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D-Brane Recoil Mislays Information

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Abstract

We discuss the scattering of a light closed-string state off a D brane, taking into account quantum recoil effects on the latter, which are described by a pair of logarithmic operators. The light-particle and D-brane subsystems may each be described by a world-sheet with an external source due to the interaction between them. This perturbs each subsystem away from criticality, which is compensated by dressing with a Liouville field whose zero mode we interpret as time. The resulting evolution equations for the D brane and the closed string are of Fokker-Planck and modified quantum Liouville type, respectively. The apparent entropy of each subsystem increases as a result of the interaction between them, which we interpret as the loss of information resulting from non-observation of the other entangled subsystem. We speculate on the possible implications of these results for the propagation of closed strings through a dilute gas of virtual D branes.

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1 Introduction.

A new era in the study of string theory and black holes has been opened up by Polchinski's realization [1] that soliton backgrounds in string theory can be described in a conformally-invariant way, in terms of world sheets with boundaries (thus incorporating open strings), on which Dirichlet boundary conditions for the collective target-space coordinates of the soliton are imposed. One of the most fruitful applications of this D-brane technology has been to black holes. In particular, many authors [2] have demonstrated that the counting of quantum states of D branes is equivalent to that of black hole states [3]. Thus, it is now generally agreed that black-hole entropy may be dissected into string states that are in principle distinguishable, as we conjectured some time ago [4, 5] on the basis of studies of two-dimensional stringy black holes [2]. One of the ways of distinguishing black-hole states is via generalized Aharonov-Bohm measurements [8], as has recently been discussed [9] in the context of D branes.

We have shown recently [10] how D branes emerge in a formulation of the string analogue of the field-theoretical path integral, based on a treatment of non-critical string theory [4] in which the role of the target time variable t is played by the Liouville field ϕ [11], treated as a local renormalization scale [4, 12]. We also pointed out in [10] several analogies between studies of D branes and two-dimensional string black holes [4]. Our interest in the latter was largely motivated by the black hole information problem and its possible implications for the effective quantum-mechanical description of light-particle degrees of freedom [4]. This problem has also been studied extensively in the context of quantum gravity. In particular, we have shown [13] that, at the one-loop level, the entropy of a scalar field in the presence of a four-dimensional black-hole background in conventional general relativity diverges logarithmically with respect to the short-distance cutoff of the model, taken to be the minimum distance from the black-hole horizon at which a 'brick-wall' boundary condition is imposed on the wave function of the scalar field. We have interpreted these quantum divergences as reflecting irreversible temporal evolution associated with entropy production as the horizon moves due to black-hole radiation.

The purpose of this paper is to initiate analogous studies in the context of D branes, with an examination of the quantum effects of D-brane recoil during the scattering of a light closed-string state, which requires a treatment of D-brane excitations. The remarkably simple construction of Polchinski [1] opens the way to a σ-model description of such D-brane excitations, in which the critical world-sheet string action is perturbed using appropriate boundary terms. We recall the form of the world-sheet boundary operators

[2] See also [6]: a similar conjecture was made later by Susskind [7] on different, heuristic grounds.

describing the excitation of a D brane (see [14] and references therein):

$$\mathcal{V}_D = \int_{\partial\Sigma} (y_i \partial_n X^i + u_i X^0 \partial_n X^i) \tag{1}$$

where n denotes the normal derivative on the boundary of the world sheet $\partial\Sigma$, which has at tree level the topology of a disk of size L, and the X^i , $i = 1, \ldots$ denote the collective excitations of the D brane, which satisfy Dirichlet boundary conditions on the world-sheet boundary:

$$X^i(\text{boundary}) = 0, \;\; i = 1, \ldots, \tag{2}$$

whilst X^0 is the target time variable which satisfies standard Neumann boundary conditions: $\partial_n X^0(\text{boundary}) = 0$. For simplicity, we will later be considering the case of a 0 brane, with the quantity u_i in (1) denoting its velocity, and y_i its initial position [3]. In this case, the operators (1) describe shifts and motion of the 0 brane, and so can be thought of as generating the action of the Poincaré group on the 0 brane, with the y_i parametrizing translations and the u_i parametrizing boosts. In the general D-brane case, these represent translations and boosts acting on the surface $\mathcal{S}$ of the D brane.

First steps were taken in refs. [16] towards the quantum theory of the scattering of string states off a D-brane background. This involves considering the motion of a closed string state towards the (initially fixed) surface $\mathcal{S}$, the latter viewed as a string soliton background, as shown in Fig. 1(a). In the general D-brane case, the surface $\mathcal{S}$ divides the target space-time into two regions. The closed string state is initially far outside the surface of the brane. At a certain moment, say $X^0 = 0$, the incoming closed string state finds itself lying partly outside and partly inside the D-brane surface. There are then two possibilities to be considered: it may be either absorbed (Fig. 1(b)) or rescattered (Fig. 1(c)). We seek below the appropriate conformal field theory description of the latter case.

In general, quantum scattering off the D brane excites an open string state on the surface of the D brane, which in the scattering case of Fig. 1(c) also emits another closed string state. The quantum excitation and emission processes are both described by closed-to-open string amplitudes, which are non-zero in a world-sheet theory with boundaries. The open-string states are excitations on the D-brane collective-coordinate surface. As was shown in ref. [17], such processes can be described in terms of data of the bulk theory. As we show in more detail below, tracing over such excitations results in a quantum modification of the Hawking-Bekenstein area law for the entropy, which has been shown to hold in tree-level treatments of D branes.

[3]We do not discuss here the more general case of curved world volumes, where the simple Dirichlet boundary conditions are known not to be conformally invariant [1, 15].

To discuss the evolution of entropy, it is essential to treat correctly the quantum recoil of the D brane during the scattering process, which we discuss in section 2. The treatment of recoil requires [16, 18, 19, 10, 20] an operator with non-zero matrix elements between different D- (in our case 0-)brane states. This can be achieved in the impulse approximation by introducing a Heaviside function factor, $\Theta(X^0)$, into the second operator in (1), which describes a 0 brane that starts moving at time $X^0 = 0$. The initial position of the 0 brane at $X^0 = 0$ is assumed to be given by the y_i. To determine the precise form of the recoil operator in our case, we observe that the leading quantum correction to the scattering of a closed string off a D brane is given by an annulus, as shown in Fig. 2(a). This is divergent in the limit where the annulus is pinched, as shown in Fig. 2(b) [16, 18, 19, 10]. As was done previously for closed strings in the context of two-dimensional black holes [4][4], we seek a deformation of the σ-model that reproduces at the tree (disc) level these infinities arising from quantum string loop corrections, which are infrared in target space. This determines the form of the quantum recoil, which is described by a pair of logarithmic operators, and one recovers momentum conservation as a consistency check of the approach[4]. In the D-brane case discussed here, the weakly-coupled string limit: $g_s \to 0$ corresponds correctly to a one-open-string-loop (annulus) analysis for a semi-classical (heavy) 0 brane (D particle), since the mass of the latter is $M_D \propto 1/g_s$ in natural string units.

We argue in section 3 that this treatment of the recoil of the D brane using logarithmic operators [23, 24, 18] entails a time evolution of the D-brane state that mirrors what we found previously in the cases of non-critical Liouville strings [4] and of a scalar field in the presence of a four-dimensional black hole [13]. The interaction between the incident light-string state and the D brane is described by a source term in the world-sheet σ model which induces an apparent departure from criticality. This is compensated by non-trivial dynamics of the Liouville field, whose zero mode is identified with target time t, i.e., the zero mode of X^0. As we demonstrate explicitly in section 4 using the D-brane equation of motion based on a world-sheet Wilson-Polchinski exact Renormalization-Group equation [25], the evolution of the D-brane state is of diffusive Fokker-Planck type, which necessarily entails entropy production. This loss of information in the D-brane sector, i.e., transition to a mixed quantum-mechanical state, is accompanied, because of quantum entanglement, by a corresponding transition to a mixed state in the truncated external light-particle system, as we discuss in section 5. The time evolution of the scattering

[4]We recall that in the similar analysis [4] of two-dimensional stringy black holes, the rôle of recoil was played by the back-reaction operator which described a change of state of the black-hole background. In the underlying conformal field-theoretical description of ref. [21], the corresponding operator was the world-sheet instanton anti-instanton vertex [22]. The instanton vertex induces (target-space infrared) infinities in correlation functions that are identical to those arising at the torus (closed-string) level [4]. In this context, the instanton coupling constant is proportional to the string coupling, and thus a one-string loop analysis corresponds to a dilute instanton gas approximation, as adopted in [4].

closed-string state does not have a simple Hamiltonian description, but provides a new explicit realization of the modified quantum Liouville equation of [26] within the general non-critical string framework of [4, 12]. As we discuss in section 6, if we speculate on the extension of these results to the propagation of a closed string state through "virtual D brane foam", the maximum possible rate of entropy growth that we find is similar in magnitude to that we estimated previously in the two-dimensional Liouville-string [4] and four-dimensional black-hole contexts [13].

2 Operator Treatment of D-Brane Recoil

We now review briefly the treatment [19] of the scattering of a closed string off a D brane, restricting ourselves for reasons of simplicity to the case of point-like 0 branes: the extension to higher-dimensional D branes is straightforward. For this purpose, we need to consider the annulus amplitude of Fig. 2(a), where the crosses denote closed-string vertex operators $V(k)$, which must be integrated over the propagating open string. This computation may be performed using the operator formalism, in which one evaluates $\mathrm{Tr}V(k_1)\Delta V(k_2)\Delta$, with $\Delta^{-1} \equiv L_0 - 1$, where L_0 is the Virasoro operator [5]. The part of this computation that is relevant for our purposes is that due to the world-sheet zero modes [19]. Writing $\Delta \equiv \int_0^1 dx x^{L_0 - 2}$, $L_0 = 2p^2 + N$, where N is the string level number, and picking out the $N = 0$ part, we find the following contribution to the annulus amplitude:

$$\mathcal{A} = \int dq < q|exp(-ik_1^0 X^0)x_1^{-2(p^0)^2} exp(-ik_2^0 X^0)x_2^{-2(p^0)^2}|q > \tag{3}$$

where the superscript 0 denotes target-time components, and q is the modular parameter of the annulus. The trace over the zero modes yields the generic form [19]

$$\mathcal{A} \ni \delta(k_1^0 + k_2^0)\sqrt{\frac{1}{\log(x_1)}} f(x_2, k_2^0) \tag{4}$$

which conserves momentum in the light-state sector, i.e., does not include any D-brane recoil momentum. The $\delta(k_1^0 + k_2^0)$ function arises from integrating over the zero modes of σ-model fields $X^0(z, \bar{z})$ in a standard fashion. There is also a corresponding $\delta(k_1 + k_2)$ over the space components, not written explicitly above, which comes from integrating over the world-sheet zero modes of the X^i fields. The amplitude (3, 4) is pathological, in the sense that it is divergent as $x_1 \to 0$ [19], and requires regularization. It is this regularization that induces recoil effects which modify the $\delta(k_1 + k_2)$ term in order to ensure momentum conservation in the presence of recoil.

[5] Alternatively, one may work in the closed-string channel, in which case one introduces a state $|B >$ into the closed-string Fock space, to impose the appropriate boundary conditions on the end of the closed-string world sheet: we return later to this approach.

The pathological behaviour in the limit $x_1 \to 0$ corresponds to the pinched-annulus configuration shown in Fig. 2(b):

$$\mathcal{A} \ni g_s \int_{x\sim 0} \frac{dx_1}{x_1\sqrt{8\pi \log(x_1)}} A_{disk}(k_1, k_2) \tag{5}$$

where $A_{disc}(k_1, k_2) =< V(k_1)V(k_2)V^iV^i >$ is the tree-level disc amplitude, with $\{V_i\}$ denoting a complete set of eigenstates of the $L - 0$ Virasoro operator, which includes D-brane Goldstone zero modes leading to dominant divergent contributions [16, 18, 19, 10]. To cancel this one-loop infrared divergence, one must add the following tree-level closed-string operator counterterm

$$\delta\mathcal{A} = \int d^2z \partial_\alpha (f(X^0)\partial^\alpha X^i) \tag{6}$$

which contributes on the boundary. Its form is determined by general properties of soliton backgrounds in string theory [18], and the function $f(X^0)$ in (6) is determined by requiring that the above operator reproduce the infinities of the annulus amplitude (5). We refer the reader to [19] for details. For our purposes, we restrict our attention to the final expression for the 'impulse' operator [19]:

$$V_{imp} \equiv \int d^2z\, \partial_\alpha([u_i X^0]\Theta(X^0)\partial_\alpha X^i) = \int d\tau\, u_i X^0 \Theta(X^0)\partial_n X^i \;; \qquad i = 1, \ldots 9. \tag{7}$$

The step-function operator in (7) needs to be defined properly, and we adopt the integral representation [20]:

$$\Theta_\epsilon(X^0) = -i \int_{-\infty}^{\infty} \frac{dq}{q - i\epsilon} e^{iqX^0} \quad , \qquad \epsilon \to 0^+ \tag{8}$$

where ϵ is an infrared regulator parameter[6]. In (7), the coupling u_i denotes a change in the velocity of the brane, which is determined by imposing overall conformal invariance of the annulus and disc amplitudes [16]. The cancellation of tree and annulus divergences requires

$$u_i = 8\sqrt{2}\pi g_s (k_1 + k_2)_i \tag{9}$$

which expresses momentum conservation. This interpretation of (9) is consistent with the fact that the soliton mass is proportional to $1/g_s$, confirming the interpretation of u_i as the D-brane velocity.

[6] The sign of ϵ is given a physical origin in the next section.

Analysis of the operator product of the operator (7) reveals that it is a *logarithmic* operator [23]

$$V_{imp}(x)V_{imp}(0) \sim \log x/|x|^2 \tag{10}$$

and can be decomposed into one of a *pair* of logarithmic operators C and D [23, 24], namely D in the notation of [20]. To identify the pair let us concentrate, for convenience, on the X^0-dependent parts. The D_ϵ recoil operator is identified as [20]

$$D_\epsilon \equiv X^0 \Theta(X^0) \tag{11}$$

The C_ϵ operator can be found by studying the operator product of D_ϵ with the stress-energy tensor:

$$T(w)D_\epsilon(z) = \frac{-\epsilon^2/2}{(w-z)^2}D_\epsilon + \frac{1}{(w-z)^2}\epsilon\Theta(X^0) \tag{12}$$

This enables us to identify the C_ϵ operator, using the general properties of such logarithmic pairs [23, 24]:

$$C_\epsilon = \epsilon\Theta_\epsilon(X^0) \tag{13}$$

Thus, the correct D-brane recoil operator is [20]:

$$\mathcal{V}_{rec} = \int d\tau \ [y_i C_\epsilon(X^0)\partial_n X^i + u_i D_\epsilon(X^0)\partial_n X^i] \tag{14}$$

The analysis of ref. [20] showed that the degenerate operators C_ϵ and D_ϵ have conformal dimension $\Delta = -\frac{\epsilon^2}{2}$, which is negative and vanishing in the limit $\epsilon \to 0^+$. This means that the impulse operators V_{rec} (14) are *relevant* in a renormalization-group sense for any non-zero ϵ, since, due to their $\partial_n X^i$ parts, their anomalous scaling dimension is $\Delta - 1 + 1 = -\frac{\epsilon^2}{2}$. This is intrinsic to the nature of logarithmic operators, which appear on the border line between conformal field theories and general renormalizable two-dimensional field theories [18],[10],[27]. These relevant deformations in the recoil problem lead to a *change* in the background 0-brane state [18, 10], with the physical consequences discussed in the next section.

Explicit expressions for the one- and two-point functions of the operators $D_\epsilon(X^0)$ and $C_\epsilon(X^0)$ appearing in (14) were derived in [20]. It is sufficient for our purposes here to quote the results for the two-point functions:

$$\begin{aligned}
&< C_\epsilon(z)C_\epsilon(0) > \sim -\epsilon^2\sqrt{\frac{\pi}{\alpha}}\int_{-\infty}^{\infty}\frac{dq}{(q^2+\epsilon^2)}e^{-2\eta q^2 \log|z/a|^2} \\
&= -\epsilon^2\pi\sqrt{\frac{\pi}{\epsilon^2\alpha}}e^{2\eta\epsilon^2\log|z/a|^2}\left(1 - \mathrm{erf}\left(\epsilon\sqrt{2\eta\log|z/a|^2}\right)\right) \\
&\stackrel{\epsilon\to 0}{\sim} 0 + O(\epsilon^2)
\end{aligned} \tag{15}$$

$$
\begin{aligned}
< C_\epsilon(z) D_\epsilon(0) > &\sim -\frac{\epsilon}{2}\sqrt{\frac{\pi}{\alpha}}\frac{\partial}{\partial\epsilon}\int_{-\infty}^{\infty}\frac{dq}{q^2+\epsilon^2}e^{-2\eta q^2 \log|z/a|^2} \\
&= \frac{\pi}{2}\sqrt{\frac{\pi}{\epsilon^2\alpha}}\Big\{e^{2\eta\epsilon^2\log|z/a|^2}\left(1-4\eta\epsilon^2\log|z/a|^2\right)\left(1-\mathrm{erf}\left(\sqrt{2\eta\epsilon^2\log|z/a|^2}\right)\right) \\
&\qquad +2\sqrt{\frac{2\eta\epsilon^2\log|z/a|^2}{\pi}}\Big\} \\
&\stackrel{\epsilon\to 0}{\simeq} \frac{\pi}{2}\sqrt{\frac{\pi}{\epsilon^2\alpha}}\left(1-2\eta\epsilon^2\log|z/a|^2\right) \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
< D_\epsilon(z) D_\epsilon(0) > &= \frac{1}{\epsilon^2} < C_\epsilon(z) D_\epsilon(0) > \\
&\stackrel{\epsilon\to 0}{\simeq} \frac{\pi}{2}\sqrt{\frac{\pi}{\epsilon^2\alpha}}\left(\frac{1}{\epsilon^2}-2\eta\log|z/a|^2\right) \qquad (17)
\end{aligned}
$$

We have denoted by η is the signature of the target metric:

$$
\mathrm{Lim}_{z\to 0} < X^0(z) X^0(0) > \simeq \eta \log|a/L|^2 \qquad (18)
$$

where a is a conventional ultraviolet regulator parameter on the world sheet and L is an infrared regulator parameter which characterizes the size of the world-sheet annulus. We see explicitly that in the limit

$$
\epsilon \to 0, \qquad \epsilon^2 \log|L/a|^2 \sim O(1) \qquad (19)
$$

we obtain the canonical two-point correlation functions of a pair of logarithmic operators [23], with one exception - the singular $1/\epsilon^2$ term in $< DD >$ [7]. The relevance of this singular term will become apparent in the next section, when we discuss the equation of motion of the semiclassical D-brane wave function [25, 28, 10].

3 Renormalization-Group Rescaling and Time

We now discuss in more detail renormalization-group rescaling on the world sheet, with the aim of establishing a connection with the target time variable. The value of the numerical constant in (19) is a free parameter which we may choose it at will, with the differences between different choices being reabsorbed in the redefinition of nonleading $\log z$ terms. It was argued in ref. [20] that the most natural choice is (19), i.e.

$$
\frac{1}{\epsilon^2} = 2\eta \log|L/a|^2 \qquad (20)
$$

[7] Note that the singularity structure at small ϵ is the same in connected correlation functions [20].

which leads to the following singularity structure:

$$
\begin{aligned}
< C_\epsilon(z) C_\epsilon(0) > &\sim 0 + O[\epsilon^2] \\
< C_\epsilon(z) D_\epsilon(0) > &\sim 1 \\
< D_\epsilon(z) D_\epsilon(0) > &\sim -2\eta \log |z/L|^2
\end{aligned}
\tag{21}
$$

up to an overall normalization factor.

We now consider a finite-size scale transformation

$$
L \to L' = L e^t \tag{22}
$$

of the only type which makes physical sense for the open string world-sheet. The relation (20) between ϵ and L entails the following transformation of ϵ [8]:

$$
\epsilon^2 \to \epsilon'^2 = \frac{\epsilon^2}{1 + 4\eta\epsilon^2 t} \tag{23}
$$

We deduce from the scale dependences of the correlation functions (21) that the corresponding transformations of C_ϵ and D_ϵ are:

$$
\begin{aligned}
D_\epsilon &\to D_{\epsilon'} = D_\epsilon - t C_\epsilon \\
C_\epsilon &\to C_{\epsilon'} = C_\epsilon
\end{aligned}
\tag{24}
$$

We emphasize that this transformation law is *unambiguous*, being in particular *independent* of the signature parameter η.

The corresponding transformation laws for the couplings y_i and u_i, which are conjugate to D_ϵ and C_ϵ in the recoil expression (14), are

$$
u_i \to u_i \quad , \quad y_i \to y_i + u_i t \tag{25}
$$

This is consistent with the interpretations of u_i as the velocity after the scattering process and y_i as the spatial collective coordinates of the brane, if and only if the scale-change parameter t is interpreted as the *target Minkowski time*[9], and is therefore to be identified with the zero mode of X^0. In this analysis we have assumed that the velocity u_i is small, as is appropriate in the weak coupling régime studied here (9). The D-brane σ-model formalism is completely relativistic, and we anticipate that a complete treatment beyond the one-loop order discussed here will incorporate correctly all relativistic effects, including Lorentz factors wherever appropriate.

[8]Note that, if ϵ is infinitesimally small, so also is ϵ' for any finite t.

[9]It is apparent from the scale transformation (24) that even if one started with the D_ϵ operator alone, a C_ϵ operator would have been induced, as required if the pair (14) is to describe correctly D-brane recoil, as in (25).

This observation that a world-sheet scale transformation leads to the target-time evolution of the D brane, once recoil is taken into account, is in the same spirit as was proposed previously in the context of Liouville strings [4, 12]. There also an identification was made between target time and a world-sheet renormalization scale, provided in that case by the zero mode of the Liouville field.

We now provide technical support for a similar identification in the present model of the zero modes of the target time variable X^0 (1) and the Liouville scale field ϕ [10], which is defined by $\gamma_{\alpha\beta} = e^{\phi}\delta_{\alpha\beta}$ for the disc topology, where $\gamma_{\alpha\beta}$ is the world-sheet metric. We first rewrite the boundary recoil operators (14) as world-sheet bulk operators

$$V_i = \int_{\Sigma} d^2z \partial_\alpha (y_i(X^0)\partial^\alpha X^i) \tag{26}$$

Since they have non-vanishing anomalous dimension $-\epsilon^2/2$, the deformed theory is non-critical. Local scale (conformal) invariance may be restored in the usual way by dressing (26) with the Liouville field $\phi(z,\bar{z})$ [29]:

$$V_i^L \equiv \int_{\Sigma} d^2z e^{\alpha_i \phi(z,\bar{z})} \partial_\alpha (y_i(X^0)\partial^\alpha X^i) \tag{27}$$

where $\alpha_i(\alpha_i + Q) = -2\delta_i$: Q is the central-charge deficit in the unperturbed theory and $\delta_i = -\epsilon^2/2$ the anomalous dimension. In our case, $Q = 0$, consistent with fixed Dirichlet boundary conditions [15], so that $\alpha_i = \epsilon$. Partial integration of (27) then leads to

$$V_i^L = \epsilon \int_{\Sigma} d^2z \int \partial_\alpha \phi \partial^\alpha X^i y_i(X^0) + \int_{\partial\Sigma} d\tau e^{\epsilon\phi(\tau)} y_i(X^0)\partial_n X^i \tag{28}$$

We now use the fact (25) that $y(X^0) = u_i X^0 \Theta(X^0)$, and the integral representation (8), to observe that the second term in (28) contains a factor $e^{i(qX^0 - i\epsilon\phi)}$. Identifying $\phi = X^0$, we may write

$$\Theta_\epsilon(X^0)e^{\epsilon X^0} = \frac{1}{i}\int_{-\infty}^{\infty} \frac{dq}{q - i\epsilon} e^{i(q-i\epsilon)\ X^0} = \frac{1}{i}\int_{-\infty - i\epsilon}^{\infty - i\epsilon} \frac{d\omega}{\omega} e^{i\omega\ X^0} \tag{29}$$

which implies that, as far as the regulated $\Theta_\epsilon(X^0)$ is concerned, the effect of the Liouville dressing and the identification of X^0 with ϕ is equivalent to setting $\epsilon \to 0$ in (8), corresponding to the restoration of conformal invariance. Thus the promotion of the local renormalization scale t to a Liouville field and its identification with the time X^0 are consistent, if the following target-space metric G_{MN} derived from (28) is considered:

$$G_{00} = 1 \qquad ; G_{ij} = \delta_{ij} \qquad G_{0i} = G_{i0} = \epsilon y_i(X^0) = \epsilon u_i X^0 \Theta(X^0) \tag{30}$$

[10] In our interpretation [4], ϕ is a local renormalization scale on the world sheet, and its zero mode is identified with $t = \log|L/a|$.

where 0 indices denote 'time' components, and latin indices denote spatial components. In our discussion so far, we have ignored the effects of the $C - D$ mixing discussed previously (24). Indeed, due to this mixing, the effects of the C_ϵ recoil operator cannot be ignored [20]. Such effects contribute a term $\epsilon y_i \Theta(X^0)$ in the world line of the 0 brane $y_i(X^0)$, with y_i the initial location of the brane. According to the discussion in ref. [20] and in the Appendix, such terms express quantum fluctuations in the location of the brane due to the excitation of *stringy* modes. Thus (30) becomes

$$G_{0i} = G_{i0} = \epsilon(\epsilon y_i + u_i X^0)\Theta(X^0) \tag{31}$$

The off-diagonal metric component G_{0i} appears to have a discontinuity at $X^0 = 0$. As we shall argue below, the ϵy_i-dependent parts of (31) lead to a curvature-singularity at $X^0 = 0$ which will be *crucial* for the information-loss problem.

The metric (31) is consistent with conformal invariance of the σ model, as can be seen by considering the derivative with respect to the (local scale) X^0:

$$\frac{dG_{0i}}{dX^0} = \epsilon[\partial_0 y_i(X^0)] = \epsilon[u_i \Theta(X^0) + (\epsilon y_i + u_i X^0)\delta(X^0)] \tag{32}$$

The right-hand side of (32) may be identified as a generalized coordinate transformation in target space of the form $\nabla_{(0}\chi_{i)}(X^0)$: $\chi_i \sim \epsilon(\epsilon y_i + u_i X^0)\Theta(X^0)$, where the parenthesized index is symmetrized. Equation (32) expresses the condition for conformal (Weyl) invariance of the respective σ model, in the usual way, and demonstrates the consistency of the identification of X^0 with the Liouville scale[11]. We note that (32) contains a singularity of the δ-function type, corresponding to the discontinuity in the metric (30). There is a corresponding singularity in the Riemann curvature scalar at $X^0 = 0$:

$$\begin{aligned} &R \ni -\epsilon^2 y^i(X^0)\partial_0 y^i(X^0) + \ldots \ni -\epsilon^2 y^i(X^0) y^i(X^0)\delta(X^0) + \ldots = \\ &-\epsilon^2(\epsilon y_i + u^i X_0)^2 \delta(X^0)\Theta(X^0) + \ldots \end{aligned} \tag{33}$$

where the temporal coordinate is X^0, and we can regard $\epsilon y_i + u_i X^0 \equiv \xi_i$ as a Galilean-transformed spatial coordinate, as discussed in more detail in section 5, where we construct the σ model that describes the low-energy matter degrees of freedom. We stress once more that, as discussed in ref. [20] and in the Appendix, the singular terms in (33) are linked to the C_ϵ (quantum) recoil operator [20], and express quantum fluctuations in the location of the recoiling D-brane due to the excitation of the virtual *stringy* effects discussed in section 4.

[11] A similar situation occurred in the $(1+1)$-dimensional black hole case [4].

After identifying X^0 with a local renormalization scale (zero mode of the Liouville field), we find that D-brane recoil *induces a back reaction on the space-time geometry which results in a temporal singularity in the curvature scalar*. In our view, this lies at the core of the information-loss problem associated with virtual D-brane excitations induced by the recoil at time X^0, as we discuss in more detail in the next sections.

4 Recoil and Information Loss

We now discuss the implications of the above recoil analysis for the time evolution of the D-brane state. We start from the formalism of the Wilson approach to the renormalization group, as refined by Polchinski and applied previously in the a σ-model approach to D branes [25, 28, 10]. We denote by $\Psi[X^i]$ the partition function of a D brane with collective coordinates X^i, computed at the disc and annulus (tree and one-loop) level, which plays the rôle of the semi-classical wavefunction of the D brane [28, 10]. This obeys a renormalization-group equation, which expresses its scaling properties with respect to a generic short-distance cutoff Λ on the world-sheet [28]:

$$\Lambda\frac{\partial}{\partial\Lambda}\Psi[X]=\frac{1}{2}\int\int_{\partial\Sigma}d\tau_1 d\tau_2\Lambda\frac{\partial}{\partial\Lambda}G(\tau_1,\tau_2)\frac{\partial^2}{\partial X^i(\tau_1)\partial X^j(\tau_2)}\Psi[X]+... \tag{34}$$

where $\partial\Sigma$ denotes the world-sheet boundary and the two-point Green function $G(\tau_1,\tau_2)=< X^i(\tau_1)\ X^j\ (\tau_2)>$. The dots denote terms of the form $(\partial\Psi[X]/\partial X)^2$, which represent interactions among D branes, and are ignored to the order of accuracy of the present work. In our case, we identify Λ with L/a and $t=\log|L/a|$ (22), so that $\Lambda\partial/\partial\Lambda=\partial/\partial t$ and the renormalization-group equation [28] may be recast in the form

$$\frac{\partial}{\partial t}\Psi[X]=\frac{1}{2}\int\int_{\partial\Sigma}d\tau_1 d\tau_2\frac{\partial}{\partial t}G(\tau_1,\tau_2)\frac{\partial^2}{\partial X^i(\tau_1)\partial X^j(\tau_2)}\Psi[X]+... \tag{35}$$

which has the form of a diffusion equation. In order to evaluate the rate of change of the entropy, we interpret $|\Psi[X]|^2$ in the natural way as the probability density distribution for the D brane to find itself in the position $X^i(t)$ at time t. Using this interpretation, (35) may be used to derive a Fokker-Planck equation for the probability distribution $\mathcal{P}\equiv|\Psi[X]|^2$:

$$\partial_t\mathcal{P}=D\ \nabla^2_{X^i}\mathcal{P}+\dots \tag{36}$$

as we discuss in detail below. In (36), D denotes a diffusion coefficient, the dots denote terms representing interactions among the D branes, which we will ignore, apart from a few speculative comments in the last section, and the X^i denote the (world-sheet-independent) zero modes of the collective coordinates of the brane.

In order to derive (36) and demonstrate that it is non-trivial, we show that both of the factors in the integrand of (35) are non-zero, using the analysis of the pair C_ϵ, D_ϵ discussed in the previous section. We start with $G(\tau_1, \tau_2)$, which depends on t only via its divergent part, which is due to the (world-sheet-independent) zero modes on the annulus. These divergent parts can readily be computed in the σ-model approach by noting that the collective coordinates of the D brane depend on the cut-off scale, as a result of the shift (25) due to C-D operator mixing (24). Thus the world-sheet-coordinate-independent logarithmic divergences in $G(\tau_1, \tau_2)$ can be studied [10] by replacing $X^i(\tau_1)$ by the boosted coordinate $u_i X^0(\tau_1)\Theta(X^0)$, i.e. the D_ϵ recoil operator itself. The calculation then reduces to the evaluation of the free two-point function of this operator, which was given in ref. [20], and quoted above (17):

$$G(\tau_1, \tau_2)|_{divergence} \sim u_i^2 < D_\epsilon(x) D_\epsilon(x') >_{divergence} = u_i^2 log|L/a|^2 \tag{37}$$

This leads to a parametrization of $G(\tau_1, \tau_2)$ by an effective diffusion coefficient D:

$$G(\tau_1, \tau_2) \simeq 2Dt : \; t \simeq log|L/a| \tag{38}$$

with

$$D = u_i u^i \tag{39}$$

which we substitute into (35) to obtain

$$\partial_t \Psi[X] = u_i^2 < \int \int_{\partial\Sigma} d\tau_1 d\tau_2 \frac{\partial^2}{\partial X^i(\tau_1) \partial X^j(\tau_2)} \Psi[X] + \ldots > \tag{40}$$

We must now verify that the integrand in (40) is also non-vanishing.

To see this, we first use the form (14) of the D-brane recoil operator to re-express this integrand as

$$\mathcal{A}_2 \equiv < \partial_n X^i(\tau_1) \partial_{n'} X^j(\tau_2) \delta_{ij} >_{annulus} \tag{41}$$

in the absence of any closed-string tachyon perturbation, where the expectation value is taken in the unperturbed D-brane σ model, without C, D deformations. The two-point function on the annulus in (41) is that of the Goldstone-mode operators $\int_{\partial\Sigma} d\tau \partial_n X^i(\tau)$ and is proportional to the shift in the mass of the Goldstone mode due to loop corrections [16]. However, since the Goldstone mode expresses the spontaneous breaking of an exact global symmetry in target space, namely translation invariance [18, 10], its mass should be strictly zero. Indeed, the contour deformation technique applied in [16] confirms this, since the only contribution to this two-point function comes from the tachyonic pathologies of the bosonic string, which are absent in supersymmetric theories. Thus, in the absence of matter deformations, the Wilson equation (35) has only trivial content.

However, equations (35) and (36) are non-trivial when matter deformations are included. To see this, we employ the effective tree-level approach to recoil, where one deforms the σ model by the world-sheet boundary term (14), and performs a perturbative expansion in powers of the tachyon deformations. Specifically, at first order we replace (41) by the three-point amplitude

$$\mathcal{A}_3 \equiv < \int_{\partial\Sigma} d\tau_1 \int_{\partial\Sigma} d\tau_2 \partial_n X^i(\tau_1)\partial_{n'} X^i(\tau_2) \int_\Sigma d^2\sigma T(k) e^{ik_M X^M(\sigma)} > \tag{42}$$

evaluated using a free σ-model action on a world sheet with boundaries, which is the leading-order contribution of matter deformations in the weak-tachyon approximation to the right-hand-side of (35). The non-zero contributions that lead to diffusion arise when the bulk tachyon deformation approaches the boundary of the world sheet. In such a case, we expect the following boundary operator product expansion [30, 17] to hold:

$$\Phi(z,\bar{z},y) \sim \sum_i (2y)^{\Delta_i - x_\Phi} C^a_{\Phi\psi_i}\psi_i(y) \tag{43}$$

provided that the set of boundary conditions a does not break conformal symmetry. In (43), $z, \bar{z}$ denote world-sheet (bulk) coordinates, the y are world-sheet boundary coordinates, $\Phi(z,\bar{z}) = e^{ik_M X^M(z,\bar{z})}$ is the bulk deformation by the closed-string tachyon, x_ϕ is its scaling dimension, and $\{\psi_i(y)\}$ is a (complete) set of boundary operators with scaling dimensions Δ_i. In theories where logarithmic operators are absent, it was shown in [17] that the open-to-closed-string O.P.E. coefficients $C^a_{\Phi\psi}$ could be re-expressed in terms of bulk data, in particular the coefficients C^i_{jk} that arise in the ordinary bulk O.P.E.. An extension of this analysis to general logarithmic operators is an important technical issue that falls beyond the scope of the present work [31], but we are able to establish that there is a non-vanishing coefficient in our case.

The boundary operator ψ of interest to us is the D operator discussed in the previous sections. To see that the $C^a_{\Phi D}$ O.P.E. coefficient is indeed non-zero, we consider the O.P.E. between two bulk operators

$$\Phi(z,\bar{z})\Phi(0,0) \sim C^I_{\Phi\Phi}V_I \tag{44}$$

where $\{V_I\}$ denotes a complete set of bulk (closed-string) operators. In the D-brane case, these include the bulk version of the D-brane recoil operator (7). The one-point function on the disc of this operator is divergent near the boundary of the disk [19] [12]. Parametrizing the world-sheet distance from the boundary by $i\zeta/2$, we have

$$< X^0(i\zeta/2)\Theta(X^0(i\zeta/2)) > \sim \int \frac{dq}{2\pi}\frac{1}{q^2}(\zeta)^{-q^2} \sim \sqrt{-\log\zeta} \tag{45}$$

[12] This is cancelled by divergences in the string amplitude on the annulus, as discussed at the beginning of section 2.

Since ζ is a short-distance scale on the world sheet, we identify $-\log\zeta \sim \log|L/a|$, and then the dominant term in the D-brane two-point function of (44) is

$$< \Phi(z,\bar{z})\Phi(0,0) >_{z\sim 0}\sim C^{D}_{\Phi\Phi}\sqrt{\log|L/a|} \tag{46}$$

Consider now the case where both Φ's in (44) lie very close to the boundary of the disc, which for convenience we parametrize by the real axis of the upper half of the complex plane. In this case, for each of the Φ one has the boundary O.P.E. expansion (43). The separation of the two operators projected on the boundary will also be small, so one may use the O.P.E. on the boundary for the boundary excitations. We see from (17) that the dominant (divergent) contributions in that expansion come from the boundary D operator, whose two-point function is divergent as $\log|L/a|$:

$$< \Phi(z,\bar{z})\Phi(0,0) >_{z\sim 0}\sim (C^{a}_{\Phi D})^2\log|L/a| \tag{47}$$

Equating the divergent contributions of (47) and (46), one obtains the relation

$$(C^{a}_{\Phi D})^2 \sim \sqrt{\frac{1}{\log|L/a|}}\; C^{D}_{\Phi\Phi} \tag{48}$$

The O.P.E. coefficient $C^{D}_{\Phi\Phi}$ on the right-hand side of (48) encodes the amplitude for an in-state tachyon to scatter off the D-brane into an out-state tachyon, including recoil, which is known to be non-zero [16, 18, 20, 10]. Equation (48) implies that there is a non-trivial 'trapping' amplitude, $C^{a}_{\Phi D}$, which therefore contributes to the diffusion coefficient in (36), and hence to the information loss via quantum recoil operators. The above calculation is incomplete, in that it was based on an analysis of the leading divergences of the recoil operators of the D brane [20], and it remains to calcuate the proportionality coefficients in (48), as well as the subleading-divergence contributions of the open-to-closed O.P.E. in a theory with logarithmic operators. This programme falls beyond the scope of the present article [31], but is not needed for our purpose: we have established that both terms in the integrand of (35) are non-vanishing, and hence that the Fokker-Planck equation (36) is non-trivial.

The diffusion interpretation of (35, 36) is contingent on the absence of a factor i between the two sides of the equations, and one might wonder whether these equations are correct as written, or whether they should be Wick rotated. We would like to emphasize that the translation laws (24, 25) were derived independently of the assumed signature η, and hence that there is no justification for any Wick rotation of the time variable in (35, 36) [13].

[13] Indeed, when $\eta = -1$, in which case X^0 has Minkowskian signature, there is a factor of i on the right-hand-side of (41), as a result of the purely imaginary normalization factor $\sqrt{1/\epsilon^2\eta Log|L/a|^2}$ of the two-point function of the operator D (17). Such factors are, therefore, crucial. Their presence implies that, in the case $\eta = -1$, the target-space physical time should be identified with it, where t denotes the change (22) in the world-sheet scale. This demonstrates that, with respect to the physical time, the equation (41) retains its diffusion form, whichever the signature η.

The appearance of a diffusion equation can be traced back to the presence in the recoil operators of a factor $\Theta(X^0)$ [19, 20]. In general, one should also take into account the possibility that the D brane is moving initially, in which case one should use the boundary operators (1) *as well*. In this case, there are trivial logarithmic operators in the two-point function $G(\tau_1, \tau_2)$ of (34), coming from the free fields X^0 in the expression for the collective coordinates $y(X^0)_i = y_i + v_i X^0$ of a D brane moving with initial velocity v_i. In the presence of both sets of operators, the right-hand side of (34) becomes *complex* for Minkowskian-signature X^0, as a result of the imaginary normalization factors appearing in the two-point functions of the $\Theta(X^0)$ operators, mentioned above. This leads to an equation of motion which contains a standard Hamiltonian term à la Schrödinger, corresponding to a freely-moving brane with finite velocity v^i, as well as a diffusion term corresponding to the virtual excitations that appear after the time $X^0 = 0$. We note in passing that the recoil treatment of [16] used only eternally-moving D branes, in contrast to the treatments in [19, 20], in which the membrane receives an impulse at the time X^0. In this article, our interest is focused on the diffusion aspect of the recoil, and we shall not discuss further the Hamiltonian terms in the wave equation of the moving brane.

In view of the important implications of (36), we now provide an alternative derivation that does not use the above identification of the σ-model partition function with the semi-classical wavefunction of the D brane. This second derivation is based on the equivalence of the D-brane σ model with a background gauge-field open string. In this formulation, the boosted collective coordinates X^i are viewed as 'couplings' of a non-critical σ-model theory. The inclusion of higher genera necessitates a summation over pinched handles (cf, the prototypes in Fig. 2(b)), which entails a natural quantization of these couplings [10], in close analogy to the wormhole calculus familiar from higher-dimensional quantum gravity. As discussed earlier, the spontaneous breaking of translational invariance by the collective coordinates of the D brane, which is a general feature of any string theory involving solitonic structures [18], is accompanied by the appearance of Goldstone zero modes. As discussed previously in the literature [16, 18, 10], the propagation of these zero modes along thin tubes yields logarithmic infinities in loop amplitudes, which manifest themselves in bilocal structures on the world sheet:

$$\mathcal{B}_{ii} = \int d^2 z V_i(z) \int d^2 w V_i(w) \frac{1}{L_0 + \overline{L}_0 - 2} \tag{49}$$

where the last factor represents the string propagator Δ_S on a degenerate handle, with the symbols $L_0, \overline{L}_0$ denoting Virasoro generators as usual. Inserting a complete set $\mathcal{E}_\alpha$ of intermediate string states, we can rewrite (49) as an integral over the parameter $q \equiv e^{2\pi i\tau}$, where τ is the complex modular parameter characterizing the world-sheet tube. The string propagator over the world-sheet tube then reads

$$\Delta_S = \sum_\alpha \int dq d\overline{q} \frac{1}{q^{1-h_\alpha}\overline{q}^{1-\overline{h}_\alpha}} \{\mathcal{E}_\alpha(z_1) \otimes (ghosts) \otimes \mathcal{E}_\alpha(z_2)\}_{\Sigma_1 \oplus \Sigma_2} \tag{50}$$

where $h_\alpha, \overline{h}_\alpha$ are the conformal dimensions of the states $\mathcal{E}_\alpha$. The sum in (50) is over all states propagating along the long, thin tube connecting Σ_1 and Σ_2, which are both equal to the sphere in the simplest case of a degenerate torus. As indicated in (50), the sum over states must include the ghosts, whose central charge cancels that of the world-sheet matter theory in any critical string model. States with $h_\alpha = \overline{h}_\alpha = 0$ may cause extra logarithmic divergences in (50) which are not included in the familiar β-function analysis on Σ [16]. This is because such states make contributions to the integral of the form $\int dq d\overline{q}/q\overline{q}$ in the limit $q \to 0$, which represents a long, thin tube. We assume that such states are discrete in the space of states, i.e., they are separated from other states by a gap. In this case, there are factorizable logarithmic divergences in (50) which depend on the background surfaces $\Sigma_{1,2}$, e.g., the sphere in the case of the degenerating torus.

The bilocal term (49) can be cast in the form of a local contribution to the world-sheet action, if one employs the trick, familiar from the wormhole calculus, of rewriting it as a Gaussian integral [32, 16]:

$$e^{\mathcal{B}_{ii}} \propto \int d\alpha e^{-\alpha_i^2 + \alpha_i \int V_i} \tag{51}$$

where the α^i are to be viewed as quantum coupling constants/fields of the world-sheet σ model. In fact, the factor $e^{-(\alpha_i)^2}$ must be replaced [32, 33, 10] by a more general Gaussian distribution of width Γ:

$$F(\alpha_i) = \frac{1}{\sqrt{2\pi}\Gamma} e^{-\frac{1}{2\Gamma^2}(\alpha_i)^2} \tag{52}$$

The extra logarithmic divergences associated with degenerate handles that we mentioned above, which are $\propto \ln\delta$ where $q \sim \delta \sim 0$, have the effect of causing the width parameter Γ also to depend logarithmically on the cutoff scale δ:

$$\Gamma \sim \ln\delta \tag{53}$$

Using the Fischler-Susskind mechanism [34] for cancelling string loop divergences, one should associate the divergence $\propto \log\delta$ with a world-sheet cut-off scale $\log\epsilon$. Upon identification of the cut-off scale with the Liouville field, and of the latter with the target-time variable [4, 12], we infer that the distributions of the couplings α_i become time-dependent [4].

In the particular case of D branes, the couplings which fluctuate are the boosted collective coordinates of the D brane, including the recoil velocities (or momenta). Redefining the 'quantum' coordinates by $X^i \equiv y^i + \alpha_i\sqrt{\log\delta}$, the above analysis shows that the quantized collective coordinates have a Gaussian 'white-noise' distribution of the generic form:

$$F(\delta X^i_q) = [\frac{1}{2\pi < \delta X^I_q >^2}]^{1/2} \exp(-\frac{(\delta X^(_q))^2}{2 < (\delta X^i_q)^2 >}) \tag{54}$$

where the δX_q^i denote quantum fluctuations of the time-dependent collective coordinates. Due to the above-mentioned divergent structure associated with zero modes, the width of this distribution has the form:

$$< (\delta X_q^i)^2 >\sim \; D \; \delta t \qquad t = D^{-1}\log\delta \tag{55}$$

where δt denotes an infinitesimal fluctuation in target time, and the relative normalization $\log\delta = D \; t$ of the diffusion coefficient is dictated by analogy with the Liouville theory [11, 4]. This structure is very similar to that of an inflationary scenario for string cosmology [35], which is understood in the context of Liouville strings [4], to which this theory is equivalent.

We are now in a position complete our second derivation of a Fokker-Planck equation for the probability distribution $\mathcal{P}(X^i, t)$ in the 'space' X^i [36, 35], using the renormalization-group equation for the coupling $X^i(t)$

$$\partial_t X^i(t) = u^i \tag{56}$$

which we interpret as an equation of motion for the classical coupling $X_c^i(t)$. We then decompose the value $X_c(t+\delta t)^i$ at a later time t as

$$X_c(t+\delta t)^i = X_c(t)^i + \partial_t X^i(t)\delta t + \delta X_q^i(t) \tag{57}$$

where the δX_q^i denote the quantum fluctuations that satisfy the white noise distribution (55). The probability distribution $\mathcal{P}$ may then be written as [36, 35]:

$$\mathcal{P}(X^i, t) = \int dX_c^i F(\delta X_q^i) P(X_c, t) \tag{58}$$

from which (36) is derived in a straightforward manner, as discussed in [35] [14]. This provides an independent *a posteriori* justification for identifying the D-brane σ-model partition function with the semi-classical wavefunction in target space.

Having derived a Fokker-Planck equation for the probability distribution, we now consider the rate of change of the entropy, defined as:

$$S = -k_B \int [DX_i]\mathcal{P}(X,t)\log\mathcal{P}(X,t) \tag{60}$$

[14] We note in passing that *if* the Fokker-Planck equation holds also in the presence of D-brane interactions, *then* target-space diffeomorphism invariance restricts [37] the form of the equation for the temporal evolution of the D-brane wave function to a non-linear Schrödinger equation:

$$i\hbar\partial_t \Psi = \mathcal{H}\Psi + i\hbar D \nabla_i^2 \Psi + iD\hbar|\nabla_i \log\Psi|^2\Psi \tag{59}$$

This equation might be useful for the quantization of interacting D-branes.

where $[DX]$ denotes an appropriate functional integral in a σ-model sense. It is straightforward using (34) to derive [35]

$$\partial_t S = \int [DX] u_j u^j \frac{|\nabla_{X^i} \mathcal{P}|^2}{\mathcal{P}} + \dots \tag{61}$$

where we work in a system of units such that $k_B = 1$, and the ... indicate terms due to interacting D branes, that have not been discussed here. The rate of change (61) is not only non-vanishing, but obviously monotonically increasing.

The rate of evolution (61) of the D-brane entropy may be estimated for asymptotically-large time t, using the analysis of the logarithmic conformal field theory describing the recoil process that we developed in sections 2 and 3. In a semi-classical treatment, we may represent $\mathcal{P} = |\Psi|^2$, where the σ-model partition function $\Psi = e^{-F}$, with F denoting the effective D-brane action:

$$\partial_t S = \int [DX] u_i u^i \frac{1}{\mathcal{P}} |\nabla_{X^i} \mathcal{P}|^2 = \int [DX] |\Psi|^2 (u_i u^i) |\nabla_{X^i} F|^2 \tag{62}$$

Then, viewing the collective coordinates X^i as couplings of the D-brane σ model, using the t dependence of X^i expressed in (56), and noting that no other couplings of the D-brane σ model depend on t to this order, we may write

$$\partial_t S = \int [DX] |\Psi[X]|^2 |\partial_t F|^2 \tag{63}$$

Using now the renormalizability of the effective action F, and the Zamolodchikov C-theorem [38] which tells us that

$$\partial_t F = -\beta^{y^i} G_{CC} \beta^{y^i} \tag{64}$$

in this case, since (25) $\beta^{y^i} = u^i$, $\beta^{u_i} = 0$, and recalling that (21) $G_{CC} \propto \epsilon^2 \sim 1/t$, we rewrite (63) as

$$\partial_t S \propto (u_i u^i)^2 \frac{1}{t^2} \tag{65}$$

where we have normalized the wavefunction $\Psi[X^i]$ in the space of the collective coordinates. The result (65), which is valid for asymptotically-large times $t \to \infty$, indicates that the change ΔS in D-brane entropy produced over the entire duration of the scattering process $\Delta\tau \lesssim 1/\epsilon$ by the quantum fluctuations of the recoiling D brane is *finite*. This is consistent with the δ-function-type temporal singularity of the effective space-time curvature scalar (33), and the finite range of effective deviations from conventional General Relativity in string theory.

5 Evolution Equation for the Effective Theory of Light States

We have shown in the previous section that the "kick" applied to the D brane by the incident light closed-string state provides it with a quantum-mechanical excitation that leaves it with finite entropy ΔS. Since the initial combined (light particle, D brane) state had no entropy, we expect the final light-particle state also to have non-zero entanglement entropy ΔS. In string world-sheet language, the interaction between the light state and the D brane implies that their effective σ models receive equal and opposite perturbations. These act as sources in the equations of motion of both subsystems, making them both appear non-critical. Thus one must introduce a renormalization scale also for the light-particle subsystem, which must also acquire non-zero entropy, according to general arguments [4].

To see this explicitly, we first review the general formalism of ref. [4] for the description of closed strings perturbed away from criticality by quantum-gravitational backgrounds. Criticality is restored by the inclusion of Liouville scaling factors and the target time is re-interpreted as the zero mode of the Liouville field ϕ, which may then be identified with that of the D-brane target-time coordinate X^0, as we have shown also in the D-brane context in section 3. It follows from the general analysis of [4] that the effective time-evolution equation for the light closed-string degrees of freedom takes the form

$$\partial_t \rho = i[\rho, H] + i\delta\!\!\!/ H \rho \tag{66}$$

where H is the light-state Hamiltonian, and the non-Hamiltonian term $\delta\!\!\!/ H \rho$ takes the generic form

$$\delta\!\!\!/ H \rho = \beta^i G_{ij} [g^j, \rho] \tag{67}$$

Here the non-zero renormalization coefficients β^i parametrize the apparent departure from criticality induced by the couplings g^j, which become quantum variables when higher-genus effects are taken into account, and $G_{ij} \propto < V_i V_j >$ is a suitable positive-definite Zamolodchikov metric in the space of the vertex operators $V_{i,j}$ corresponding to these couplings. The presence of the non-Hamiltonian term (67) then induces entropy production in the effective light-particle theory:

$$\frac{dS^{light}}{dt} = -\beta^i G_{ij} \beta^j \tag{68}$$

causing S^{light} to vary monotonically if any $\beta^i \neq 0$ and G_{ij} has definite sign [38]. As we have argued previously [4], the appearance of a term (67) is necessarily accompanied by entropy production (decoherence) in the effective light-particle system, at a rate given in general by (68) [4].

In the D-brane case at hand, we specialize to the subspace spanned by the tachyon modes (lowest-lying closed-string states), so that (68) reduces to

$$\frac{dS^{light}}{dt} = -\beta^T G_{TT} \beta^T \tag{69}$$

where T denotes a tachyon coupling. In this application, the dominant contribution to the conformal anomaly for the tachyon β functions comes from the interaction of the tachyon with the Goldstone modes D_ϵ, C_ϵ. We work in a renormalization scheme [38] where

$$G_{TT} \sim 1 + O(T^2) \tag{70}$$

We now use the general expression $\beta^i \sim C^i_{jk} g^j g^k + \ldots$ for the β function, where the $\ldots$ denote higher-order terms and the $C^i_{jk} = G^{im} C_{mjk}$, where the C_{mjk} are completely symmetric trilinear O.P.E. coefficients, i.e., three-point correlation functions $< V_i V_j V_k >$. In our D-brane case the contributions of recoil to the conformal anomaly are of the form

$$\beta^T \simeq C^T_{C_g C_g} (u_i)^2 + \ldots \tag{71}$$

to leading order in the matter deformations, where C_g denotes a bulk operator that represents the virtual excitations appearing in D-brane recoil as discussed in sections 2 and 3, having as couplings the recoil velocities u_i, and the $C^T_{C_g C_g}$ denote appropriate O.P..E. coefficients [15]. In this weak-tachyon perturbative treatment, the dominant contributions to the change in entropy (69) of the light-particle subsystem is of order

$$dS^{light}/dt \propto \sum_T (u_i u^i)^2 (C^T_{C_g C_g})^2 \qquad t \to \infty \tag{72}$$

Below we shall express the bulk amplitude $C^T_{C_g C_g}$ in (72) in terms of open-to-closed scattering amplitudes of the open-string sector. This enables us to show that (72) is, to this order in the matter deformations, identical to (65) but *opposite* in sign, thereby providing an explicit consistency check of the overall conformal invariance of the matter-plus-quantum-D-brane recoil system.

As a step towards this demonstration, we first develop the proper definition of the light-state σ model, so as to represent correctly the motion of low-lying closed-string modes in an 'environment' of quantum-fluctuating D branes. It is technically convenient for this purpose first to re-express the σ-model for the 0 brane itself in the Liouville-dressed formalism, using a Galilean transformation of the collective coordinates of the 0 brane. This, as we shall see, facilitates the analysis, as it provides a simple expression for C_g in

[15] An explicit expression for C_g is given later in (77).

terms of a compact representation of the D-brane boundary excitations (the C,D pair). We rewrite the full recoil operator (14) as

$$V_{rec} = \epsilon \int_{\partial\Sigma} d\tau \xi_i \partial_n \xi^i \Theta_\epsilon(X^0) \qquad \xi_i \equiv y_i + \frac{u_i}{\epsilon} X^0 \tag{73}$$

where the ξ^i *do not* satisfy fixed Dirichlet boudnary conditions. Using the analysis of quantum-mechanical uncertainties of the 0-brane subsystem [20], which are briefly discussed in the Appendix, c.f., eq. (85) in particular, we see that $\xi_i - y_i$ in (73) expresses quantum uncertainty in the initial location of the D brane, since u_i/ϵ is the minimum uncertainty obtained as a result of the action of the D_ϵ quantum-recoil operator. This interpretation of (73) may also be expressed in the language of renormalization. We see from eq. (9) that the effects of ϵ may be absorbed in an appropriate renormalization of the string coupling constant g_s, and a corresponding renormalized recoil velocity $u_i^R \equiv u_i/\epsilon$. Eq. (85) of the Appendix tells us that these renormalization effects result from quantum fluctuations in the location of the 0-brane collective coordinates. In this picture, which from a physical point of view corresponds to a Galilean transformation of the collective (spatial) coordinates in a co-moving frame of velocity u_i^R, consistency with the renormalization-group approach requires u_i^R to be exactly marginal, whilst u_i will now have an anomalous dimension $du_i/dt = -(1/2t)u_i$, $t \sim 1/\epsilon^2$, in agreement with the fact that the anomalous scaling dimension of the C and D operators is $-\epsilon^2/2 \sim -1/(2t)$: $t \sim Log|L/a|^2$. In what follows, unless explicitly stated, we will suppress the superscript when denoting the renormalized recoil velocity: $u_i^R \to u_i$. In this interpretation, the $\epsilon\xi_i$ factor in (73) may be considered as the minimum bound on the (quantum) uncertainties in the location of the 0-brane in a co-moving (Galilean-transformed) frame. According to the analysis in the Appendix (c.f., eq. (86)), the uncertainty represented by the world-sheet zero mode of $\epsilon\xi_i$ is due to the action of the C_ϵ quantum-recoil operator, and is thereby related to the *stringy modes* of the 0-brane [16].

Passing to the bulk formalism as in (27), expressing the coordinate X^i in terms of ξ^i, and identifying ϕ with X^0, one finds the following Liouville-dressed σ-model action:

$$S_\sigma^D = \int_\Sigma d^2 z \frac{1}{4\pi\alpha'}[-(1-(u_i^R)^2)\partial_\alpha X^0 \partial^\alpha X^0 + \partial_\alpha \xi^i \partial^\alpha \xi^j \delta_{ij} + \\ (-u_R^i + \epsilon^2 \xi_i \Theta(X^0))\partial_\alpha X^0 \partial^\alpha \xi^i] + \int_{\partial\Sigma} d\tau \epsilon \xi_i \partial_n \xi^i \Theta(X^0) \tag{74}$$

where the ξ_i, X^0 are independent σ-model fields, and $\Theta(X^0)$ is the Liouville-dressed Θ operator (29), which is conformal [17]. Notice that in the above approach ϵ is viewed as a pa-

[16] Note that within our interpetation of ϵ^{-2} as the target time t, this uncertainty become time dependent. This is important for the correct definition of the light-particle subsystem in this framework, as we discuss later on.

[17] One may renormalize the X^0 field in (74) so that its kinetic term assumes the canonical form, in which case the off-diagonal metric term is multiplied by a factor $1/(1-(u_i^R)^2)$, which is almost unity for

rameter that is independent of the zero mode of the Liouville field X^0. The identification of $1/\epsilon^2$ with X^0 should be made only at the very end of the computations. The off-diagonal metric in (74) satisfies the generalized Weyl-invariance conditions for a transformation that is singular at $X^0 = 0$. Moreover, the curvature scalar constructed out of the coordinates ξ_i, X^0, exhibits the temporal singularity (33) at $X^0 = 0$, $R \ni -\epsilon^4 \xi_i \xi^i \delta(X^0)\Theta(X^0)$ due to the effects of D-brane recoil, which induces the above-mentioned uncertainty $\epsilon\xi$ in the location of the brane.

In the formalism (74), the recoiling boundary quantum state is characterized by a single Goldstone-mode operator $\mathcal{C}$ of the C type:

$$S_\sigma^D \ni \int_{\partial\Sigma} d\tau \mathcal{C}(\tau) \equiv \int_{\partial\Sigma} d\tau \epsilon \xi_i \partial_n \xi^i \Theta(X^0) \tag{75}$$

which incorporates the C,D operator mixing discussed in section 2, by virtue of the transformed ξ variable. On the other hand, a generic macroscopic D-brane boundary state $|a>$ is described by an operator of the form (75) without the $\Theta(X^0)$ term and with $\epsilon\ \xi$ replaced by a macroscopic (fixed) coordinate y_i. This is not exhibited explicitly in (74), but it should be understood in the following. We recall that in the modern approach to D branes [1], the Dirichlet boundary conditions for the collective coordinates are viewed as reflecting appropriate gauge-field background excitations of ordinary open-string σ models, with all the coordinates of the string obeying standard Neumann (closed-string) boundary conditions [1, 39]. This will always be implicit in our subsequent manipulations based on the action (74). The σ-model action S_σ^D (74) describes the 0 brane, viewed as a subsystem for which the scattered matter has been integrated out. Hence the Liouville field/time X^0 appearing in (74) represents collectively the matter effects which induce violations of conformal invariance that are restored by the gravitational (Liouville) dressing.

We now use the above formalism (74) to construct the matter-subsystem σ model, which enables us to evaluate the right-hand side of (69). In the bulk formalism, our σ model describing the matter subsystem is *defined* by (74), upon *dropping* the boundary term (75) which encodes the environment of quantum-recoil fluctuations of the D brane, and *adding* the closed-string tachyon deformation:

$$T(\xi, X^0) = \int d^D k d\omega e^{ik_i\xi^i} e^{i\omega X^0} \tag{76}$$

where D is the target-space dimension. This definition makes physical sense for the following reason: As we mentioned previously, in the closed-string formalism one may view a D brane located originally at y_i as being created by an appropriate operator

slow-moving 0 branes, or for recoiling branes in a low-energy scattering experiment.

which creates the macroscopic boundary state of the brane $|a>$. On the other hand, a *low-energy* observer is obliged to 'average out' the time-dependent *stringy* effects due to quantum uncertainties in the location of the brane associated with the virtual recoil excitations, that are expressed via the boundary term (75). In σ-model language, such an averaging-out procedure is equivalent to ignoring the corresponding deformations of the world-sheet action. In this framework, the bulk operator C_g appearing in (71), which describes the quantum D-brane recoil excitations from a matter-theory point of view, and hence is retained in the action for the light-matter subsystem, may be identified with the following deformation:

$$S_\sigma^D \ni \int_\Sigma d^2z C_g(z,\bar{z}) \equiv \epsilon^2 \int_\Sigma d^2z[\xi_i \Theta(X^0)\partial_\alpha X^0 \partial^\alpha \xi^i] \tag{77}$$

This completes our specification of the matter σ model.

We now check the mathematical consistency of this definition for the matter subsystem by reproducing the entropy change (65) through (69). First we show that the operator C_g (77) contains the bulk form of the recoil D operator discussed in section 2, by splitting the field ξ^i into zero-mode and fluctuation parts:

$$\xi_i = \xi_{0,i} + \hat{\xi}_i(z,\bar{z}) \tag{78}$$

where $\xi_{0,i}$ is independent of the world-sheet coordinates. The operator C_g is now seen to be equivalent to

$$\int C_g \simeq \epsilon^2 \xi_0 \int_\Sigma d^2 \partial_\alpha (\int \frac{dq}{iq^2} e^{iq\ X^0})\partial^\alpha \xi^i + \ldots \tag{79}$$

where $\ldots$ denote the $\hat{\xi}$-dependent parts. Clearly, if the world sheet has a boundary, the operator exhibited in (79) becomes the D-brane recoil operator $\epsilon^2\xi_0 \int_{\partial\Sigma} d\tau D(\tau) + \ldots$, where the $\ldots$ denote world-sheet terms that vanish on shell. In this spirit, the 'coupling' $\epsilon^2\xi_0$ in (79), which has the dimensions of velocity, plays the rôle of the renormalized recoil velocity u_i of the brane. These ξ_0 zero-mode parts of C_g contribute the dominant divergent terms in the conformal anomaly (71), and we now concentrate on them.

There is a formal connection between the closed (bulk) and open (boundary) string theories [17], which leads to a determination of (72) in terms of quantities in the open-string formalism, which provides the advertized connection with (65). We recall that the matrix element $C^T_{C_gC_g}$ describes - to leading order in the matter deformation - the interaction of the bulk gravitational modes of the D brane, expressing quantum recoil, with the light matter in the closed-string channel [19, 10]. According to ref. [17], such bulk amplitudes may be expressed as 'squares' of open-to-closed string amplitudes containing excitations on the (conformal) boundary state $|a>$. The pertinent relation is of the form [18]:

$$\sum_T C^T_{C_g C_g} \propto (C^a_{C_g \mathcal{C}})^2 \qquad (80)$$

with C_g given by (77) and $\mathcal{C}$ by (75).

We now notice that the sum over tachyon modes in (80) actually reduces to a single term in the string case, as a result of the path integration of the zero-mode of X^0. This can be seen easily by using (76) for the tachyon mode and taking into account the specific form of the dominant ξ_0 zero-mode part of the C_g operators (79): $C_q \propto \int dq \frac{1}{q^2} e^{iqX^0}$. It is then straightforward to see that the integration over the zero mode of X^0 simply imposes energy conservation: $\omega + q + q' = 0$ for the three-point amplitudes in (80), thereby reducing the sum over tachyon modes T (i.e., over ω) to a single term which is uniquely specified in terms of the C_g. This allows (80) to be inserted in the right-hand side of (72).

We now recall the fact, mentioned above, that the dominant contributions to (71) arise from the zero-mode parts of C_g. Equivalently, in the open-string formalism, the dominant contributions appear when the operator C_g is near the boundary, and its one-point function diverges [19, 20, 10]. Using (79), we then see easily that the amplitude $C^a_{C_g \mathcal{C}}$ is essentially $< CCD >$. Since the one-point function of the D operator diverges like $1/\epsilon$ [19, 20] near the boundary (45), and the two-point O.P.E. of CC vanishes as [20] ϵ^2 (15), it follows that the leading short-distance singularity of the above-mentioned three-point function, when the arguments of the three operators are all behaving $\mathcal{O}(\epsilon)$, is:

$$C^a_{C_g \mathcal{C}} \sim \epsilon \qquad (81)$$

which implies that

$$C^T_{C_g C_g} \sim (C^a_{C_g \mathcal{C}})^2 \sim \epsilon^2 \qquad (82)$$

Identifying $1/\epsilon^2 \sim t$, where t is the target time/renormalization-group scale, we obtain the following result for the rate of change (72) of the entropy of the light subsystem:

$$dS^{light}/dt \propto -(u_i u^i)^2 \frac{1}{t^2} \qquad t \to \infty \qquad (83)$$

which is *identical* to the result (65), but *opposite* in sign. Thus, for the light-particle system, the direction of time t must be taken *opposite* to that of the Liouville renormalization-group flow, in order to have increasing entropy. This is consistent with the fact that the total entropy of the brane-plus-matter system remains constant as a consequence of conformal invariance. A similar situation has been argued to hold in the non-critical Liouville

[18] For a rigorous proof of (80) in the presence of logarithmic operators, see [31].

analysis of the string black hole. The above analysis completes the consistency check on our definition of the light-state subsystem, to lowest non-trivial order in matter deformations.

The expression (83) is finite and independent of the area of the D-brane collective-coordinate surface. Hence, the Bekenstein-Hawking area law is not valid for the information-theoretic entropy associated with the quantum fluctuations of a recoiling D brane. We recall that the area law is semiclassical, being associated with a tree-level treatment of the gravitational background, which is described in our case by a D-brane configuration. The semiclassical treatment ignores recoil, and simply expresses the entropy of the subsystem of tachyons obtained by integrating over the tachyon degrees of freedom behind the collective-coordinate surface of a classical D-brane configuration. This is a geometric term which is also present in standard flat-space local field theories [40]. Our analysis goes beyond this by considering quantum fluctuations in the background induced by recoil during a scattering process. The situation is analogous to that studied in [13], where the information-theoretic content of quantum fluctuations in a scalar field at the horizon of a four-dimensional black hole in General Relativity also violated the area law. Recent claims [7] to have derived the Bekenstein-Hawking law for the string black-hole entropy, considering the horizon of a stringy black hole as a D brane, apply only at the semiclassical level.

The order of magnitude of the decoherence term (83) is the same as that of the non-Hamiltonian term $\not\delta H$ in the evolution equation of the density matrix of the light subsystem. This is due to the fact that, for asymptotic $t \to \infty$ where the non-critical string is near its equilibrium (critical) point, the commutator $i[g^i, \rho] \sim i[g^i, e^{-H}] = O[\partial_t g^i = \beta^i]$, as a result of the canonical formalism for the couplings g^i [35, 10]. According to the discussion above, the decoherence of the light-state subsystem is determined by the recoil velocity of the collective-coordinate surface of the D brane induced during scattering. This is given by (9), and can be at most of the same order of magnitude as $g_s E$, where E is a typical energy of the propagating closed-string state, which plays the rôle of the low-energy propagating particle treated in [4]). Equation (83) tells us that, for long times $t \to \infty$, the dominant decoherence effects in the evolution equation of the density matrix are of order $\frac{g_s^4}{M_s^4}E^4$, where M_s is the string scale. To make this estimate, we take into account the fact that the evolution parameter on the world sheet scale is measured in string units $\sqrt{\alpha'}$, where $\alpha' = 1/M_s^2$ is the string Regge slope.

6 Conclusions

We have argued in this paper that D branes provide another example of a model of quantum gravity in which the effective theory of the propagating light-particle states obeys a modified quantum Liouville equation of the type (66), following the previous examples of $1+1$-dimensional string black holes [4] and a scalar field outside a four-dimensional black hole with Yang-Mills interactions. As in these previous cases, time is interpreted as a renormalization-group scale parameter, which may be identified in the D-brane and string black-hole cases with the zero mode of the Liouville field on the world sheet. In these two string models, there is no question of modifying quantum mechanics on the world sheet. In both cases, the light degrees of freedom are to be regarded as truncated open systems, and the modification (66) of the quantum Liouville equation can be traced to the need to treat the interaction with the black-hole environment as a source term in the world-sheet equation of motion for the light-particle system, pushing it away from criticality [10].

We take this opportunity to clarify where we differ from views espoused elsewhere [41, 9] on the possible maintenance of quantum coherence. In our view, the fact that the conventional Hawking-Bekenstein black-hole entropy can be identified with the number of distinguish*able* D-brane states does not mean that these states are necessarily distinguish*ed* in a realistic experiment: for example, the infinite set of Aharonov-Bohm phase measurements described in [8, 9] are *not* feasible in practice. This is why one must sum over the *un*observed degrees of freedom, which are represented in the present case by the *quantum* excitations of the D brane, that cause a deviation from the Hawking-Bekenstein area law and were not studied in [9]. It is convenient technically to perform the sum over unseen states using the Liouville field, whose zero mode we interpret as time, as confirmed in this D-brane analysis.

During the course of this analysis, we have indicated various steps where a more rigorous treatment is desirable and in preparation [31]. There are also some important physical issues that should be addressed in future work on this D-brane model. One is the possible rôle of infinite-dimensional symmetries such as the W_∞ discussed in connection with the $1+1$-dimensional black-hole model. We have argued previously [42, 4] that the infinite-dimensional Cartan subalgebra of W_∞ provides an infinite set of conserved charges that are available to label black-hole states with W hair. We have further argued that a finite low-energy experiment is unable to measure all the W hair [4], and that the corresponding inevitable leakage of W quantum numbers between the light-particle subsystem and the black-hole background is a measure of the rate of information loss inherent in (66). Previously, we have presented formal evidence for this leakage in the $1+1$-dimensional black-hole model [4], and exhibited analogues of these W charges in a higher-dimensional

D-brane model. The possible rôle of such charges in the model studied here deserves further investigation. We are optimistic that such W charges do indeed exist, since it is known [43] that a world-sheet W_∞ algebra [44] is present in any model with logarithmic operators C, D, such as this one. Moreover, the singularity (33) at $X^0 = 0$ in this model is effectively $1+1$ dimensional, leading us to expect that it may have an underlying algebraic structure similar to the $1+1$-dimensional black-hole example. However, these points require further analysis.

Another area worthy of deeper analysis is the creation and annihilation of D branes. In this paper, we have a pre-existing D-brane background, and discussed the formalism for quantum excitations on its surface. However, we expect that much of the machinery developed here could also be used to discuss the production, annihilation and decay of D branes. In particular, the $\Theta(X^0)$ and singularity structure of the recoil operator (14) is well adapted to the creation of a D-brane pair. However, an extension in necessary to describe "D-brane foam", i.e., the treatment of virtual D-brane fluctuations in the space-time background [19]. We have developed previously [45] the corresponding description of $1+1$-dimensional black holes in terms of monopoles on the world sheet. Extending the present analysis to virtual D branes is, however, a non-trivial technical problem.

In the absence of an appropriate treatment, we can nevertheless offer some intriguing speculations. Let us suppose, as was found in this paper, that the information lost in any encounter with a microscopic D brane is $\propto E^4 M_P^{-4}$, where E is a typical energy scale of the light state. Let us further hypothesize that the density of virtual D-brane excitations of the space-time background is $\mathcal{O}(M_P^3)$ [20]. On the other hand, the low-energy particle cross section σ is assumed to be proportional to $1/E^2$ [46]. Thus, if the speculations in the two preceding sentences were correct, we would infer that $\not{\delta}H \propto 1/M_P$, where by dimensional analysis the numerator would be $\mathcal{O}(E^2)$, where E is a characteristic energy scale of the light-particle system. This heuristic argument is consistent in order of magnitude with estimates made previously in the contexts of the $1+1$-dimensional black hole and the four-dimensional quantum gravity model. Remarkably, it is also of the same order as the sensitivity of the neutral kaon [47] system to any such possible deviations from the canonical quantum Liouville equation. Therefore we cannot yet exclude the possibility that the type of decoherence effect discussed in this paper might have some phenomenological relevance, however far-fetched this may seem.

[19] This was achieved in the $(1+1)$-dimensional-black-hole case using a world-sheet valley approach [45].

[20] We do not distinguish for this heuristic argument between $M_P, M_s = 1/\sqrt{\alpha'}$, or any other gravitational scale in string theory.

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Appendix: D-brane Decoherence and Position Uncertainty

Additional support for our picture of information loss and decoherence induced by the recoil of D branes is provided by a closer look at the modified uncertainty principle for D particles which was discussed previously in ref. [20]. As was discussed there, the rôle of the logarithmic operators describing recoil is crucial in a derivation of an uncertainty principle for D branes. The D operator provides the conventional quantum-mechanical part of the uncertainty, whilst the C operator is associated with intrinsically stringy parts. To be more precise, if one considers the scattering of light closed-string state, playing the rôle of a 'detector', off a D brane, and assumes that the momentum of the detector is known precisely before and after the scattering, one arrives at the conclusion that there is an uncertainty in the energy of the D brane which is of order $\hbar\epsilon$.

This conclusion was reached [20] by first observing that the regulated $\Theta_\epsilon(X^0)$ operator appearing in the definition of the recoil operators (14) may be written as $\Theta_\epsilon \sim \Theta(X^0)e^{\epsilon X^0}$; from this it follows that the recoil (measurement) takes a finite time $\Delta\tau \sim 1/\epsilon$, thereby implying an uncertainty in the total energy of order $\hbar\epsilon$. The kinematical analysis of ref. [20] established the following relation:

$$\Delta P^i \sim \hbar\epsilon/u^i \tag{84}$$

where ΔP^i denotes the uncertainty in the D-brane momentum. Given that the collective coordinate of the D brane starts growing as $u_i t$ (25), the uncertainty (84) implies an uncertainty due to the D_ϵ recoil operator in the position of the collective coordinate of the D brane of order

$$\Delta_{D_\epsilon} X^i \sim u_i \Delta t = u_i \frac{1}{\epsilon} = \frac{\hbar}{\Delta P^i} \tag{85}$$

which is a quantum-mechanical point-like particle uncertainty betweeen coordinates and momenta.

The essentially stringy parts of the uncertainty are obtained from the C_ϵ operator, which, due to its specific form (13), yields a lower bound on the uncertainty of the coordinate y^i of order $y^i\epsilon$. Using (84) one obtains

$$\Delta_{C_\epsilon} X^i \sim \frac{y^i u_i}{\hbar} \Delta P^i \tag{86}$$

Thus, the total uncertainty in the collective spatial coordinate of a recoiling D brane, due to the combined action of the C and D logarithmic pair , is

$$\Delta_{total} X^i \sim \frac{\hbar}{\Delta P^i} + \frac{y^i u_i}{\hbar} \Delta P^i \tag{87}$$

It is important to stress once again that, due to the mixing between the operators C and D, one cannot evade the influence of the C operator, and hence a 'stringy' form of the uncertainty principle for a D brane [20].

For our purposes, it is important to notice the dependence of the second term in (87) on the coordinate y^i. Minimizing the right-hand-side of the uncertainty (87) with respect to ΔP^i, one obtains a lower bound on ΔX^i of the form:

$$(\Delta X^i)_{min} \sim 2\sqrt{u_i y^i} = 4\sqrt{2\sqrt{2}\pi g_s \ (k_1 + k_2)_i \ y^i} \tag{88}$$

The $\sqrt{y^i}$ dependence is reminiscent of the measurability bounds occuring in the non-critical Liouville string approach to target time [48]. As discussed there, the result (88) is compatible with the decoherence (65) implied by the effects of the recoil operator C_ϵ. This similarity should not have come as a surprise, given the discussion above on the marginal perturbation of the recoiling D brane from a conformal point, and the generic connection of soliton backgrounds in string theory with Liouville strings [10].

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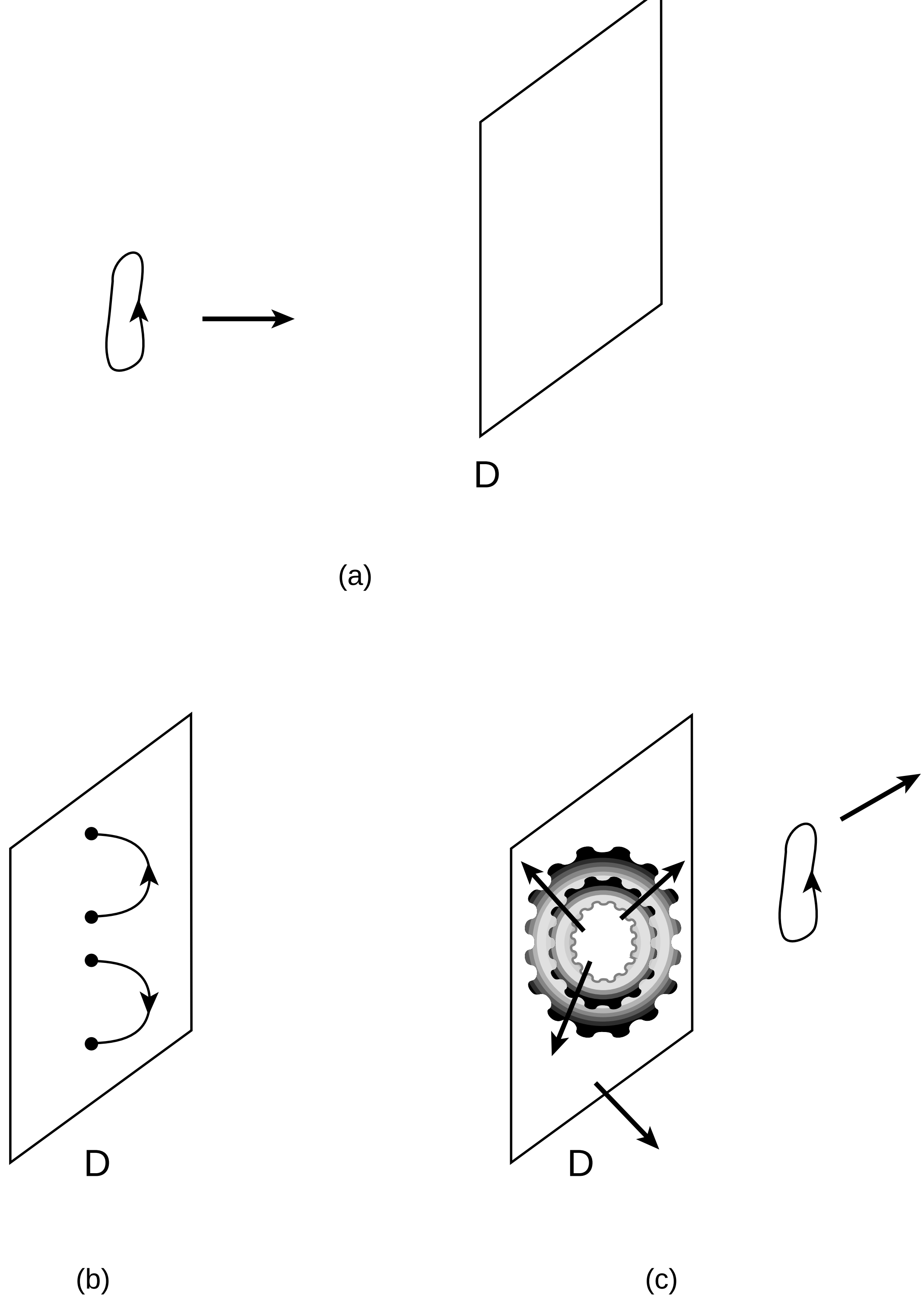

Figure 1: Scattering of a low-energy closed-string state off a D brane: (a) asymptotic past, (b) time of impact ($X^0 = 0$) with trapping of the string state on the D-brane surface by a split into two open-string excitations, and (c) asymptotic future, after two open strings recombine to emit a closed-string state, while the D brane recoils with finite velocity and its internal state fluctuates

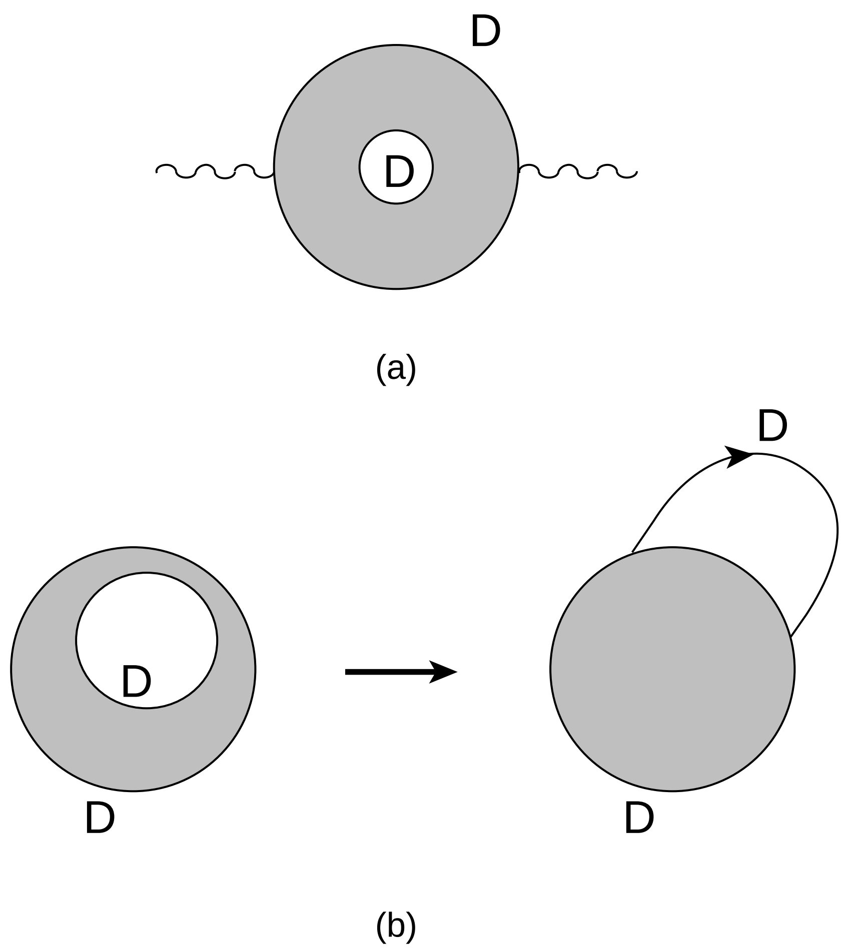

Figure 2: (a) : World-sheet annulus diagram for the leading quantum correction to the propagation of a string state in a D-brane background, and (b) the pinched annulus configuration which is the dominant divergent contribution to the quantum recoil.