A simple approach to describe hadron production rates in e^+e^- annihilation [*]

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We show that, based on the idea of string fragmentation, the production rates of light flavored mesons and baryons originating from fragmentation can be described by the spin, the binding energy of the particle, and a strangeness suppression factor. Apart from a normalization factor, e^+e^- data at different center-of-mass energies can be described simultaneously. Applying to the heavy flavor production, we find that our predictions are in good agreement with data.

1 Introduction

The soft processes of hadronization are not calculable with a perturbative approach and instead rely currently on phenomenological models. The most successful of these models are the string[1] and the cluster[2] fragmentation models, implemented in the Monte Carlo programs JETSET[3] and HERWIG[4], respectively. However, these models either require a large number of free parameters in order to reproduce the measured hadron production rates (JETSET), or do not give a satisfactory description of baryon data (HERWIG). A review may be found in Ref.[5,6].

In this paper, we first show some regularities in the hadron production rates measured at LEP. Then based on the idea of string fragmentation, we deduce a simple formula to describe the LEP data. The formula is also applied to data obtained at center-of-mass energies around 10 GeV and 29-35 GeV, and to the heavy flavor production.

2 Overview of LEP data

Thanks to excellent performance of the detectors and high statistics available, very careful work by all four LEP experiments has given a very complete picture of the production of identified particles from e^+e^- annihilation. The measured production rates per hadronic Z event for the identified particles at LEP[7,8] are listed in Table 1.

Studies of general features of particle production, such as the fraction $V/(V+P)$ of mesons produced in spin-1 states and the strangeness suppression factor $\gamma_s = \mathrm{s/u}$, provide useful information about the fragmentation process. The ratio $V/(V+P)$ for primary mesons is expected to be equal to 0.75 from simple spin counting. From the rates and the primary fractions given in Table 1 and Ref.[9], we obtain a value of 0.46 ± 0.04, 0.42 ± 0.03, 0.56 ± 0.04 and 0.75 ± 0.04 for u(d)-, s-, c- and b-mesons respectively. The low value of $V/(V+P)$ for light flavored mesons could be explained by mass differences between the vector and pseudoscalar mesons, i.e. by the relatively larger binding energy of pseudoscalar mesons. For c-mesons the measured ratio of $V/(V+P)$ is also low, while for b-mesons it agrees well with the expected value of 0.75. We will discuss this later in more detail.

Figure 1 shows the production rates of primary hadrons (measured rate × fraction from JETSET as listed in Table 1), divided by the spin factor (2J+1), as a function of the hadron mass. We see similar behavior[a] (steps) for hadrons belonging to the same multiplet, such as $(\rho/\omega, \mathrm{K}^*, \phi)$, $(\mathrm{p}, \Lambda, \Sigma, \Xi)$, $(\Delta, \Sigma^*, \Xi^*, \Omega)$ and $(\mathrm{f}_2, \mathrm{K}_2^*, \mathrm{f}_2')$. The mass difference of hadron pairs which are in the same multiplet but differ by one in the strangeness is in the range of 0.1–0.15 GeV (except $m_\mathrm{K} - m_\pi$). This is close to the mass difference between the s and the u(d) quark, showing that the binding energy of hadrons in each pair is about the same. Unlike the case of $V/(V+P)$, one could expect the γ_s value determined from different hadron pairs is about the same.

In the string fragmentation model, one expects the strangeness suppression factor γ_s to be around 0.3. This parameter can be measured from the production rates of strange compared with non-strange hadrons, and from the momen-

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[a] From Fig. 1 one can see that it is not possible to describe all the data with just an exponential function of the hadron mass (or mass squared).

Table 1: Average particle production rates (excluding charge conjugates and antiparticles), compared to the calculated values (see next section). The fraction of primary hadrons obtained from the fit and JETSET is also shown.

Particle	Rate Measured	Rate Calc.	Prim. Frac. Calc.	Prim. Frac. JETSET
π^0	9.19 ± 0.73	9.77	0.16	0.14
π^+	8.53 ± 0.22	8.70	0.18	0.16
K^0	1.006 ± 0.017	1.008	0.25	0.30
K^+	1.185 ± 0.065	1.100	0.23	0.27
η	0.95 ± 0.11	0.85	0.30	0.33
η'	0.22 ± 0.07	0.11	0.59	0.79
$f_0(980)$	0.140 ± 0.034	0.080	0.99	0.93
ρ^0	1.29 ± 0.13	1.12	0.47	0.54
K^{*0}	0.380 ± 0.021	0.390	0.49	0.60
K^{*+}	0.358 ± 0.034	0.395	0.50	0.60
ω	1.11 ± 0.14	1.05	0.48	0.57
ϕ	0.107 ± 0.009	0.092	0.64	0.70
$f_2(1270)$	0.25 ± 0.08	0.19	0.78	0.96
$K_2^*(1430)^0$	0.095 ± 0.035	0.051	1.00	0.98
$f_2'(1525)$	0.020 ± 0.008	0.018	1.00	0.98
p	0.49 ± 0.05	0.54	0.12	0.56
Λ	0.186 ± 0.008	0.165	0.12	0.44
Σ^0	0.0355 ± 0.0065	0.0387	0.40	0.86
Σ^+	0.044 ± 0.006	0.037	0.42	0.88
Ξ^-	0.0129 ± 0.0007	0.0118	0.41	0.75
Δ^{++}	0.064 ± 0.033	0.069	0.69	0.95
$\Sigma(1385)^+$	0.011 ± 0.002	0.016	0.91	0.92
$\Xi(1530)^0$	0.0031 ± 0.0006	0.0050	0.94	0.94
Ω^-	0.0008 ± 0.0003	0.0015	0.88	0.92
$\Lambda(1520)$	0.0107 ± 0.0014	0.0129	0.71	—

tum spectrum of strange mesons. The results are very consistent with the expectation (a review may be found in Ref.[9]). This suggests that the strangeness suppression occurs at the quark level. In the following we consider that the particle production proceeds in two stages, namely quark pair production in the color string field and successive recombination.

3 Analysis

Quark pair production in the color string field can be considered as a tunneling process. The probability to produce a $q\bar{q}$ pair is proportional to $\exp(-\pi m_q^2/\kappa)$, where m_q is the (constituent) quark mass, and κ the string constant. We assume that the probability of quarks recombined to a hadron with the mass M_h is proportional to $\exp(-E_{bind}/T)$, where $E_{bind} = M_h - \sum_i m_{q_i}$ is the hadron binding energy[b], and T the effective temperature in hadronization. The production rates of light flavored mesons and baryons from fragmentation can be described as

$$< N >= C \cdot \frac{2J+1}{C_B} \cdot (\gamma_s)^{N_s} \cdot \mathrm{e}^{-\frac{E_{bind}}{T}}, \quad (1)$$

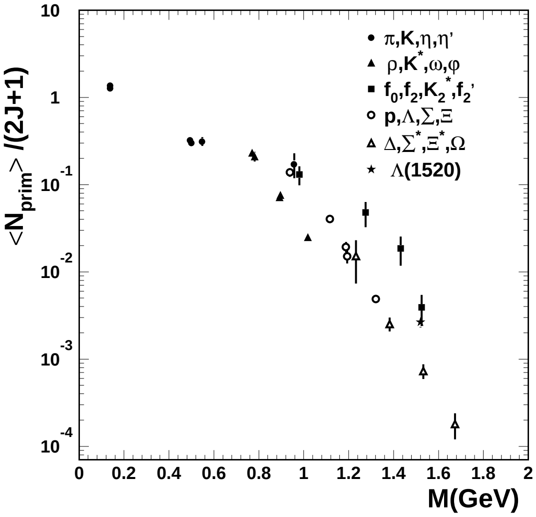

Figure 1: Production rates of primary light flavored hadrons at LEP energy, divided by the spin factor (2J+1), as a function of the hadron mass.

where $\gamma_s = \exp(-\pi(m_s^2 - m_u^2)/\kappa)$ is the strangeness suppression factor, N_s the number of strange quarks contained in the hadron, and J the spin of the hadron. C is an overall normalization factor, and C_B is the relative normalization factor between mesons and baryons (for mesons $C_B=1$). Equation 1 can also be applied to mixed states of the SU(3) octet and singlet, such as η and η', by adding up the $u\bar{u}(d\bar{d})$ and $s\bar{s}$ contributions. For this purpose we use the mixing formulae and angles given in Ref.[11].

Total hadron rates are calculated as follows. At first the number of light flavored hadrons produced from fragmentation is calculated by using Eq.1. For hadrons which contain a primary quark q, we use Eq.1 to determine their relative ratios, and then get their rates by normalizing the sum of the rates to the $q\bar{q}$ fraction, $\Gamma_{q\bar{q}}/\Gamma_{had}$, which can be calculated by the Standard Model. All light flavored hadrons up to a mass of 2.5 GeV in the meson and baryon summary table of Ref.[11] are included in the calculation. In the next step we let all these primary hadrons decay according to their decay channels and branching ratios given in Ref.[11]. The decay chain stops when μ, π, $K^\pm$, K_L^0 or stable particles are reached.

In the fit we choose γ_s, $\Delta m = m_s - m_u$, T, C and C_B as free parameters[c] (we assume $m_u = m_d$).

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[b] It can be ascribed to the hyperfine interaction[10].

[c] Since $\exp(-E_{bind}/T) = \exp(\sum_i m_u/T) \cdot \exp(-(M_h - \sum_i (m_{q_i} - m_u))/T)$, the factor $\exp(\sum_i m_u/T)$ can be absorbed in C and C_B. The error function of the fit is mainly

Table 2: Results of the fit to LEP data and to data obtained at different center-of-mass energies.

Parameters	$\sqrt{s} = 91$ GeV	Simultaneous Fit
γ_s	0.29 ± 0.03	0.29 ± 0.02
Δm (GeV)	0.150 ± 0.029	0.161 ± 0.024
T (GeV)	0.289 ± 0.020	0.298 ± 0.015
C	0.209 ± 0.041	$C_{91} = 0.218 \pm 0.034$ *
C_B	10.0 ± 1.0	11.0 ± 0.9
χ^2/dof	56.2/19	155.8/57

* $C_{30} = 0.124 \pm 0.020$, $C_{10} = 0.049 \pm 0.008$

The results of the fit to the LEP data[d] for a typical s quark mass of 0.5 GeV are listed in Table 2. The calculated total rate and primary fraction for different hadrons are listed in Table 1. We see a good agreement between the measured and calculated rates except for some decuplet baryons. As mentioned in Ref.[7,6], experimental errors for the decuplet baryons are still large and there are discrepancies between experiments. The large χ^2 value of the fit is mainly due to the decuplet baryons. If they are excluded in the fit, the χ^2/dof is then equal to 22.6/15 while the fit results remain essentially unchanged. However, the difference between data and our predictions for the decuplet baryons might suggest that baryon production is not described properly in our approach, since baryons are considerably more complicated objects than mesons. If diquark production[1] is the main mechanism for baryon production, it is then more suitable to use the effective diquark masses than the quark masses in Eq.1. This will, however, increase considerably the number of free parameters.

We also perform a simultaneous fit to data at different center-of-mass energies with 7 free parameters: γ_s, Δm, T, C_B, C_{91}, C_{30} and C_{10}, where C_{91}, C_{30} and C_{10} are the overall normalization factor at the center-of-mass energy 91, 29-35 and 10 GeV, respectively. The fit results are listed in Table 2 and shown in Fig. 2. The value of γ_s, Δm, T and C_B is very consistent with that obtained from the LEP data alone, showing that the value of these parameters is independent of the center-of-mass energy.

The γ_s value determined from the fit is in an excellent agreement with the expectation. The value of $\Delta m = m_s - m_u$ required from the fit is consistent with that obtained from hadron masses[10] and from QCD calculations[12]. While the values of γ_s, Δm and T do not depend on the choice of the m_s value, the values of C and C_B are correlated with the m_s value. However, the calculated baryon rates do not depend on the choice of the m_s value since they are determined by both m_s and C_B. From our analysis we find that baryons contribute 10.9% of the primary produced hadrons at LEP energy.

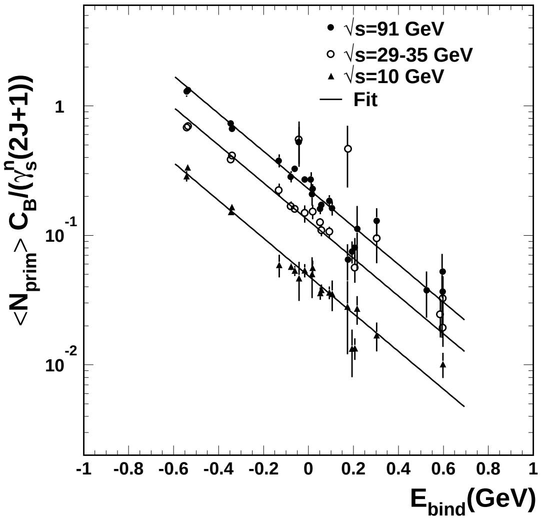

Figure 2: Production rates of light flavored hadrons originating from fragmentation at different center-of-mass energies, multiplied by the factor $C_B/(\gamma_s^{N_s}(2J+1))$, as a function of the binding energy of hadrons.

Fractions of primary hadrons determined from the fit are listed in Table 1. They are in general lower than those predicted by JETSET due to the inclusion of orbitally excited states up to a mass of 2.5 GeV in our analysis. JETSET includes some of orbitally excited meson states (only those with $L = 1$), but no orbitally excited baryon states. From our analysis we obtain the fraction of primary produced states with orbital excitation to be 27% for mesons and 39% for baryons at LEP energy.

Using the T value obtained from the fit, the ratio $V/(V+P)$ can be calculated according to the mass difference of vector and pseudoscalar mesons. We obtain a value of 0.38, 0.44, 0.65 and 0.72 for u(d)-, s-, c- and b-mesons respectively, which are in good agreement with the measurements.

4 Predictions of heavy flavor composition

Using the fit results obtained with light flavored hadrons, the relative production rate of heavy flavored hadrons can be predicted by our method. The absolute value of the rates can be determined by normalizing the sum of the rates to the

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sensitive to the change in the mass difference $m_s - m_u$.

[d] The new result for $\Lambda(1520)$ is not included in the fit.

Table 3: Predictions of average production rates for heavy flavored hadrons, compared with LEP data.

Hadrons	Prediction	Measurement
D^0	0.242	0.221 ± 0.012
D^+	0.092	0.087 ± 0.008
D^{*+}	0.114	0.088 ± 0.006
D_s^+	$0.054(\pm 0.005)$ [a]	0.041 ± 0.008
Λ_c^+	0.026	0.037 ± 0.009
B_s^0	0.091	0.097 ± 0.026
$D^*/(D^* + D)$ [b]	0.66	0.56 ± 0.4
$B^*/(B^* + B)$	0.70	0.75 ± 0.04
$< D_1^0 + D_2^{*0} >_c \mathrm{Br}(D^{**0} \to D^{*+}\pi^-)/ < D^{*+} >_c$ [b]	0.124	0.10 ± 0.03
$B_{u,d}^{**}/B_{u,d}$	0.39	$(0.24 \pm 0.03)f$ [c]
$(B_1 + B_2^*)_{u,d}/B_{u,d}$	0.228	0.216 ± 0.033
$(B_{s1} + B_{s2}^*)/B_{u,d}^{**}$	0.110	0.142 ± 0.055
$(B_{s1} + B_{s2}^*)/B^+$	0.086	0.052 ± 0.016
D^0/c-jet	0.593	0.570 ± 0.046
D^+/c-jet	0.237	0.249 ± 0.026
D^{*+}/c-jet	0.272	0.241 ± 0.015
D_s^+/c-jet	$0.101(\pm 0.025)$ [a]	0.128 ± 0.027
B_s^0/b-jet	$0.108(\pm 0.030)$ [d]	0.122 ± 0.031
Λ_c^+/c-jet	0.069	0.076 ± 0.044
Λ_b/b-jet	0.073	0.076 ± 0.019 [e]
$(\Sigma_b + \Sigma_b^*)$/b-jet	0.060 [f]	0.048 ± 0.016
D_{s1}/c-jet	0.012	0.016 ± 0.006
$(D_{s1} + D_{s2}^*)$/c-jet	0.028	—
$(B_{s1} + B_{s2}^*)$/b-jet	0.035	0.037 ± 0.012
$(D_1 + D_2^*)_{u,d}$/c-jet	0.170	0.173 ± 0.053
$(B_1 + B_2^*)_{u,d}$/b-jet	0.188	—
D^{**}/c-jet	0.38	—
B^{**}/b-jet	0.38	—
c-baryon/c-jet	0.089	—
b-baryon/b-jet	0.091	0.115 ± 0.040

[a] for $\mathrm{Br}(D_s^{**} \to D_s X) = 0.5(\pm 0.5)$

[b] excluding D, D^* and D^{**} from B decays

[c] $f = 1$–2 (see Ref. [9])

[d] for $\mathrm{Br}(B_s^{**} \to B_s X) = 0.5(\pm 0.5)$

[e] for $\mathrm{Br}(\Lambda_b \to \Lambda_c^+ l^- \bar{\nu} X) = 0.1$

[f] $= 0.052$ if $m_{\Sigma_b, \Sigma_b^*} - m_{\Lambda_b}$ measured in Ref. [14] are used

$\Gamma_{c\bar{c}}/\Gamma_{had}$ or $\Gamma_{b\bar{b}}/\Gamma_{had}$, which can be calculated by the Standard Model. We use all heavy flavored hadrons in the JETSET 7.4 table, which includes orbitally excited mesons with $L = 1$, such as D^{**} and B^{**}, but no orbitally excited heavy flavored baryons (for more discussions see Ref. [9]).

As an example we consider the heavy flavor composition at LEP energy. Our predictions, together with the corresponding measurements [9,13], are listed in Table 3. There is a good agreement between the predicted and measured values. The main uncertainty on the prediction is due to the limited knowledge on properties of orbitally excited heavy flavored hadrons, such as mass, decay modes, and branching ratios [9]. Without taking into account the production of orbitally excited c- and b-baryons, the fractions c-baryon/c-jet and b-baryon/b-jet are calculated to be around 9%. The calculated values would be 30-40% higher [9] if orbitally excited c- and b-baryons could be included, which are however barely known.

5 Conclusions

We have shown that the production rates of light flavored mesons and baryons in e^+e^- annihilation, which span a range of four orders of magnitude, can be described by a simple approach based on the idea of string fragmentation. Data at different center-of-mass energies can be described simultaneously apart from a normalization factor, which reflects the rise of multiplicities with increasing energy. From the fit we determine the strangeness suppression factor to be 0.29 ± 0.02. Applying to the heavy flavor production, we find that our predictions are in good agreement with data. A comparison of our approach with other recently proposed approaches can be found in Ref. [9].

The proposed approach may provide insight into the hadron production mechanism. To further check the approach, precise data on the production of the decuplet baryons and orbitally excited hadrons are needed. In particular, with better understanding of the baryon production mechanism, the description of baryon production in our approach can be improved.

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