Basis-Independent Measures of R-Parity Violation

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Abstract

We construct basis-independent expressions that measure the magnitude of R-parity breaking due to possible superpotential terms in the Minimal Supersymmetric extension of the Standard Model, in the absence of soft supersymmetry-breaking terms and spontaneous gauge symmetry breaking. We also discuss briefly their application to a consistent treatment of cosmological constraints on R-parity violation.

Supersymmetry (SUSY) [1] is a popular extension of the Standard Model, which renders natural its large mass hierarchy: $M_P/M_W >> 1$. Since the number of particles in even the Minimal Supersymmetric extension of the Standard Model (MSSM) is more than doubled, a number of new tree-level interactions are allowed, some of which violate baryon and/or lepton number. A discrete symmetry called R parity is often imposed to forbid all such interactions. Alternatively, one may consider a broader class of theories containing such interactions, and constrain their coupling constants to be small enough that they do not conflict with experimental observations [2] and cosmological bounds [3].

The R-non-conserving interactions that violate lepton number have the important feature that they can be moved around the Lagrangian by field redefinitions [4], introducing a possible ambiguity that should be taken into account when evaluating these constraints. This is not in general a serious problem with accelerator bounds, because there it is clear that one is working in the mass-eigenstate basis. However, in cosmology the correct choice of basis is less clear, because the mass-eigenstate basis changes with temperature, and an interaction-eigenstate basis would be more natural. A desirable resolution of this ambiguity is to define expressions that are independent of the basis chosen for the fields to measure the amount of R violation, which is the purpose of this letter[1].

In the Minimal Supersymmetric extension of the Standard Model (MSSM) with R conservation, the superpotential can be written in the form

$$W = \mu_H \bar{H}H + h_u^{ij} \bar{H}Q_i U_j^c + h_d^{ij} HQ_i D_j^c + h_e^{ij} HL_i E_j^c \tag{1}$$

where $i, j = 1, ..., 3$ are quark and lepton generation indices. In addition to the supersymmetric interactions parametrized by (1), one must introduce soft SUSY-breaking terms which include

$$\text{mass terms} + B_H \bar{H}H + A_u^{ij} \bar{H}Q_i U_j^c + A_d^{ij} HQ_i D_j^c + A_e^{ij} HL_i E_j^c \tag{2}$$

as well as gaugino mass terms. We use capitalized fields to represent both the superfields (as in eqn (1)) and their scalar components (as in eqn (2)). Equation (2) and the Lagrangian derived from (1) are invariant under the multiplicatively-conserved Z_2 global symmetry $R = 3B + L + 2S$, where B is the baryon number of any field, L is its lepton number, and S is its spin.

Once we allow for interactions that do not conserve R, we can add four more B- or L-violating terms to the superpotential:

$$W_{\not{R}} = \epsilon_i HL_i + \lambda_1^{ijk} L_i L_j E_k^c + \lambda_2^{ijk} L_i Q_j D_k^c + \lambda_3^{ijk} U_i^c D_j^c D_k^c \tag{3}$$

In this letter we do not consider λ_3, and concentrate on the L-violating interactions $\epsilon, \lambda_{1,2}$. To appreciate the meaning of these terms, it is convenient to recall that the Higgs field H and the lepton doublets L_i have the same gauge quantum numbers. In the Standard Model, they are distinguished by the fact that the Higgs is a scalar, and the leptons are fermions carrying global quantum number that are automatically conserved if the neutrinos are massless. In the MSSM with the minimal superpotential (1), lepton number is still conserved. However, SUSY removes the distinction between H and the L_i based on their spins, and once we allow for R violation, there is no unambiguous distinction between the H and the L_i, and we can assemble them in a vector

$$\phi_I = (H, L_i) \tag{4}$$

whose index I runs from 0 to 3. A Lagrangian contains R violation if different interactions choose conflicting directions in ϕ space to be the Higgs.

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[1]Our approach is similar in philosophy to the analysis by Jarlskog and others [5] of basis-independent measures of CP violation in the Standard Model.

λ_3 as already mentioned, we may write the superpotential as

$$W = \mu^I \bar{H} \phi_I + \lambda_e^{IJk} \phi_I \phi_J E_k^c + \lambda_d^{Ijk} \phi_I Q_j D_k^c + h_u^{ij} \bar{H} Q_i U_j^c \tag{5}$$

This parametrization could be extended to discuss soft SUSY-breaking terms, but we shall not do so in this paper. The appearance of R violation can be moved around the Lagrangian by unitary transformations on the fields, depending on what interaction one uses to define the Higgs. One must therefore be careful to specify the basis in which one is working, or else discuss R violation in a basis-independent formalism.

In this letter, we opt for the latter, and construct combinations of coupling constants which are zero when R is conserved, and which are independent of the basis transformations in ϕ_I space. We normalise these invariants relative to the magnitudes of the superpotential couplings in (5), so that they provide some measure of the relative magnitude of R violation present. We present geometrical interpretations of these invariants, as well as generic diagrams that give rise to them. We then discuss briefly cosmological constraints on R violation, as an example how the invariants can be used. In this letter, we work in the absence of soft supersymmetry breaking and gauge symmetry breaking; a more complete analysis will be presented elsewhere [6].

2   Invariants in Simplified Models

As warm-up exercises which help provide some intuition, we first discuss two toy examples: first a one-generation model, and then a model with two generations of leptons, but only one generation of quarks.

Consider a one-generation version of the superpotential (5): this means that upper-case indices run from 0 to 1, and there are no lower-case indices. Since λ_e^{IJ} is antisymmetric in the indices I, J, there is no LLE^c interaction in this model, and the Yukawa coupling HLE^c is invariant under rotations in ϕ space. However, by its presence, the λ_e^{IJ} interaction forces one direction in ϕ space to be a lepton. The Higgs can be defined either from the $\mu^I \phi_I \bar{H}$ term, or from the $\lambda_d^I \phi_I Q D^c$ term. If, for instance, the basis in ϕ space is chosen such that $\mu^1 = 0$, then the superpotential becomes

$$W = \mu_H \bar{H} H + h_u \bar{H} Q U^c + h_d H Q D^c + h_e H L E^c + \lambda_2 L Q D^c \tag{6}$$

where the R violation is contained in

$$\lambda_2 = \frac{\mu^{0*} \lambda_d^1 - \mu^{1*} \lambda_d^0}{[|\mu^0|^2 + |\mu^1|^2]^{1/2}} \tag{7}$$

Alternatively, one could choose a (primed) basis such that $\lambda_d^{1'} = 0$, but $\mu^{1'} = -(\mu^{0*}\lambda_d^1 - \mu^{1*}\lambda_d^0)/[|\lambda_d^0|^2 + |\lambda_d^1|^2]^{1/2}$.

It is clear that R parity is broken in this model if two conditions are satisfied. Firstly, λ_e must be present, so that one direction in ϕ space is a lepton, and secondly μ and λ_d must choose different directions in ϕ space to be the Higgs. A measure of R_p violation is then the length of the component of λ_d that is orthogonal to μ: $\mu^{0*}\lambda_d^1 - \mu^{1*}\lambda_d^0$, as illustrated in Fig. 1(a). It is helpful to normalize this measure by dividing it by the lengths of the vectors μ and λ_d:

$$\epsilon = \frac{\mu^{0*}\lambda_d^1 - \mu^{1*}\lambda_d^0}{|\mu||\lambda_d|} \tag{8}$$

The basis-dependent R-violating coupling $\mu^{1'}$ is then $|\mu| \times \epsilon$, and similiarly $\lambda_2 = |\lambda_d| \times \epsilon$.

Another way to express the invariant measure of R violation in this model is

$$\epsilon = \frac{\mu^{I*} \lambda_e^{IJ} \lambda_d^{J*}}{|\mu||\lambda_e||\lambda_d|} \tag{9}$$

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of including also the Yukawa coupling, which must be present for the model to violate R. We prefer this formulation, because it can readily be generalized to more generations.

In fact, it is more appropriate to use the square of this expression as a measure of R violation, because it corresponds to a closed-loop Feynman diagram (see Fig. 1(b)), such as would appear in a zero temperature rate calculation. We therefore advocate

$$\delta_1 = \frac{|\mu^{I*}\lambda_e^{IJ}\lambda_d^{J*}|^2}{|\mu|^2|\lambda_e|^2|\lambda_d|^2} \tag{10}$$

as the invariant measure of R violation in this simple one-generation model.

We now discuss the second toy model. It has two lepton generations, so the matrices λ_e^{IJp} enter in a non-trivial way, but the ϕ space is three-dimensional, so that it is easy to understand geometrically the invariants found below. Suppose one has a superpotential of the form (5), with upper case indices that run from 0 to 2 and lower case lepton indices that run from 1 to 2. The coupling constants define a number of directions in ϕ space: μ and λ_d correspond to directions that one could choose to be the Higgs, and each λ_e determines a plane spanned by a Higgs and a lepton. The intersection of the two planes chooses a direction for the Higgs. Each plane can therefore be written as the cross product of the Higgs and an orthogonal vector corresponding to the lepton:

$$\lambda^{IJp} = L^{Ip}H^J - H^I L^{Jp} \tag{11}$$

As vectors in ϕ space, the L^{Ip} depend on the choice of basis for the right-handed leptons. To see this, imagine choosing the Higgs to correspond to the direction ϕ^0; the index I on L^{Ip} then runs from 1 to 2, like p, and L^{Ip} has the same form as the 2×2 Yukawa matrix. It can be diagonalised by independent unitary transformations on the left- and right-handed fields (i.e., on the I and p indices), which means that the two L^{Ip}, regarded as vectors in ϕ^I space, are orthogonal only for a specific choice of basis for the p indices.

This model violates R parity if the direction chosen by one interaction to be the Higgs has a non-zero inner product with the direction chosen by another interaction to be a lepton. There are therefore three scalars one can construct that measure $\not R$. If one projects μ onto the plane associated with one of the Yukawa couplings, it picks out the direction in the plane to be identified with the Higgs. The orthogonal direction is therefore a lepton, and its inner product with λ_d: $\mu^I \lambda_e^{IJp} \lambda_d^J$, is a measure of R violation, see Fig. 1(c). This expression is not yet completely satisfactory, because it carries a right-handed lepton index p, and so is not fully basis-independent. This defect can be remedied by multiplying it by its complex conjugate, and summing over p:

$$\delta_1 = \frac{(\mu^{I*}\lambda_e^{IJp}\lambda_d^{J*})(\mu^M \lambda_e^{MNp*}\lambda_d^N)}{|\mu|^2|\lambda_e|^2|\lambda_d|^2} \tag{12}$$

This measure of R violation is invariant under all basis transformations and corresponds to the closed supergraph shown in Fig. 1(b). However, we should emphasize that one disadvantage of summing over right-handed lepton indices is that we now only have a measure of total L violation, and are not able to determine whether one of the individual L_i might be conserved.

We see also that R is not conserved if the direction chosen to be the Higgs by the two Yukawa couplings has a component perpendicular to μ or λ_d, as seen in Fig. 2(a). Let us consider μ (the invariant for λ_d will be a straightforward modification). We wish to know the length of the component of μ in the plane spanned by the two leptons L^p. It is clear that, up to an overall normalization, this is

$$|L^1|^2|\mu \cdot L^2|^2 + |L^2|^2|\mu \cdot L^1|^2 - 2(\mu \cdot L^1)(L^1 \cdot L^2)(\mu \cdot L^2) \tag{13}$$

where the ϕ indices are suppressed. Note that L^1 and L^2 are not orthogonal in a generic basis. Writing this expression in terms of the matrices λ_e^{IJp}, we get

$$\delta_2 = \frac{\mu^{I*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_e^{KLq}\lambda_e^{LMp*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJp}\lambda_e^{JKq*}\mu^K \lambda_e^{LMp*}\lambda_e^{LMq})}{|\mu|^2 Tr[\lambda_e^{p*}\lambda_e^p \lambda_e^{q*}\lambda_e^q]} \tag{14}$$

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tarised indices, and the lower case indices are also summed. A similar expression for χ_d, corresponding to the difference between the supergraphs of Fig. 3, gives

$$\delta_3 = \frac{\lambda_d^{Irs*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_e^{KLq}\lambda_e^{LMp*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_d^{Krs}\lambda_e^{LMp*}\lambda_e^{LMq})}{|\lambda_d|^2 Tr[\lambda_e^{p*}\lambda_e^{p}\lambda_e^{q*}\lambda_e^{q}]} \tag{15}$$

which completes our enumeration of measures of R violation in this second toy model, to this order in the couplings.

3 Three-Generation Model

We are now in a position to generalize to the physical situation of three lepton generations. In this case, not only are the Yukawa coupling matrices λ_e^{IJp} non-invariant under ϕ basis transformations, but they can also contain R violation independently of the other interactions present. In the limit where there is no R, they define a plane in ϕ space that is spanned by the Higgs and a lepton. The μ^I and λ_d^{Ijp} again define directions in ϕ space that one would like to interpret as Higgs fields. In the absence of R violation, these two directions coincide with the Higgs direction chosen by the planes of the three leptonic Yukawa matrices λ_e^{IJp}.

The simultaneous presence of at least three interactions is required in order to provide R violation. 1) At least one Yukawa coupling must be present, so as to force one plane in ϕ space to contain a lepton and a Higgs. 2) The second interaction can choose which direction in this plane is the Higgs. 3) The third interaction must then specify a direction that conflicts with this choice. Since one must square the amplitude in order to obtain a rate, our invariants involve at least six coupling constants.

For simplicity, we do not consider expressions involving more than the minimum three coupling constants required to obtain R violation. The list we now present consists of all the invariants that can be constructed out of three coupling constants, assuming that at least one of them is a λ_e and summing over all three generations. This list is complete, in the sense that at least one of the invariants is non-zero if the theory contains R violation at this order.

In an exactly supersymmetric theory, the following squares of combinations of three coupling-constant matrices are invariant under transformations in ϕ space, and zero in an R-conserving theory[2]:

$$\delta_1 = \frac{(\mu^{I*}\lambda_e^{IJp}\lambda_d^{Jqr*})(\mu^K\lambda_e^{KLp*}\lambda_d^{Lqr})}{|\mu|^2|\lambda_e|^2|\lambda_d|^2} \tag{16}$$

$$\delta_2 = \frac{\mu^{I*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_e^{KLq}\lambda_e^{LMp*}\mu^M - 1/2(\mu^{I*}\lambda_e^{IJp}\lambda_e^{JKq*}\mu^K\lambda_e^{LMp*}\lambda_e^{LMq})}{|\mu|^2 Tr[\lambda_e^{p*}\lambda_e^{p}\lambda_e^{q*}\lambda_e^{q}]} \tag{17}$$

$$\delta_3 = \frac{\lambda_d^{Irs*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_e^{KLq}\lambda_e^{LMp*}\lambda_d^{Mrs} - 1/2(\lambda_d^{Irs*}\lambda_e^{IJp}\lambda_e^{JKq*}\lambda_d^{Krs}\lambda_e^{LMp*}\lambda_e^{LMq})}{|\lambda_d|^2 Tr[\lambda_e^{p*}\lambda_e^{p}\lambda_e^{q*}\lambda_e^{q}]} \tag{18}$$

$$\delta_4 = \frac{Tr[\lambda_e^{p}\lambda_e^{q*}\lambda_e^{r}\lambda_e^{r*}\lambda_e^{q}\lambda_e^{p*}] + Tr[\lambda_e^{p}\lambda_e^{q*}\lambda_e^{r}\lambda_e^{p*}\lambda_e^{q}\lambda_e^{r*}] - Tr[\lambda_e^{p}\lambda_e^{q*}\lambda_e^{r}\lambda_e^{p*}]Tr[\lambda_e^{q}\lambda_e^{r*}]}{Tr[\lambda_e^{p}\lambda_e^{p*}\lambda_e^{r}\lambda_e^{r*}\lambda_e^{q}\lambda_e^{q*}]} \tag{19}$$

$$\delta_5 = \frac{(\lambda_d^{Ist*}\lambda_e^{IJp}\lambda_d^{Jqr*})(\lambda_d^{Kst}\lambda_e^{KLp*}\lambda_d^{Lqr}) + (\lambda_d^{Ist*}\lambda_e^{IJp}\lambda_d^{Jqr*})(\lambda_d^{Kqt}\lambda_e^{KLp*}\lambda_d^{Lsr})}{|\lambda_e|^2|\lambda_d|^4} \tag{20}$$

The traces are over the capitalized ϕ indices. The lower-case indices correspond to quark and right-handed lepton generations, and are also summed. The numerators correspond to the closed super-graphs of Figures 1, 2, 3, 4 and 5, respectively. The magnitudes of the matrices and vectors in the

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[2]They also give the above two-generation expressions in the limit where one generation's interactions are set to zero.

$$|\mu|^2 = \sum_I \mu_I \mu_I^* \tag{21}$$

$$|\lambda_d|^2 = \sum_{Ipq} \lambda_d^{Ipq} \lambda_d^{Ipq*} \tag{22}$$

$$|\lambda_e|^2 = \sum_{IJr} \lambda_e^{IJr} \lambda_e^{IJr*} \tag{23}$$

There are two new invariants in this three-generation model, shown in Figures 4 and 5. The Yukawa couplings λ_e now contain R violation all by themselves, which is measured by λ_4. It is also possible that the various λ_d (there are nine of them in the case of three quark generations) might choose different directions in ϕ space to be the Higgs: δ_5 measures the component of λ_d^{st} perpendicular to λ_d^{qr}.

One can see by inspection that these combinations are zero in an R-conserving theory, and contain the minimal (because the invariants correspond to squares of matrix elements) two powers of the R-violating coupling constants from equation (3).

4 Cosmological Applications of the Invariants

The uses of the invariants we have defined above are limited by the fact that we have neglected soft supersymmetry breaking and have not included the Higgs vev. However, these restrictions do not hinder application in the early Universe above the electroweak phase transition (EPT). A limitation that is more serious for this application is that we have also summed over the quark and right-handed lepton indices. We plan to address this and the previous deficiencies in a subsequent publication [6]. However, we can already use the above invariants to discuss cosmological constraints on R violation, providing we restrict ourselves to sums over quarks and right-handed leptons. We will first review these cosmological constraints, then calculate them in a one-generation model, where there are no unwanted sums, and finally discuss the constraints on our invariants in the three-generation case.

In many cosmological scenarios, the survival of the baryon asymmetry can provide strong bounds on R violation. If the asymmetry was created before the EPT, it would have to contend with the anomalous electroweak $B+L$-violating processes known to be present in the thermal bath before the EPT. For a baryon asymmetry to survive, there would have to be a $B-L$ asymmetry. If R-violating interactions were also present in the thermal bath, the asymmetry would be washed out if they took $B-L$ to zero. Requiring all R-violating interaction rates to be less than the expansion rate, so that they are unable to perpetrate this catastrophe, may yield strong bounds on the coupling constants of (3). However, there are two loopholes in this argument: the baryon asymmetry may be made at the EPT, or the R-violating interactions that are in thermal equilibrium may only wash out one lepton number, e.g., L_1, and not wash out $B-L$. For the purposes of this section, we neglect these loopholes.

We require all R-violating interactions to be out of thermal equilibrium at $T \sim 100$ GeV, which we assume to be of the same order as the supersymmetric particle masses and the EPT temperature. We assume that the supersymmetric particles are present with a density $\sim T^3$ in the thermal bath, and neglect mass effects, whose inclusion would change the bounds only slightly [3]. The supersymmetry-breaking scalar masses are not present in the invariants, so it is consistent to neglect the contribution of these masses to R-violating rates.

As an example of the estimation of such a rate, we consider δ_1 (16), first in the one-generation model, and then with three generations. In the one-generation case, it is clear that if all of the rates associated with the interactions μ, λ_e and λ_d are not in equilibrium, i.e., greater than the expansion rate H, the corresponding coupling constant can be effectively neglected. In fact, all the

In the one-generation toy model, the R-violating rate is

$$\Gamma_{\not R} \simeq \delta_1 \times \Gamma_{min} \tag{24}$$

where Γ_{min} is the smaller of the rates associated with $|\mu|$ and $|\lambda_d|$. As discussed in section 2, the R-violating coupling constants in the one-generation model are μ^1 and/or λ_d^1: λ_e enters into δ_1 only to ensure that some direction in ϕ space is a lepton. We constrain (24) to be out of equilibrium.

The rates associated with $|\mu|$ and $|\lambda_d|$ are estimated [3] to be:

$$\Gamma_\mu \simeq 10^{-2} \frac{\mu^2}{T}, \qquad \Gamma_{\lambda_d} \simeq 10^{-2} \lambda_d^2 T \tag{25}$$

Taking $\mu \sim 100$ GeV, $\lambda_d \sim h_b \sim 3 \times 10^{-2}$, the bound $\delta_1 \Gamma_{\lambda_d} < H$ yields

$$\frac{(\mu^1 \lambda_d^0 - \lambda_d^1 \mu^0)^2}{|\mu|^2} < 10^{-11} \tag{26}$$

If we assume self-consistently that R violation is small, we can work in the MSSM thermal mass eigenstate basis, as is appropriate just before the electroweak phase transition. In this basis (26) gives the bounds

$$\frac{\mu^1}{\mu^0} < 3 \times 10^{-6}, \qquad \lambda_d^1 < 10^{-7} \tag{27}$$

Note that this bound on μ^1 is considerably weaker than what one would obtain naively by requiring the rate associated with a mass term [3] to be out of equilibrium:

$$\Gamma \sim 10^{-2} \left(\frac{\mu^i}{\mu^0} \right)^2 T \lesssim H \tag{28}$$

which would give $\mu^i \lesssim 10^{-5}$ GeV.

In our invariant approach, it is easier to understand when R violation is (or is not) in equilibrium in the early Universe. In the basis-dependent approach, one can move the R violation from the mass terms to the Yukawa terms. Interaction rates associated with masses and Yukawa couplings scale differently with temperature, so the temperature at which the R violation goes in or out of equilibrium appears to depend on the basis one chooses. This confusion is avoided by noting that all the interactions that enter into the definition of an invariant must be in equilibrium, as well as the invariant times one of the rates, e.g., (24).

In the case of three generations, the invariants we have defined cannot be used to estimate rates so easily, because we have summed over the quark and right-handed lepton-generation indices. These sums made the invariants independent of basis transformations on all the fields. However, since it is only the rotations in ϕ space that move the R violation around, for practical purposes we can drop the sums over the quark and right-handed lepton indices. The "invariants" are then basis-dependent, and we compute them in the thermal mass-eigenstate basis for quarks and right-handed leptons, which should be the right basis in which to compute rates in the early Universe. In principle, it would be better to quote bounds on the "invariants", as in (26),rather than on terms from the sum making them up, as in (27) because there could be cancellations between the different contributions. However, assuming such cancellations do not take place, the bounds we now present are easier to interpret.

We consider again, as an example, δ_1. In the absence of the sums over lower-case indices, we now have 27 invariants: there are three λ_e, and nine λ_d, because the leptonic Yukawa-coupling matrices are diagonal in the thermal mass-eigenstate basis, but the quark Yukawa couplings are not. The R-violating entries in λ_e do not contribute to δ_1 at lowest order, so we neglect them. In the mass-eigenstate basis, λ_e^p will therefore pick out the plane in ϕ space spanned by the Higgs and the lepton

that how there are nine possible vectors λ_d in ϕ space. For each r and s, we have

$$\frac{(\mu^p \lambda_d^{0rs} - \mu^0 \lambda_d^{prs})^2}{|\mu|^2 |\lambda_d^{rs}|^2} \times 10^{-2} |\lambda_d^{rs}|^2 T < H \tag{29}$$

at $T \sim 100$ GeV. If one chooses to work in the MSSM thermal mass-eigenstate basis, i.e., assume that R violation is small, one gets the following generic basis-dependent bounds:

$$\frac{\mu^p}{\mu^0} < 3 \times 10^{-6} \;, \qquad \lambda_d^{prs} < 10^{-7} \tag{30}$$

where p, r, s run from 1 to 3, and we have assumed that $\lambda_d^{011} \simeq h_b \simeq 3 \times 10^{-2}$, where h_b is the bottom-quark Yukawa coupling.

Similarly, we do not sum over the quark and right-handed lepton indices in the invariants $\delta_2, \delta_3, \delta_4$ and δ_5, but instead compute "invariants" with the quarks and right-handed leptons in the thermal mass-eigenstate basis. Requiring that the rates associated with these "invariants" be smaller than the expansion rate of the Universe at $T \sim 100$ GeV, we obtain the generic bound

$$\lambda \lesssim 10^{-7} \tag{31}$$

where λ is some R-violating coupling in the MSSM thermal mass-eigenstate basis. We also get a bound on μ^i/μ^0 from δ_2, but it is weaker than (30) because the bottom-quark Yukawa coupling is larger than that of the τ.

Our generic bounds on the R-violating dimensionless coupling constants λ_1, λ_2 (see equation (3)) are the same as previously calculated [3]. The constraint on μ^i is weaker, because the bound had previously been estimated by requiring that $(\mu^1/\mu^0)^2 \alpha T < H$. However, since the gauge interactions are diagonal in ϕ space, they do not mix the different ϕ^I, so the bound on μ^i comes from the mass corrections to the Yukawa interactions, as demonstrated by our analysis of invariants[3].

5   Conclusions

We have presented in this paper an analysis of R violation in supersymmetric models that is formulated in terms of basis-independent measures, in analogy with the treatment of CP violation in [5]. The enumeration of these measures is quite complicated in general, and we have presented here a simplified analysis in which soft supersymmetry-breaking masses and Higgs v.e.v.'s are neglected. This restriction still permits R-violating rates in the early Universe to be discussed, and we have presented an analysis of the generic bounds obtained in one- and three-generation models, using all the invariant measures of R-violating rates that appear up to sixth order in the couplings. Our upper limits on generic Yukawa couplings are similar to those obtained previously [3], but our bounds on the supersymmetric mass-mixing parameters μ^i are less restrictive.

We have not studied in this paper loopholes in these cosmological bounds, which may be exploited if the baryonic asymmetry is generated at the EPT, or if there is a conserved quantum number or flavour symmetry that preserves a pre-existing baryon asymmetry, or if there are cancellations among the different contributions to our invariants. These and the inclusion of soft supersymmetry breaking and Higgs v.e.v.'s remain to be analyzed in future work [7].

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[3]Note that this would not be the case for cosmological bounds on an R-violating soft supersymmetry-breaking mass. In this case, the mass-eigenstate bases for the fermions and scalars are different, so the D-term interactions violate R and the bound on the R-violating soft supersymmetry-breaking masses would be of order (28). If μ^i is rotated away in a theory with soft supersymmetry-breaking masses, it generates R-violating scalar masses proportional to soft mass differences, so the bound (28) would still be overly optimistic, assuming that these mass differences are small.

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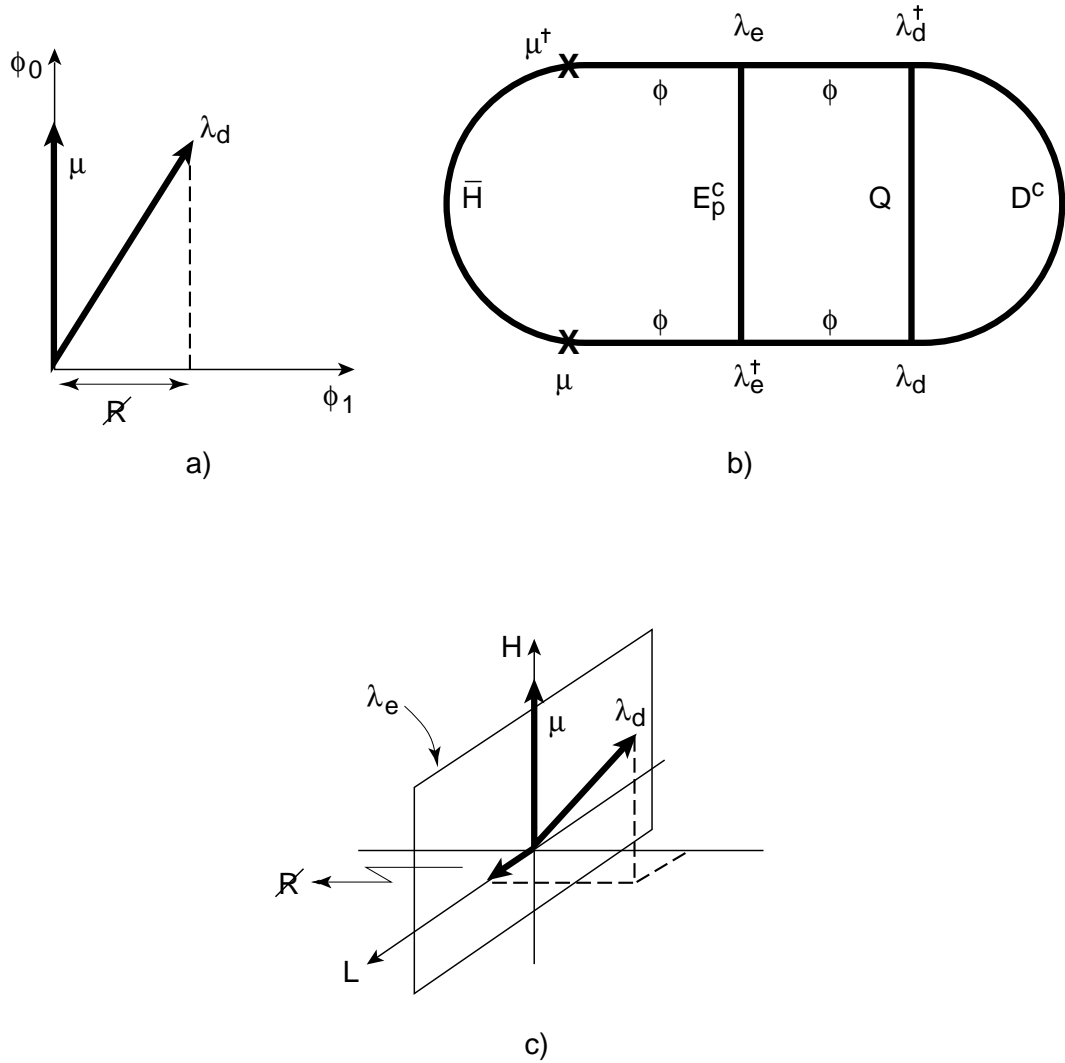[7] B.A. Campbell, S. Davidson, J. Ellis and K.A. Olive, work in progress.

Figure 1: A geometric interpretation of the invariant $\mu^\dagger \lambda_e \lambda_d^*$, and the associated supergraph: (a) is for a one generation model - R parity is not conserved if μ and λ_d do not choose the same direction in ϕ space to be the Higgs. (b) The supergraph corresponding to the invariant δ_1. With multiple generations, there is a sum of diagrams, corresponding to the sum over the right-handed lepton index p. (c) A geometric interpretation of $\mu^\dagger \lambda_e^p \lambda_d^*$ in a two-generation model. The matrix λ_e^p chooses a plane in ϕ space, spanned by a Higgs and a lepton. The projections of μ and λ_d onto that plane each choose directions that should be the Higgs - there is R violation if these directions are not parallel.
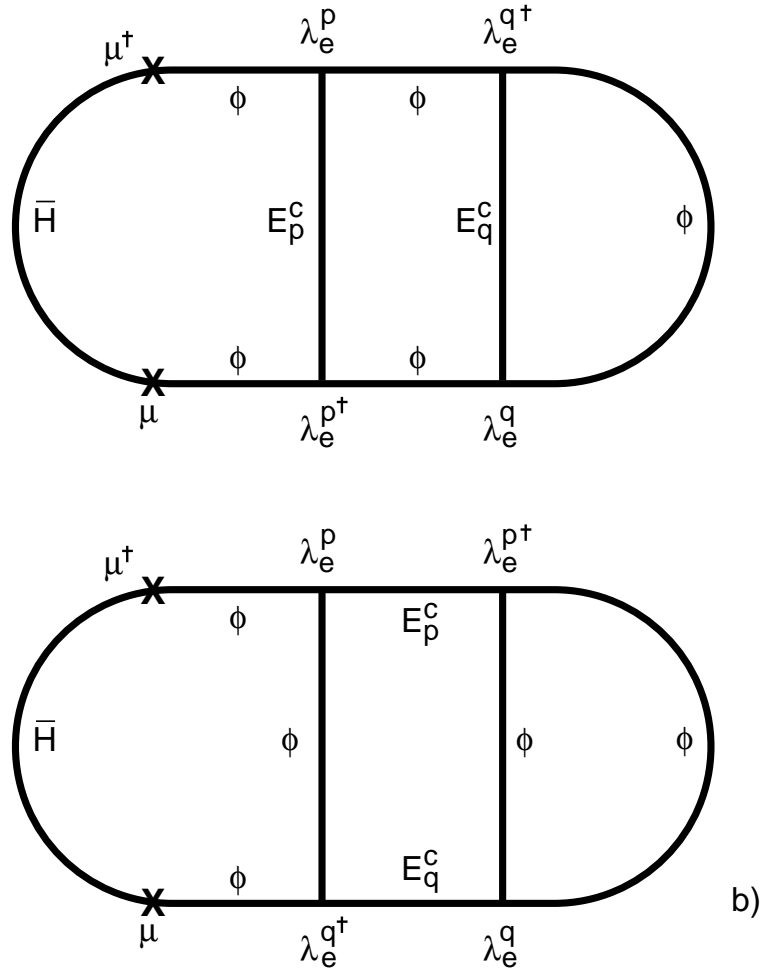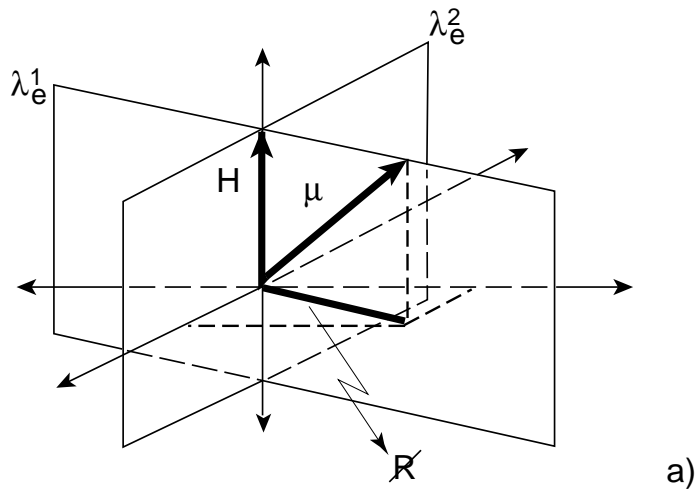
Figure 2: (a) A geometric interpretation of the invariant δ_2 in a two-generation model, before the sum over right-handed lepton indices. Each matrix λ_e^p corresponds to a plane in ϕ space. The intersection of the planes corresponds to the direction that is chosen to be the Higgs by the λ_e matrices. If this direction is not parallel to the direction chosen by μ, there is R-parity violation. (b) The supergraphs for δ_2: the right-handed lepton indices p and q are summed.
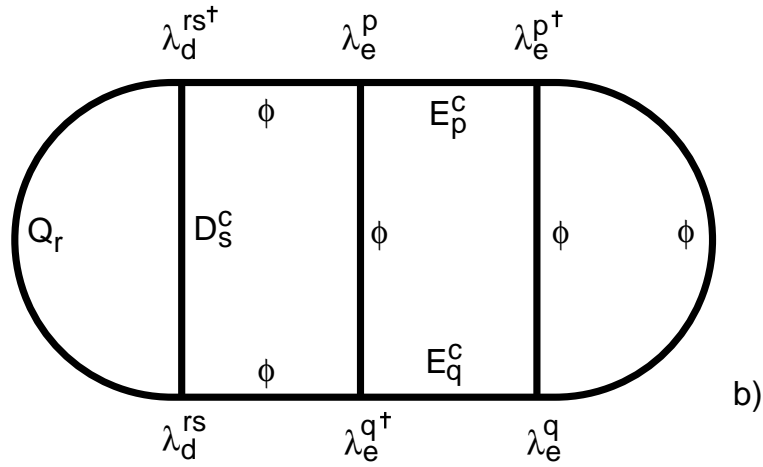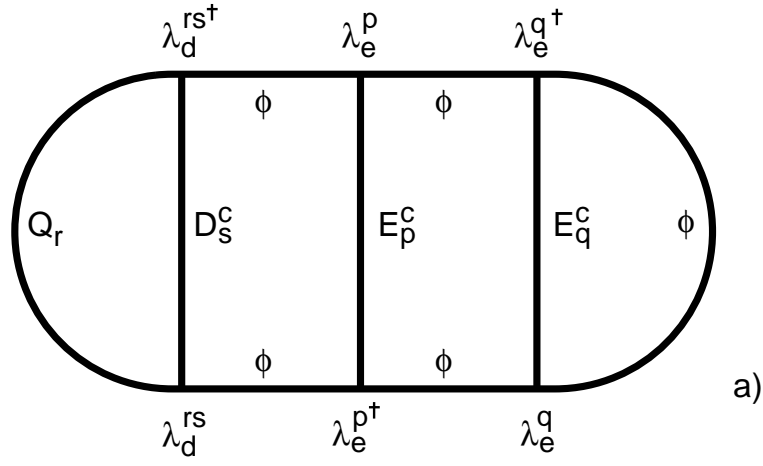
Figure 3: Supergraphs corresponding to the invariant δ_3: the right-handed lepton indices p and q and the quark indices r and s are summed.
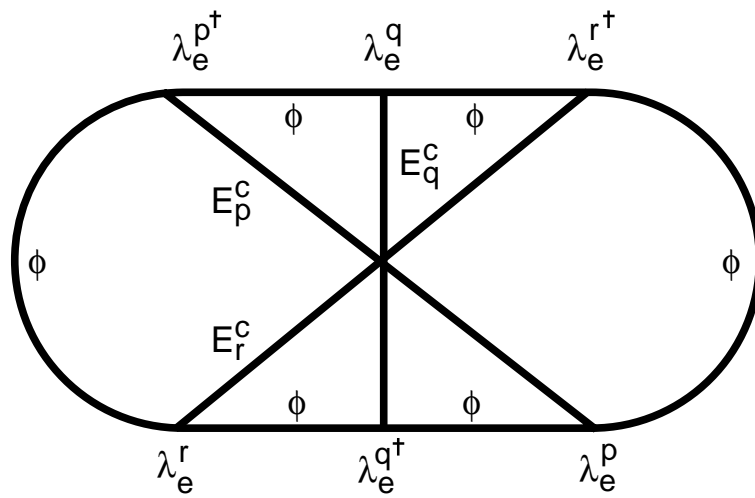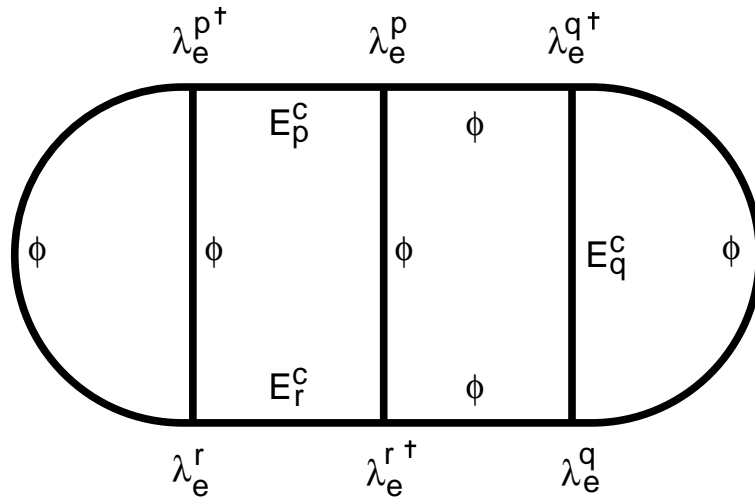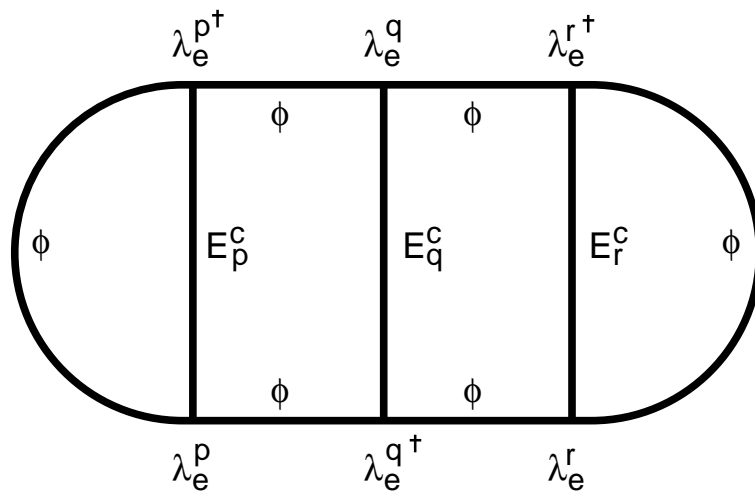
Figure 4: Supergraphs corresponding to the invariant δ_4: the right-handed lepton indices p, q and r are summed.
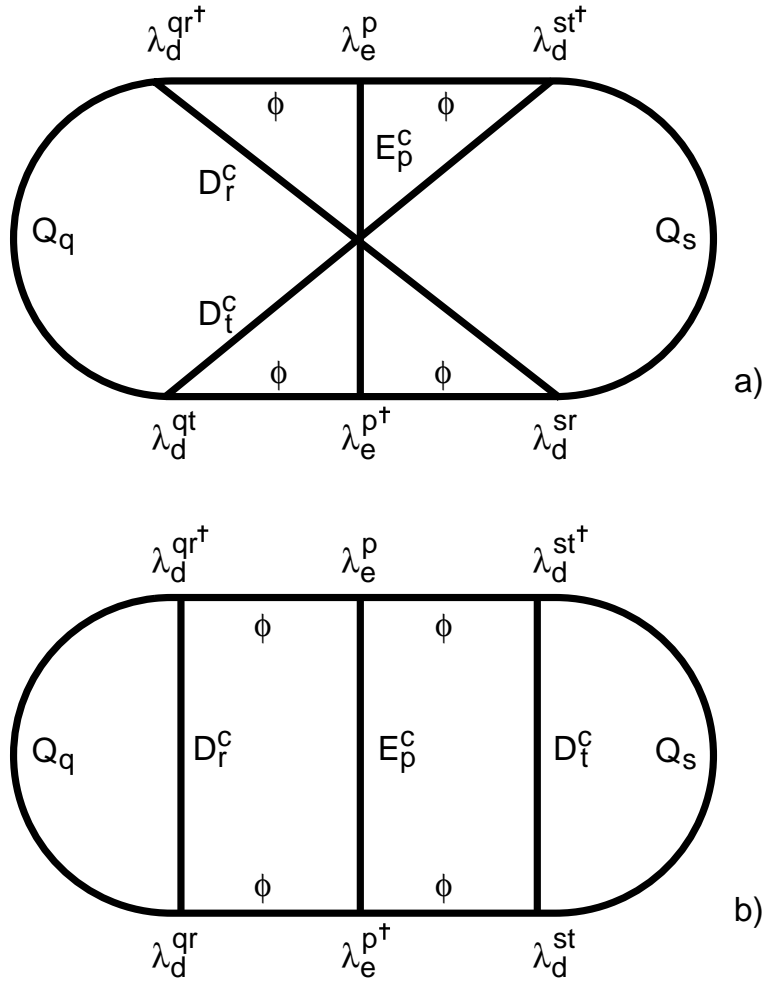
Figure 5: Supergraphs corresponding to the invariant δ_5: the right-handed lepton index p and the quark indices q, r, s and t are summed.