

# MULTIBUNCH BNS DAMPING AND WAKEFIELD ATTENUATION IN HIGH FREQUENCY LINACS

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## Abstract

In high frequency linacs, where the wakefields are strong, the stability of a train of bunches is critical. It was therefore important for the Compact Linear Collider study (CLIC) to investigate numerically and theoretically this question. Basically, two methods of controlling beam break-up have been considered; firstly a multibunch generalization of the BNS damping principle and secondly the attenuation of the long-range fields as it results from damping or staggered tuning of the accelerating sections. Simulation codes have been written for both checking the theoretical predictions and investigating the requirements associated with a possible application to the CLIC main linac.

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# 1 Introduction

The question of the stability of a train of bunches in a high frequency linac is important in the framework of the Compact Linear Collider (CLIC) study. Indeed, though the single bunch mode of CLIC already provides an usable luminosity, a large fraction of the research planed in the physics experiments requires even higher luminosities with small energy spread, that can only be reached in a multibunch mode, if we want to restrict the necessary RF power. The more recent lists of parameters [1,2] therefore take this into account and are based on the use of at least five to ten bunches per pulse or more.

In these new conditions, the possible instability of the bunch train often called beam break-up and associated with the long-range dipole modes of transverse wakefields has to be cured. If not, the transverse beam modulation is carried along the linac from accelerating section to accelerating section, through the beam. Beam blow-up then occurs and manifests itself along the linac as an amplitude growth from the head to the tail of the bunch train, decreasing thereby the luminosity. Different remedies can be considered.

The first two possibilities envisaged in most designs consist of attenuating the long-range fields seen by the trailing bunches, either by damping or by detuning of the RF cavities [3]. In damped structures there are modified disk-loaded wave-guides in which the power is coupled out to lossy regions, whereby the quality-factor of the indeseirable modes is strongly lowered. In staggered tuned sections, the cell dimensions are varied in order to spread the dipole mode frequencies and to induce a roll-off of the wake fields behind each bunch. In the design of such structures, it is essential to have an estimate of the field attenuation required to stabilize a given bunch train. In order to obtain this information for the CLIC parameters of LC95 [4], it was necessary to develop a numerical code with long-range wakefields in addition to the short-range ones and with multibunch treatment, using different wakefield models as described below.

Another possibility [3], never seriously considered because of its apparent difficulty to be implemented, is based on the generalization of BNS damping to a train of bunches. Since the train in CLIC is limited to a relatively small number of bunches, the chance of stabilizing the beam is a priori better and it was therefore interesting to study this idea in spite of the expected difficulties. Such a principle study obliged us to first develop the necessary theory dealing with multibunch BNS damping using microwave quadrupoles and strong focusing. Then, the specific code developed for multibunch tracking could be used for checking the principles of this method as well as the validity of the theoretical predictions.

In this report, we first deal with the general theory of BNS damping and its extension to multibunch operation. Then the multibunch numerical simulation codes MBTRACK and WAKET are described. Later, the numerical tests on BNS damping and on wakefield attenuation effects with a simplified model of a multibunch train are reported, so that we can draw out some conclusions about the limitations of the former approach and the attenuation requirements in the latter.

## 2 Short recapitulation of the general theory of BNS damping

Following Ref. 5, we write the integro-differential equation of second order for a 'slice of charge' in a bunch traveling at relativistic speed along a linac and feeling the focusing magnetic forces as well as the force of the tranverse wakefields inside the bunch.

$$\gamma(s)x''(s, z) + \gamma'(s)x'(s) + k^2(s, z)\gamma(s)x(s, z) = \frac{e^2}{m_0c^2} \int_{-\infty}^z \rho(z^*)W(z - z')x(s, z^*)dz^* \quad (1)$$

In this equation  $x$  denotes the transverse displacement of a slice of charge in the bunch. The independent variables  $s$  and  $z$  are there for the position of the bunch inside the linac and the

relative position inside the bunch respectively; the origin  $z = 0$  corresponds to the center of the bunch. The quantities  $e$ ,  $m_0$  and  $c$  represent the elementary charge, the rest mass of the electron and the light velocity while  $\gamma(s)$  is the energy of the bunch as function of the position in the linac and therefore is given by

$$\gamma(s) = \gamma_0 + Gs \quad (2)$$

Here  $\gamma_0$  is the energy of the injected bunch and  $G$  the accelerating gradient of the linac. The derivative  $'$  is always taken with respect to  $s$ . The function  $\rho(z)$  describes the particle density as function of the position inside the bunch while  $W$  stands for the transverse wake potential per unit length produced by a point charge of unit value [5]. The 'homogeneous part' of equation (1) contains a damping term proportional to  $\gamma'(s)$  which can be suppressed by using the following ansatz for  $x$ :

$$x(s, z) = a(s, z) \exp[b(s, z)] \quad (3)$$

Inserting (3) into (1) we find

$$\gamma a'' + [2\gamma b' + \gamma']a' + [\gamma b'' + \gamma b'^2 + \gamma' b' + \gamma k^2]a = \frac{e^2 \exp(-b)}{m_0 c^2} \int_{-\infty}^z \rho(z^*) W(z - z^*) a(s, z^*) \exp(b) dz^* \quad (4)$$

Now we determine the function  $b$  such as to suppress the  $a'$  term of this equation, hence

$$b' = -\frac{1}{2} \frac{\gamma'}{\gamma} \implies b(s) = \ln \frac{1}{\sqrt{\gamma}} \quad (5)$$

Then the coefficient of  $a$  becomes

$$\dots + [\gamma k^2 + \frac{\gamma^2}{4\gamma}] a \dots \quad (6)$$

Note that for obtaining this expression we have to evaluate

$$b'' = -\frac{d}{ds} \left( \frac{\gamma'}{2\gamma} \right) = -\frac{2\gamma\gamma'' - 2\gamma'^2}{4\gamma^2} = \frac{1}{2} \frac{\gamma'^2}{\gamma^2} \quad (7)$$

since from (2) follows that  $\gamma'' = 0$ . The function  $b$  just depends on  $\gamma$  and (because of (2)) only on the independent variable  $s$ . So we may extract  $\exp(b)$  from the integral in (4) where it is canceled by the factor  $\exp(-b)$ . After a division by  $\gamma$  Eq. (4) becomes eventually,

$$a'' + \left[ k^2(s, z) + \frac{1}{4} \frac{\gamma'^2}{\gamma^2} \right] a = \frac{e^2}{\gamma m_0 c^2} \int_{-\infty}^z \rho(z^*) W(z - z^*) a dz^* \quad (8)$$

The basic idea of BNS-damping [6] is to cancel the wakefield effect by a proper choice of the focusing force along the single bunch. As we can see for instance from [5] this is in principle possible. Using our undamped version of the equation for  $a$  we write the focussing force  $k^2(s, z)$  as

$$k^2(s, z) = k_0^2(s) [1 + f(z)] \quad (9)$$

so that Eq. (8) becomes

$$a'' + \left[ k_0^2(s, z) + \frac{1}{4} \frac{\gamma'^2}{\gamma^2} \right] a = \frac{e^2}{\gamma m_0 c^2} \int_{-\infty}^z \rho(z^*) W(z - z^*) a(s, z^*) dz^* - k_0^2(s) f(z) a(s, z) \quad (10)$$

A coherent oscillation of all bunchslices can be obtained if the right hand side of the above equation becomes identical to zero i.e.

$$f(z) = \frac{e^2}{\gamma m_0 c^2 k_0^2(s) a(s, z)} \int_{-\infty}^z \rho(z^*) W(z - z^*) a(s, z^*) dz^* \quad (11)$$

In the coherent limit we may cancel  $a(s, z)$  and  $a(s, z^*)$  in the above equation because in this limiting case  $a$  only depends on  $s$ . Hence,

$$f(z) = \frac{e^2}{\gamma(s)m_0c^2k_0^2(s)} \int_{-\infty}^z \rho(z^*)W(z - z^*)dz^* \quad (12)$$

In order to cause no contradiction with Eq. (9) which states that  $f$  should only depend on the position in the bunch  $z$  we have to cancel the explicit  $s$  dependence in Eq. (14). This leads to a necessary condition for BNS-damping

$$k_0^2(s) = \frac{C}{\gamma(s)} = \frac{1}{\gamma_0 + Gs} \quad ; \quad C = \text{Constant} \quad (13)$$

which is also given in [5,7]. The final condition for  $f(z)$  then becomes

$$f(z) = \frac{e^2}{m_0c^2C} \int_{-\infty}^z \rho(z^*)W(z - z^*)dz^* \quad (14)$$

If we find means to realize this function  $f(z)$  in a real linac we may exactly cancel the wake effect and we remain with a bunch in which all slices of charge oscillate in a coherent way.

### 3 Extension of BNS damping to multibunch operation

We now show how the method of BNS damping can be extended to the situation of a train of bunches moving along the LINAC and acting on each other via the long range wakefields. The underlying model is the one of a 'superbunch' consisting of a certain number of equally spaced Delta Function charges representing the single bunches. Of course we may apply the same equation (1) to this problem but now the integration along the bunch coordinate  $z$  is transformed into a finite sum over the different single bunch contributions. Then we define a new correcting focusing function  $f(z)$  which should be chosen such as to cancel as well as possible the noncoherent part of the multibunch oscillations. Assuming equidistant bunches separated by  $\Delta z$  and with identical charge  $q$ , we can write the value that should be taken by  $f(z)$  at every bunch  $i$  of the train

$$f(z_i) = f_i = \frac{Qe}{\gamma m_0 c^2 C} \sum_{n=1}^{i-1} W_T[n\Delta z] \quad (15)$$

For  $i = 1$  (Bunch #1) this sum has to be set equal to zero since this bunch is not exposed to any longrange wakefield. The quantity  $Q$  stands for the total charge of one bunch. It has to appear because the integral over one Delta Function bunch

$$\int_{z_i-\epsilon}^{z_i+\epsilon} \rho(z)dz = N \quad ; \quad Ne = Q \quad (16)$$

where  $N$  is the number of particles in one bunch. With the further assumption that the bunch separation  $\Delta z$  is a multiple of the RF wavelength  $\lambda_{RF}$ , say 20, Equation (15) becomes

$$f_i = \frac{eQ}{\gamma m_0 c^2 C} \sum_{n=1}^{i-1} W_T[20n\lambda_{RF}] \quad (17)$$

In order to have numerical estimates of the righthand side of equation (17), we can for instance use the development of the wake potential in terms of the structure normal modes

$$W_T(z) = 2 \sum_{m=1}^M \frac{k_{1m}c}{\omega_{1m}a^2} \sin(\omega_{1m} \frac{z}{c}) \quad (18)$$

where  $k_{1m}$  and  $\omega_{1m}$  are the loss factors and frequencies of the M modes retained and obtained from a field calculation code [8], while  $a$  is the iris radius. Of course, focusing does not make it possible to exactly fulfill the condition for  $\bar{f}(z)$  to be independent of  $s$

$$k_0^2(s) = \frac{C}{\gamma(s)} \quad (19)$$

but allows to reach in average a situation close to the coherent oscillation. Supposing this is done, remains the major task of achieving a differential focusing function  $\bar{f}(z)$  able to stabilize the multibunch mode and avoid the so-called beam breakup. Extending the principle applied in CLIC of using (high-frequency) RF quadrupoles in order to provide single bunch BNS damping, we can use different sets of (lower-frequency) RF quadrupoles to generate the function  $f$  required. The idea is to represent the ideally required function  $f$ , i.e. more precisely the values it takes at the bunch positions  $z_i$ , by using a function arbitrarily called  $f^*$ , obtained by the addition of a small number of frequency components. Such a function can be realized in the form

$$f^*(z) = \sum_k A_k \sin \omega_k \frac{z}{c} \quad (20)$$

where the index  $k$  runs from 1 to the number of bunches minus 1. This form automatically gives  $f(z = 0) = 0$  (no effect on the first bunch). If we choose the driving frequencies for the RF-quadrupoles as given, we may compute the amplitudes  $A_k$  necessary to equalize  $f^*(z_i)$  with the required values  $f_i$  in Eq. (17). This leads to a linear set of equations in the amplitudes  $A_k$ :

$$M \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \end{pmatrix} \quad (21)$$

with

$$M = \begin{pmatrix} \sin \omega_1 \frac{z_1}{c} & \sin \omega_2 \frac{z_1}{c} & \sin \omega_3 \frac{z_1}{c} & \dots \\ \sin \omega_1 \frac{z_2}{c} & \sin \omega_2 \frac{z_2}{c} & \sin \omega_3 \frac{z_2}{c} & \dots \\ \sin \omega_1 \frac{z_3}{c} & \sin \omega_2 \frac{z_3}{c} & \sin \omega_3 \frac{z_3}{c} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \implies \vec{A} = M^{-1} \vec{f} \quad (22)$$

### 3.1 Strong focusing

The theory developed in the previous section is based on the integro - differential equation Eq. (1) describing the dynamics of a bunch with focusing and transverse wakefields and supposing a weak focusing situation. (The  $s$  dependence in  $K^2(z, s)$  is only due to the scaling of the focusing with energy and is adiabatic (slowly changing within one FODO cell)). The expression for the correcting focusing function  $f^*(z)$  given in Eq. (20) has been derived using this weak focusing assumption. However in the real main LINAC of CLIC focusing is achieved using a strong focusing structure (FODO lattice). So we expect that a correction to the function  $f^*(z)$  has to be applied in order to cope with the fact that focusing and correction are not continuously distributed along the LINAC. In order to derive the necessary correction we start with Hill's equation describing the betatron motion in a strong focusing periodic lattice:

$$x'' + K(s)x = 0 \quad ; \quad K(s) = K(s + L) \quad (23)$$

The exact solution can be written using the betafunction  $\beta(s)$  as

$$x(s) = a \sqrt{\beta(s)} \cos \left[ \int_0^s \frac{ds}{\beta(s)} + b \right] \quad (24)$$

Since  $\beta(s)$  is periodic in  $L$  we may write it as Fourier series

$$\beta(s) = \bar{\beta} + \sum_{n=1}^{\infty} \left[ a_n \cos \left[ \frac{2\pi n}{L} s \right] + b_n \sin \left[ \frac{2\pi n}{L} s \right] \right] \quad (25)$$

The quantity  $\bar{\beta}$  represents the average value of  $\beta(s)$  over the cell length  $L$ .

$$\bar{\beta} = \frac{1}{L} \int_0^L \beta(s) ds \quad (26)$$

The weak focusing approximation consists in neglecting the  $s$  - dependence of the betafunction and representing it just by its average value.

$$\beta(s) \approx \bar{\beta} \quad (27)$$

With this approximation the solution becomes

$$x_w(s) = a\sqrt{\bar{\beta}} \cos \left[ \int_0^s \frac{ds}{\bar{\beta}} + b \right] = a\sqrt{\bar{\beta}} \cos \left[ \frac{s}{\bar{\beta}} + b \right] \quad (28)$$

The differential equation having this general solution is

$$x'' + \frac{1}{\bar{\beta}^2} x = 0 \quad (29)$$

from which we deduce that the weak focusing force  $k$  in Eq. (1) is equal to the inverse of the average betafunction of the associated strong focusing lattice.

$$x'' + k^2 x = 0 \implies k = \frac{1}{\bar{\beta}} \quad (30)$$

Following Eq. (9) we find

$$k^2 = k_0^2 [1 + f_w(z)] = \frac{1}{\bar{\beta}^2} [1 + f_w(z)] \quad (31)$$

In order to compute  $\bar{\beta}$  for the case of the CLIC main LINAC we use the thin lens model of the standard FODO lattice. While between the thin focusing and defocusing quadrupoles  $\beta(s)$  follows a quadratic parabola -  $\beta(s) = a + bs + cs^2$  - its values at the positions of the quadrupoles as functions of the phase advance per cell  $\mu$  are given by

$$\beta_{F,D} = \frac{L}{\sin \mu} \left[ 1 \pm \sin \frac{\mu}{2} \right] \quad (32)$$

where the indices  $F$  and  $D$  stand for focusing and defocusing quadrupoles (Ref. "Erice School"). The slope  $\beta'_F$  is given by the expression

$$\beta'_F = -\frac{2}{\cos \frac{\mu}{2}} \left[ 1 + \sin \frac{\mu}{2} \right] \quad (33)$$

If we identify the positions of  $QF$  with  $s = 0$  and the one of  $QD$  with  $s = L/2$ ,  $\beta(s)$  reads as

$$\beta(s) = \beta_F + \beta'_F s + \frac{4s^2}{L^2} \left( \beta_D - \beta_F - \frac{\beta'_F L}{2} \right) \quad (34)$$

Since the betafunction of a thin lens FODO lattice is symmetric w.r.t. the defocusing quadrupol, the average  $\bar{\beta}$  is given by

$$\bar{\beta} = \frac{2}{L} \int_0^{\frac{L}{2}} \beta(s) ds \quad (35)$$

which results in

$$\bar{\beta} = \frac{2}{3}\beta_F + \frac{1}{3}\beta_D + \frac{1}{12}L\beta'_F \quad (36)$$

Inserting the formulae (32) and (33) and application of some trigonometric identities finally leads to

$$\bar{\beta} = \frac{L}{\cos \frac{\mu}{2}} \left[ \frac{1}{2 \sin \frac{\mu}{2}} - \frac{1}{6} \sin \frac{\mu}{2} \right] \quad (37)$$

The integrated focusing strength of a thin lens quadrupole can be written as follows in terms of the cell length  $L$  and the phase advance per cell  $\mu$  (Ref. "Erice School")

$$\delta_0 = (Kl)_0 = \frac{4}{L} \sin \frac{\mu}{2} \quad (38)$$

Now we use

$$k_0^2 = \frac{1}{\bar{\beta}^2} = \frac{\cos^2 \frac{\mu}{2}}{L^2} \frac{1}{\left[ \frac{1}{2 \sin \frac{\mu}{2}} - \frac{1}{6} \sin \frac{\mu}{2} \right]^2} \quad (39)$$

From Eq. (38) we deduce

$$\sin \frac{\mu}{2} = \frac{L}{4} \delta_0 \quad (40)$$

$$\cos^2 \frac{\mu}{2} = 1 - \frac{L^2}{16} \delta_0^2 \quad (41)$$

so that

$$k_0^2 = \delta_0^2 \frac{1 - \frac{L^2 \delta_0^2}{16}}{\left( 2 - \frac{\delta_0^2 L^2}{24} \right)^2} \quad (42)$$

This establishes the equivalent weak force for a FODO lattice with a thin lens strength  $\delta_0$  given in Eq. (38). Going from  $k_0^2$  to a modulated weak force  $k_0^2[1 + f_w(z)]$  ( $f_w(z)$  is the function  $f^*(z)$  computed in the previous section) we are interested in the equivalent modulation  $1 + f_s(z)$  to be applied to the strong force. We therefore must solve the following equation in the unknown  $\delta = \delta_0[1 + f_s(z)]$

$$\delta_0^2 \frac{1 - \frac{L^2 \delta_0^2}{16}}{\left( 2 - \frac{\delta_0^2 L^2}{24} \right)^2} [1 + f_w(z)] = \delta^2 \frac{1 - \frac{L^2 \delta^2}{16}}{\left( 2 - \frac{\delta^2 L^2}{24} \right)^2} \quad (43)$$

The main LINAC of CLIC operates with a FODO lattice around  $\mu = \pi/2$ . The thin lens strength (38) behaves fairly linear in the interval  $0 < \mu < \pi/2$  so that we may expect to obtain a good approximation by linearizing (43) in  $\delta^2$  and  $\delta_0^2$ . Setting  $u = \delta^2$  and performing a Taylor Expansion w.r.t.  $u$  gives

$$u \frac{1 - \frac{L^2 u}{16}}{\left( 2 - \frac{u L^2}{24} \right)^2} = \frac{1}{4} u - \frac{1}{192} L^2 u^2 + O(u^3) \quad (44)$$



Truncating after the linear term and resubstituting for  $\delta$  leads to

$$\delta_0^2[1 + f_s(z)]^2 = \delta_0^2[1 + f_w(z)] \quad (45)$$

which results in

$$f_s(z) = \sqrt{1 + f_w(z)} - 1 \quad (46)$$

If  $f_w(z)$  is sufficiently small, we may expand the squareroot to obtain

$$f_s(z) = \frac{1}{2}f_w(z) + O(f_w(z)^2) \quad (47)$$

Hence, given a certain value for the function  $f^*(z) = f_w(z)$  which can be computed using the wakefield formalism described in the previous section, we obtain the necessary modulation of the strong focussing based on a FODO lattice by dividing the weak force modulation by 2.

## 4 Multi-bunch simulation codes

### 4.1 The code MBTRACK

For linear accelerators that are supposed to contain a large number of bunches, it often looks judicious to study the multi-bunch stability of a train independently of the single bunch behaviour, by assuming that each bunch can be represented by a macro-particle. Merging eventually both mechanisms may be tedious but necessary to understand the properties of the whole beam. In CLIC, where the beam should not contain a very large number of bunches, we decided to include from the beginning the single bunch treatment in the multi-bunch simulations. However, other approximations described below have initially been used to limit the computing time. Consequently, the multibunch codes of the MBTRACK series have been based on the MTRACK group of codes [7] written for single-bunch stability studies. The main characteristics of the MTRACK family are therefore recalled hereafter.

#### 4.1.1 Recall about the MTRACK codes

The group of codes of MTRACK type (written in FORTRAN) solves the equation of motion of a single bunch travelling through drifts, accelerating cavities, position monitors, magnetic and microwave quadrupoles, in using a matrix formalism with the following points:

- solution derived in the two transverse directions  $x$  and  $y$  for both trajectories and emittances.
- realistic strong focusing of FODO type which can be arbitrarily scaled with the energy, i.e. with the distance along the linac (assuming thin lenses in the simpler version).
- transverse and longitudinal self (short-range) wakefields either approximated by close formulae (resistive cylindrical pipes in drifts) or obtained from a separate calculation providing a large number of loss factors and frequencies of synchronous modes (accelerating structures).
- longitudinal motion containing acceleration and action of the longitudinal wakefield.
- coupling between the transverse and the longitudinal motions through the energy dependence of the focusing.
- random transverse misalignments of all the linacs components if required.
- different kinds of trajectory corrections [10].
- longitudinal division of the bunch in slices, populated according to a gaussian distribution and having same initial transverse emittances as well as injection energy. The beam is assumed to be fully relativistic so that there is no possible longitudinal redistribution of the different slices ("frozen" beam).

The most recent version of this code allows a division of the linac in sectors (at constant focusing) linked through matching insertions, linear coupling between transverse motions (tilt effect) and the use of thick lenses [11]. However, these options were not considered to be necessary for the present multibunch study, though they might be used later for more detailed investigations.

The specific points of the MTRACK codes are the strong focusing, BNS damping with microwave quadrupoles [12] rather than with energy spread, independent scaling of quadrupole strength and cell length to balance dispersion and wakefield effects [13], the use of Green's functions for the short-range wakefields and the beam model. The effective emittance of the bunch is calculated by projection into the transverse phase plane of the individual ellipses of each slice which are independently off-centered by the betatron motion. Main inputs are the lattice data, beam parameters, misalignment amplitudes and the frequencies and loss factors of the wake-field modes. The first three groups of data are contained in a common file with namelists while the wakefield data appear in a separated file. Main outputs are the trajectory deviations, transverse effective emittances obtained by projection in the phase plane of each slice contribution, optics parameters and the characteristics in energy, all as functions of the position in the linac. The

first two are listed in a common file, that is distinct from the file containing the other two. A third file contains a brief summary of the input data and of the output values at the end of the linac. Special files can be created on demand with the necessary information for plotting the wakefield functions or the beam energy distribution at the linac end.

#### 4.1.2 Description of the codes MBTRACK

Starting from MTRACK codes, an extension to the multibunch mode has been worked out in the group of codes MBTRACK in three steps, with increasing complexity:

- At first, the particle distribution was duplicated in order to track two identical bunches, both experiencing the same short-range wakefields. The long-range wakefield applied to the second bunch was approximated by a field constant along the bunch and depending only on the average position of the first bunch. First indications of long-range transverse wakefield effects were obtained in this way.

- Facing the need for a better model, we worked out a second version still tracking two bunches only (large number of slices permitted) but with more realistic fields and offering the possibility to approximately estimate the behaviour of any bunch in a train, as explained below. A routine was added, which computes the long-range effects on the following bunch, by applying slice by slice and proportionally to each slice-position and -current, the fields generated by the first bunch. At this point, the approximation made for convenience resides in the fact that the long-range field due to one short slice was taken equal to the integrated field of a short (but still longer) gaussian bunch. This approximation is certainly acceptable when considering the effect of bunch 1 on bunch 2. Turning to the trailing part of the train, if long-range fields are sufficiently attenuated or multi-bunch BNS damping is successfully applied, the motions of the first two bunches will be so similar, i.e. coherent, that their motion can be simulated through one head-bunch only and the direct addition of their wakefields can be done with the code WAKET (Section 4.2) for getting the effect on bunch 3. The code gives then a reasonable (preliminary) approximation of the behaviour of bunch 3, even if it tracks only two bunches simultaneously. Generalizing the argument, it is possible to get an idea about the stability of any bunch in the train near coherent conditions. This code has been used for obvious reasons of simplification and computing time saving.

- It is clear that we ultimately need a code that allows to track simultaneously an arbitrary number of bunches, with all their identical short-range fields and self-consistent long-range fields computed within MBTRACK, using Green's functions and according to the actual slice positions. The less coherent is the motion of the first bunch, however submitted to BNS damping, the more important it is to have the entire interdependence of the single-bunch motion and of the long-range effects. Unfortunately, since the number of cross dependences increases quadratically with the total number of slices in a train, the computing time rises in the same way. This may oblige us to work in particular cases with a reduced number of slices per bunch and to run the program on fast computers. A version of MBTRACK has been written with such a full bunch-to-bunch interaction [14], allowing in general for an arbitrary number of bunches. This version called MBTR is described in the quoted reference and now produces animated graphics showing the evolution of the bunches. It will be intensively used in the future.

In the first two versions of the codes MBTRACK, both the single bunch and multibunch BNS damping are included and simulated in the same way. Indeed, thin microwave quadrupoles are supposed to be installed next to each magnetic quadrupole of the FODO lattice, with their appropriate amplitudes, frequencies and phases. It is worth to mention that the present MBTRACK family assumes uncoupled vertical and horizontal motions, as well as thin quadrupoles and a continuous scaling of the focusing properties with the beam energy. It uses a one-to few

(more beam position monitors than quadrupoles) algorithm of trajectory correction with possible iterations of this correction. Comparing with MTRACK, the input file contains an additional namelist, with multibunch parameters like the long-range field attenuation and the possible BNS damping quadrupole strengths. The output file with trajectories and emittances is duplicated for the lagging bunch. On option, it is also possible to get the effective emittance of the whole train by including the bunch off-sets relative to the average beam position.

In the following sections, the results presented and discussed concern the three versions of MBTRACK, though mostly the first two. All these codes have been written in FORTRAN (on the CERN VM system) and have been transferred to the new CERN platform SP2. In addition, for development purposes, working convenience and interactive procedures that include the possibility to have animated graphics showing the evolution of the whole bunch train (utility written in QBASIC), an MBTRACK version has been created for running in a stand-alone mode on a personal computer (DOS system). Given the speed and power of the recent computers, this version makes it possible to track through the main linac in a still reasonable time a train of more than 20 bunches subdivided in more than 10 slices.

## 4.2 The code WAKET

Parallel to the development of MBTRACK we worked on a second code capable to compute short and longrange longitudinal and transverse wakefields generated by a train of bunches which travels through the accelerating and focusing structure of a LINAC. Input parameters are the basic properties of the structure and the bunches (bunchlength, bunch population, frequency of the accelerating field, bunch separation) as well as a list of frequency components and loss factors describing the wakefields. The latter are calculated in a field computation program [8,15] which uses as input the geometry of the accelerating structure as well as the shape and the population of a bunch of charged particles.

The code WAKET has been written in FORTRAN 77 using an IBM compatible PC. A mathematical coprocessor is strongly recommended for running the program on PC. Of course the (interactively organized) program can be installed on any computer containing a FORTRAN compiler.

The basic functions of the program are shortly described in the following subsections.

### 4.2.1 Long-range transverse wakefields

In this part of the code the transverse wakefields are computed along a train of bunches (up to five). In this case we apply the following approximation:

- A coherent motion of the leading bunches is assumed. This implies that the contributions of all these bunches can simply be added (taking also into account the given bunch separation)

In order to compute the contribution of a bunch for every slice of this bunch (from head to tail) the Green functions transverse wake field is computed and weighted with the actual population according to a Gaussian bunch. Then the so obtained Green function fields are superimposed according to the relative positions of the generating slices. In Fig.1 we see the longrange transverse wakefield across a train of five CLIC bunches with a relative separation of  $20\lambda_{RF} \approx 20cm$ .

### 4.2.2 Computation of the RF focusing function for multibunch BNS damping

Following the formalism developed in section 2, this part of the program first computes the necessary values of the RF focusing function  $f(z)$  (in the weak focusing approximation) at the

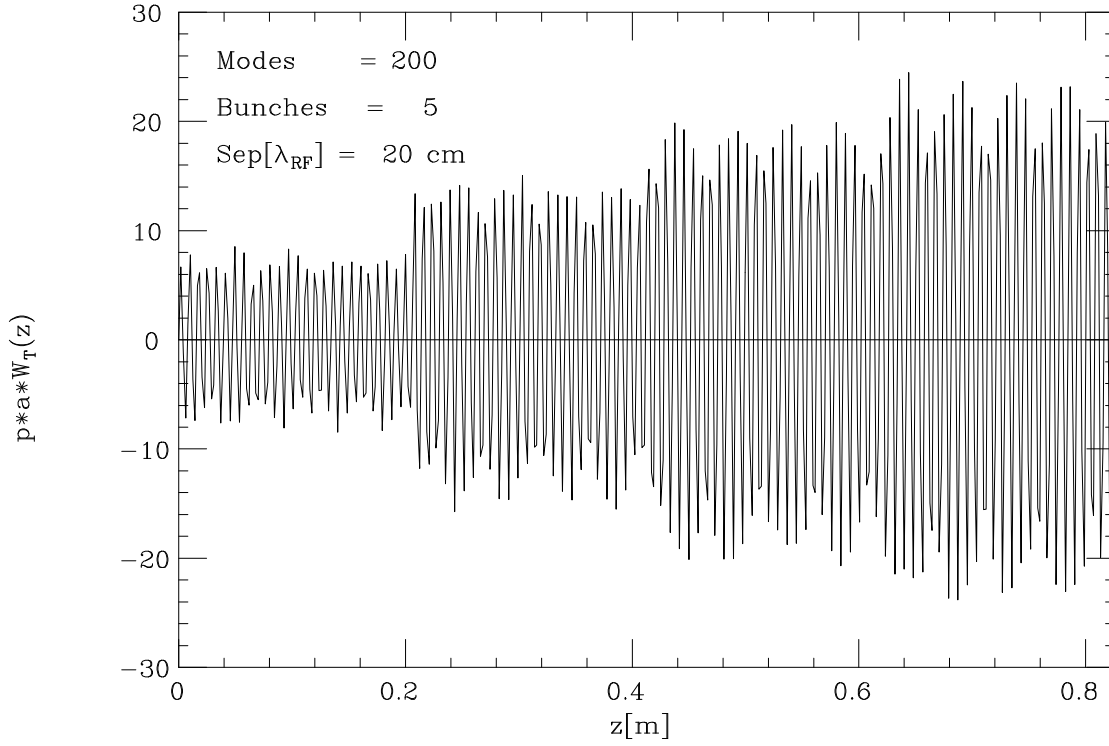


Figure 1: Transverse wakefield across a train of five CLIC bunches.

bunch center positions in order to minimize the noncoherent relative bunch oscillations. For that it uses Eq. (15). Finally a correction scheme for five bunches is set up using four given frequencies and four amplitudes which follow from the linear system (22). These amplitudes (weighted with a factor close to 0.5 in order to cope with the realistic strong focusing situation in the main LINAC) can then be used to modulate the FODO lattice in the desired way. Fig. 2 shows this function for the four frequencies  $f_{1,2,3,4} = 10, 11, 12$  and  $13$   $Ghz$  and the according amplitudes along a train of five bunches (no effect on the first bunch).

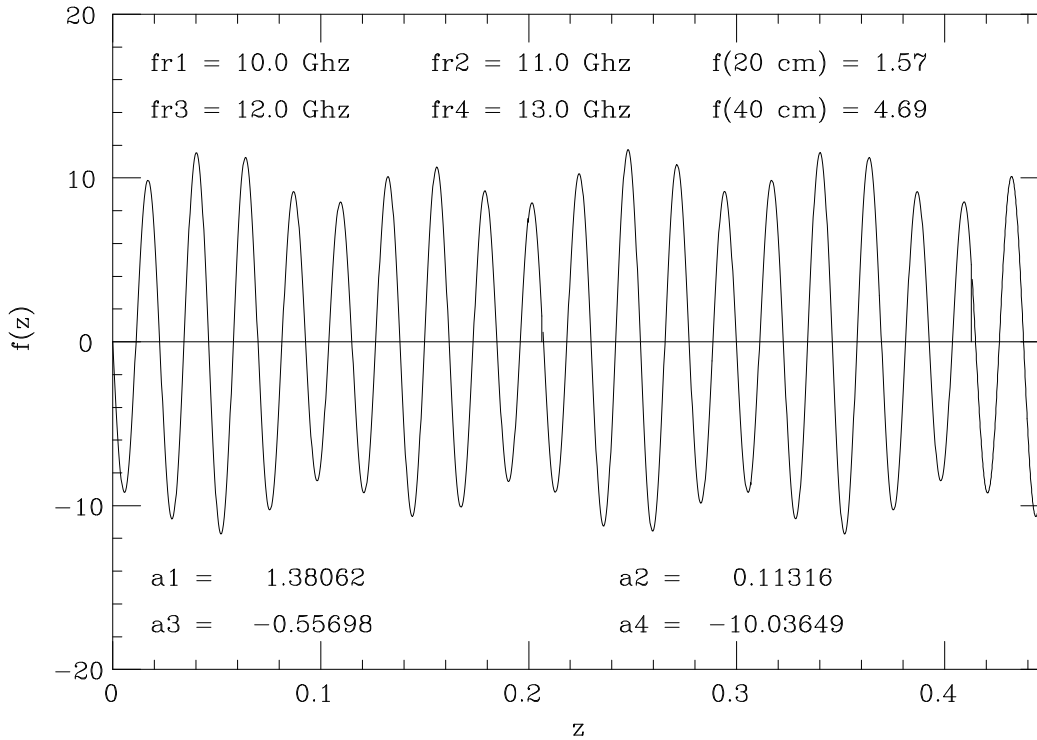


Figure 2: RF focusing function  $f(z)$  along two CLIC bunches out of a train of five.

## 5 Numerical Tests on Multibunch BNS damping

For this series of tests the program code MBTRACK (2nd version) as described in section 4 has been used together with the code WAKET. WAKET is used to compute the longrange wakes of a train of bunches as well as the RF focusing function  $f(z)$  (in the weak focusing model) to be applied to the FODO lattice of the main LINAC. The program MBTRACK tracks two bunches through the LINAC structure containing accelerating cavities, focusing and the presence of short and longrange wakefields. The longrange transverse and longitudinal wakes are provided by the code WAKET. Since many runs have been performed we used the relatively small number of 11 slices for representing one bunch (slice number 6 represents the central slice). Although we know from experience that with such a low number of slices we do not obtain very good quantitative results, the qualitative behaviour of the bunches is well represented.

### 5.1 General tests with two bunches

At first we make a test with two bunches where the second bunch feels the transverse wakefields of the first one. We use no attenuation of the propagating fields and we follow the motion of the two bunches over a total length of 500 m. Using the code WAKET in the described way for given frequencies  $f_{1,2,3,4} = 10, 11, 12, 13$  Ghz we find the particular value at 20 cm of the focusing function  $f(20\text{cm}) = 1.57$  after the application of the following Fourier's development (in the weak

focusing model):

$$f_w(z) = A_1 \sin[20\pi z/c] + A_2 \sin[22\pi z/c] + A_3 \sin[24\pi z/c] + A_4 \sin[26\pi z/c] \quad (48)$$

where  $A_{1,2,3,4} = 1.38, 0.11, -0.55, -10.04$ . The function  $f_w(z)$  is always computed to interpolate correctly through 5 bunches. Then we apply the modulating function

$$f_s(z) = \alpha f_w(z) \quad (49)$$

to the strong focusing quadrupoles of the FODO lattice. In Fig.3 we plot the second bunches emittance  $E_y$  as function of position inside the LINAC. We compare the case of  $\alpha = 0$  (no correction) to the cases of  $0.46 < \alpha < 0.52$  and we see clearly that we obtain a strongly reduced emittance blow up of the second bunch for values of  $\alpha$  close to the predicted value of 0.5.

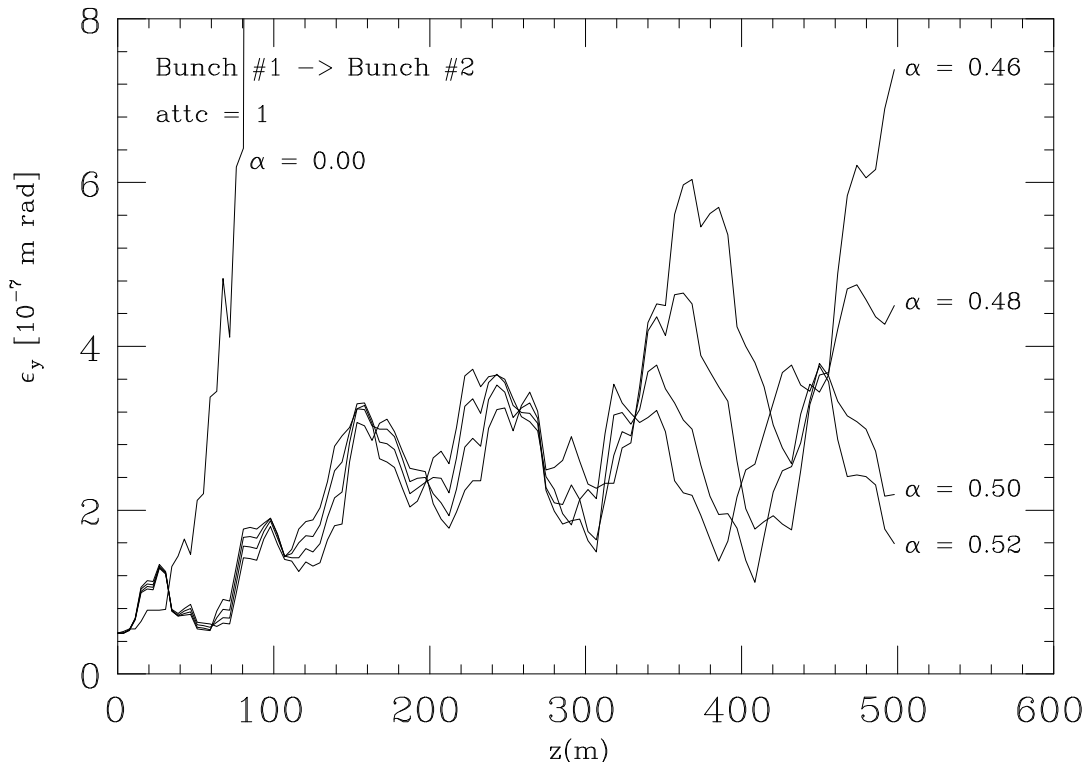


Figure 3: Emittance of the second bunch without and with BNS correction using different focusing factors around 0.5, as function of  $s$ .

To demonstrate more clearly that in fact we obtain a minimum emittance blow up close to  $\alpha = 0.5$  as predicted from theory we plot the emittance of bunch 2 after  $1000m$  as function of  $\alpha$ . The horizontal line in Fig.4 represents the emittance of the first bunch and we see that the emittance blow up of bunch 2 w.r.t. bunch 1 for a small interval of  $\alpha$  close to  $\alpha = 0.5$  becomes smaller than unity.

In Fig. 5 we see a more detailed picture of the minimum emittance region for bunch 2.

As it could already be seen in the previous figure, the emittance of bunch 2 is even smaller than the one of bunch 1 when  $0.58 < \alpha < 0.60$ .

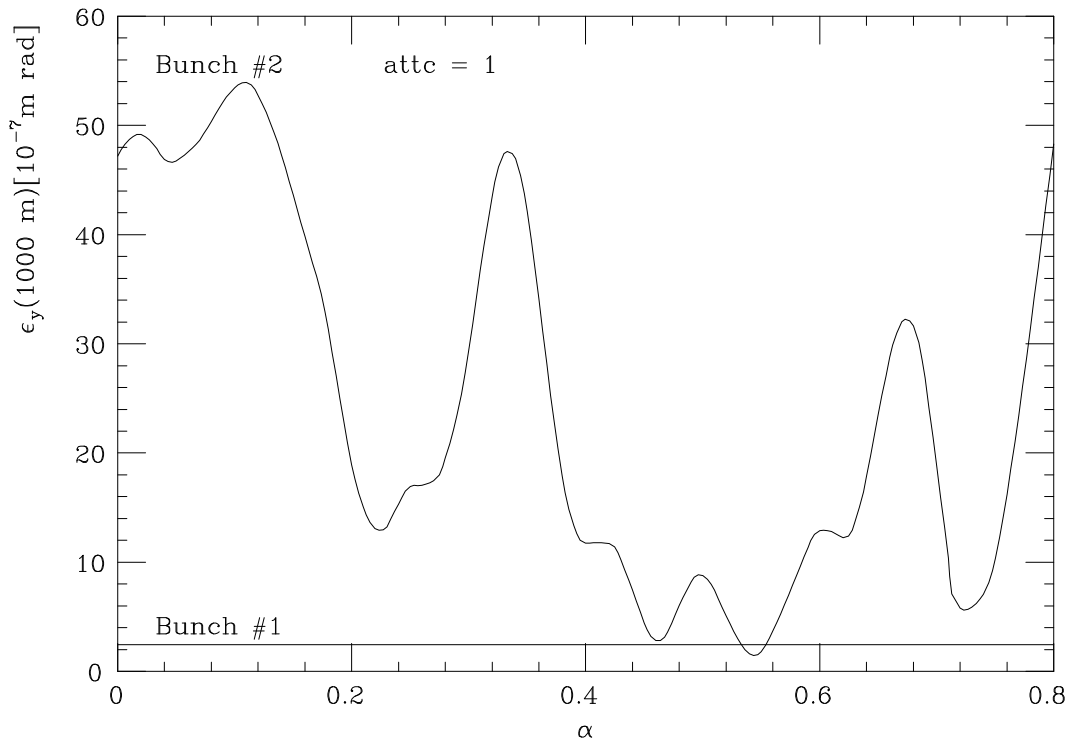


Figure 4: Emittance of the second bunch as function of  $\alpha$  for  $0.0 < \alpha < 0.80$ .

### 5.1.1 The slope effect

The theory of multibunch BNS damping only applies to the central value of the transverse wake inside the bunch. However, we expect a residual effect if the transverse wake has a finite slope. In order to study the effect we continue considering a two bunch situation but now having in addition an attenuation of the long range transverse wakefield by a factor of 5. The actual wake function across bunch 2 is shown in Fig.6. The corresponding focusing function for providing multibunch BNS damping is now given by the same function (48), but with  $A_{1,2,3,4} = 0.00717, 0.05525, -0.0518, -1.80291$ .

As can be seen, the wake across bunch 2 is close to be flat (zero slope). In order to produce a clean situation we replace this wakefunction by an exactly constant one leaving the central value (slice 6) unchanged. Then we 'rotate' this reference case around the central slice to produce different 'test wakes' with given slopes. The slopes are measured in changes of  $W$ , namely  $\delta W_{trans}/slice$ .



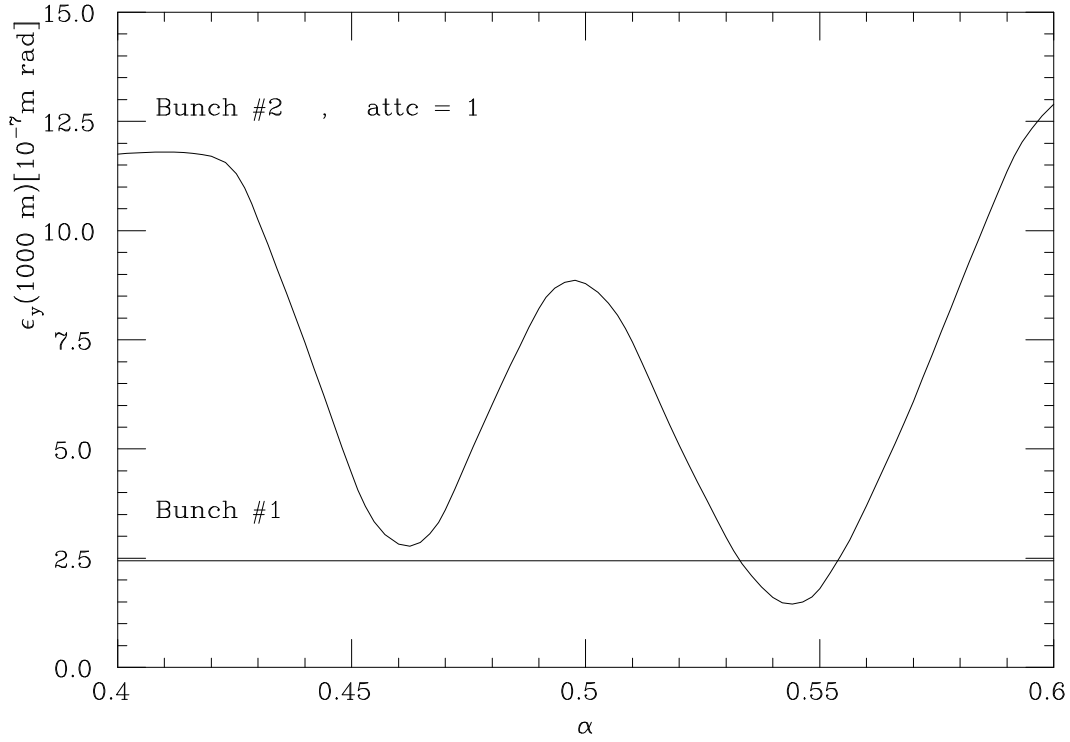


Figure 5: Emittance of the second bunch as function of  $\alpha$  for  $0.4 < \alpha < 0.6$ .

Fig.7 shows the vertical emittance blow up in percents w.r.t. the zero slope case as function of the slope. The zero slope case has been corrected using two sectors with different RFQ's (0 - 1500 m,  $\alpha_1 = 0.54$ , 1500 - 3000 m,  $\alpha_2 = 0.52$ ). The maximum change of the transverse wakefield across bunch 2 (with a field attenuation of 5) can be obtained by looking at the amplitude of the transverse wake between bunch 1 and 2 and approximating the wake by its lowest frequency component as

$$W_T(z) = A \sin \omega_1 z/c \quad ; \quad c = 3 \cdot 10^8 m/s \quad (50)$$

From Fig.8 we see that in the given unit  $V/pCm$ ,  $A \approx 2$  between bunch 1 and 2.

The lowest frequency component of the transverse wakefield inside the CLIC structure is  $38GHz$ , hence  $\omega_1 = 2\pi \cdot 38 \cdot 10^9$ . The CLIC bunches are truncated at  $+1.3$  and  $-1.8\sigma_z$  hence they extend over about  $3\sigma_z$ . With  $\sigma_z \approx 0.2mm$  this means that the effective bunch length  $\Delta z \approx 6 \cdot 10^{-4}m$ . Therefore the maximum possible change of  $W_T$  per slice of charge - if we model one bunch by 11 slices - becomes:

$$\frac{\Delta W_T}{\text{slice}} = \frac{A\omega_1 \Delta z}{11 c} \approx 0.1 \quad (51)$$

From Fig. 7 we find that the corresponding vertical emittance blow up over the entire 3 km LINAC is about equal to 10% w.r.t. the case of zero slope which is not a very critical effect.

### Bunch Nr. 2

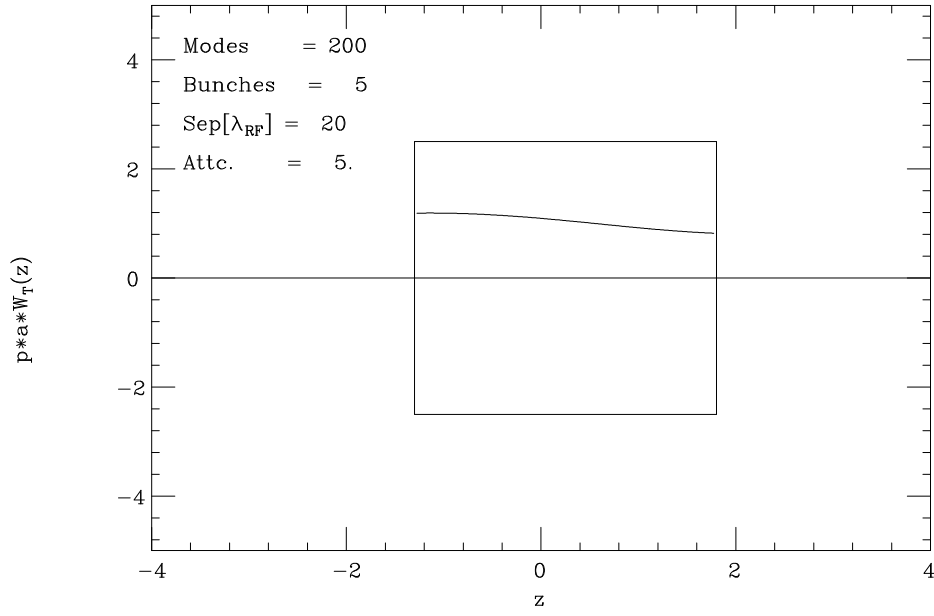


Figure 6: Transverse wakefield in bunch 2, generated by bunch 1 and attenuated by a factor 5.

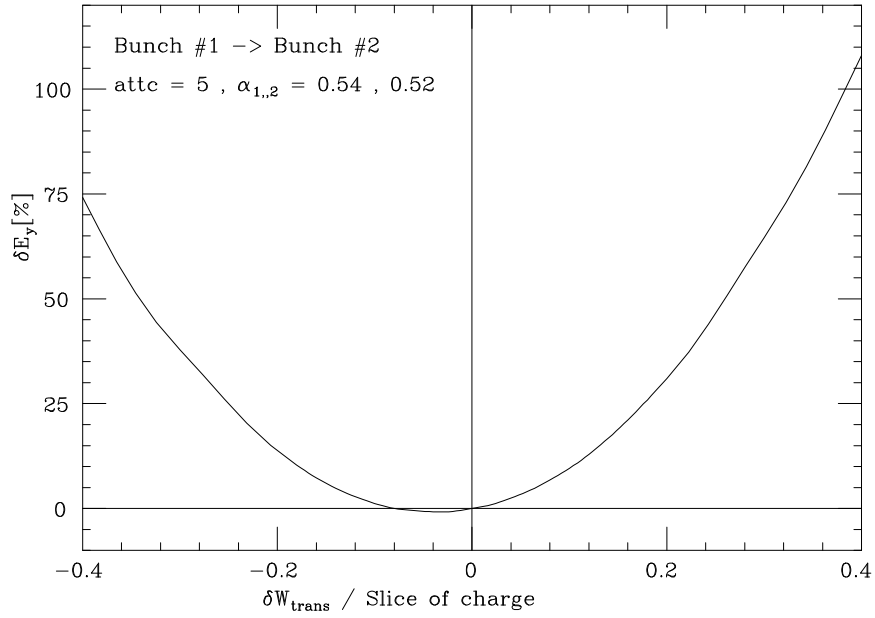


Figure 7: Emittance blow-up due to finite slopes of the transverse wakefields in bunch 2.

### 5.1.2 Random machines

The behaviour of LINACS of CLIC type depends strongly on misalignments of different machine components. These misalignments are always present and they are randomly distributed along the machine. In the CLIC case three types of misalignments are important, namely the ones of quadrupoles, accelerating sections as well as pickups. This is why - as has been described in the section concerning tracking codes - we always have to deal with a set of different random machines using a Gaussian random generator which adds random misalignments to the critical components. Hence if we wish to check multi bunch BNS damping we have to make sure it works for a random set of LINACS inside the specified tolerances. In Fig.9 we see such a test for a set of 10 random machines. As before we concentrate on the action of bunch 1 on bunch 2 without attenuating the field. We follow the bunches over 1 km and plot the RF focusing factor  $\alpha$  against the vertical emittance of bunch 2 after the 1000 m. The horizontal line represents the emittance of bunch 1.

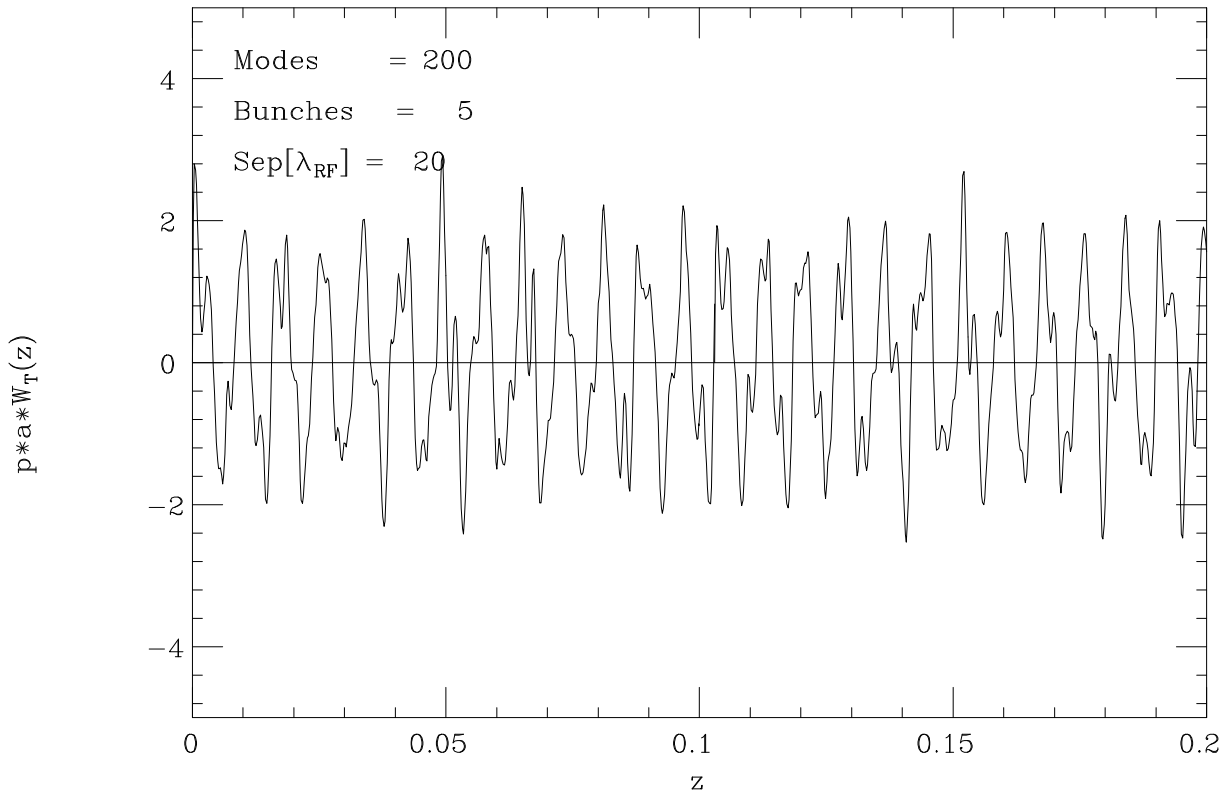


Figure 8: Transverse Wakefield between bunch 1 and 2 (attenuation of 5).

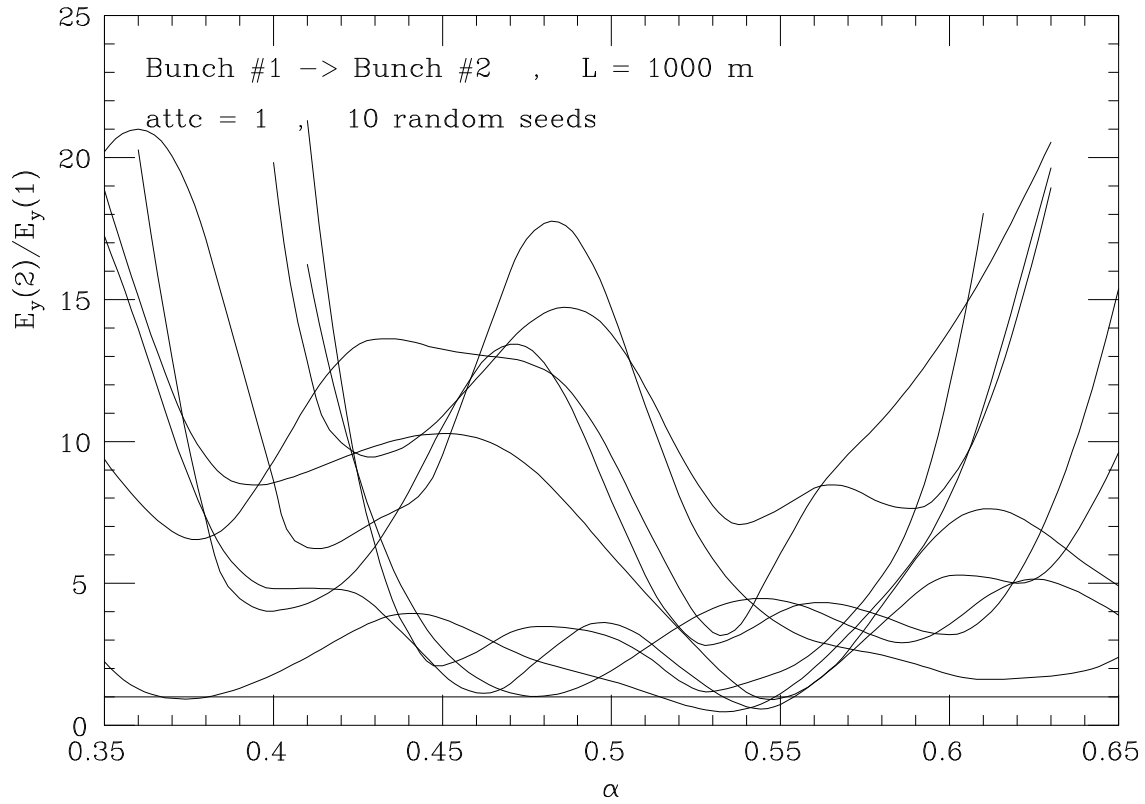


Figure 9: Test of BNS damping for multibunch mode, on 10 machines.

It is clearly visible that all the emittance curves have local minima in the given range of  $\alpha$  ( $0.35 < \alpha < 0.65$ ) which means the method works in princip. However there are non negligible differences in the quality of the correction of different machines. Table 1 shows the statistics.

Item	Ocurrencies	Percents
Local Minima	10	100 %
$E_y(2) \leq E_y(1)$	6	60 %
$E_y(1) \leq E_y(2) \leq 3E_y(1)$	3	30 %
$E_y(2) \geq 3E_y(1)$	1	10 %

Table 1: Statistical behaviour of 10 random LINACS with a two bunch BNS damping.

## 5.2 BNS damping in a train of bunches

Since the LC95 design parameters of CLIC mentions up to 10 bunches [4] we are interested to extend our investigations to more than two bunches. Here we only report the simulation results obtained from the ‘two bunch approximation’ simulation code described in subsection 4.1. Using the code WAKET (see subsection 4.2), we are able to compute the combined longitudinal and transverse wake effects of an ensemble of bunches. We therefore use our two bunch program MB-TRACK with externally precomputed wakefields (assuming coherently moving exciting bunches) and apply these wakes to a test bunch. We use two examples. In the first only the action of bunch 1 on bunch 2 (separated by  $20\lambda_{RF}$ ) is studied with no field attenuation. In Fig. 10 the result of the emittances with and without correction is presented for this case.

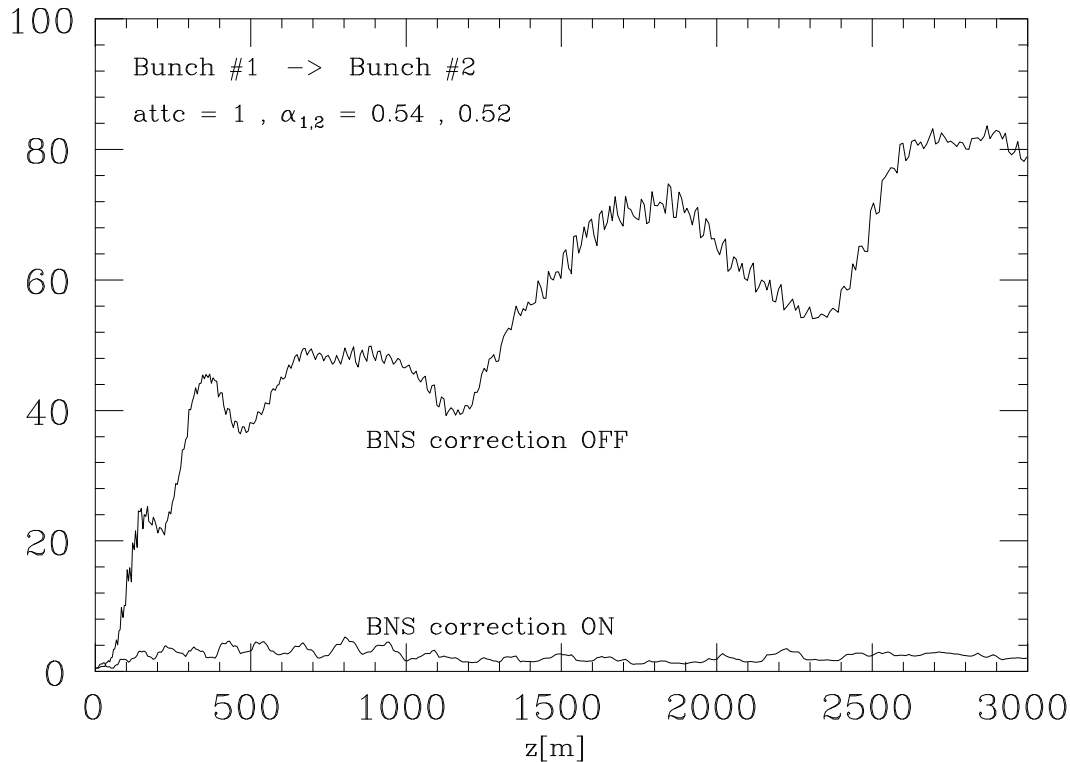


Figure 10: Vertical Emittance of bunch 2 along the linac without and with BNS damping (no attenuation).

The correction, being very powerful in this case, has been realized in two sections, from 0 – 1500 m with  $\alpha_1 = 0.54$  and from 1500 – 3000 m with  $\alpha_2 = 0.52$ . The procedure is first to minimize the emittance at the end of the first section varying  $\alpha_1$  and then to minimize on the second section using  $\alpha_2$ .

Considering the effect of the first four bunches on the fifth one it turns out that it is not possible to work without an additional field attenuation. There is a simple explanation for this fact. Computing the necessary values of  $f_w(z)$  for this case and for bunch 5 we obtain  $f_5 = f_w(z_5) = 9.15$ . Applying the factor  $\alpha = 0.5$  we still obtain a modulation to be applied to the strong focusing magnets equal to about 4.6. This means that  $f_s(z)$  upsets completely the

focusing by inverting the focusing force from focusing to defocusing and vice versa during the passage of the bunch #5. This evidently leads to an unstable betatron motion and the beam gets lost. A detailed theory about motion of bunches through modulated FODO lattices remains to be developed. However, using a field attenuation of 3 is sufficient to obtain a perfect correction scheme as we see from Fig. 11. As before, we used a two sections correction, each 1500 m long, with focusing modulations of  $\alpha_1 = 0.55$  and  $\alpha_2 = 0.51$ . As can be seen a close to constant vertical emittance for bunch 5 can be achieved throughout the whole LINAC.

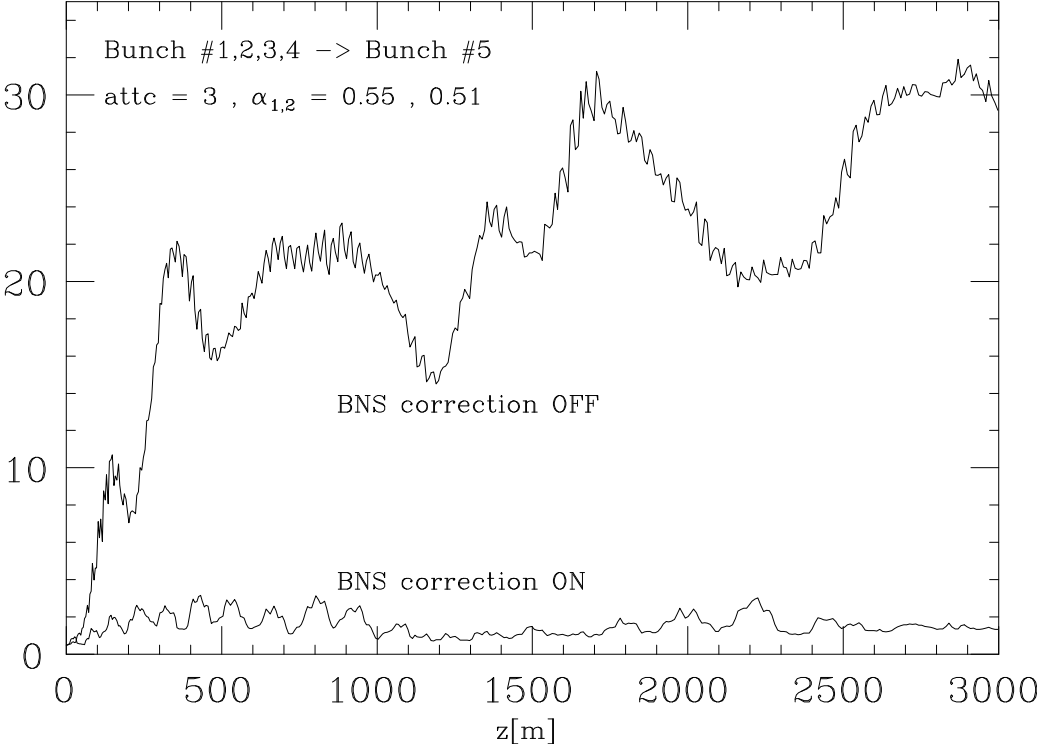


Figure 11: Vertical emittance of bunch 5 along the linac without and with BNS damping (attenuation by a factor 3)

## 6 Test of wakefield attenuation effects in multibunch mode

The way usually considered in order to take care of the long-range wakefield effect consists of attenuating the field seen by the following bunch either by producing a wake roll-off in staggered tuned sections or by using damped structures [16,17]. The resulting difference between these two approaches on the emittance growth mainly comes from the way the attenuation is distributed over the different wakefield modes. In the first method, mainly the fundamental mode is decreased while the control of the next ones is difficult, but in the second method, the high frequency modes are consequently damped when the first one has been reduced. The "attenuation factor" of the long-range wakefield at the position of the trailing bunches is a critical design parameter and the numerical tests performed with MBTRACK are aiming to an estimate of the required attenuation for the different modes.

At first, we used the code based on the coherency assumption ("two-bunch" tracking described in section 4.1) to simulate the long-range wakefield effect on a train of up to 10 bunches separated by 20 RF periods. For all the bunches sitting beyond bunch 2, the coherent addition of the wakefield modes is done in the code WAKET (section 4.2) externally to the tracking program and simulations have been carried on for some of them in the train. Considering the kind of attenuation resulting from staggered tuning of the accelerating sections [18], the following model was retained for the wakefields:

- about 200 modes are used to describe short- and long-range wakefields,
- all modes are kept at full intensity for the short-range wakefields of both bunches,
- the first long-range longitudinal mode is cancelled in order to simulate beam loading compensation,
- the first five long-range transverse modes are attenuated by a variable factor, the others remaining unchanged.

The obvious aim is that the emittance of the following bunch is as close as possible to the emittance of the first bunch, at the end of the acceleration, in order to avoid a significant loss of luminosity.

Firstly, tests have been done in this direction considering initially the bunch 2 and a short distance along the linac, equal to 200 m. Fig. 12 (case a) shows that in this case the individual emittances of the bunches 1 and 2 differ by a factor 4 without attenuation and becomes almost identical after an attenuation by 50.

Next came tests over the whole length of the linac, i.e. 3200 m corresponding to 250 GeV. Considering bunch 2, an attenuation of 50 for the first dipole mode, of 10 for the next 4 modes and no attenuation for the others give the following results for a machine with r.m.s. misalignments of  $50 \mu\text{m}$  for the quadrupoles and  $5 \mu\text{m}$  for the other components and after three iterations of the correction:

- final emittance of bunch 1 equal to  $1.43 \cdot 10^{-7}$  radm
- final, total emittance of the two bunches equal to  $1.92 \cdot 10^{-7}$  radm

The ratio of the final emittance of the 2 bunches to the bunch 1 emittance is of the order of 1.4 and the absolute difference equal to  $0.5 \cdot 10^{-7}$  radm approximately.

For bunch 3, Fig. 12 (case b) indicates now that the emittance ratio between bunches 1 and 3, equal to 6 without attenuation, goes down exponentially to reach an asymptotic value of about 1.3 in this particular exemple. The absolute difference between the two emittances is about  $1.5 \cdot 10^{-7}$  radm.

Let us mention for the record that these simulations were done with  $5 \mu\text{m}$  r.m.s. misalignments of all components,  $\alpha_{RF} = 0.49$ ,  $\phi_{RF} = 12$  degrees,  $8 \cdot 10^9$  particle per bunch and a bunch length of 0.195 mm. The curve of Fig. 12 shows that the asymptotic value is almost reached for an attenuation factor of 50 to 100.

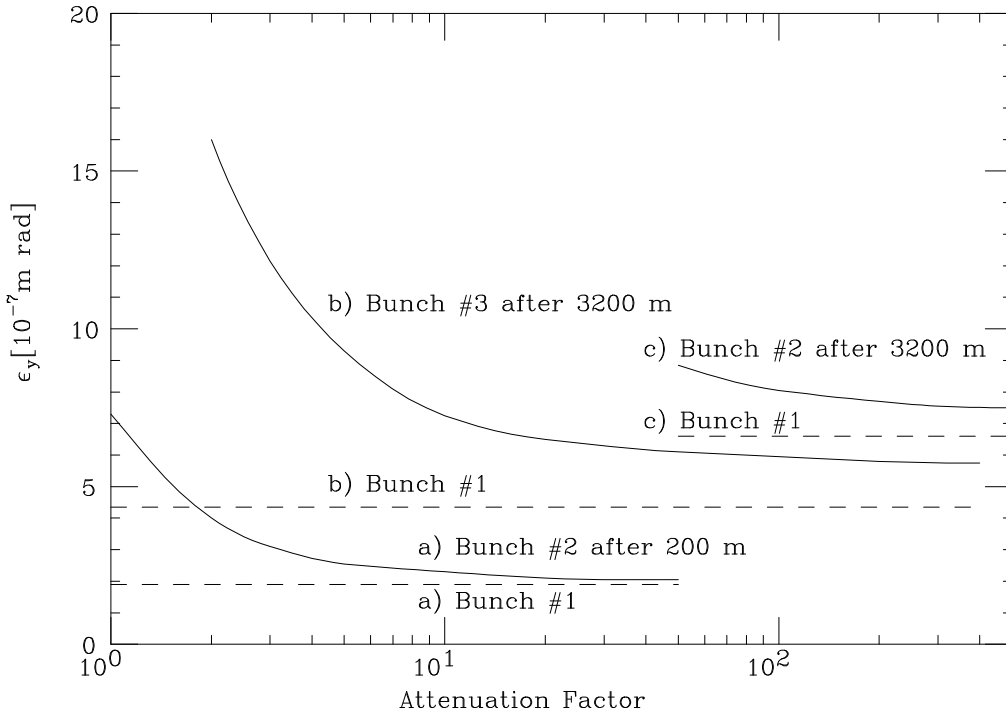


Figure 12: Emittance as function of the long-range wakefield attenuation factor (no multibunch BNS).

For bunch 10, in similar conditions as for bunch 3 but with a different set of actual misalignments and a longer linac of 12750 m (about 1 TeV), two-bunch tracking with externally computed wakefields indicates final emittances of  $1.58 \cdot 10^{-7}$  radm for the head-bunch and of  $4.19 \cdot 10^{-7}$  radm for the follower. The emittance behaviour indicates that the ratio of these two emittances regularly increases along the linac (starting from 1) and eventually approaches 2.5 in the last part of the linac.

These simulations prove that an attenuation of the order of 50 to 100 for the first mode provides approximately the asymptotic value of the following bunch emittance, at least in the CLIC situation where the tolerated single-bunch emittance growth is by a factor of the order of 3. This asymptotic value however remains too high for it implies a significant reduction of luminosity. Since it was checked that long-range longitudinal wakefields are not responsible alone for the fact that the asymptotic value never reaches the bunch 1 value, results suggest that a further gain can only be obtained by also attenuating the higher transverse modes in the same proportion as the first dipole mode.

We then turned to the most recent version MBTR of MBTRACK [13] written for dealing with several bunches simultaneously and many slices per bunch, but initially applied to two bunches only (see Section 4.1.2). The same beam parameters and the so-called wakefield model are used as above, but the wakefields are now calculated within the program in a consistent way for both the short- and the long-range effects, with the complete interdependence between all the slices. R.m.s. misalignments of all components are taken equal to  $5 \mu\text{m}$  and the first 5 dipole modes are



attenuated by the same factor, more like in a damped structure though the higher modes should also be reduced in this case.

With a linac not fully optimised for single-bunch BNS damping, i.e. with a final emittance of bunch 1 larger than desired ( $6.4 \cdot 10^{-7}$  radm), tracking in this way reproduced the curve obtained with the simplified version of the code. The asymptotic emittance value of bunch 2 for a large attenuation differs from the bunch 1 emittance by an amount which is compatible with the other results (Fig. 12, case c). If both the relative effect and the absolute difference ( $\sim 1.0 \cdot 10^{-7}$  radm) are small, it is due to the fact that the bunch 1 emittance is not minimized. This result advocates an attenuation of 100 approximately and the residual emittance difference is simultaneously due to both dipole and longitudinal high-order modes.

In the next series of simulations, we have re-optimized the microwave quadrupoles for single bunch BNS damping and the final emittance of bunch 1 (250 GeV,  $8 \cdot 10^9$  particules, 0.2 mm bunch length, RF at 12 degree) reaches now the value of  $2.35 \cdot 10^{-7}$  radm. As a by-product, this re-optimization gives back the result already obtained [19] for the single bunch energy spread, i.e. 0.24 %. Considering now the beam made of the two bunches as a whole and computing the total emittance (that also includes the deviations in the trajectories of one bunch with respect to the other) with a field attenuation (or damping) of 100, trackings have been done while varying the number of transverse modes that are damped. With an attenuation of 100 over 15 modes, for instance, the total emittance is only  $0.4 \cdot 10^{-7}$  larger than the emittance of bunch 1. The behaviour of bunch #2 is very similar to the one of bunch #1 all along the linac (with only localized differences of the order of 20 %), while the optimization of the trajectories is done on bunch 1 only. The same exponential decrease of the total emittance is observed as in Fig. 12 and indicates that with a bunch separation of 20 RF wave lengths the asymptotic value is reached when about 20 modes are attenuated (Fig. 13).

In order to have a broader knowledge of the effects, the sensitivity of the multibunch blowup to the bunch separation and to the frequency of the first dipole mode have been investigated by varying both these parameters and by repeating multibunch tracking. Fig. 13 shows the total emittance growth for bunch separation equal to 19 or 21 RF periods and variable number of attenuated modes. The effect is more important and the asymptotic value is reached with 50 modes say. With 19 RF periods, the remaining emittance difference when virtually all the dipole modes are attenuated (by 100) has been checked to be due to the longitudinal higher modes. Varying next the dipole mode frequency by  $\pm 1.5$  GHz around the nominal value of 37.90 GHz exhibits only a weak dependence of the total emittance on this parameter, with an attenuation 100 of 20 modes (maximum emittance increase is 8 %).

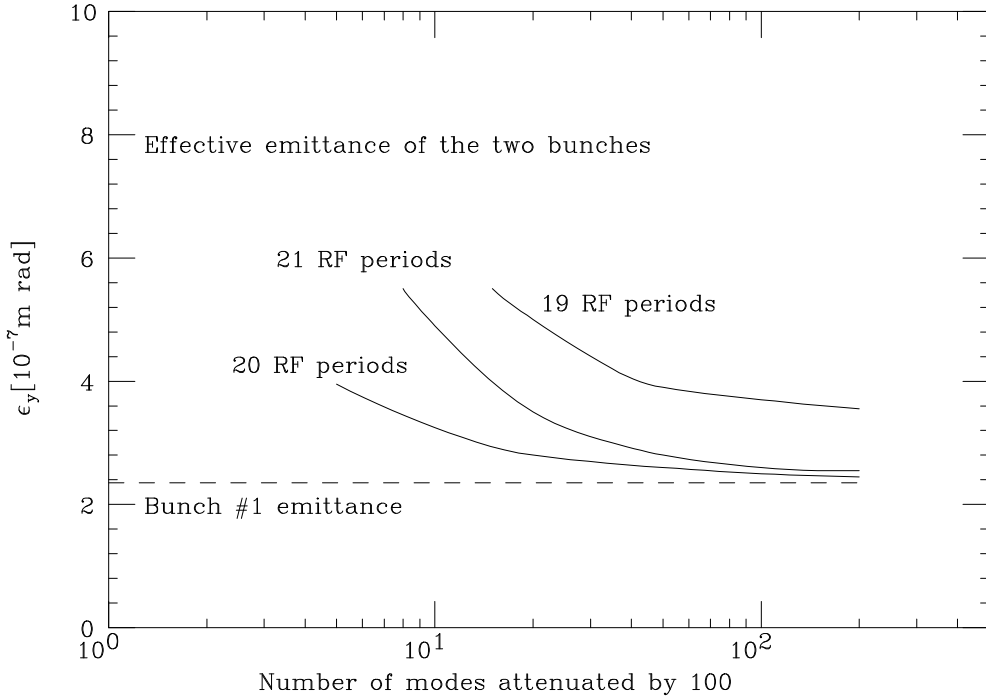


Figure 13: Emittance as function of the number of attenuated modes and RF separation (no multibunch BNS).

## 7 Summary and Comments

A multibunch tracking code family, called MBTRACK, has been created on the basis of different assumptions which are given in Section 4.1.2 together with a description of the program. It is issued from the single bunch tracking code MTRACK (Section 4.1.1) and specific to relativistic bunches. The most advanced version MBTR of the program contains a fully self-consistent calculation of both the short-range and the long-range wakefields for a train of bunches, It is an indispensable tool for the study of beam break-up in the CLIC main linac and it has been used to simulate different cases related to either multibunch BNS damping or field attenuation through staggered tuned or damped structures.

The principles of a multibunch BNS damping have been investigated. The theory is recapitulated and has been extended to the case of a train of bunches equidistant in time. In addition, an approximate theory for translating the weak focusing modulation function into the modulation required in a strong focusing FODO lattice has been provided. They allow to predict the correction to apply, in using a small number of families of microwave quadrupoles running at different frequencies and sitting near each magnetic quadrupole. The simulations confirm the theoretical results and the bunch train can be stabilized, but the practical application of this method remains problematic.

The strongest limitations for multibunch BNS damping are the following:

- Without some (limited) attenuation of the long-range wakefield, the amplitude of the focus-

ing modulation with RF quadrupoles is so large that the overall focusing can be upset in some cases. This implies to combine attenuation with a hypothetical BNS damping. A theory about the stability of the betatron motion in a modulated FODO lattice would be required for better understanding modulation effects.

- The simulations presently assume a distribution of the microwave quadrupoles all along the linac which means additional power sources in these positions. More investigations would indicate if focusing modulation over a limited distance toward the end of the linac is possibly sufficient.

- The required power per microwave quadrupole (around 11 GHz) looks prohibitive after a rapid estimate. By optimizing the choice of the RF frequencies of the microwave quadrupole families used for multibunch damping, it is possible to minimize their required strength and subsequently the required power. However, in the most optimistic case, the peak power to fill such a quadrupole in 25 nsec would be 6 MW say, that corresponds to an average power of 0.45 kW [20].

Nevertheless, the basis for a possible multibunch BNS damping has been established theoretically and numerically in the case of a small number of bunches (5 to 10) though the application is questionable. The additional variation of the wakefields within the bunches, that we call slope effect, appears not to be very important for the first 3 to 4 bunches but becomes significant for the bunches 5 to 10 and this represents another possible limitation.

The probably most important outcome which was expected from the multibunch numerical simulations concerns the long-range field attenuation that is necessary to damp the beam break-up as well as the number of dipole modes that have to be included in this attenuation. The results obtained so far with the MBTRACK codes indicate that it is not sufficient to attenuate 1 to 5 transverse modes, but that not less than the first 20 deflecting modes must be attenuated; the necessary attenuation factor should be contained between 50 and 100. This first conclusion tends to indicate that it is preferable to use damped accelerating structures rather than staggered tuned sections.

These results appear to be coherent with those obtained in other designs [21,22,23] after scaling quantities like the wakefields, the r.m.s. misalignments, the charge per bunch, the bunch length and the linac length. It looks therefore very interesting to exploit the unique facilities based on MBTR and on the utility program MBUNCH [14] which have been thoroughly checked and provide an extremely useful tool. They can now be ran to build up some statistics in the simulations, and to test new possible sets of parameters and characteristics of accelerating structures.

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