look at more complicated 11-dimensional solutions which can be compactified on Calabi–Yau three-folds. For a generic Calabi–Yau three-fold, the intersection of three of the 4-cycles is determined by the classical intersection numbers C_{ABC}. This leads us to make the following ansatz for the 11-dimensional metric, in analogy to (4.1),

$$ds_{11}^2 = \frac{1}{(\frac{1}{6}C_{ABC}H^A H^B H^C)^{\frac{1}{3}}}\Big[du\,dv + H_0\,du^2 + \tfrac{1}{6}C_{ABC}H^A H^B H^C\,d\vec{x}^2 + H^A\omega_A\Big]\,, \quad (4.2)$$

where ω_A $(A = 1,2,3)$ are the 2-dimensional line elements, which correspond to the intersection of two of the 4-cycles. Below, we will fix the harmonic functions H^A for the solution (3.40) in the double extreme limit.

After compactifying the internal coordinates in (4.2), we obtain a magnetic string solution in $D = 5$ dimensions. Similarly to the extreme Reissner-Nordstrom black hole in $D = 4$ dimensions, this magnetic solution has a non-singular horizon with the asymptotic geometry $AdS_3 \times S_2$ [40]. In order to obtain a regular solution in $D = 4$ dimensions as well, we first have to perform a boost along this string (parameterized by H_0), which will keep the compactification radius G_{uu} finite everywhere. This boost will induce momentum modes propagating along the magnetic string. Turning off these modes has the consequence that this radius shrinks to zero size on the horizon and that the solution becomes singular. Thus, performing the boost adds one electric charge to the three magnetic charges. Then, all the radii of the Calabi–Yau 2- and 4-cycles as well as of the string will also stay finite on the horizon. The resulting 4-dimensional metric defines an extreme Reissner-Nordstrom geometry given by

$$ds_4^2 = -\frac{1}{\sqrt{-\frac{1}{6}H^0\,C_{ABC}H^A H^B H^C}}\,dt^2 + \sqrt{-\tfrac{1}{6}H_0\,C_{ABC}H^A H^B H^C}\,d\vec{x}^2\,. \quad (4.3)$$

Next, let us consider the dual heterotic string solution with fields S, T and U. This will allow us to determine the harmonic functions H^A. We will restrict ourselves to the classical solution, that is to (3.42) with $b = 1, a = 0$.

First, we will have to change the symplectic basis. That is, we will have to go from the basis corresponding to (2.17) to the perturbative basis preferred by the heterotic string. This requires a symplectic reparametrization, after which p^1 is no longer a magnetic, but an electric charge: $p^1 \to -q_1$. Hence in the heterotic string basis the solution is now characterised by 2 magnetic (p^2, p^3) and 2 electric (q_0, q_1) charges. The classical S-T-U black hole can then be obtained from the 6-dimensional solution [38]

$$ds_6^2 = \frac{1}{H_1}\Big(du\,dv + H_0\,du^2\Big) + H_2\Big(\frac{1}{H_3}(dx_4 + \vec{V}d\vec{x})^2 + H_3\,d\vec{x}^2\Big) \quad (4.4)$$

$(\epsilon_{ijk}\partial_j V_k = \partial_i H_3)$. It describes a fundamental string lying in a solitonic 5-brane. Again, in order to keep the compactification radii finite, we need to perform a boost along the string and put a Taub-NUT soliton in the transversal space. From the resulting solution, we can immediately read off the S, T and U fields. By compactifying over u and x_4, we obtain for the internal metric that

$$G_{rs} = \begin{pmatrix} \frac{H_0}{H_1} & 0 \\ 0 & \frac{H_2}{H_3} \end{pmatrix} = \frac{H_2}{H_3} \begin{pmatrix} (\mathrm{Re}\, U)^2 & 0 \\ 0 & 1 \end{pmatrix} , \tag{4.5}$$

and thus we find for the scalar fields that

$$\begin{aligned} S &= e^{-2\phi} = e^{-2\hat{\phi}} \sqrt{|G_{rs}|} = \sqrt{\tfrac{H_0 H_1}{H_2 H_3}} , \\ T &= \sqrt{|G_{rs}|} = \sqrt{\tfrac{H_0 H_2}{H_1 H_3}} \quad , \quad U = \sqrt{\tfrac{H_0 H_3}{H_1 H_2}} \end{aligned} \tag{4.6}$$

($\hat{\phi}$ is the 6-dimensional dilaton). In the double extreme limit, we have to fix the values of the scalars at infinity so that they are constant everywhere. For the harmonic functions this means that

$$\begin{aligned} H_0 &= \sqrt{2} q_0 \Big(c + \frac{1}{r}\Big) , \qquad H_1 = \sqrt{2} q_1 \Big(c + \frac{1}{r}\Big) , \\ H_2 &= \sqrt{2} p^2 \Big(c + \frac{1}{r}\Big) , \qquad H_3 = \sqrt{2} p^3 \Big(c + \frac{1}{r}\Big) , \end{aligned} \tag{4.7}$$

with $c^{-4} = 4 q_0 q_1 p^2 p^4$ (in order to obtain asymptotic Minkowski geometry in $D = 4$). The limit of large q_0 now has the consequence that the boost or momentum along the string becomes large. Hence, this direction decompactifies and we obtain the 5-dimensional string solution. The metric (in the Einstein frame) is in this case again given by (4.3) with $C_{123} = 6$ as the only non-vanishing element.

We can now insert the harmonic functions (4.7) into the metric (4.3) with general coefficients C_{ABC}. In this way we precisely recover the metric (3.3). Note that in our notation $-q_0 \frac{1}{6} C_{ABC} p^A p^B p^C > 0$. When approaching the horizon $r \to 0$, we obtain the Bertotti-Robertson geometry which is non-singular ($AdS_2 \times S_2$) and restores all supersymmetries. The radius of the S_2 is given by the mass and so the area of the horizon is $A = 4\pi M^2 = 8\pi \sqrt{-q_0 \frac{1}{6} C_{ABC} p^C p^B p^C}$. This is the metric in the Einstein frame. The string, however, couples to the string-frame metric, which can easily be given on the heterotic side. Replacing $-\frac{1}{6} C_{ABC}$ by d_{ABC} and using the dilaton value $S = -iz^1$ with z^1 given by (3.39), we find that

$$\begin{aligned} ds^2_{str} &= \frac{\sqrt{q_0\, d_{ABC}\, p^A p^B p^C}}{|q_0 p^1|}\, ds^2 \\ &= -\frac{1}{|q_0 p^1|} \Big(c + \frac{1}{r}\Big)^{-2} dt^2 - \frac{d_{ABC}\, p^A p^B p^C}{p^1} \Big(c + \frac{1}{r}\Big)^2 d\vec{x}^2 \end{aligned} \tag{4.8}$$

(for $q_0 < 0$ and $p^A > 0$). This again has a throat geometry for $r \to 0$

$$ds^2_{str} \to -e^{2\eta/R}\, dt^2 + d\eta^2 + R^2\, d\Omega^2 \quad , \quad R^2 = -\frac{d_{ABC}\, p^A p^B p^C}{p^1} \tag{4.9}$$

($r \sim \exp(\eta/R)$). Since the curvature has its maximum inside the throat, we can keep higher curvature corrections ($\sim \mathcal{O}(\alpha')$) under control if the radius of the throat is sufficiently large: $-d_{ABC}\, p^A p^B p^C \gg p^1$. This means that sufficiently large magnetic charges ensure that all higher curvature terms can be suppressed.

5 Summary

Supersymmetric black holes provide us with a tool to probe the properties of the future fundamental theory which will describe non-perturbative quantum gravity. This theory is expected to explain in a quantum-mechanical context the existence of all non-perturbative states, or solitons, in string theory and in supergravity and also to control the interaction between these states. Meanwhile, in the absence of such a theory, it is important to study supersymmetric black holes and their properties as the most particular representatives of the non-perturbative states of quantum gravity. One of the remarkable property of all ($N = 2, D = 4$) supersymmetric black holes is the topological nature of the area of the black hole horizon in the sense that the area does not depend on the values of the moduli fields at spatial infinity [2]. The explanation of the entropy via the counting of string states [1] was thus established for a class of black holes for which the entropy depends only on electric and magnetic charges. In this paper we have found various new area formulae for a class of $N = 2, D = 4$ supersymmetric theories. The choice of the prepotentials is motivated by various versions of string theory at the classical level as well as by string-loop corrections.

On the heterotic side we have studied the prepotentials which include the contributions from string-loop corrections. At the perturbative level, we could prove that the entropy takes precisely the same form as the tree-level entropy [4, 13], where the tree-level coupling constant $S+\bar{S}$ is replaced by the perturbative coupling constant, which originates entirely from one-loop effects and contains the Green-Schwarz modification:

$$\frac{\mathcal{S}}{\pi} = (S + \bar{S} + V_{GS})\Big|_{\mathrm{hor}} \, (p^0 q_1 + p^a \eta_{ab} p^b). \tag{5.1}$$

Therefore, we confirmed the conjecture [13] that the string loops will affect the area formula only via a perturbative modification of the string coupling. In case that the one-loop heterotic prepotential can be approximated by a cubic polynomial, as it is true for large moduli values, the one-loop string coupling can be explicitly expressed in terms of the magnetic/electric charges for non-axionic black-hole solutions. For solutions that are not axion-free, the explicit expressions depend on solving some higher-order polynomial equation, as exhibited in section 3.1.

On the type-IIA side, our new area formulae imprint also the topological data of the Calabi-Yau manifold, in particular the intersection numbers C_{ABC}. The fact that this symmetric tensor enters the area formula for the five-dimensional black holes was known before [3]. However, in five dimensions the area formula is implicit, as one still has to minimize it in the moduli space. Here, for the first time, we have found the area formulae of four-dimensional black holes which depend on charges and on arbitrary intersection numbers C_{ABC}. In addition, we have found an interesting dependence of the area on the second Chern class of the three-fold c_2 . Here we deal with the Witten-type shift [36] of the electrical charge via magnetic charge in the presence of axions. Finally the entropy will in general depend on the Euler number χ of the Calabi-Yau three-fold, but this contribution is small compared to the other effects we studied.

In the simplest case, when all the moduli z^A are imaginary (the axion-free solution with $p^0 = q_A = 0$ and $c = 0$), the entropy is given by

$$\mathcal{S} = 2\pi\sqrt{\left(-q_0 + \frac{c_2 \cdot J_A}{24} p^A\right) \frac{C_{BCD}}{6} p^B p^C p^D} \; . \tag{5.2}$$

This formula reproduces some previously known solutions, in particular for the S-T-U black holes [12], where $C_{123} = 6$ and $c_2 = 0$, and where the entropy of the simplest non-axion solution was found to be $\mathcal{S} = 2\pi\sqrt{|q_0 p^1 p^2 p^3|}$. One can now address the following issue: which fundamental theory is capable of giving a microscoping interpretation to (5.2)?

An interesting feature of the new area formulae is the possibility to relax some of the electric charges due to the above mentioned shift effect via $c_2 \cdot J_A$ terms. A simple example of such a relaxation is as follows. When applying the theta-angle shift to the S-T-U black hole [12] with one magnetic and three electric charges, which has the area formula $\mathcal{S} = 2\pi\sqrt{|p^0 q_1 q_2 q_3|}$, we obtain an entropy formula which is non-vanishing for $q_1 = q_2 = q_3 = 0$, even though there now is only one magnetic charge present: $\mathcal{S}/\pi = 2(p^0)^2\sqrt{\frac{1}{24^3}|(c_2 \cdot J_1)(c_2 \cdot J_2)(c_2 \cdot J_3)|}$.

In conclusion, we have found various qualitatively new features of supersymmetric black holes in $N = 2, D = 4$ supergravity theories motivated by string theory.

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