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## Multi-Messenger Theories of Gauge-Mediated Supersymmetry Breaking

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## Abstract

We study gauge-mediated theories containing several messengers with the most general SU(5)-invariant mass and supersymmetry-breaking parameters. We show that these theories are predictive, containing only two relevant parameters more than the minimal gauge-mediated model. Hypercharge D-terms can contribute significantly to the right-handed charged sleptons and bring them closer in mass to the left-handed sleptons. The messenger masses must be invariant under either SU(5) or a "messenger parity" to avoid spontaneous breaking of charge conservation.

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Theories in which supersymmetry breaking is communicated to the observable fields through gauge interactions [1], instead of gravity, have the advantage of insuring a correct suppression of flavour-changing neutral currents. Another interesting feature of these theories is their sharp predictivity of the new particle mass spectrum. Let us first consider the most restrictive case [2] in which the messenger fields responsible for the communication of supersymmetry breaking form a single representation of a GUT model. Let us also define M as the mass of the messenger supermultiplet and  $\sqrt{F}$  as the supersymmetry-breaking scale, or, in other words, as the mass splitting inside the messenger supermultiplet. In this minimal case, the complete mass spectrum of squarks, sleptons, and gluinos is described only by the parameter F/M, beside a mild logarithmic dependence on M.

A complication arises in the Higgs sector, as the Higgs mass parameters which violate a Peccei-Quinn symmetry cannot be generated by gauge interactions alone. In order to parametrize some new unknown interactions, two free inputs have to be introduced, denoted by  $\mu$  and  $B_{\mu}$  with standard conventions. One of these two parameters is determined by the electroweak symmetry breaking condition. Notice however that if  $\mu$  and  $B_{\mu}$  are generated radiatively by some new interaction, then it is fairly generic [3] to obtain new contributions to the Higgs mass parameters, usually denoted by  $m_{H_1}^2$  and  $m_{H_1}^2$ . Thus, unless one relies on specific models for the Peccei-Quinn violating interactions, the Higgs sector introduces three new free parameters to the theory. However the predictions for slepton, squark, and gluino masses do not depend on these parameters.

As these mass predictions may soon face experimental test, it is important to establish how much they depend on the minimality of the model or how much they descend from the gauge-mediation principle. Variations of the minimal model have already been considered in refs. [4, 5]. Direct superpotential couplings between messenger and observable fields have been analysed in ref. [4]. Although these couplings spoil the natural flavour conservation of gauge-mediated supersymmetry-breaking theories, they have the advantage of insuring a fast decay of the lightest messenger, thus avoiding cosmological problems [6]. Because of an accidental cancellation<sup>1</sup>, the new flavour-violating contributions to squark and slepton masses are suppressed by the ratio  $F^2/M^4$  and are therefore less dangerous than one may have naively

<sup>&</sup>lt;sup>1</sup>This cancellation was first pointed out in ref. [3], in the case of couplings between Higgs and messenger superfields.

expected [4]. The author of ref. [5] has investigated the case in which the messenger fields do not form complete GUT representations. Although the successful gauge-coupling unification is usually lost, it is interesting to know that several of the mass predictions are approximately preserved, despite the large number of discrete choices for the messenger representations.

In this paper we want to study a generalization of the minimal model. We will consider the case in which the messengers form n copies of the same real GUT representation and their supersymmetric and supersymmetry-breaking mass matrices have a completely generic structure invariant under GUT symmetry. For definiteness, we will take  $n \ \mathbf{5} + \overline{\mathbf{5}}$  of SU(5), called  $\Phi_i$  and  $\overline{\Phi}_i$  (i = 1, ..., n), and the extension of our results to other representations is completely straightforward. The messenger mass matrix is defined by the superpotential term

$$W = \bar{\Phi}_i M^{\Phi}_{ij} \Phi_j \qquad \quad i, j = 1, \dots, n \tag{1}$$

and by a supersymmetry-breaking term in the scalar potential

$$V = \bar{\Phi}_i F^{\Phi}_{ij} \Phi_j + \text{h.c.}$$
<sup>(2)</sup>

Here  $\Phi_i$  label either the SU(2) doublet or the SU(3) triplet components of the *i*-th **5**-plet messenger. After GUT-symmetry spontaneous breaking, the mass parameters for the doublets and the triplets, distiguished by the index  $\Phi$  on  $M^{\Phi}$  and  $F^{\Phi}$  in eqs. (1)–(2), renormalize differently. Our basic hypothesis is that the mass scales  $M^{\Phi}$  and  $F^{\Phi}$  are generated by a sector neutral under GUT interactions, and therefore at the GUT scale  $M^{\Phi}$  and  $F^{\Phi}$  are the same for both doublets and triplets. We will refer to this as the "GUT singlet hypothesis". If we define M and F as the common values of the mass matrices at the GUT scale  $M_{GUT}$ , a one-loop renormalization scaling gives the values of the different  $M^{\Phi}$  and  $F^{\Phi}$  at the energy scale Q:

$$\frac{F^{\Phi}(Q)}{F} = \frac{M^{\Phi}(Q)}{M} = \prod_{r=1}^{3} \left[ \frac{\alpha_r(Q)}{\alpha_r(M_{GUT})} \right]^{-2\frac{C_r^{\Psi}}{b_r}} .$$
(3)

Here  $C^{\Phi}$  depends on the Standard Model quantum numbers,  $C = \frac{N^2 - 1}{2N}$  for the N-dimensional representation of  $SU_N$ , and  $C = Y^2$  ( $Y = Q - T_3$ ) for the  $U_1$  factor. Also  $b_r$  are the  $\beta$ -function coefficients

$$b_3 = -3 + n, \quad b_2 = 1 + n, \quad b_1 = 11 + \frac{5}{3}n,$$
 (4)

and n counts the messenger contribution. Equation (3) shows that the ratio  $F^{\Phi}(Q)/M^{\Phi}(Q)$  is independent of the energy scale Q. The "GUT singlet hypothesis" implies also that this

ratio is independent of  $\Phi$ , *i.e.* it is equal for the doublet and triplet messenger components,  $F^{\Phi}(Q)/M^{\Phi}(Q) = F/M.$ 

We will take here  $M_{ij}$  and  $F_{ij}$  as general  $n \times n$  matrices, because of our lack of knowledge on the sector which originally breaks supersymmetry. A simpler case can be considered, in which both  $M_{ij}$  and  $F_{ij}$  originate from the couplings of messengers to a single superfield which acquires vacuum expectation values in the scalar and auxiliary components. Then  $M_{ij}$  and  $F_{ij}$ are proportional, and the model simply reduces to n replicas of the minimal model considered above. We believe that there are no strong reasons to make the restrictive assumption that  $M_{ij}$  and  $F_{ij}$  are proportional. Moreover we will show that the case of generic  $M_{ij}$  and  $F_{ij}$  has a richer structure and leads to important differences in the physical mass spectrum.

Let us start analyzing the supersymmetry-breaking masses in the observable sector induced by radiative corrections. After a redefinition of the messenger superfields, we can choose, without loss of generality,  $M_{ij}$  to be diagonal with real and positive eigenvalues  $M_i$  (i = 1, ...n). The gaugino masses arise at one loop

$$m_{\lambda_r} = k_r \; \alpha_r \frac{\Lambda_G}{4\pi} \; \left[ 1 + \mathcal{O}(F^2/M^4) \right] \; , \qquad r = 1, 2, 3 \; ,$$
 (5)

where  $k_1 = 5/3$ ,  $k_2 = k_3 = 1$ , the gauge coupling constants are normalized such that  $k_r \alpha_r$ (r = 1, 2, 3) are all equal at the GUT scale, and

$$\Lambda_G = \sum_{i=1}^n \frac{F_{ii}}{M_i} \,. \tag{6}$$

Here and in the following, we use the approximation that all entries of the F matrix are smaller than the entries of  $M^2$ . This is justified because some of the scalar messenger square masses can become negative if some eigenvalues of F are larger than the corresponding one in  $M^2$ . The deviations from the leading-order expansion are generally small [5, 6] unless  $F/M^2$  is extremely close to 1. Notice also that  $\Lambda_G$  does not depend on the energy scale at which is defined, as a consequence of the non-renormalization of the ratio F/M, see eq. (3). In eq. (5)  $\alpha_r$  is evaluated at the scale M. The one-loop renormalization group running of  $m_{\lambda_r}$  just amounts to using eq. (5) with  $\alpha_r$  evaluated at  $m_{\lambda_r}$ .

The two-loop contributions to squark and slepton masses are given by

$$m_{\tilde{f}}^2 = 2\sum_{r=1}^3 C_r^{\tilde{f}} k_r \, \alpha_r^2 \left(\frac{\Lambda_S}{4\pi}\right)^2 \, \left[1 + \mathcal{O}(F^2/M^4)\right] \,. \tag{7}$$

 $C^{f}$  depends on the quantum number of squarks or sleptons as explained after eq. (3). In the limit in which M is proportional to the identity  $(M = M_0 \times 1)$ ,  $\Lambda_S$  is given by

$$\Lambda_S = \left(\sum_{i,j=1}^n \frac{|F_{ij}|^2}{M_0^2}\right)^{1/2} \,. \tag{8}$$

In models where the matrices M and F are both proportional to the identity,  $\Lambda_G/\Lambda_S = \sqrt{n}$ , but, in the general case,  $\Lambda_G$  and  $\Lambda_S$  are independent values. Eq. (7) is valid at the scale M. Throughout the paper we follow the convention that  $\alpha_r$  is always evaluated at M, unless indicated otherwise. After the appropriate one-loop renormalization running,  $m_{\tilde{f}}^2$  becomes

$$m_{\tilde{f}}^{2} = 2\sum_{r=1}^{3} C_{r}^{\tilde{f}} k_{r} \alpha_{r}^{2} \left[ \left( \frac{\Lambda_{S}}{4\pi} \right)^{2} + \frac{k_{r}(1-\xi_{r}^{2})}{b_{r}} \left( \frac{\Lambda_{G}}{4\pi} \right)^{2} \right] , \qquad (9)$$

where  $\xi_r \equiv \alpha_r(m_{\tilde{f}})/\alpha_r$  and  $b_r$  are given in eq. (4) taking n = 0.

It is known that squark and slepton masses can also be generated at one loop, but these contributions are proportional to hypercharge and therefore not positive definite. It is usually assumed that, in a realistic model, such contributions have to vanish [2]. In the theories under consideration, they are given by

$$\Delta m_{\tilde{f}}^2 = \frac{\alpha_1}{4\pi} Y_{\tilde{f}} \operatorname{Tr} Y_{\Phi} \bar{\Lambda}_D^{\Phi 2} , \qquad (10)$$

where, at the leading order in  $F/M^2$ ,  $\bar{\Lambda}_D^{\Phi 2}$  is independent of the specific component of the GUT multiplet  $\Phi$ ,

$$\bar{\Lambda}_D^{\Phi 2} \equiv \bar{\Lambda}_D^2 = \frac{1}{2} \sum_{i,j=1}^n \frac{|F_{ji}|^2 - |F_{ij}|^2}{M_i^2} f\left(\frac{M_j^2}{M_i^2}\right) , \qquad (11)$$

$$f(x) = \frac{2}{(1-x)} + \frac{(1+x)}{(1-x)^2} \ln x .$$
(12)

For simplicity, in the following we will focus mainly on the case n = 2, in which eq. (11) reduces to

$$\bar{\Lambda}_D^2 = \frac{|F_{21}|^2 - |F_{12}|^2}{M_1^2} f\left(\frac{M_2^2}{M_1^2}\right) .$$
(13)

As proved in ref. [3], the hypercharge D-term contributions to scalar masses vanish up to two loops, if F is hermitian in the basis in which M diagonal, real, and positive. This is a consequence a symmetry transforming the messenger superfields  $\Phi$ ,  $\overline{\Phi}$ , and the gauge vector superfields V as follows [3]

$$\Phi \to U \bar{\Phi}^{\dagger} , \quad \bar{\Phi} \to \Phi^{\dagger} \bar{U} , \quad V \to -V ,$$
(14)

where U and  $\overline{U}$  are arbitrary  $n \times n$  unitary matrices. The messenger sector is invariant under this "messenger parity" transformation provided that, independently of the specific basis, one can find some matrices U and  $\overline{U}$  such that

$$M^{\dagger} = \bar{U}MU , \quad F^{\dagger} = \bar{U}FU . \tag{15}$$

As "messenger parity" is broken by ordinary particles, hypercharge D-terms mass contributions can arise at higher-loop order, but these are too small to play any significant role in the supersymmetric particle mass spectrum. As a consequence of "messenger parity" eq. (11) vanishes when  $F_{ij} = F_{ji}^{\star}$ . Notice also that eq. (11) vanishes if M is proportional to the identity. Indeed if  $M \propto 1$ , we can always rotate the messenger scalar fields and make F hermitian.

In the absence of "messenger parity", eq. (10) in general leads to negative square masses for either  $\tilde{e}_L$  or  $\tilde{e}_R$ . However if the "GUT singlet hypothesis" is valid, then eq. (10) vanishes simply because  $\text{Tr}Y_{\Phi} = 0$  [3]. This cancellation is guaranteed by the universality of  $\bar{\Lambda}_D^{\Phi}$  within the GUT multiplet, or, in other words, by the non-renormalization of the ratio F/M. It is then clear that this cancellation will not persist to higher orders in  $F/M^2$ . At next order,  $\bar{\Lambda}_D^{\Phi}$  depends on the different Standard Model quantum numbers of the messenger fields, and therefore it acts non-trivially under the trace in eq. (10):

$$\bar{\Lambda}_{D}^{\Phi 2} = \sum_{i,j,k,l} \left[ (F^{\Phi^{\dagger}})_{ij} F_{jk}^{\Phi} (F^{\Phi^{\dagger}})_{kl} F_{li}^{\Phi} - F_{ij}^{\Phi} (F^{\Phi^{\dagger}})_{jk} F_{kl}^{\Phi} (F^{\Phi^{\dagger}})_{li} \right] T_{ijkl} , \qquad (16)$$

$$T_{ijkl} = \frac{i}{\pi^2} \int d^4k \frac{1}{(k^2 - M_i^{\Phi 2})^2} \frac{1}{k^2 - M_j^{\Phi 2}} \frac{1}{k^2 - M_k^{\Phi 2}} \frac{1}{k^2 - M_l^{\Phi 2}} \frac{1}{k^2 - M_l^{\Phi 2}} .$$
(17)

In the case n = 2, eq. (16) reduces to

$$\bar{\Lambda}_{D}^{\Phi 2} = \frac{|F_{12}^{\Phi}|^{2} - |F_{21}^{\Phi}|^{2}}{M_{1}^{\Phi 6}} \left[ |F_{11}^{\Phi}|^{2} g_{1} \left( \frac{M_{2}^{\Phi 2}}{M_{1}^{\Phi 2}} \right) - |F_{22}^{\Phi}|^{2} \frac{M_{1}^{\Phi 6}}{M_{2}^{\Phi 6}} g_{1} \left( \frac{M_{1}^{\Phi 2}}{M_{2}^{\Phi 2}} \right) \right. \\
\left. + \left( |F_{12}^{\Phi}|^{2} + |F_{21}^{\Phi}|^{2} \right) g_{2} \left( \frac{M_{2}^{\Phi 2}}{M_{1}^{\Phi 2}} \right) \right],$$
(18)

$$g_1(x) = \frac{x^2 - 8x - 17}{6(1-x)^3} - \frac{1+3x}{(1-x)^4} \ln x , \qquad (19)$$

$$g_2(x) = \frac{x^2 + 10x + 1}{2x(1-x)^3} + \frac{3(1+x)}{(1-x)^4} \ln x .$$
(20)

With the help of the expressions for  $M^{\Phi}$  and  $F^{\Phi}$  in terms of their boundary conditions at the GUT scale M and F, see eq. (3), we can explicitly evaluate the trace in eq. (10):

$$\operatorname{Tr}Y_{\Phi}\bar{\Lambda}_{D}^{\Phi2} = \left[ \left(\frac{\alpha_{3}}{\alpha_{X}}\right)^{\frac{16}{3b_{3}}} \left(\frac{k_{1}}{\alpha_{X}}\right)^{\frac{4}{9b_{1}}} - \left(\frac{\alpha_{2}}{\alpha_{X}}\right)^{\frac{3}{b_{2}}} \left(\frac{k_{1}}{\alpha_{X}}\right)^{\frac{1}{b_{1}}} \right] \bar{\Lambda}_{D}^{(GUT)2} .$$
(21)

Here  $\bar{\Lambda}_D^{(GUT)^2}$  is given by eq. (16) evaluated at the GUT energy scale or, in other words, by these equations with the index  $\Phi$  suppressed. Also  $\alpha_X = k_r \alpha_r(M_{GUT})$  for any r = 1, 2, 3.

For n = 2 eq. (21) gives  $\text{Tr}Y_{\Phi}\bar{\Lambda}_D^{\Phi 2} \simeq -0.3\bar{\Lambda}_D^{(GUT)2}$ . Nevertheless  $\bar{\Lambda}_D^{(GUT)2}$  can be quite smaller than  $\Lambda_S^2$ , because of the extra  $F^2/M^4$  factor. Actually, if  $\sqrt{F} \lesssim (\alpha_3/\pi)^{1/4}M \simeq 0.4M$ , the most important contribution from the hypercharge D-term comes at the two-loop order, from the diagrams shown in fig. 1. Evaluation of these diagrams, for the case n = 2, gives

$$\bar{\Lambda}_D^{\Phi 2} = \sum_{r=1}^3 \frac{\alpha_r}{\pi} C_r^{\Phi} \frac{|F_{21}|^2 - |F_{12}|^2}{M_1^2} f\left(\frac{M_2^2}{M_1^2}\right) , \qquad (22)$$

where the function  $f(M_2^2/M_1^2)$  is given in eq. (12). The trace over the fundamental SU(5) messenger representation gives

$$\operatorname{Tr}Y_{\Phi}\bar{\Lambda}_{D}^{\Phi 2} = \left(\frac{4}{3}\frac{\alpha_{3}}{\pi} - \frac{3}{4}\frac{\alpha_{2}}{\pi} - \frac{5}{36}\frac{\alpha_{1}}{\pi}\right)\frac{|F_{21}|^{2} - |F_{12}|^{2}}{M_{1}^{2}}f\left(\frac{M_{2}^{2}}{M_{1}^{2}}\right) .$$
(23)

The two-loop contribution to squark and slepton square masses from the hypercharge D-term can then be written as

$$\Delta m_{\tilde{f}}^2 = Y_{\tilde{f}} \alpha_1 \left(\frac{16}{3}\alpha_3 - 3\alpha_2 - \frac{5}{9}\alpha_1\right) \left(\frac{\Lambda_D}{4\pi}\right)^2 , \qquad (24)$$

$$\Lambda_D^2 \equiv \frac{|F_{21}|^2 - |F_{12}|^2}{M_1^2} f\left(\frac{M_2^2}{M_1^2}\right) \ . \tag{25}$$

For definiteness let us choose  $M_1 < M_2$ . The function  $f(M_2^2/M_1^2)$ , given in eq. (12), varies between 0 and about 0.1. Therefore, we expect that  $\Lambda_D^2/\Lambda_S^2$  can be at most equal to about 0.1. The effect of  $\Lambda_D$  is therefore rather small for squark masses. However, in the case of sleptons, the  $\alpha_3$  present in eq. (24) can compensate the smallness of the ratio  $\Lambda_D^2/\Lambda_S^2$ , and the hypercharge D-term contribution can be of the same order of the usual gauge contribution.

As an example, we have plotted in fig. 2, as functions of  $\Lambda_D^2/\Lambda_S^2$ , the masses of the lefthanded and right-handed selectron, derived from eqs. (9) and (24). For simplicity we ignore the tree-level contribution of the D terms coming from electroweak symmetry breaking, which are generally quite small. We have fixed  $\Lambda_S$  such that  $m_{\tilde{e}_R} = 100$  GeV if  $\Lambda_D = 0$ , and both selectron masses scale linearly with  $\Lambda_S$ . We have chosen  $\alpha_3 = 0.08$  at the messenger scale M, and  $\Lambda_G = \Lambda_S$ . Results depend only very weakly on the choice of  $\Lambda_G$ , unless  $\Lambda_G \gg \Lambda_S$ . The effect of non-vanishing  $\Lambda_D$  is important for  $m_{\tilde{e}_R}$ . For  $\Lambda_D^2/\Lambda_S^2 = 0.1$  the ratio  $m_{\tilde{e}_L}/m_{\tilde{e}_R}$  is 1.4, instead of its value of 2.1 at  $\Lambda_D = 0$ . This leads to similar production cross sections for  $\tilde{e}_L$ and  $\tilde{e}_R$  at the Tevatron and it could be important for the interpretation of the  $ee\gamma\gamma E_T$  event reported by the CDF collaboration [7, 8]. Notice that the sign of  $\Lambda_D^2$  is not determined, and  $m_{\tilde{e}_R}$  is substantially reduced if  $\Lambda_D^2/\Lambda_S^2$  is negative. Indeed large negative values of  $\Lambda_D^2/\Lambda_S^2$  can be excluded by the experimental lower bound on the selectron mass.

The new mass contribution from hypercharge D term will also affect the Higgs mass parameters  $m_{H_1}^2$  and  $m_{H_2}^2$  at the scale M:

$$m_{H_1}^2 = \left(\frac{3}{2}\alpha_2^2 + \frac{5}{6}\alpha_1^2\right) \left(\frac{\Lambda_S}{4\pi}\right)^2 - \alpha_1 \left(\frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{5}{18}\alpha_1\right) \left(\frac{\Lambda_D}{4\pi}\right)^2 , \qquad (26)$$

$$m_{H_2}^2 = \left(\frac{3}{2}\alpha_2^2 + \frac{5}{6}\alpha_1^2\right) \left(\frac{\Lambda_S}{4\pi}\right)^2 + \alpha_1 \left(\frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{5}{18}\alpha_1\right) \left(\frac{\Lambda_D}{4\pi}\right)^2 \ . \tag{27}$$

As mentioned above, any radiative mechanism which generates the Peccei-Quinn violating terms  $\mu$  and  $B_{\mu}$  will also give new contributions to  $m_{H_1}^2$  and  $m_{H_2}^2$  [3]. We assume here for simplicity that these contributions are negligible with respect to those in eqs. (26)–(27).

The running below the messenger scale M does not considerably modify  $m_{H_1}^2$  but, because of the stop and gluino contributions, has a very important effect on  $m_{H_2}^2$ . Keeping only the leading terms proportional to the strong gauge coupling constant and to the top Yukawa coupling, the effect of running is to induce an extra contribution to eq. (27):

$$\delta m_{H_2}^2 = -\alpha_3^2 \frac{\alpha_h t}{\pi} \left\{ \left[ \frac{\alpha_3 t}{\pi} \xi_3 - \frac{9}{8} \frac{\alpha_h t}{\pi} \left( 1 - I(\xi_3) \right)^2 \right] \left( \frac{\Lambda_G}{4\pi} \right)^2 + 4I(\xi_3) \left( \frac{\Lambda_S}{4\pi} \right)^2 \right\} , \qquad (28)$$

$$I(\xi_3) \equiv \frac{9}{7} \left( \frac{1 - \xi_3^{-7/9}}{\xi_3 - 1} \right) \qquad \xi_3 \equiv \frac{\alpha_3(\tilde{m})}{\alpha_3} = \left( 1 - \frac{3\alpha_3}{4\pi} t \right)^{-1} . \tag{29}$$

Here  $t = \ln(M^2/\tilde{m}^2)$  where  $\tilde{m}$  is the typical stop (or gluino) mass scale;  $\alpha_h = h_t^2/(4\pi)$ , and  $h_t$  is the top-quark Yukawa coupling at the scale  $\tilde{m}$  related to the running top-quark mass  $m_t$  and to the ratio of Higgs vacuum expectation values  $\langle H_2 \rangle / \langle H_1 \rangle = \tan \beta$  by the equation

$$h_t = \left(\frac{m_t}{174 \text{ GeV}}\right) \frac{1}{\sin\beta} . \tag{30}$$

The electroweak symmetry breaking condition determines the parameter  $\mu$  in terms of  $m_{H_1}^2$ ,  $m_{H_2}^2$ , and  $\tan \beta$ 

$$\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2} .$$
(31)

Using eqs. (26), (27), and (28), we can now express  $\mu$  as a function of  $\Lambda_S$ ,  $\Lambda_G$  and  $\Lambda_D$ . The hypercharge D term has the effect to reduce or increase the value of  $\mu$ , if  $\Lambda_D^2$  is positive or negative respectively. A reduction in  $\mu$  is very welcome, as it reduces the amount of fine tuning of the theory [9]. Indeed, in gauge-mediated supersymmetry breaking models, the steep running of  $m_{H_2}^2$  in the electroweak symmetry breaking condition, see eq. (28), can only be compensated by a large  $\mu$ . Therefore the physical value of  $M_Z$  is obtained at the price of a rather accurate cancellation between the term proportional to  $\Lambda_S^2$  and  $\mu^2$ . The hypercharge D term can however reduce  $\mu$  only up to 5%, if  $\Lambda_D^2/\Lambda_S^2 < 0.1$ . The effect is therefore too small to solve the finetuning problem. On the other hand, the hypercharge D term can increase the value of  $m_{\tilde{e}_R}$ , allowing lower values of  $\Lambda_S$  consistent with experimental limits. In this respect it is possible, for  $\Lambda_D^2/\Lambda_S^2 = 0.1$  and for a fixed  $m_{\tilde{e}_R}$ , to obtain values of  $\mu$  30–40% smaller than for  $\Lambda_D = 0$ , and therefore alleviate the fine-tuning problem.

In conclusion we have studied theories in which supersymmetry breaking is mediated by a multi-messenger sector with GUT-invariant mass parameters. The mass spectrum of the supersymmetric particles is described by three parameters  $\Lambda_G$ ,  $\Lambda_S$ , and  $\Lambda_D$ . These three mass scales are in general independent, but all of the same order of magnitude  $\mathcal{O}(100 \text{ TeV})$ . The relations between the physical masses and the scales  $\Lambda_{G,S,D}$  are completely specified by the particle gauge quantum numbers.  $\Lambda_G$  determines the gaugino masses, while  $\Lambda_S$  contributes to the squark and slepton masses.  $\Lambda_D$  corresponds to the hypercharge D-term contribution to slepton and squark masses, and has no analogue in the single-messenger case. It generates scalar masses at one-loop, but its contribution is suppressed by a factor  $F^2/M^4$ . It also gives a two-loop contribution, which can be of the same order of magnitude as the  $\Lambda_S$  contribution for the right-handed charged sleptons, and smaller for the other supersymmetric particles. Therefore the minimal model mass relations are robust among supersymmetric particles with the same spin, with the exception of the right-handed slepton masses which are a good measure of possible hypercharge D-term contributions. These conclusions rely on the assumption that the strong dynamics responsible for messenger masses is invariant under the SU(5) symmetry or, in other words, that M and F are the same for the doublet and triplet components of the messenger five-plet at  $M_{GUT}$ . If this is not the case, the messenger sector must have a new "parity" symmetry, which forbids dangerous mass square contributions to scalar particles at one loop. This symmetry then guarantees the same mass relations among supersymmetric particles with the same spin as in the minimal model, while scalar and fermion masses depend on two different parameters.

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## References

- M. Dine, W. Fischler, and M. Srednicki, Nucl. Phys. B189 (1981) 575;
   S. Dimopoulos and S. Raby, Nucl. Phys. B192 (1981) 353;
   M. Dine and W. Fischler, Phys. Lett. B110 (1982) 227;
   M. Dine and M. Srednicki, Nucl. Phys. B202 (1982) 238;
   M. Dine and W. Fischler, Nucl. Phys. B204 (1982) 346;
   L. Alvarez-Gaumé, M. Claudson, and M. Wise, Nucl. Phys. B207 (1982) 96;
   C.R. Nappi and B.A. Ovrut, Phys. Lett. B113 (1982) 175;
   S. Dimopoulos and S. Raby, Nucl. Phys. B219 (1983) 479.
- M. Dine, A.E. Nelson, and Y. Shirman, *Phys. Rev.* D51 (1995) 1362;
   M. Dine, A.E. Nelson, Y. Nir, and Y. Shirman, *Phys. Rev.* D53 (1996) 2658.
- [3] G. Dvali, G.F. Giudice, and A. Pomarol, preprint CERN-TH/96-61, hep-ph/9603238.
- [4] M. Dine, Y. Nir, and Y. Shirman, preprint SCIPP 96/30, hep-ph/9607397.
- [5] S.P. Martin, preprint hep-ph/9608224.
- S. Dimopoulos, G.F. Giudice, and A. Pomarol, preprint CERN TH/96-171, hepph/9607225.
- S. Park (for the CDF Coll.), Proc. 10th Topical Workshop on Proton-Antiproton Collider Physics, eds. R. Raha and J. Yoh (AIP Press, New York, 1995).
- [8] S. Dimopoulos, M. Dine, S. Raby and S. Thomas, *Phys. Rev. Lett.* **76** (1996) 3494;
   S. Ambrosanio, G.L. Kane, G.D. Kribs, and S.P. Martin, *Phys. Rev. Lett.* **76** (1996) 3498;

S. Dimopoulos, S. Thomas, and J.D. Wells, preprint SLAC-PUB-7148, hep-ph/9604452;

S. Ambrosanio, G.L. Kane, G.D. Kribs, and S.P. Martin, preprint hep-ph/9605398.

[9] R. Barbieri and G.F. Giudice, Nucl. Phys. B306 (1988) 63.

## **Figure Captions**

Fig. 1: Two-loop Feynman diagrams contributing to sfermion  $(\tilde{f})$  masses.  $\Phi$ ,  $\bar{\Phi}$  and  $\psi_{\Phi}$ ,  $\psi_{\bar{\Phi}}$  denote the messenger scalar and fermionic components respectively.  $\lambda$  is the gaugino and the wavy lines correspond to gauge bosons.

Fig. 2: Masses of the right-handed  $(\tilde{e_R})$  and left-handed  $(\tilde{e_L})$  sleptons as a function of  $\Lambda_D^2/\Lambda_S^2$ . We have chosen a normalization such that  $m_{\tilde{e_R}} = 100$  GeV when  $\Lambda_D = 0$ .





Fig. 1



m<sub>r</sub> [GeV]