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Constraints on Supersymmetric Models from the Muon Anomalous Magnetic Moment

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Abstract

We study the impact of present and future $(g - 2)_\mu$ measurements on supersymmetric models. The corrections to $(g - 2)_\mu$ become particularly relevant in the presence of light sleptons, charginos and neutralinos, especially in the large $\tan \beta$ regime. For moderate or large values of $\tan \beta$, it is possible to rule out scenarios in which charginos and sneutrinos are both light, but nevertheless escape detection at the LEP2 collider. Furthermore, models in which supersymmetry breaking is transferred to the observable sector through gauge interactions can be efficiently constrained by the $(g - 2)_\mu$ measurement.

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1 Constraints from a_μ

The measurement of the anomalous magnetic moment of the muon [1]

$$a_\mu \equiv \frac{g_\mu - 2}{2} \equiv \frac{\mu_\mu}{(e\hbar/2m_\mu)} - 1 = (11\,659\,230 \pm 84) \times 10^{-10} \quad (1)$$

has provided an extremely precise test to QED (for a review, see [2] and references therein). The theoretical prediction for a_μ in the context of the Standard Model has different contributions which are usually divided into

$$a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}(vac\ pol) + a_\mu^{had}(\gamma \times \gamma) . \quad (2)$$

a_μ^{QED} contains the pure QED contribution which is known to order α^5 [2]. Extracting the value of α from $g_e - 2$ [3], one obtains

$$a_\mu^{QED} = (11\,658\,470.6 \pm 0.2) \times 10^{-10} . \quad (3)$$

a_μ^{EW} contains the electroweak corrections which are now fully known up to two loops [4]

$$a_\mu^{EW} = (15.1 \pm 0.4) \times 10^{-10} . \quad (4)$$

The largest source of uncertainty comes from the hadronic contributions, which cannot be computed by perturbation theory alone. $a_\mu^{had}(vac\ pol)$ includes hadron vacuum polarization corrections which enter at order α^2 . These corrections can be extracted from $e^+e^- \rightarrow$ hadrons data by use of dispersion relations. A recent calculation gives [5]

$$a_\mu^{had}(vac\ pol) = (702 \pm 15) \times 10^{-10} , \quad (5)$$

in agreement with other modern evaluations (see ref. [5] for a comparison of different estimates presented in the literature). Future measurements of the cross sections for $e^+e^- \rightarrow$ hadrons at BEPC in Beijing, at DAΦNE in Frascati, and at VEPP-2M in Novosibirsk are expected to reduce the error in $a_\mu^{had}(vac\ pol)$ to about 7×10^{-10} [6].

The hadronic light by light amplitudes cannot be related to observables and $a_\mu^{had}(\gamma \times \gamma)$ must be estimated theoretically. Hayakawa *et al.* [7] give

$$a_\mu^{had}(\gamma \times \gamma) = (-5.2 \pm 1.8) \times 10^{-10} , \quad (6)$$

while Bijnsens *et al.* [8] give

$$a_\mu^{had}(\gamma \times \gamma) = (-12.4 \pm 5.0) \times 10^{-10} . \quad (7)$$

Summing the different contributions and combining errors in quadratures, we obtain two theoretical estimates for a_μ

$$a_\mu = (11\,659\,183 \pm 15) \times 10^{-10} \quad (8)$$

$$a_\mu = (11\,659\,175 \pm 16) \times 10^{-10}, \quad (9)$$

depending on which of the two values of $a_\mu^{had}(\gamma \times \gamma)$ in eqs. (6)-(7) we use.

The experimental precision in the measurement of a_μ , presently at the level of 84×10^{-10} is expected to be improved by the E821 experiment at Brookhaven National Laboratory to the level of 4×10^{-10} , and possibly to $1-2 \times 10^{-10}$ if large statistics is accumulated [9]. If theoretical errors in a_μ^{had} are reduced, the E821 result will allow a direct test of the electroweak corrections and therefore will also be sensitive to new physics effects. Indeed already the present sensitivity can constrain some new physics contributions, as it was shown in the case of supersymmetry [10, 11], light-gravitino interactions [12], compositeness [13], lepto-quarks [14], and light non-minimal Higgs bosons [15].

Constraints on new physics can be obtained by requiring that the new contribution δa_μ lies within the difference between experimental result and theoretical prediction. From eqs. (1) and (8)–(9), combining the theoretical and experimental errors in quadratures, we find at 90% C.L.

$$-90 \times 10^{-10} < \delta a_\mu < 190 \times 10^{-10} \quad (10)$$

On the other hand, after the E821 experiment it will be possible to test the value of a_μ at the level of 4×10^{-10} .

In this paper we want to apply these constraints on δa_μ to specific cases of interest, in the context of supersymmetric models. We will first show how, in a fairly model-independent way, δa_μ can rule out regions of parameters with light charginos and sneutrinos which cannot be covered by direct LEP2 searches. Then we will show how, in models with gauge-mediated supersymmetry breaking, the bounds from δa_μ translate into strong bounds on all supersymmetric particle masses.

2 Chargino mass limits from LEP1.5 and LEP2

The negative searches in the LEP runs at 130 and 136 GeV have allowed to set a lower bound on the chargino mass of 67.8 GeV [16], if the chargino is gaugino-like and the sneutrino is heavy. This bound can be relaxed in two cases we want to consider here. If the sneutrino is light and its mass is chosen appropriately, the chargino production cross section suffers from a destructive interference and it can be considerably smaller than in the case of heavy sneutrino. The LEP1.5 limit on the chargino mass is then reduced, especially if $\tan \beta$, the ratio of the two Higgs vacuum expectation values, is large [16]. The second case occurs when $m_{\chi^\pm} > m_{\tilde{\nu}} \gtrsim m_{\chi^\pm} - 3$ GeV [17]. The chargino decay is then dominated by the two-body decay $\chi^\pm \rightarrow \ell^\pm \tilde{\nu}$, but the final-state charged lepton is too soft to be efficiently detected. In this case the LEP1.5 bound is completely lost, and the chargino could still be as light as $m_Z/2$. This can happen in regions of parameters which cannot be excluded by independent searches for neutralinos. Notice that this problem will also remain in the LEP2 analyses at higher \sqrt{s} . It is therefore important to understand if this region of parameters, difficult for LEP searches, can be excluded by other experiments. Recently the authors of ref. [18] have argued that this region of parameters can lead to an

appropriate amount of cold dark matter but cannot be excluded by cosmological constraints. Here we want to study whether both regions where the LEP chargino limit is reduced can be excluded by the experimental data on a_μ .

As emphasized in ref. [11] the supersymmetric contributions to a_μ coming from smuon-neutralino and sneutrino-chargino loops are significant and the present experimental bound already sets important constraints on the parameters, especially if $\tan\beta$ is large. For $\tan\beta \gg 1$, the supersymmetric contribution is approximately given by

$$\delta a_\mu \simeq \frac{\alpha}{8\pi \sin^2 \theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan\beta \simeq 15 \times 10^{-10} \left(\frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \tan\beta, \quad (11)$$

where \tilde{m} represents the typical mass scale of weakly-interacting supersymmetric particles. It is evident from eq. (11) that, if $\tan\beta \gg 1$, the experimental constraint on δa_μ can set bounds on the supersymmetric particle masses which are competitive with the direct collider limits. Indeed, the case $\tan\beta \simeq m_t/m_b \gg 1$ has some special theoretical appeal. First of all, it allows the unification of the bottom and tau Yukawa couplings at the same energy scale at which gauge couplings unify, consistently with the prediction of the minimal $SU(5)$ GUT model. Also it allows a dynamical explanation for the top-to-bottom mass ratio, with approximately equal top and bottom Yukawa couplings at the GUT scale, consistently with the minimal $SO(10)$ GUT [19].

The supersymmetric contribution to a_μ is

$$\begin{aligned} \delta a_\mu^{\chi^0} &= \frac{m_\mu}{16\pi^2} \sum_{mi} \left\{ -\frac{m_\mu}{6m_{\tilde{\mu}_m}^2 (1-x_{mi})^4} (N_{mi}^L N_{mi}^L + N_{mi}^R N_{mi}^R) \right. \\ &\times \left(1 - 6x_{mi} + 3x_{mi}^2 + 2x_{mi}^3 - 6x_{mi}^2 \ln x_{mi} \right) \\ &\left. - \frac{m_{\chi_i^0}}{m_{\tilde{\mu}_m}^2 (1-x_{mi})^3} N_{mi}^L N_{mi}^R (1 - x_{mi}^2 + 2x_{mi} \ln x_{mi}) \right\} \end{aligned} \quad (12)$$

$$\begin{aligned} \delta a_\mu^{\chi^\pm} &= \frac{m_\mu}{16\pi^2} \sum_k \left\{ \frac{m_\mu}{3m_{\tilde{\nu}}^2 (1-x_k)^4} (C_k^L C_k^L + C_k^R C_k^R) \right. \\ &\times \left(1 + 1.5x_k + 0.5x_k^3 - 3x_k^2 + 3x_k \ln x_k \right) \\ &\left. - \frac{3m_{\chi_k^\pm}}{m_{\tilde{\nu}}^2 (1-x_k)^3} C_k^L C_k^R \left(1 - \frac{4x_k}{3} + \frac{x_k^2}{3} + \frac{2}{3} \ln x_k \right) \right\} \end{aligned} \quad (13)$$

where $x_{mi} = m_{\chi_i^0}^2/m_{\tilde{\mu}_m}^2$, $x_k = m_{\chi_k^\pm}^2/m_{\tilde{\nu}}^2$,

$$\begin{aligned} N_{mi}^L &= -\frac{m_\mu}{v_1} U_{3i}^N U_{Lm}^{\tilde{\mu}} + \sqrt{2} g_1 U_{1i}^N U_{Rm}^{\tilde{\mu}} \\ N_{mi}^R &= -\frac{m_\mu}{v_1} U_{3i}^N U_{Rm}^{\tilde{\mu}} - \frac{g_2}{\sqrt{2}} U_{2i}^N U_{Lm}^{\tilde{\mu}} - \frac{g_1}{\sqrt{2}} U_{1i}^N U_{Lm}^{\tilde{\mu}} \\ C_k^L &= \frac{m_\mu}{v_1} U_{k2} \\ C_k^R &= -g_2 V_{k1} \end{aligned} \quad (14)$$

Here U_{ij}^N , $U_{(R,L)m}^{\tilde{\mu}}$, U_{kl} and V_{kl} are the neutralino, smuon and chargino mixing matrices, $i, j = 1, 4$; $k, l = 1, 2$ and $m = 1, 2$; $m_{\chi_i^0}$, $m_{\tilde{\mu}_m}$, $m_{\tilde{\nu}}$ and $m_{\chi_k^\pm}$ are the neutralino, smuon, sneutrino and chargino mass eigenstates, m_μ is the muon mass and g_i are the electroweak gauge couplings.

The value of δa_μ depends on M , μ and $\tan\beta$ in the chargino and neutralino sectors (we are assuming unification of gaugino masses), and on the parameters \tilde{m}_{LL} , \tilde{m}_{ER} , A , which determine the smuon mass matrix

$$m_{\tilde{\mu}}^2 = \begin{pmatrix} \tilde{m}_{LL}^2 + m_\mu^2 + (-\frac{1}{2} + \sin^2\theta_W) \cos 2\beta M_Z^2 & m_\mu(A - \mu \tan\beta) \\ m_\mu(A - \mu \tan\beta) & \tilde{m}_{ER}^2 + m_\mu^2 - \sin^2\theta_W \cos 2\beta M_Z^2 \end{pmatrix} \quad (15)$$

The parameter A appears only in the left-right smuon mixing, which is dominated by the μ term in the large $\tan\beta$ region. As we will mainly focus to this case, we can safely set $A = 0$. Moreover the total result is usually dominated by the sneutrino-chargino contribution, which is independent of A . Finally the sneutrino mass square is given by

$$m_{\tilde{\nu}}^2 = \tilde{m}_{LL}^2 + \frac{1}{2} \cos 2\beta M_Z^2 \quad (16)$$

As δa_μ is sensitive only to the mass of the muon sneutrino, while the LEP1.5 bound is affected by any sneutrino, in order to proceed we have to assume universality of sneutrino masses, $m_{\tilde{\nu}_e} = m_{\tilde{\nu}_\mu} = m_{\tilde{\nu}_\tau}$. This hypothesis is usually invoked to avoid unwanted lepton flavor violations (as in $\mu \rightarrow e\gamma$) and it is satisfied by the minimal supersymmetric model. It should be noticed however that this assumption is not a necessary requirement for the suppression of flavor-changing neutral current processes, as this can also be guaranteed by additional global symmetries [20] or by a dynamical principle [21], even in presence of large mass splittings among squarks and sleptons with different flavours.

Figure 1 shows the present experimental limit on the chargino mass, as a function of the sneutrino mass [17] in the large $\tan\beta$ region. A value of $\tan\beta = 20$ has been chosen in the figure, but the mass bounds are stable under changes of $\tan\beta$, for $\tan\beta \geq 10$. For large values of the sneutrino mass $m_{\tilde{\nu}} \gg M_Z$, the chargino mass bound is close to the kinematical limit and it is insensitive to the sneutrino mass. However, for lighter sneutrinos, the destructive interference in the chargino production cross section causes a reduction of the chargino mass bound. The results for two different values of μ are displayed, $\mu = -100$ GeV (shaded area) and $\mu = -500$ GeV (dark shaded area). In the case $\mu = -100$ GeV the chargino mass limit is reduced by more than 5 GeV, as the sneutrino destructive interference effects are maximal for $\mu \simeq -\mathcal{O}(M_2)$.

For $m_{\tilde{\nu}} < m_{\chi^\pm}$, the two-body decay channel $\chi^\pm \rightarrow l^+\tilde{\nu}$ is kinematically accessible and its branching ratio becomes of order one. However for small values of $m_{\chi^\pm} - m_{\tilde{\nu}}$, the charged leptons are too soft to be detectable. This is the origin of the gap in the chargino bound shown in fig. 1. One might expect this gap to be covered by neutralino searches, especially in the higgsino region, where the neutralino production cross section is sizable. Instead, the light sneutrino insures that the next-to-lightest neutralino predominantly decays into invisible final states, $\chi_2^0 \rightarrow \chi_1^0 \nu \bar{\nu}$. As an extreme case, three sneutrinos, two neutralinos, and one chargino could be just above the LEP1 threshold, but escape searches at LEP2.

For a given set of chargino parameters and sneutrino mass, we have chosen the value of \tilde{m}_{E_R} which minimizes the effect on δa_μ , in order to obtain the most conservative bound. This bound is shown in fig. 1 and superimposed to the experimental limit. The present constraint on δa_μ already closes the “hole” left by LEP1.5, if $\tan\beta$ is large enough. Indeed, for $|\mu| = \mathcal{O}(100 \text{ GeV})$, the hole, which would survive after the final LEP2 run if no chargino is found, can be closed through the δa_μ constraints for $\tan\beta \geq 10$ (20) for negative (positive) values of μ . For $|\mu| = \mathcal{O}(500 \text{ GeV})$, for which the lightest chargino and neutralino are mostly gauginos, $|\delta a_\mu|$ is slightly suppressed, leading to somewhat weaker bounds. In this case, the hole is closed for $\tan\beta \geq 20$ (40) for negative (positive) values of μ . Indeed, these bounds may be inferred from fig. 1, by taking into account the approximate linear dependence of δa_μ with $\tan\beta$ in the large $\tan\beta$ regime and the fact that the sign of δa_μ is given by the sign of the μ parameter. The bounds become also somewhat weaker deep into the Higgsino region ($M_{1/2} \gg M_Z$). The lightest neutralino becomes almost degenerate in mass with the lightest chargino in this region, and, independently on the sneutrino mass, no experimental limit may be set if their mass difference is below 5 GeV [23].

In the above we have minimized δa_μ by scanning over the right handed smuon mass, up to values of order 1 TeV. If, instead, the value of the right handed smuon is restricted to be of order of the left handed one, for instance below 200 GeV, the above results will be modified, depending on the gaugino and Higgsino components of the light chargino and neutralinos. The dependence on the maximum right-handed smuon mass is significant when μ is large. Indeed, in this case, the chargino diagram contributions are suppressed and bino-exchange diagram provides the dominant contribution. The contribution of this diagram is minimized for large values of the right handed smuon mass. Hence, more stringent bounds than the ones obtained in the case of very heavy smuons appear in this case. Numerically, the minimal value of $\tan\beta$ for which the hole can be closed for $\mu \simeq -500 \text{ GeV}$ changes from 20 to 14. In the case of small μ , $|\mu| = \mathcal{O}(100 \text{ GeV})$, both the chargino and neutralino contributions are relevant and a partial cancellation takes place between them. In this case, the minimal value of δa_μ is obtained for low values of the right handed smuon mass and hence no variations in the previous bounds are obtained by restricting the value of the right-handed smuon mass.

We also want to point out that a similar analysis can help in closing “holes” in the LEP2 search for charged sleptons. In particular the selectron production cross section can vary by more than one order of magnitude because of interference among the different contributions [22]. This makes the search harder, as the production rate can become very small. Future LEP2 analyses can benefit from the δa_μ bound, as this narrows considerably the allowed variation of the relevant parameters.

3 Gauge-mediated supersymmetry breaking

Theories with gauge-mediated supersymmetry breaking [24] have recently received renewed attention [25]–[31], because of their property of naturally suppressing flavour violations. These theories have also the attractive feature of predicting the supersymmetric mass spectrum in

terms of few parameters. Assuming that the messenger particle which communicate supersymmetry breaking belong to complete GUT multiplets, the gaugino and squark or slepton masses are respectively

$$m_{\lambda_j} = k_j \frac{\alpha_j}{4\pi} \Lambda_G \left[1 + \mathcal{O}(F^2/M^4) \right] \quad (17)$$

$$\tilde{m}^2 = 2 \sum_{j=1}^3 C_j k_j \left(\frac{\alpha_j}{4\pi} \right)^2 \Lambda_S^2 \left[1 + \mathcal{O}(F^2/M^4) \right] , \quad (18)$$

where $k_1 = 5/3$, $k_2 = k_3 = 1$, and $C_3 = 4/3$ for colour triplets, $C_2 = 3/4$ for weak doublets (and equal to zero otherwise), $C_1 = Y^2$ ($Y = Q - T_3$). In the simplest models with minimal messenger structure, the gaugino and scalar scales Λ_G and Λ_S are related by

$$\Lambda_G = \sqrt{n} \Lambda_S = n \frac{F}{M} . \quad (19)$$

Here M is the messenger mass scale and \sqrt{F} is the original supersymmetry-breaking scale; n is the effective number of messenger fields. Perturbativity of gauge coupling constants up to the GUT scale requires that the integer number n satisfies $n \leq 4$. The masses in eqs. (17)–(18) are defined at the messenger mass scale M , and we have rescaled them to the physical mass value using the one-loop renormalization group equations. We have chosen the messenger mass scale $M = 100$ TeV, but our results depend only mildly on this choice, because of the slow logarithmic dependence. It is just the ratio F/M which really sets the supersymmetric particle masses, and it will be taken as a free parameter in our analysis. For a generic messenger sector, the energy scales Λ_G and Λ_S are independent and hypercharge D-term contributions can significantly affect the mass of the right-handed smuon [29]. For simplicity we will restrict our consideration to the minimal case in which eq. (19) holds.

Besides the parameters F/M and n which describe the gaugino and scalar spectrum, we also need to introduce the parameters μ and $\tan \beta$ which define the higgsino mass and mixings as in the ordinary supersymmetric model considered in sect. 2. In gauge-mediated supersymmetric theories, μ originates from new interactions beyond the usual gauge forces [27], and its relation with F/M depends on unknown constants.

We present our results of δa_μ as a function of μ and the weak gaugino mass M_2 (which determines F/M), for fixed values of n and $\tan \beta$. As discussed previously, the stronger limits from δa_μ come in the region $\tan \beta \gg 1$. In this region, an important constraint comes from the requirement that the determinant of the stau square mass matrix is positive. In gauge-mediated supersymmetric theories, the trilinear term A vanishes at the messenger mass scale. Therefore the slepton left-right mixing is dominated by the μ term, which can become dangerously large if $\tan \beta \gg 1$.

Figure 2 and 3 show the bounds which may be obtained in these models in the $M_2 - \mu$ plane for two different values of $\tan \beta$ and for $n = 1, 3$, respectively. Large values of $|\mu|$ are restricted by the lower bound on the stau mass, while low values of M_2 lead to unacceptable values of δa_μ . The bounds on M_2 become particularly strong when $\tan \beta$ is close to its extreme value $\tan \beta \simeq m_t(m_t)/m_b(m_t) \simeq 60$. Notice that the limits become more stringent as n is

increased. Indeed, as apparent from eq. (19), for a given value of the gaugino mass, larger n correspond to lighter sleptons, thus to larger contributions to δa_μ . Because of the mass relations in eqs. (17)–(18), a bound on M_2 can be easily translated into bounds on the various supersymmetric particle masses. For instance, the gluino mass is $M_{\tilde{g}} \simeq (2.9\text{--}2.5) M_2$, where the variation in the ratio $M_{\tilde{g}}/M_2$ comes from the scale dependence of the gaugino masses. Analogously, the right-handed squark mass is $m_{\tilde{q}} \simeq (4.1\text{--}3.5) M_2/\sqrt{n}$.

Future limits on δa_μ will put very strong constraints on models of gauge mediated supersymmetry breaking. Indeed, the foreseen experimental sensitivity is of the order of the effects which are obtained for values of the gluino mass $M_{\tilde{g}}$ as high as 1.4 TeV (3 TeV) for $n=1$ and $\tan\beta = 10$ (60) respectively. For $n=3$, the experimental sensitivity is of the order of the effects obtained for $M_{\tilde{g}} \simeq 2$ TeV (4 TeV) respectively. Figure 4 shows the values of the gluino and right-handed squark masses which can be tested assuming the bound $|\delta a_\mu| < 4 \times 10^{-10}$. This is just the future sensitivity of the E821 experiment. Of course the actual bounds will depend on how much the theoretical error can be reduced, and on the central values of the experimental measurement and the theoretical prediction. The limits on the sparticle masses corresponding to a bound on δa_μ different from 4×10^{-10} can be obtained from fig. 4 by noticing that, in the large $\tan\beta$ regime, the supersymmetric contribution to δa_μ is proportional to $\tan\beta$. The limits are obtained by minimizing the effect on δa_μ as a function of μ , for fixed values of M_2 and $\tan\beta$. As seen in the figure even for values of $\tan\beta$ as low as one, values of the gluino masses of order 450, 600 GeV may be tested for gauge mediated supersymmetry breaking models with $n = 1, 3$ respectively. Hence, δa_μ will represent a crucial test of these models, even for moderate values of $\tan\beta$.

Recently it has been argued [30] that, in theories with gauge-mediated supersymmetry breaking, large $\tan\beta$ could be a natural option. This is because the different Higgs mass parameter may arise at different order in perturbation theory, allowing therefore a natural hierarchy which leads to large values of $\tan\beta$. We have found here that constraints from δa_μ strongly bound the large $\tan\beta$ region in these theories, and future measurements of a_μ will give a definite test of the proposal in ref. [30]. On the other hand, it should be mentioned that the motivation for large $\tan\beta$ coming from b – τ unification is weakened in these theories. As shown in ref. [31], the messenger particles slow down the running of α_s as the energy scale is increased. This has the effect of enhancing the ratio m_b/m_τ at low energies, and therefore b – τ unification can be achieved only at the price of a low $\alpha_s(M_Z)$. This effect becomes more important as n increases. Present LEP determinations of $\alpha_s(M_Z)$ are already cornering b – τ unification in gauge-mediated scenarios [31].

In gauge-mediated theories the original scale of supersymmetry breaking can be rather low, of the order of 100 TeV. If this is the case, the gravitino is very light, of the order of the eV. The Goldstino component of the gravitino has couplings much stronger than gravitational ones. The contribution to δa_μ from gravitino-smuon loops is [12]

$$\delta a_\mu = \frac{G_N}{36\pi} \frac{m_\mu^2}{m_{3/2}^2} \text{Tr} m_{\tilde{\mu}}^2, \quad (20)$$

where G_N is the Newton constant, $m_{3/2}$ is the gravitino mass, and the trace is taken over

the 2×2 smuon square mass matrix. Relating the gravitino mass to the original scale of supersymmetry breaking, $m_{3/2} = F\sqrt{(4\pi/3)G_N}$, and using the expression of the smuon mass in eq. (18), we can write eq. (20) in terms of the messenger mass scale M :

$$\delta a_\mu = \frac{n}{2} \left(\frac{\alpha m_\mu}{2\pi^2 \sin 2\theta_W M} \right)^2 \sim n \left(\frac{100 \text{ TeV}}{M} \right)^2 \times 10^{-19}. \quad (21)$$

This contribution is too small to give a significant constraint to the model.

In conclusion, we have studied how the measurement of the muon anomalous magnetic moment constrains supersymmetric models in two different scenarios. We have first discussed the case in which a light chargino evades detection at LEP2 because of the presence of a light sneutrino. Present bounds on a_μ can rule out this scenario if $\tan \beta$ is sufficiently large and therefore provide an important tool complementary to direct collider searches. Then we have considered the case of theories with gauge-mediated supersymmetry breaking. Because of the mass relations among sleptons, charginos, and neutralinos, the bound on δa_μ gives a very definite constraint on the whole supersymmetric mass spectrum. Future measurements on a_μ can conclusively test these models, particularly in the moderate and large $\tan \beta$ regions.

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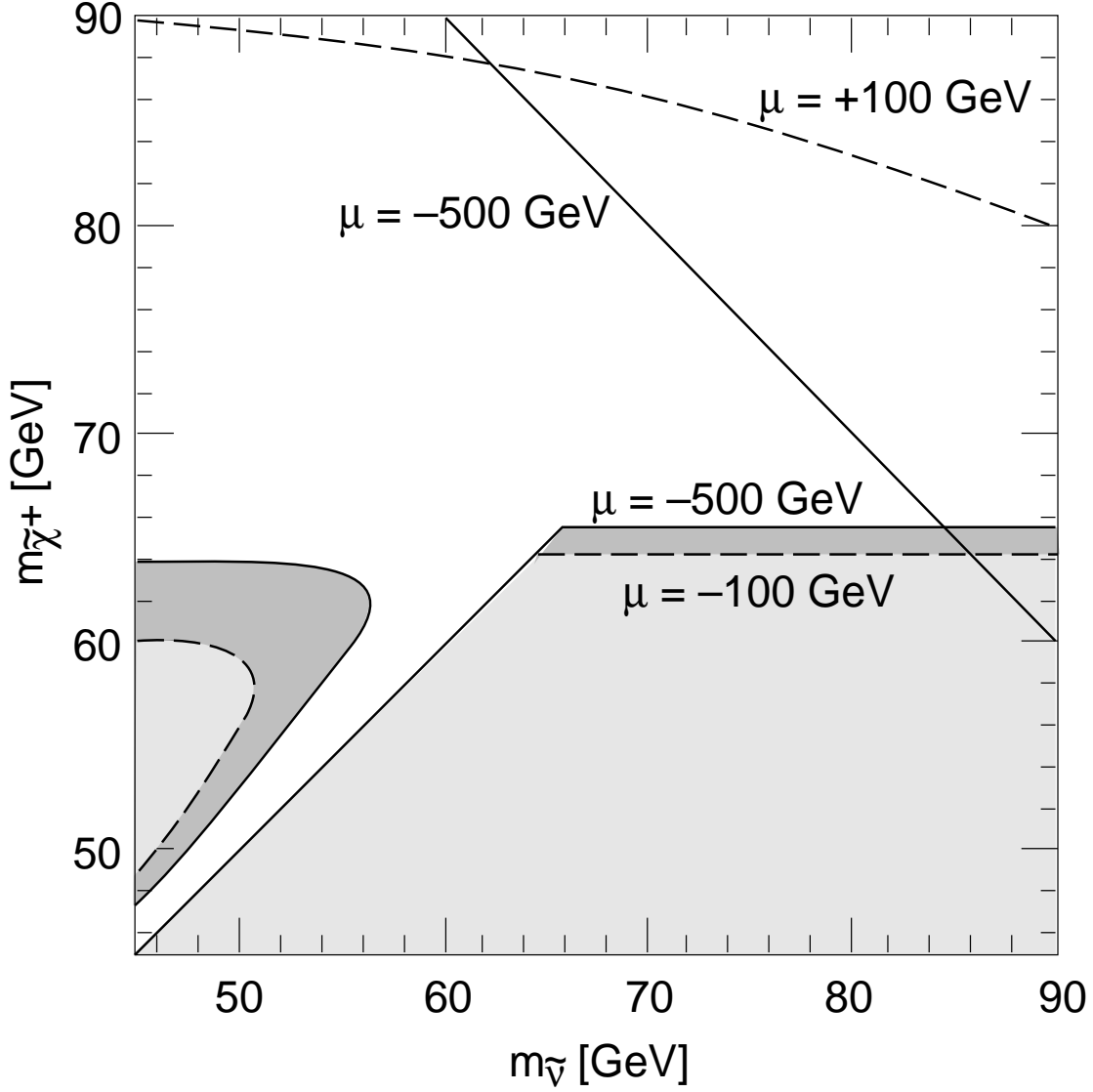


Figure 1: Present experimental limit on the lightest chargino mass as a function of the sneutrino mass $m_{\tilde{\nu}}$ for $\tan\beta = 20$, and two values of μ , $\mu = -500$ GeV (dark shaded region) and $\mu = -100$ GeV (light shaded region). Also displayed in the figure are the present limits coming from constraints on δa_μ for $\tan\beta = 20$ and for $\mu = -500$ GeV and $\mu = 100$ GeV, respectively.

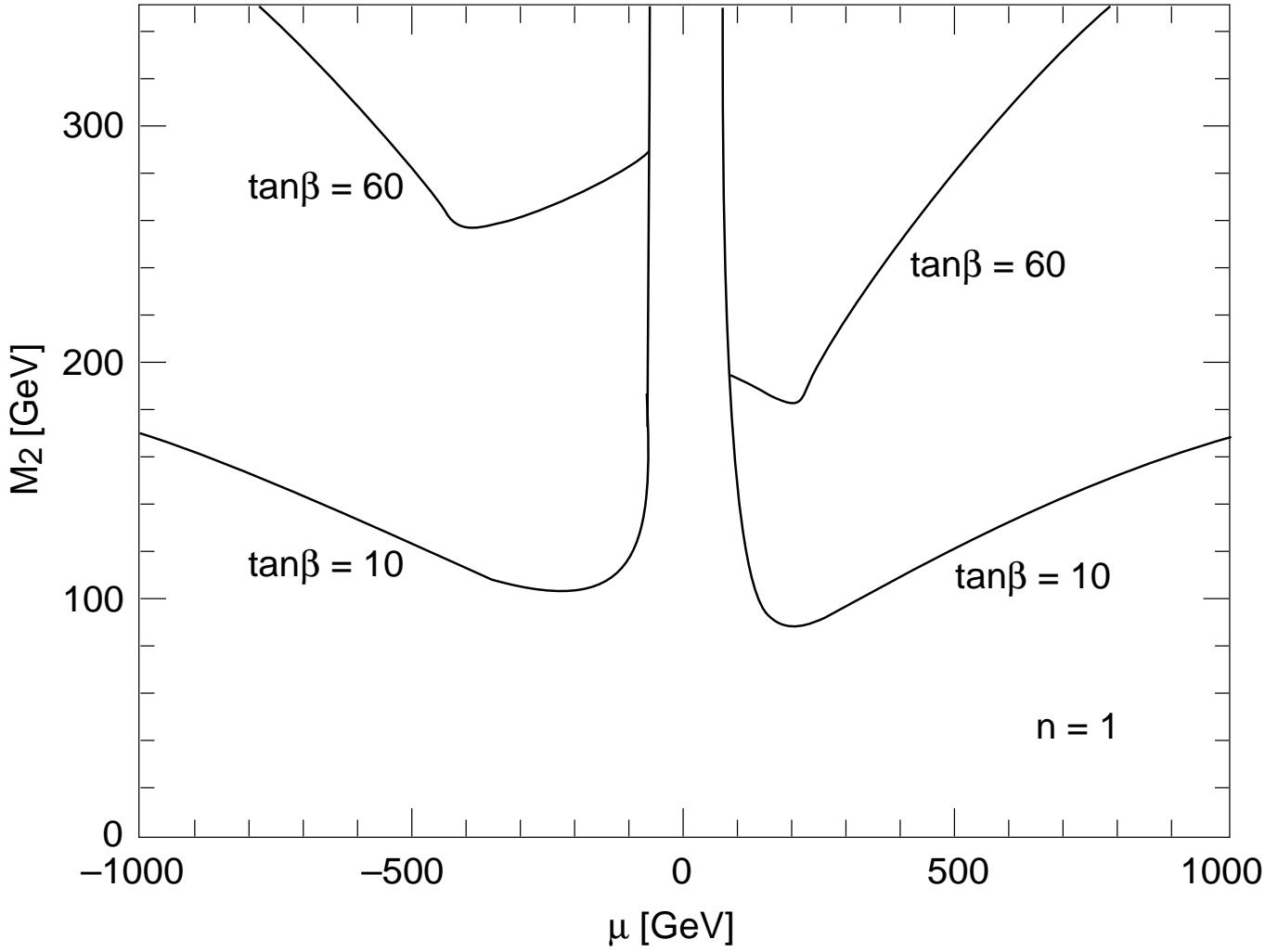


Figure 2: Present limits on the gaugino mass parameter M_2 as a function of the Higgsino mass parameter μ , in gauge mediated supersymmetry breaking models with $n = 1$. The upper curve represents the limit for $\tan\beta = 60$, while the lower curve is the result for $\tan\beta = 10$.

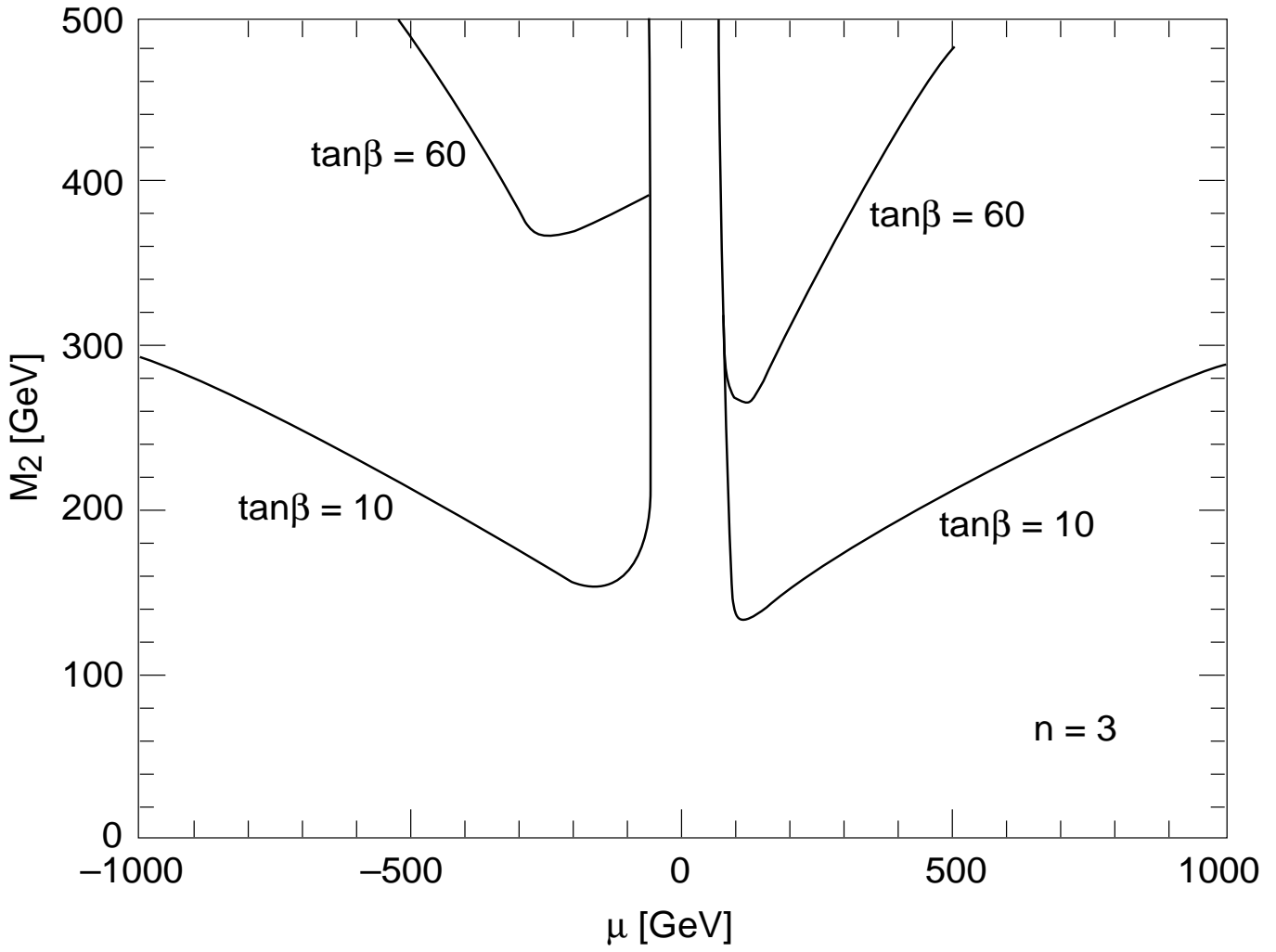


Figure 3: The same as fig. 2, but for $n = 3$.

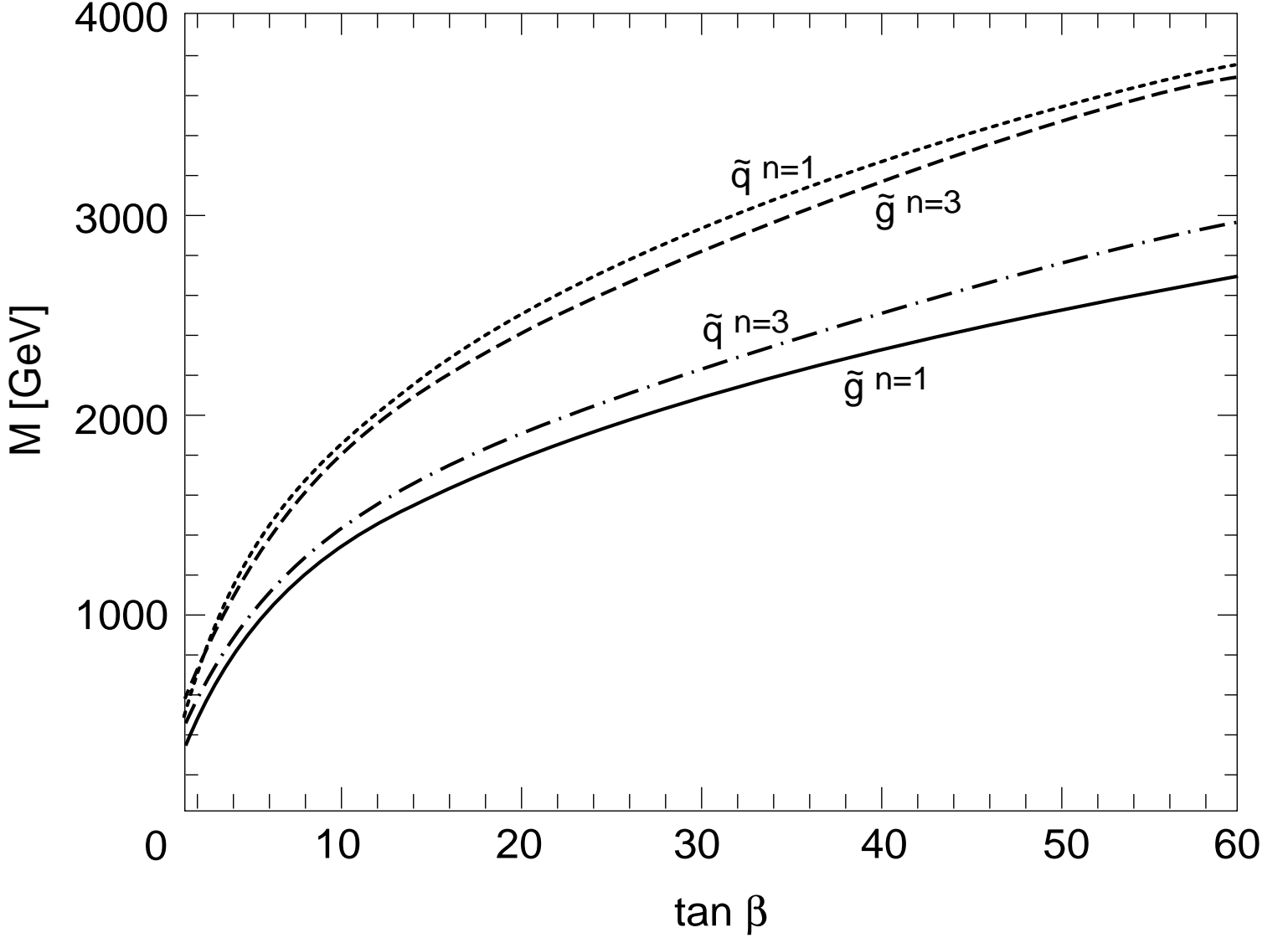


Figure 4: Limits on the gluino and right-handed squark masses as a function of $\tan \beta$ for gauge-mediated supersymmetry-breaking models with $n = 1$ (solid and dotted lines) and $n = 3$ (dashed and dot-dashed lines), assuming that the future experimental sensitivity and the future theoretical estimates will allow to constraints new physics effects at the level $|\delta a_\mu| < 4 \times 10^{-4}$.