OUTP-96-53P
CERN-TH/96-246
hep-ph/9610202
September 1996

Disoriented Chiral Condensates in Hadron-Hadron Collisions[1]

G. Amelino-Camelia[(a,b)], **J.D. Bjorken**[(a,c)], **and S.E. Larsson**[(a)]

(a) Theoretical Physics, University of Oxford, 1 Keble Rd., Oxford OX1 3NP, UK
(b) CERN, TH-Division, CH-1211 Geneva 23, Switzerland
(c) SLAC, Stanford University, Stanford, California 94309, USA

ABSTRACT

We review recent progress in the description and understanding of *disoriented chiral condensates*. Certain important unsolved issues are underlined, and the preliminary results of our program of investigation of these issues in the framework of the classical linear sigma model are reported. We also briefly review a formalism which could be useful at the full non-equilibrium quantum field theory level of analysis.

[1]Presented by G. Amelino-Camelia at the *10th International Conference on Problems of Quantum Field Theory*, Alushta, Crimea, Ukraine, May 13-18, 1996.

1 Introduction

Recently, in order to explain rare events with a deficit or excess of neutral pions observed in cosmic ray experiments, there has been increased interest in the conjecture that it might be possible to produce *disoriented chiral condensates* (DCCs), *i.e.* correlated regions wherein the quark condensate, $\langle 0|q_L\bar{q}_R|0\rangle$, is chirally rotated from its usual orientation in isospin space.

On the theoretical side there has been great interest (see, *e.g.*, Refs. [1-14]) both in the development of technical tools suitable for the description of this new phenomenon, and in the exploration of the possibilities opened by DCCs as probes of the structure of chromodynamics, most notably in relation to the chiral phase transition.

The idea that such DCCs might be produced in high energy collisions at existing or planned hadron or heavy-ion accelerators has originated several experimental proposals. In particular, one of us is co-spokesman for a Fermilab experiment [15] looking for DCCs in hadron-hadron collisions. In high energy p-$\bar{p}$ collisions which lead to a sizeable multiplicity of produced particles, but not necessarily with high-p_T jets in the final state, the time evolution is quasi-macroscopic, because the hadronization time can be rather large, 3-5f. At times t before hadronization, the initial state partons, produced in a volume much smaller than a cubic fermi, stream outward at essentially the speed of light in all directions, occupying the surface of a sphere of radius t (in units such that the speed of light is 1). Most of the outgoing energy/momentum is expected to be concentrated near the light cone, *i.e.* on the fireball surface. However, the interior of the fireball is also an interesting place. If its energy density is low enough, the interior should look very similar to the vacuum, with an associated non-vanishing quark condensate. Since the energy density from the intrinsic chiral symmetry breaking is small [6, 17], and the fireball surface isolates the interior from the exterior of the light cone, it is reasonable to consider the possibility that well inside the light cone the quark condensate might be chirally rotated from its usual orientation. At late times this disoriented vacuum would relax back to ordinary vacuum, radiating its collective modes, pions. The properties of the radiated pions would be strongly affected by the semiclassical, coherent nature of the process; in particular, one could expect anomalously large event-by-event fluctuations in the ratio of the number of charged pions to neutral pions produced. Importantly, assuming that the event-by-event deviation of the quark condensate from its usual orientation be random, one finds [1-4,16,17] that the distribution $P(f)$ of the neutral fraction

$$f \equiv \frac{N_{\pi^0}}{N_{\pi^0}+N_{\pi^+}+N_{\pi^-}} \equiv \frac{N_{\pi^0}}{N_{tot}} , \qquad (1)$$

is given by

$$\frac{dP}{df} = \frac{1}{2\sqrt{f}} , \qquad (2)$$

at large N_{tot}. Most notably, this implies that for "DCC pions" the probability of finding extreme values of f is very different from ordinary pion production (which is given by a binomial distribution), in which the fluctuations are expected to be peaked at $f=1/3$ and fall exponentially away from the peak. Recent experimental DCC searches [15] are largely based on the structure of Eq. (2).

2 Description at the Classical Level

Since the DCC is essentially a classical pion field, the description of its space-time evolution is most naturally described using classical or semiclassical techniques. The $O(4)$ σ-model is typically used as a model of chromodynamics in DCC studies. The σ-model is simple enough to be treatable, has the correct chiral symmetry properties, and describes the low energy phenomenology of pions. The Lagrangian of the (linear) σ-model is (in the chiral limit $m_\pi=0$)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - f_\pi^2)^2 \ . \tag{3}$$

A meaningful σ-model description can start at some small proper time, of order 0.2-0.3f, near the light cone, when the collective coordinates σ and π become relevant [10, 18]. At this early proper time the distribution of the chiral field

$$\Phi \equiv \sigma + i\vec{\pi}\vec{\tau} \ , \tag{4}$$

can be expected to be noisy, but with $\langle\Phi\rangle=0$.

As proper time increases the field Φ rolls into a minimum with $\Phi^+\Phi=f_\pi^2$, and during this "rolling phase" the pion mass is imaginary, leading to unstable growth of the Goldstone modes [3, 10]. Since, as mentioned in the Introduction, the energy density from the intrinsic chiral symmetry breaking is small [6, 17], and the fireball surface isolates the interior from the exterior of the light cone, it is reasonable to expect that the interior ends up in a disoriented vacuum.

At late times one such region of disoriented vacuum with a given isospin orientation would relax back to ordinary vacuum $\langle\Phi\rangle = \langle\sigma\rangle = f_\pi$, radiating pions with the same isospin orientation.

In modelling these stages of evolution, the chiral limit can be safely taken as long as the proper time is small compared to m_π^{-1}, while at times of order 1-2m_π^{-1} the pion mass can no longer be neglected and one should [10] decompose the DCC field into physical-pion normal modes and let them propagate out to infinity as free states.

It is also reasonable to expect that approximations based on the replacement of the full linear σ-model (3) by the simpler non-linear σ-model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 \ , \quad \text{with } \sigma^2 + \vec{\pi}^2 = \mathrm{f}_\pi^2 \ , \tag{5}$$

could be reliably used at times late enough for the chiral field to have already rolled into a minimum with $\Phi^+\Phi\equiv\sigma^2 + \vec{\pi}^2=f_\pi^2$.

Actually the entire picture of DCC evolution given above can be implemented (although, at least at early times, the quantitative aspects are not accurately evaluated) within the non-linear σ-model in the framework of the set of classical solutions identified by Anselm [1] and others [4, 17], which have the form

$$\Phi = f_\pi \, V_L^+ e^{i\theta\tau_3} V_R \ , \tag{6}$$

where V_L and V_R are constant but otherwise arbitrary matrices [1], and θ is such that

$$\Box\theta = 0 \ . \tag{7}$$

In particular, by taking $V_L = V_R$ and using the following solution of (7)

$$\begin{aligned} \theta(r) \;\; = \;\; & \frac{\pi}{4} \, \frac{2T + r - t}{r} \, \Theta(2T + r - t) \, \Theta(t - r) \\ & - \frac{\pi}{4} \, \frac{2T - r - t}{r} \, \Theta(2T - r - t) \, \Theta(t + r) \ , \end{aligned} \tag{8}$$

where Θ is the step function, one can describe several stages of the evolution of an ideal spherically symmetric DCC formed at the collision centre. For $t \leq 2T$ the DCC field described by (8) expands at the speed of light (mimicking the "rolling phase" of the DCC evolution). For $2T \leq t \leq 4T$ the DCC field keeps expanding but the true vacuum starts breaking into the interior of the light cone. For $t \geq 4T$ the true vacuum is everywhere apart from a bump of DCC field propagating outward. The final stage of evolution, the one in which the classical DCC field radiates pions, cannot be reproduced within (8); it requires, as stated above, taking into account the non-vanishing pion mass.

There are two more aspects of the simple solution (8) which are worth emphasizing. First we notice that (8) is a solution of (7) in the presence of a DCC source term on the light cone at small times; such a source term is a straightforward way to mimic the unstable growth of the Goldstone modes associated with the "rolling phase", and could also be useful in more detailed analyses. [Note that taking $V_L = V_R$ is very important, since otherwise the source term on the light cone will not vanish at large times.] A second aspect of (8) which could be instructive for other developments is the dominant role played by the Minkowski geometry. This remark also gives us a chance to state our belief that geometry should play a central role in DCC physics, and therefore the investigation of the possibility of DCC formation in hadron-hadron collisions should proceed differently from the corresponding heavy-ion case.

Going back to the study of the space-time evolution of the DCC, let us now consider the full linear σ-model, which is necessary for a description starting at early proper times. A good starting point is the generalization to the linear σ-model of the solutions (6) and (7) of the non-linear σ-model. One finds solutions of the form [13]

$$\Phi = \rho \, V^+ e^{i\theta\tau_3} V \ , \tag{9}$$

with θ and ρ such that

$$\Box\theta = 0 \ , \quad \Box\rho - \rho\,(\partial_\mu\theta)^2 + \lambda\,\rho\,(\rho^2 - f_\pi^2) = 0 \ . \tag{10}$$

The equation for ρ is not easy to handle analytically, but progress can be achieved by numerical techniques.

We are at present using numerical and analytic techniques in a study of the space-time evolution of DCCs within the linear σ-model (3). The simulations start from a given configuration for the pion/sigma fields at an initial proper time, and we observe the dynamics of the transition to the ordered broken symmetry phase as the system expands. We plan to study different types of initial conditions, with the chiral field starting either from the top or from the brim of the "Mexican hat" and with various assignments for the initial velocities in the $\vec{\pi}$ and σ directions. One of the issues that we are interested in investigating is the possibility of correlations in the evolution of two (or more) pieces of DCC in different regions of the lego plot. It would be interesting to check whether these pieces of DCC "attract" or "repel", and whether the presence of DCC in one region of the lego plot stimulates production in neighbouring regions. Ultimately, we hope to create simulations realistic enough to be used in the interpretation of hadron-hadron experimental results relevant for the DCC, but at present our research program is still in its preliminary stages. We have started by investigating the linear σ-model evolution of a spherically symmetric DCC formed at the collision centre, and observed that, as expected [7, 10], the evolution proceeds through the same stages of the simple non-linear σ-model solution associated with (8); however, also depending on the initial conditions, we find some evidence of a possible secondary wave front resulting from implosion of the true vacuum as it occupies the interior region of the fireball. Such a secondary wave could in principle be very important for experimental purposes, but much more numerical work is needed to check whether in fact it can be expected to occur. Other preliminary results concern simulations of scenarios with two pieces of DCC and, for certain initial conditions, they indicate that the evolution of one piece of DCC is substantially affected by the presence of the other piece. Also for this effect much more numerical work is needed in order to establish its significance for experimental purposes. Our analytic and numerical work is also aimed at describing, within the proper framework of Minkowskian geometry, the way a piece of DCC affects the evolution of another piece of DCC by acting as a source.

3 Quantum Field Theory

The expression in quantum terms of the ideas discussed classically in the previous section is most natural in the framework of coherent states or squeezed states. We refer the reader to Refs. [8, 9, 16, 17] for analyses within these formalisms. Here we will briefly review a formalism that could be useful in investigating whether such quantum states are actually created as a consequence of the relevant non-equilibrium quantum dynamics. We are not aware of much progress in the direction of exploiting this formalism in the investigation of DCCs, at least in association with hadron-

hadron collisions[2], but a few conceptual points can be made even before actual calculations.

One can describe a physical system via its density matrix ρ, $\rho \equiv e^{-\beta H}/(tr\, e^{-\beta H})$, and average values of observables O are determined by the density matrix: $\langle O \rangle = tr \rho O$. In equilibrium physics the density matrix is time-independent; in general, however, the density matrix is time-dependent, and the task of non-equilibrium quantum field theory is to study the time evolution of ρ. Non-equilibrium quantum physics is a vast subject, and there is no canonical approach to its investigation. Usually the approach is suggested by the specifics of the physical system that one wants to describe. We review an approach [21, 22] that has proved useful in early Universe cosmology [23], and might also be useful in the investigation of DCCs. This approach is set up in the framework of the field theoretic Schrödinger picture, which is particularly suitable to time-dependent problems that require an initial condition for a specific solution. The (functional) density matrix is given by a superposition of wave functionals

$$\rho(\phi_1, \phi_2) = \sum_n p_n \Psi_n(\phi_1) \Psi_n^*(\phi_2) \ , \qquad (11)$$

where $\{\Psi_n\}$ is a complete set of wave functionals, and p_n is the probability ($\sum_n p_n = 1$) that the system is in the state Ψ_n. In general the Ψ_n's and the p_n's are time-dependent.

In equilibrium the dynamics is time-translation invariant and energy is conserved. Then the complete set of wave functionals $\{\Psi_n\}$ can be chosen to be the set of the (time-dependent) energy eigenstates, while the p_n's are time-independent and are given by the canonical Boltzmann distribution, $p_n = e^{-\beta E_n}/(\sum_n e^{-\beta E_n})$, where E_n is the energy eigenvalue of the state Ψ_n. The time evolution of the corresponding density matrix is trivial: it remains constant in time because the p_n's and the $\Psi_n \Psi_n^*$'s are constant (N.B.: the time dependence of the Ψ_n's is just a phase).

For non-equilibrium physics the time evolution of the density matrix is instead non-trivial. In fact, it might not be possible to choose the Ψ_n's as energy eigenstates, and the p_n's need not be Boltzmann factors and can change in time. Under the assumption that the time dependence of the Ψ_n's be determined by a time-dependent Schrödinger equation, the density matrix ρ satisfies the following differential equation

$$\frac{d\rho}{dt} = \sum_n p_n \frac{d}{dt}(\Psi_n \Psi_n^*) + \sum_n \frac{dp_n}{dt}(\Psi_n \Psi_n^*) = i[\rho, H] + \sum_n \frac{dp_n}{dt}(\Psi_n \Psi_n^*) \ . \qquad (12)$$

In order for Eq. (12) to describe a well defined initial value problem for the time evolution of ρ, it is necessary to give the form of H and a model for dp_n/dt.

[2]Related studies have been reported in Refs. [11, 12]; however, the formalism adopted there was somewhat different. In particular, the Liouville-vonNeumann equation (which in this section will arise only after a suitable approximation) was taken as a starting point. Moreover, as a result of the assumptions made about the geometry of the problem, previous DCC analyses using non-equilibrium quantum field theory are more relevant to the case of heavy-ion collisions.

One important simplification is allowed when the evolution of interest is entropy-conserving; in fact, in this case one can work with time-independent p_n's, which indeed can be shown [21] to correspond to entropy-conserving time evolution. In these hypotheses, Eq. (12) takes the form of the quantum Liouville-vonNeumann equation with time-dependent Hamiltonian:

$$\frac{d\rho}{dt} = i[\rho, H] \ . \tag{13}$$

While the investigation of the Liouville-von Neumann equation is still extremely difficult, it is certainly remarkably simpler than the original Eq. (12). [We are not aware of any fruitful investigations of Eq. (12).] It is therefore quite crucial to establish whether the assumption of entropy-conserving evolution (time-independent p_n's) is justified in the study of DCCs. In the *baked Alaska* scenario for the formation of DCC in hadron-hadron collisions that has been considered here, it appears that the assumption of entropy-conserving evolution could be quite accurate; in fact, the interior of the fireball is a rather "peaceful" place. Instead, in the case of heavy-ion collisions the relevant time scales are such that the formation of DCCs might be affected by non-entropy-conserving stages of the evolution of the quark-gluon plasma (*i.e.* one might be led to a definition of *the system* such that the evolution is entropy non-conserving). Even when it is safe to assume that the time evolution is entropy-conserving, and therefore the Liouville-von Neumann equation can be meaningfully taken as a starting point, for non-trivial Hamiltonians, such as the ones relevant for DCC physics, progress requires further approximations. Interestingly, one can obtain approximate solutions using the observation [21, 22] that the Liouville-von Neumann equation can be derived by varying an action-like quantity within a variational principle introduced by Balian and Veneroni [24]. An approximate application of the variational principle, with a restricted variational *ansatz*, leads to approximate equations for the density matrix[3].

4 Outlook

The recent increased interest in DCC physics has led to substantial progress, but much work is still needed. The challenge is quite strong both for theorists, who must find their way through the complicated models involved and provide experimentalists with a reasonable (quantitative!) picture of what to look for, and for experimentalists, who must disentangle the (possibly faint) DCC signal from substantial backgrounds.

We are confident that our program of numerical and analytical investigation of the linear σ-model will soon lead to simulations realistic enough to be used in the

[3] Alternatively, as recently observed in Ref.[25], an *ansatz* can be made directly at the level of the Liouville-von Neumann equation, without advocating the Balian-Veneroni variational riformulation of the problem.

interpretation of hadron-hadron experimental results relevant for the DCC.

Acknowledgements

Much ot this work was initiated some time ago, before the onset of the MiniMax experiment together with C. Taylor and K. Kowalski, which have made similar investigations independently. It is a pleasure to acknowledge conversations with D. Boyanovsky, O. Eboli, F. Cooper, R. Jackiw, E. Mottola, S.-Y. Pi, K. Rajagopal, S. Sarkar, and P. White. This work was supported in part by funds provided by the European Union under contract #ERBCHBGCT940685, the American Trust for Oxford University (George Eastman Visiting Professorship), CSN and KV (Sweden), ORS and OOB (Oxford), and the Sir Richard Stapley Educational Trust (Kent).

References

[1] A. A. Anselm, Phys. Lett. **B217** (1989) 169; A. A. Anselm and M. C. Ryskin, Phys. Lett. **B266** (1991) 482.

[2] J. D. Bjorken, Int. J. Mod. Phys. **A7** (1992) 4189.

[3] K. Rajagopal and F. Wilczek, Nucl. Phys. **B379** (1993) 395; **B404** (1993) 577.

[4] J.-P. Blaizot and A. Krzywicki, Phys. Rev. **D50** (1994) 442.

[5] J. D. Bjorken, K. L. Kowalski and C. C. Taylor, SLAC-PUB-6109, in the proceedings of *Les Rencontres de Physique de la Vallée D'Aoste*, La Thuile (1993); hep-ph/9309235, in the proceedings of *Workshop on Physics at Current Accelerators and the Supercollider*, Argonne (1993).

[6] S. Gavin, A. Gocksch and R. D. Pisarski, Phys. Rev. Lett. **72** (1994) 2143;

[7] M. Asakawa, Z. Huang and X.-N. Wang, Phys. Rev. Lett. **74** (1995) 3126; Z. Huang and X.-N. Wang, Phys. Rev. **D49**, 4335, (1994); Z. Huang, I. Sarcevic, R. Thews and X.-N. Wang, Phys. Rev. **D54** (1996) 750.

[8] I.V. Andreev, Pisma v JETP **33** (1981) 384; S. Pratt, Phys. Lett. **B301** (1993) 159.

[9] I.I. Kogan, JETP Lett. **59** (1994) 307; R.D. Amado and I.I. Kogan, Phys. Rev. **D51** (1995) 190.

[10] J.D. Bjorken, SLAC-PUB-6488, in the proceedings of *Workshop on Continuous Advances in QCD*, Minneapolis (1994).

[11] D. Boyanovsky, H. J. de Vega and R. Holman, Phys. Rev. **D51** (1995) 734.

[12] F. Cooper, Y. Kluger, E. Mottola and J.P. Paz, Phys. Rev. **D51** (1995) 2377.

[13] A.A. Anselm, M.C. Ryskin and A.G. Shuvaev, Z. Phys. **A354** (1996) 333.

[14] F. Cooper, Y. Kluger and E. Mottola, *Anomalous Transverse Distribution of Pions as a Signal for the Production of DCCs*, hep-ph/9604284.

[15] Fermilab test/experiment T-864 (J. Bjorken and C. Taylor, spokesmen), "MiniMax"; for further information consult **http://fnmine.fnal.gov**.

[16] V. Karmanov and A. Kudrjavtsev, *The Centauro Events as a Result of Induced Pion Emission*, ITEP-88 (1983).

[17] K. L. Kowalski and C. C. Taylor, *Disoriented Chiral Condensate: A White Paper for the Full Acceptance Detector*, CWRUTH-92-6 (1992) hep-ph/9211282.

[18] H. Georgi and A. Manohar, Nucl. Phys. **B234** (1984) 189.

[19] A. Kerman and R. Jackiw, Phys. Lett. **A71** (1979) 159; R. Jackiw, Int. J. Quantum Chem. **17** (1980) 41.

[20] R. Jackiw, Canad. Math. Soc. Conference Proceedings **9** (1988) 107.

[21] O. Eboli, R. Jackiw and S.-Y. Pi, Phys. Rev. **D37** (1988) 3557; R. Jackiw, Physica **A158** (1989) 269.

[22] R. Jackiw and G. Amelino-Camelia, hep-ph/9311324, in the proceedings of the *Third Workshop on Thermal Field Theories and Their Applications*, Banff (1993).

[23] For a review, see L. F. Abbott and S.-Y. Pi, *Inflationary Cosmology* (World Scientific, Singapore, 1986).

[24] R. Balian and M. Veneroni, Phys. Rev. Lett. **47** (1981) 1353 and 1765(E); Ann. Phys. **164** (1985) 334.

[25] S. Habib, Y. Kluger, E. Mottola, J.P. Paz, Phys. Rev. Lett. **76** (1996) 4660; F. Cooper, S. Habib, Y. Kluger and E. Mottola, in preparation.