

Colour-octet contributions to exclusive charmonium decays

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Abstract

We investigate the theoretical uncertainties of P -wave charmonium decays into two pions, $\chi_{cJ} \rightarrow \pi^+\pi^-, \pi^0\pi^0$. Constraining the pion distribution-amplitude from the recent precise data on $F_{\pi\gamma}(Q^2)$, we find an order-of-magnitude discrepancy between data and prediction. The disagreement persists even after inclusion of transverse degrees of freedom and Sudakov suppressions. We propose the colour-octet mechanism as the solution to the puzzle. The colour-octet decay contribution arising from the higher Fock component $|c\bar{c}g\rangle$ of the χ_{cJ} wave function is actually not power suppressed with respect to the usual colour-singlet decay arising from the dominant $|c\bar{c}\rangle$ Fock state. An explicit calculation yields an agreement with the data for a very reasonable value for the single extra non-perturbative parameter.

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Up to very recently, hard exclusive reactions involving pions could be believed to be successfully described within the hard-scattering approach. The reactions to be mentioned are the pion–photon transition form factor, the pion electromagnetic form factor, pion-pair production by two photons, as well as the two-pion decays of the charmonium states J/ψ , χ_{c0} , and χ_{c2} . The hard-scattering approach (HSA) provides the framework of calculations of exclusive reactions at large momentum transfers [1]: a full amplitude factors into two parts, a hard-scattering amplitude T_H , calculable in perturbative QCD, and parton-distribution amplitudes Φ_H for each hadron H . The amplitude T_H describes the scattering of clusters of collinear partons from the hadron. The leading-twist contribution, i.e. the one with the weakest fall-off with Q , the typical momentum transferred in the process, is given by valence-parton scatterings only. Hence the only non-perturbative input required are the probability amplitudes (or quark distribution amplitudes) $\Phi_H(x_i, Q)$ for finding valence quarks in the hadron, each carrying some fraction x_i of the hadron’s momentum.

In the case of mesons, the leading Fock state is the $|q\bar{q}\rangle$ state, which, for the pion in the isotopic limit, can be described by a symmetric wave function $\Phi_\pi(x) = \Phi_\pi(1-x)$. Upon expansion over Gegenbauer polynomials $C_n^{(3/2)}$, the distribution amplitude at a scale μ_F can be characterized by non-perturbative coefficients B_n (see [1] and references therein):

$$\Phi_\pi(x, \mu_F) = \frac{f_\pi}{2\sqrt{6}} \phi_{\text{as}}(x) \left[1 + \sum_{n=2,4,\dots}^{\infty} B_n \left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^{\gamma_n} C_n^{(3/2)}(2x-1) \right]. \quad (1)$$

Here f_π is the pion decay constant, $f_\pi = 130.7$ MeV, α_s the strong coupling constant, and μ_0 a hadronic scale, $0.5 \lesssim \mu_0 \lesssim 1$ GeV. Since the γ_n in (1) are positive fractional numbers increasing with n , higher-order terms are gradually suppressed and any distribution amplitude evolves into $\phi_{\text{as}}(x) = 6x(1-x)$ asymptotically, i.e. for $\ln(Q^2/\Lambda_{\text{QCD}}^2) \rightarrow \infty$.

Lowest-order perturbative QCD (pQCD) calculations in leading-twist of the above-mentioned reactions involving pions were all in reasonable agreement with data, provided the quark distribution amplitude in the pion is strongly end-point-concentrated (in sharp contrast to the asymptotic distribution amplitude). Such a distribution amplitude is given by the Chernyak–Zhitnitsky (CZ) [2] one defined by $B_2 = 2/3$ and $B_n = 0$ for $n > 2$. So far, there existed basically only one exception, namely the pion form factor in the time-like region: the HSA prediction with the CZ distribution amplitude fails by about a factor of 2 as compared with the data. The experimental information on the time-like form factor comes from two sources, $e^+e^- \rightarrow \pi^+\pi^-$ and $J/\psi \rightarrow \pi^+\pi^-$, which provides, to a very good approximation, the form factor at $s = M_{J/\psi}^2$. Although the e^+e^- annihilation data of Ref. [3] suffer from low statistics, they agree very well with the result obtained from the J/ψ decay.

Meanwhile, the situation has changed drastically. On the experimental side, very precise new data for fairly large momentum transfer [4] have shown that $F_{\pi\gamma}(Q^2)$ is smaller than predicted from the CZ distribution amplitude. On the theoretical side, the HSA has been modified through the incorporation of transverse degrees of freedom and Sudakov suppressions [5, 6]. This more refined treatment (termed the “modified HSA”) allows the calculation of the genuinely perturbative contribution self-consistently, in the sense that the bulk of the perturbative contribution is accumulated in regions of reasonably small values of the strong coupling constant α_s . This approach hence overcomes arguments [7] against the applicability of the standard HSA (sHSA: the lowest-order pQCD approach in the collinear approximation using valence Fock states only) to experimentally accessible regions of momentum transfers. It is to be stressed that the effects of the transverse

degrees of freedom taken into account in the modified HSA (mHSA) represent soft contributions of higher-twist type. Still, mHSA calculations are restricted to the dominant (valence) Fock state.

A detailed analysis of the photon–pion transition form factor $F_{\pi\gamma}(Q^2)$ within the mHSA [8, 9] shows that the pion distribution amplitude is close to the asymptotic form $\phi_{\text{as}}(x)$. In order to give a quantitative estimate of the allowed deviations from the asymptotic distribution amplitude, one may assume that B_2 is the only non-zero expansion coefficient in (1). The truncated series suffices to parametrize small deviations. Moreover, it has the advantage of interpolating smoothly between the asymptotic distribution amplitude and the frequently used CZ distribution amplitude. For large momentum transfer our assumption is justified by the properties of the anomalous dimensions γ_n . Quantitatively, choosing $\mu_0 = 0.5 \text{ GeV}$, a best fit to the $F_{\pi\gamma}$ data above 1 GeV^2 yields

$$\begin{aligned} B_2(\mu_0) &= -0.006 \pm 0.014 && \text{mHSA} \\ &= -0.39 \pm 0.05 && \text{sHSA} . \end{aligned} \quad (2)$$

The non-perturbative pion distribution amplitude (i.e. the coefficient B_2) is, in fact, best extracted from $F_{\pi\gamma}(Q^2)$, since the pseudoscalar meson–photon transition form factors are pure QED processes with QCD corrections of only 10–20% and higher Fock-state contributions suppressed by powers of α_s/Q^2 and expected to be small too [10]. Uncertainties due to these effects may be absorbed into a theoretical error of B_2 , which we generously estimate to amount to ± 0.1 for the mHSA and ± 0.39 for the sHSA. However, with a distribution amplitude close to the asymptotic form, calculations for the other processes involving pions disagree strongly with data, at least when calculated in the collinear approximation. In this paper we shall investigate in detail the χ_{cJ} decays into two pions, by calculating the decay rates and branching ratios in the modified HSA, using the information on the process-independent expansion coefficient B_2 obtained from the $\pi\gamma$ form factor.

Using the method proposed by Duncan and Mueller [11], the decay rate calculated in the sHSA can be written as

$$\Gamma[\chi_{cJ} \rightarrow \pi^+\pi^-] = f_\pi^4 \frac{1}{m_c^8} |R'_P(0)|^2 \alpha_s(\mu_R^{\text{excl}})^4 \left| a_J + b_J B_2(\mu_0) + c_J B_2(\mu_0)^2 \right|^2 , \quad (3)$$

where a_J , b_J , and c_J are (analytically) calculable real numbers, see table 1. The evolution factor $[\ln(\mu_F^2/\Lambda_{\text{QCD}}^2)/\ln(\mu_0^2/\Lambda_{\text{QCD}}^2)]^{\gamma_2}$ (see (1)) is absorbed into the coefficients b_J and c_J (quadratically in the latter case), and $\gamma_2 = 50/81$. μ_R^{excl} is an appropriate renormalization scale, which is of the order of the charm-quark mass m_c [1, 12, 13]. In the sHSA analysis, it is customary to identify factorization scale μ_F and renormalization scale. Finally, $R'_P(0)$ denotes the derivative of the non-relativistic $c\bar{c}$ wave function at the origin (in coordinate space) appropriate for the dominant Fock state of the χ_{cJ} , a $c\bar{c}$ pair in a colour-singlet state with quantum numbers $^{2S+1}L_J = {}^3P_J$.

In the following we will choose as central values $R'_P(0) = 0.22 \text{ GeV}^{5/2}$ and $m_c = 1.5 \text{ GeV}$, which is consistent with a global fit of charmonium parameters [14] as well as results for charmonium radii from potential models[15]. With this choice, $\mu_R = m_c$, and $B_2(\mu_0) = -0.39$ (see (2)) the sHSA results for the $\chi_{c0(2)} \rightarrow \pi^+\pi^-$ decay widths are almost two orders of magnitude below the experimental data, see table 2. To assess the uncertainty of the result we vary the charm-quark mass between 1.35 GeV and 1.8 GeV . Of course, the value of $R'_P(0)$ has to be adjusted accordingly: using the well-known

Approach	a_0	b_0	c_0
sHSA	19.60	37.43	26.23
mHSA	$-14.67 + 23.36 i$	$15.36 + 73.22 i$	$27.18 + 26.39 i$
	a_2	b_2	c_2
sHSA	4.589	8.479	7.719
mHSA	$-3.420 + 5.141 i$	$6.681 + 13.42 i$	$5.744 + 1.384 i$

Table 1: Coefficients defined in (3) obtained within the standard HSA (sHSA) for $\mu_R^{excl} = m_c = 1.5$ GeV and coefficients (10) of the modified HSA (mHSA).

B_2	m_c [GeV]	Λ_{QCD} [GeV]	$\Gamma(\chi_{cJ} \rightarrow \pi^+\pi^-)$ [keV]		$\text{BR}(\chi_{cJ} \rightarrow \pi^+\pi^-)$ [%]	
			$J = 0$	$J = 2$	$J = 0$	$J = 2$
Standard HSA						
-0.39	1.5	0.2	0.872	0.065	0.011	0.003
0	1.35	0.25	15.3	0.841	0.113	0.020
Modified HSA						
0	1.5	0.2	8.22	0.41	0.102	0.017
0.1	1.5	0.2	12.13	0.53	0.151	0.021
-0.1	1.5	0.2	5.61	0.33	0.070	0.013
0	1.8	0.2	2.54	0.12	0.050	0.008
0	1.35	0.2	15.3	0.78	0.144	0.024
0	1.5	0.15	4.34	0.21	0.071	0.011
0	1.5	0.25	13.1	0.68	0.128	0.022
Standard HSA plus colour-octet contributions						
0	1.5	0.2	49.85	3.54	0.36	0.177
Experiment						
PDG[16]			105 ± 30	3.8 ± 2.0	0.75 ± 0.21	0.19 ± 0.10
BES[17]			62.3 ± 17.3	3.04 ± 0.73	0.427 ± 0.064	0.152 ± 0.034

Table 2: Decay widths and branching ratios of $\chi_{cJ} \rightarrow \pi^+\pi^-$ for various choices of the parameters compared with experimental data. The standard HSA results are obtained with $\mu_R^{excl} = m_c$, the branching ratios are evaluated with $\mu_R^{incl} = m_c$ and $F_1 = 1$.

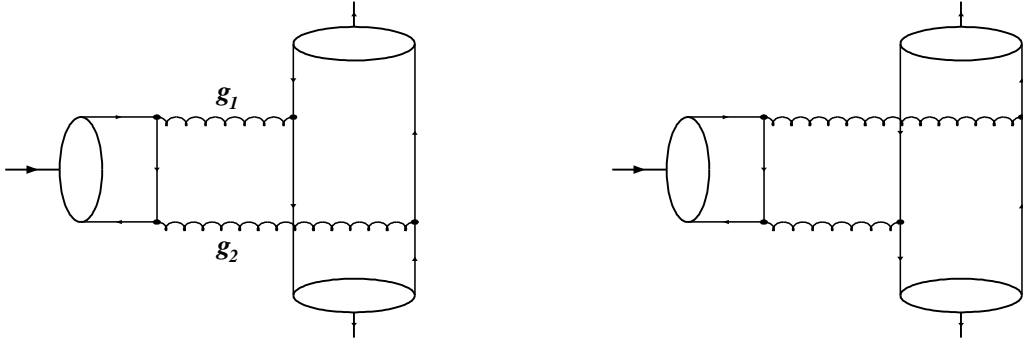


Figure 1: Feynman diagrams for the colour-singlet decay $\chi_{cJ} \rightarrow \pi \pi$ ($J = 0, 2$).

scaling properties of quarkonium potential models [18] we take $R'_P(0) = 0.274 \text{ GeV}^{5/2}$ for $m_c = 1.8 \text{ GeV}$ and $R'_P(0) = 0.194 \text{ GeV}^{5/2}$ for $m_c = 1.35 \text{ GeV}$. However, even when stretching all parameters to their extreme values, the predictions stay a factor 3–6 below the data. Note that¹ $\alpha_s(m_c) = 0.447$ for $m_c = 1.35 \text{ GeV}$ and $\Lambda_{\text{QCD}} = 0.25 \text{ GeV}$.

The starting point of the calculation of the χ_{cJ} decays within the modified HSA is the convolution formula

$$M(\chi_{cJ} \rightarrow \pi^+ \pi^-) = \frac{32 \sqrt{2} \pi^{3/2} R'_P(0)}{3 \sqrt{3} m_c^{7/2}} \sigma_J \int_0^1 dx dy \int \frac{d^2 \mathbf{b}}{(4\pi)^2} \hat{\Psi}_\pi^*(y, \mathbf{b}) \hat{T}_{HJ}(x, y, \mathbf{b}) \hat{\Psi}_\pi(x, \mathbf{b}) \exp[-S(x, y, \mathbf{b}, t_1, t_2)], \quad (4)$$

which adapts the methods proposed by Botts and Sterman [5] to our case ($\sigma_0 = 1$; $\sigma_2 = \sqrt{3}/2$). Note that we work in the transverse coordinate space (the quark–antiquark separation \vec{b} is canonically conjugated to the usual transverse momentum \vec{k}_\perp). \hat{T}_{HJ} is the Fourier-transformed hard-scattering amplitude to be calculated from the graphs shown in Fig. 1:

$$\hat{T}_{HJ}(x, y, \mathbf{k}_\perp - \mathbf{k}'_\perp) = \frac{\alpha_s(t_1) \alpha_s(t_2)}{(g_1^2 + i\epsilon)(g_2^2 + i\epsilon) N} \left(1 + \frac{(-2)^{J/2} (x - y)^2}{2 N} \right), \quad (5)$$

and with $\mathbf{k} = (\mathbf{k}_\perp - \mathbf{k}'_\perp)/2m_c$

$$\begin{aligned} N &= x(1 - y) + (1 - x)y + 2 \mathbf{k}^2, \\ g_1^2 &= xy - \mathbf{k}^2, \\ g_2^2 &= (1 - x)(1 - y) - \mathbf{k}^2. \end{aligned} \quad (6)$$

In analogy to previous applications of the modified perturbative approach, the Sudakov exponent is given by

$$\begin{aligned} S(x, y, \mathbf{b}, t_1, t_2) &= s(x, b, 2m_c) + s(1 - x, b, 2m_c) + s(y, b, 2m_c) + s(1 - y, b, 2m_c) \\ &\quad - \frac{4}{\beta} \log \frac{\log(t_1/\Lambda_{\text{QCD}}) \log(t_2/\Lambda_{\text{QCD}})}{\log(1/(b\Lambda_{\text{QCD}}))^2}, \end{aligned} \quad (7)$$

¹We calculate α_s in the one-loop approximation with $\Lambda_{\text{QCD}} = 200 \text{ MeV}$ and $n_f = 4$.

where the function $s(x, b, Q)$ can be found, for instance, in Ref. [19].

The renormalization scales $t_i = \max(2\sqrt{x_i y_i} m_c, 1/b)$ ($i = 1, 2$) appearing in α_s and in the Sudakov exponent are determined by the virtualities of the intermediate gluons, which depend non-trivially on the integration variables. This choice of the renormalization scale avoids large logs from higher-order pQCD. The factorization scale is given by the quark-antiquark separation \vec{b} , $\mu_F = 1/b \equiv 1/|\vec{b}|$. The ratio $1/b$ marks the interface between non-perturbatively soft momenta, which are implicitly accounted for in the pion wave function Ψ_π , and the contributions from semi-hard gluons, incorporated in a perturbative way in the Sudakov factor.

The last object appearing in (4) is the full (soft) wave function of the pion describing also the dependence on the transverse momentum. Following [20] we make the ansatz

$$\begin{aligned}\hat{\Psi}_\pi(x, b; \mu_F) &= \frac{f_\pi}{2\sqrt{6}} \Phi_\pi(x, \mu_F) \hat{\Sigma}_\pi(x, b), \\ \hat{\Sigma}_\pi(x, b) &= 4\pi \exp\left[-\frac{x(1-x)b^2}{4a_\pi^2}\right].\end{aligned}\quad (8)$$

Note, however, that a_π is not a free parameter since it is fixed from $\pi^0 \rightarrow \gamma\gamma$ [21]. That constraint leads to the closed formula $1/a_\pi^2 = 8(1+B_2)\pi^2 f_\pi^2$. This additional B_2 dependence is taken into account in our calculation by expanding the Gaussian in $\hat{\Sigma}$ over B_2 . For the asymptotic form of the wave function ($B_2 = 0$) the transverse size parameter a_π takes the value 0.861 GeV^{-1} .

In terms of the amplitude (4) the decay widths are given by

$$\begin{aligned}\Gamma(\chi_{c0} \rightarrow \pi^+ \pi^-) &= \frac{1}{32\pi m_c} \left| M(\chi_{c0} \rightarrow \pi^+ \pi^-) \right|^2 \\ \Gamma(\chi_{c2} \rightarrow \pi^+ \pi^-) &= \frac{1}{240\pi m_c} \left| M(\chi_{c2} \rightarrow \pi^+ \pi^-) \right|^2.\end{aligned}\quad (9)$$

The final results in the modified HSA, tables 1 and 2 can be obtained numerically only, but can be cast into a form similar to (3)

$$\Gamma[\chi_{cJ} \rightarrow \pi^+ \pi^-] = f_\pi^4 \frac{1}{m_c^8} |R'_P(0)|^2 \alpha_s(m_c)^4 \left| a_J + b_J B_2(\mu_0) + c_J B_2(\mu_0)^2 \right|^2. \quad (10)$$

The coefficients a_J , b_J , and c_J are now complex-valued. It is still convenient to divide out the fourth power of α_s at the fixed scale m_c in (10), since the main effect of the strong coupling is thus made explicit. The actual effective renormalization scale μ_R in the modified HSA differs from m_c and depends on B_2 . We find

$$\begin{aligned}\mu_R^{\text{eff}} &= 1.15 \text{ GeV} \quad , \quad \alpha_s^{\text{eff}} = 0.43 \quad (B_2 = 0) \\ &= 0.85 \text{ GeV} \quad , \quad = 0.52 \quad (B_2 = 2/3) .\end{aligned}\quad (11)$$

Note that part of the increase of the decay widths compared to the sHSA results follows from the larger values of the strong coupling constant α_s , cf. (11) with $\alpha_s(m_c^2) = 0.374$.

The calculation of the charmonium decay is theoretically self-consistent in so far as only 2% (20%) for $B_2 = 0$ ($B_2 = 2/3$) come from soft regions where the use of pQCD is inconsistent. The soft region is defined by $\alpha_s(t_1^2) \alpha_s(t_2^2) > 0.5$.

Uncertainties in our predictions of the χ_c decay rates into pions arise from uncertainties in the values of B_2 , Λ_{QCD} , $R'_P(0)$ and the charm-quark mass m_c . In order to show the dependence on m_c we give our results in table 2 for the above three choices of m_c (with $R'_P(0)$ adjusted accordingly). We also show the dependences on Λ_{QCD} and B_2 , where we stretch B_2 up to the maximal value allowed by the analysis of the photon–pion transition form factor, see above. As shown in table 2 the predicted decay widths for both χ_{c0} and χ_{c2} are well below the data, even if the parameters are pushed to their extreme values.

In order to eliminate the dependence on $R'_P(0)$ and reduce the one on m_c one can consider the branching ratios into $\pi^+\pi^-$ rather than the absolute widths. When normalising to the total width we correct for the fraction ρ_J of χ_{cJ} decays that goes into light hadrons (rather than decaying electromagnetically or into lighter charmonium states, and which can be determined from data). The branching ratio has a form similar to (10):

$$\text{BR}[\chi_{cJ} \rightarrow \pi^+\pi^-] = F_1 \frac{\alpha_s(m_c)^4}{\alpha_s(\mu_R^{\text{incl}})^2} \frac{f_\pi^4}{m_c^4} d_J \left| a_J + b_J B_2(\mu_0) + c_J B_2(\mu_0)^2 \right|^2, \quad (12)$$

where $d_0 = \rho_0/6$, $d_2 = 5\rho_2/8$, and $F_1 = 1$ (see below). The results for the branching ratios, see table 2, are indeed less sensitive to the parameter choice (and independent of $R'_P(0)$), and confirm our conclusion based on the decay widths: The calculation based on the assumption that the χ_{cJ} is a pure $c\bar{c}$ state, is not sufficient to explain the observed rates. The necessary corrections would have to be larger than the leading terms. A new mechanism is therefore called for.

In the quark-potential model, any charmonium state is a pure $c\bar{c}$ state, where the quarks have angular momentum L and spin S such that these match the quantum numbers by which any meson is characterized in QCD, total spin J , parity P and charge conjugation C : $\mathbf{J} = \mathbf{L} + \mathbf{S}$, $P = (-1)^{L+S}$, $C = (-1)^{L+S}$. The χ_{cJ} states are therefore made out of 3P_J $c\bar{c}$ states (in the spectroscopic notation $({}^{2S+1}L_J)$), which, obviously, are in colour-singlet ($c = 1$) states. Although the HSA acknowledges higher Fock components in the meson’s wave function, these do not contribute in either sHSA or mHSA calculations since their contributions are assumed to be suppressed by powers of the hard scale [1].

Recently, the importance of higher Fock states in understanding the production and the *inclusive* decays of charmonia has been pointed out [13]. The heavy-quark mass allows for a systematic expansion of both the quarkonium state and the hard, short-distance process and, hence, of the inclusive decay rate or the production cross section. The expansion parameter is provided by the velocity v of the heavy quark inside the meson. In the case of the χ_{cJ} , the Fock-state expansion starts as

$$|\chi_{cJ}\rangle = O(1) |c\bar{c}_1({}^3P_J)\rangle + O(v) |c\bar{c}_8({}^3S_1)g\rangle + O(v^2) \quad (13)$$

where the subscript c specifies whether the $c\bar{c}_c$ is in a colour-singlet ($c = 1$) or colour-octet ($c = 8$) state.

The crucial observation is now that, for inclusive decays (and also production rates), both states in (13) contribute at the same order in v and, hence, the inclusion of the “octet mechanism”, i.e. the contribution from the $|c\bar{c}_8g\rangle$ state, is necessary for a consistent description. (Without its inclusion the factorization of the decay width into long- and short-distance factors is spoiled by the presence of infrared-sensitive logarithms.) In simple terms this can be understood by realizing that the annihilation of the $c\bar{c}$ pair of the higher Fock state into light hadrons is an S -wave process compared to the P -wave annihilation of the leading Fock state. The inclusion of the colour-octet contribution to the total χ_{cJ}

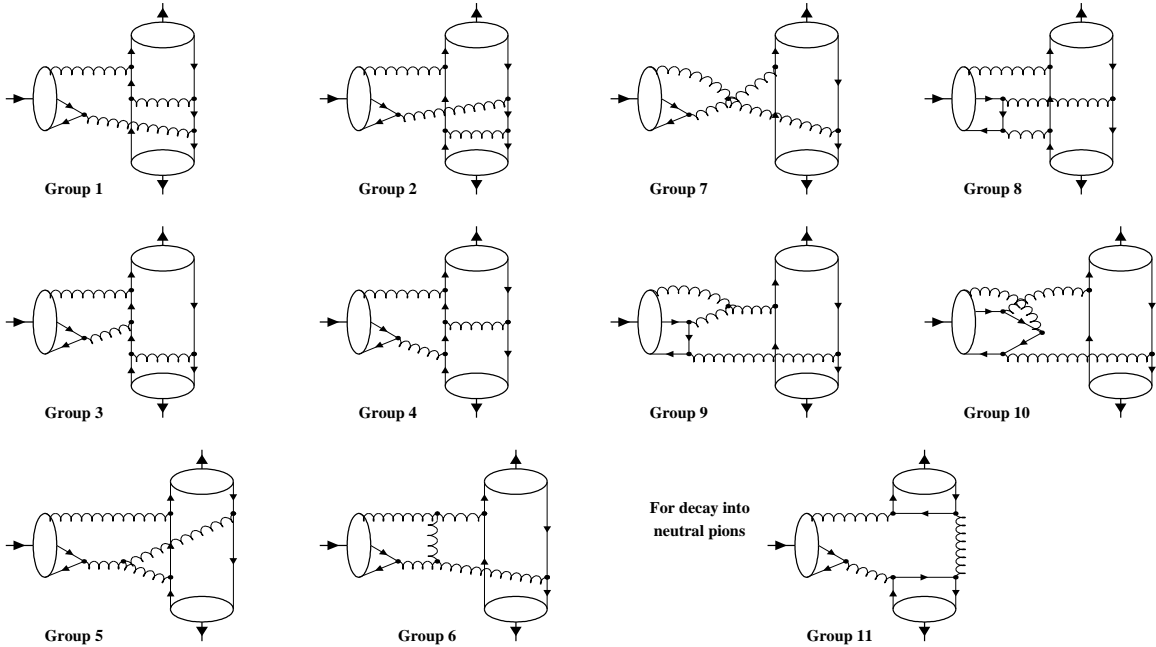


Figure 2: Representatives of the various groups of colour-octet decay graphs.

width does, however, worsen the comparison of (12) with data since the fraction F_1 of the total hadronic width, which proceeds through the $c = 1$ contribution, is less than unity.

Here we propose the inclusion of contributions from the $|c\bar{c}_8(^3S_1)g\rangle$ Fock state to the *exclusive* χ_{cJ} decays as the solution to the failure of the HSA. We shall show that this colour-octet contribution is indeed not suppressed by powers of $1/m_c$ compared with the conventional, colour-singlet contribution. Let us first, however, discuss the problem of colour conservation. This is a new feature, specific to exclusive decays.

The obvious solution to colour conservation seems to be to demand one of the final-state pions to be also in a higher Fock state, i.e. to consider $|c\bar{c}g\rangle \rightarrow |q\bar{q}g\rangle + |q\bar{q}\rangle$. The hard process consists of the Feynman diagrams of Fig. 1 plus the diagram in which c and \bar{c} annihilate into a single gluon, which, in turn, makes a $q\bar{q}$ pair. Yet, we disregard this possibility. The reason has to do with the importance of higher Fock states in the pion. Since the $q\bar{q}$ pair of the dominant Fock state is in a 1S_0 state, it is in a 1P_1 state for the next-higher state. The first S -wave state $|q\bar{q}_8(^3S_1)g\rangle$ is reached at $O(v^2)$. Contributions from higher Fock states of the pion therefore seem to be suppressed.

Our solution to colour conservation is the following: We attach the gluon of the $|c\bar{c}_8g\rangle$ state (in all possible ways) to the hard process leading to the Feynman diagrams shown in Fig. 2. We do, however, realize that the virtualities of the quark propagators to which the soft gluon couples are typically (much) smaller than the virtualities associated with the hard gluons. Therefore we treat the coupling of the soft gluon as a parameter separate from α_s and denote it by α_s^{soft} . It will turn out that the final result depends on just a single non-perturbative parameter, in which α_s^{soft} appears as a factor.

It is important to realize that in the $|c\bar{c}_8g\rangle$ Fock state not only the $c\bar{c}$ pair is in a colour-octet state, but also the three particles, c , \bar{c} and g , are in an S -state. The covariant spin

wave functions we are using for the χ_{cJ} read

$$S_{0\nu}^{(8)} = \frac{1}{\sqrt{6}} (\not{p} + M_0) (p_\nu/M_0 - \gamma_\nu), \quad S_{2\nu}^{(8)} = \frac{1}{\sqrt{2}} (\not{p} + M_2) \varepsilon_{\mu\nu} \gamma^\mu. \quad (14)$$

The colour-octet component of the χ_{cJ} is given by

$$|\chi_{cJ}^{(8)}\rangle = \frac{t_{c\bar{c}}^a}{2} f_J^{(8)} \int dz_1 dz_2 \Phi_J^{(8)}(z_1, z_2, z_3) S_{J\nu}^{(8)}, \quad (15)$$

where $t = \lambda/2$ is the Gell-Mann colour matrix and a the colour of the gluon. Since orbital angular momenta are not involved, the transverse degrees of freedom are already integrated over. Therefore, we only have to operate with a distribution amplitude that is, as usual, subject to the condition $\int dz_1 dz_2 \Phi_J^{(8)} = 1$, and an octet decay constant $f_J^{(8)}$. In the following, we will make the plausible assumption that the colour-octet χ_{cJ} states differ only by their spin wave functions, i.e. the distribution amplitudes as well as the decay constants are assumed to be the same for the two χ_{cJ} states.

Usually higher Fock state contributions to exclusive reactions are suppressed by powers of $1/Q^2$ [1] where $Q = m_c$ in our case. However, for the decays of P -wave charmonia this suppression does not appear as a simple dimensional argument reveals: the colour-singlet and octet contributions to the decay amplitude defined in (9) behave as

$$M_J^{(c)} \sim f_\pi^2 f_J^{(c)} m_c^{-n_c}. \quad (16)$$

The singlet decay constant, $f_J^{(1)}$, represents the derivative of a two-particle coordinate space wave function at the origin. Hence it is of dimension GeV^2 . The octet decay constant, $f_J^{(8)}$, as a three-particle coordinate space wave function at the origin, is also of dimension GeV^2 . Since, according to (9), $M_J^{(c)}$ is of dimension GeV , $n_c = 3$ in both cases. We note that the χ_{cJ} decay constants may also depend² on m_c .

We estimate the colour-octet contributions to the χ_{cJ} decays by calculating the set of Feynman graphs of which representatives are shown in Fig. 2. The graphs of group 11 only contribute to decays into $\pi^0 \pi^0$. In order to simplify matters we carry through the calculation in the collinear approximation and assume a δ -function-like $c\bar{c}g$ distribution amplitude ($z_1 = z_2 = (1-z)/2$; $z_3 = z \simeq (1-2m_c/M_c) \simeq 0.15$, where M_c is the average charmonium mass and z_3 is the momentum fraction carried by the constituent gluon). Most of the details of the calculation, as well as the application of the modified HSA, are left to a forthcoming paper [22]. We note that in some of the graphs shown in Fig. 2 the virtuality of the quark propagator adjacent to the χ_{cJ} constituent gluon is small. It is these diagrams that constitute, to leading order in α_s and z , the higher Fock state $|q\bar{q}g\rangle$ of the pion. However, for arbitrary z , these diagrams alone do not lead to a gauge-invariant amplitude.

In the calculation of the colour-octet contribution, a number of singular integrals appear as a consequence of the collinear approximation. If the transverse momenta are kept, all integrals are well defined. The difficulty arises from the propagator $D_{ca} = [(z-x)(z-y) + i\epsilon]^{-1}$ of the gluon exchanged between the light quarks for which the usual $i\epsilon$ prescription fails. In order to regularize these integrals we replace D_{ca} by

$$D = [(z-x)(z-y) + \varrho^2 + i\epsilon]^{-1}. \quad (17)$$

²In the χ_{c0} case for instance the spin wave for the $c\bar{c}$ Fock state reads $S_0^{(1)} = \frac{1}{\sqrt{2}} [\not{p} + M_0 + 2\not{K}] \not{K}$. The decay constant is then defined as $f_0^{(1)} = \frac{4\pi}{3} \int \frac{dk k^4}{16\pi^3 M_0} \Psi_0^{(1)}(k) = -i |R_P'(0)| / \sqrt{16\pi m_c}$.

	$\Gamma(\chi_{cJ} \rightarrow \pi^0 \pi^0)$ [keV]		$\text{BR}(\chi_{cJ} \rightarrow \pi^0 \pi^0)$ [%]	
$B_2 = 0$	25.7	1.81	0.18	0.091
Exp. PDG [16]	42 ± 18	2.2 ± 0.6	0.31 ± 0.06	0.110 ± 0.028

Table 3: Decay widths and branching ratios of $\chi_{cJ} \rightarrow \pi^0 \pi^0$ (colour-octet contributions included; $m_c = 1.5$ GeV, $\Lambda_{\text{QCD}} = 0.2$ GeV).

The integrals can then be worked out straightforwardly [23]. The regulator ϱ represents a mean transverse momentum of the quarks inside the pions. Indeed,

$$\varrho^2 = \langle k_{\perp}^2 \rangle / (4m_c^2) . \quad (18)$$

We checked that for appropriate values of ϱ this regularisation recipe provides, to a reasonable approximation, similar numerical results as one obtains when all transverse quark momenta are kept and, weighted by Gaussian k_{\perp} -dependences, integrated over.

The final results for the colour-octet contribution to the decay amplitudes can again be written in the form (10) with

$$a_J^{(8)} = \kappa \tilde{a}_J(z, \varrho) , \quad b_J^{(8)} = \kappa \tilde{b}_J(z, \varrho) , \quad c_J^{(8)} = \kappa \tilde{c}_J(z, \varrho) , \quad (19)$$

where \tilde{a}_J , \tilde{b}_J and \tilde{c}_J are only weakly dependent on the exact values of the parameters z and ϱ . Thus, it appears reasonable to take κ , defined by

$$\kappa = \sqrt{\alpha_s^{\text{soft}} f_0^{(8)} / \varrho^2} \quad (20)$$

as the only fit parameter. Evaluating the colour octet contribution from the asymptotic π wave function ($B_2 = 0$) with $z = 0.15$, ϱ about 0.1, which corresponds to some typical transverse momentum of 300 MeV, and, guided by our results for the singlet contribution obtained within the modified HSA, $\alpha_s(\mu_R^{\text{excl}}) = 0.45$, we find

$$\tilde{a}_0 = (151.1 + i25.9) \text{ GeV}^{-2} , \quad \tilde{a}_2 = (156.1 + i6.6) \text{ GeV}^{-2} . \quad (21)$$

The quantities \tilde{b}_J and \tilde{c}_J , necessary for $B_2 \neq 0$ case, will be given in [22].

The χ_{cJ} decay widths into pions are now obtained by adding coherently the respective colour-singlet and colour-octet contributions. Since both contributions have the same scaling with $1/c$ they have to be evaluated in the same scheme, i.e. the colour-singlet result in the collinear approximation has to be used. The four widths, $\chi_{cJ} \rightarrow \pi^+ \pi^-$, $\pi^0 \pi^0$ for $J = 0, 2$ are given in terms of the single non-perturbative factor κ . A fit yields $\kappa = 0.161 \text{ GeV}^2$ and the individual widths are shown in tables 2 and 3. We present also predictions for the respective branching ratios using the experimentally measured total widths. As shown in tables 2 and 3 the inclusion of the colour-octet mechanism brings predictions and data in generally good agreement.

Finally we address the question whether the value found for the parameter κ is sensible. For this purpose we estimate the probability for the χ_{c0} to be in the colour-octet Fock state from a plausible wave function:

$$\begin{aligned} \Psi_0^{(8)}(z_i, \mathbf{k}_{\perp i}) = & N z_1 z_2 z_3^2 \exp \left\{ -2a_{\chi}^2 m_c^2 \left[(z_3 - z)^2 + (z_1 - z_2)^2 \right] \right\} \\ & \times \exp \left\{ -a_{\chi} \sum k_{\perp i}^2 \right\} . \end{aligned} \quad (22)$$

This ansatz combines the known asymptotic behaviour of a distribution amplitude for a $q\bar{q}g$ Fock state [24] with mass-dependent exponentials and a Gaussian k_\perp -dependence in analogy to the Bauer-Stech-Wirbel parametrization of charmed-meson wave functions [25]. The mass-dependent exponentials guarantee a pronounced peak of the distribution amplitude at $z_1 \simeq z_2 \simeq m_c/M_c$. The δ -function-like distribution amplitude used in the estimate of the colour-octet contribution appears as the peaking approximation to this function. Since the $c\bar{c}g$ Fock state is an S -wave state we assume its radius to be equal to that of the S -state charmonia, namely 0.42 fm [15]. In this case the oscillator parameter a_χ takes the value 1.23 GeV^{-1} . The probability of the colour-octet Fock state is

$$P_{c\bar{c}g} = \left(f_0^{(8)} / 2.1 \times 10^{-3} \text{ GeV}^2 \right)^2 . \quad (23)$$

This relation is obtained with $z = 0.15$; it is however practically independent of the exact value of z . Taking $\alpha_s^{\text{soft}} \simeq \pi$ we obtain from (20) $f_0^{(8)} \simeq 0.9 \times 10^{-3} \text{ GeV}^2$ and $P_{c\bar{c}g} \simeq 19\%$. Hence, the probability for the χ_{cJ} to be in the $c\bar{c}g$ state comes out much more reasonable than one might have hoped for.

In summary, we have shown that the constraints on the pion-distribution amplitude from $F_{\pi\gamma}(Q^2)$ data lead to predictions for the exclusive decay $\chi_{cJ} \rightarrow \pi\pi$ that fall well (a factor of ten) below the data when restricting to the dominant, colour-singlet $|c\bar{c}(^3P_J)\rangle$ Fock state. The discrepancies persist after the inclusion of transverse degrees of freedom and Sudakov suppressions in the modified HSA approach. The deviations are, in fact, so large that they cannot sensibly be explained by higher-order perturbative QCD (α_s) or genuine higher-twist ($1/m_c$) corrections.

We have shown that contributions to the exclusive decay from the $O(v)$ -suppressed Fock component $|c\bar{c}g\rangle$ are actually of the same power in $1/m_c$ as the conventional colour-singlet contribution. Predictions including this colour-octet mechanism basically require only a single additional non-perturbative parameter. With quite a reasonable value for it, we obtain decay widths and branching ratios for the exclusive decays $\chi_{cJ} \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$ which are in good agreement with data.

We do not expect a significant colour-octet contribution to J/ψ (and Υ) decay into pions, since contributions from the higher-Fock state $|c\bar{c}(^3P_J)g\rangle$ are truly suppressed by $1/m_c^2$. The solution to a similar discrepancy between data and theory for $J/\psi \rightarrow \pi^+\pi^-$ is therefore still open and needs further study. Our findings for exclusive χ_{cJ} apply to P -wave bottomonium decays $\chi_{bJ} \rightarrow \pi\pi$ as well: a large fraction of the width will originate from the $b\bar{b}g$ state. One may speculate that the colour-octet mechanism is relevant to other exclusive decays as well, e.g. $B \rightarrow \chi_{cJ}K$. Work along these lines is in progress.

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