# $B$ MIXING AT LEP

P. COYLE
*CERN, Geneva, Switzerland and*
*Centre de Physique des Particules De Marseille, France*

Recent results from LEP on mixing in the $B$ sector are reviewed. The new LEP 95% CL lower limit on $B_s$ mixing of $\Delta m_s > 9.5\ \mathrm{ps}^{-1}$ combined with the LEP average for $\Delta m_d$ of $0.469 \pm 0.017\,\mathrm{ps}^{-1}$, starts to significantly constrain the unitarity triangle for the first time.

## 1 Introduction

The observed flavour eigenstates of the neutral $B_q$ mesons ($B_d$ or $B_s$) are linear combinations of their corresponding mass eigenstates. The mass difference ($\Delta m_q$) between the mass eigenstates gives rise to a time dependent phase difference between their wavefunctions and thus a probability to observe a $B^0$ or $\bar{B}^0$ which oscillates as a function of time. Assuming CP is conserved and negligible difference in decay widths between the two mass eigenstates, the probability of observing a $B_q$ meson in a state the same ($P_+$, unmixed) or different ($P_-$, mixed) from that at production is given by

$$P_{\pm}(t) = \frac{1}{2\tau_{B_q}} e^{-t/\tau_{B_q}} (1 \pm cos(\Delta m_q t))$$

where $t$ is the proper time and $\tau_{B_q}$ is the relevant $B_q$ lifetime. A measurement of the oscillation frequency therefore allows $\Delta m_q$ to be determined, in analogy to the neutral kaon system.

Within the Standard Model, mixing is mediated by box diagrams involving two $W$-bosons and two quarks of charge $\frac{2}{3}e$. Taking only the dominant top quark exchange into account, the mass difference is given by

$$\Delta m_q = \frac{G_F^2}{6\pi^2} m_{B_q} m_t^2 F\left(\frac{m_t^2}{m_W^2}\right) \eta_B B_{B_q} f_{B_q}^2 |V_{tb}^* V_{tq}|^2 \tag{1}$$

where $G_F$ is the Fermi constant, $F$ gives the functional dependence on the ratio of the top quark and $W$ masses ($m_t$ and $m_W$), $\eta_B$ is a known QCD correction factor (0.55), $m_{B_q}$ is the mass of the $B_q$ meson, $B_{B_q}$ is the 'bag' factor and $f_{B_q}$ the decay constant for the $B_q$ meson. Substantial theoretical uncertainties in the parameters hinder the extraction of the product of the CKM matrix elements $|V_{tb}^* V_{tq}|$ from a measurement of the oscillation frequency.

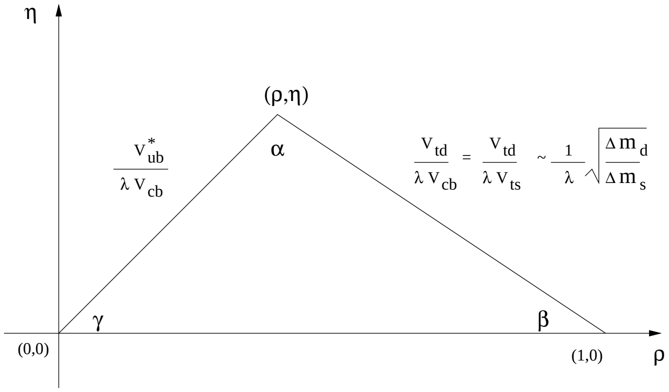

Figure 1: The unitarity triangle represented in $\rho - \eta$ space; $\rho$ and $\eta$ are parameters of the Wolfenstein parameterization of the CKM matrix and $\lambda$ is the sin of the Cabbibo angle (0.2205). The side opposite the angle $\gamma$ is constrained by $\Delta m_s/\Delta m_d$.

The ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}}\xi^2 \left|\frac{V_{ts}}{V_{td}}\right|^2 \tag{2}$$

is under better control as all dependence on the top mass drops out and the ratio of the theoretical quantities $\xi = \frac{\sqrt{B_{B_s}} f_{B_s}}{\sqrt{B_{B_d}} f_{B_d}}$ is better known ($1.15 \pm 0.05$ [1]). The ratio $\Delta m_s/\Delta m_d$ is of particular interest, as assuming $|V_{cb}| = |V_{ts}|$, it is can be used to constrain the length of a side of one of the most useful unitarity triangles of the CKM matrix (Fig. 1). Assuming the Standard Model to be correct, existing measurements of the CKM matrix elements can be used [1] to predict a preferred range for $\Delta m_s$ of $8.6\ \text{ps}^{-1} < \Delta m_s < 17.0\ \text{ps}^{-1}$.

## 2 Experimental Methods

At LEP, both time-integrated and time-dependent methods can be used to extract information on $\Delta m_d$ and $\Delta m_s$. As time-integrated measurements have no sensitivity to large $\Delta m$, for the fast $B_s$ oscillation one must rely on measuring the time dependence of the mixing. At the $\Upsilon(4S)$ only the total time integrated probability of $B_d$ mixing can be measured as the $B$'s are produced at rest and $B_s$ mesons are not created.

Experimental measurement of the oscillation frequency necessitate tagging the production and decay states of the $B_q$ meson and a measurement of its proper time.

The decay tag is obtained by looking at the charge of some of the final state decay particles for example, a high $p_t$ lepton or the charge of a reconstructed charm particle ($D^{*\pm}$ for $B_d$ mixing or $D_s^{\pm}$ for $B_s$ mixing).

The production tag is more involved and is inferred from studying the properties of the $b$ hadron in the opposite thrust hemisphere to the decay tag for example, the charge of a high $p_t$ lepton or the momentum weighted hemisphere jet charge. Many analyses also use information from the same hemisphere, such as the charge of the leading fragmentation kaon (just for $B_s$) or the same hemisphere jet charge. Although the rate of mistag ($\eta$) from a combination of the same and opposite jet charges ($\eta \approx 28\%$) is usually higher than that from a high $p_t$ lepton ($\eta \approx 20\%$) or fragmentation kaon ($\eta \approx 26\%$) this is compensated by its high efficiency. Using a method ("optimal tagging") of combining information from variables choosen to have some discriminating power to identify the initial state, (for example the $p_t$ of a lepton, the value of the jet charge etc.) ALEPH[2] has obtained an effective (event-by-event) mistag of $\approx 27\%$ with $100\%$ efficiency.

The proper time is calculated as $t = g\ell$ where $g$ is 1/boost and $\ell$ is the decay length. The decay length is the distance from the primary vertex to the $B$ decay vertex projected onto the estimated $B$ flight direction. The $B$ vertex is reconstructed by combining the charged track(s) thought to come directly from the $B$ with a fully or partially reconstructed charm particle. The resolution on the proper time can be expressed as

$$\sigma_t = \sqrt{(t\sigma_g/g)^2 + (g\sigma_\ell)^2}.$$

The first term corresponds to the effect of the boost resolution ($\sigma_g/g \approx 10 - 20\%$) and increases with $t$. The second term, approximately constant, is due to the decay length resolution ($\sigma_\ell \approx 230 - 400\mu m$) and dominates the resolution at small t.

The final sensitivity of a particular analysis to mixing depends on the "properties" of the analysis in the following way[3]

$$signal/noise \approx \sqrt{\frac{N}{2}}(1-2\eta)p\exp\left(-\frac{1}{2}(\Delta m_q\sigma_t)^2\right)$$

where $N$ is the total number of selected events and $p$ is the purity of the sample. As the exponential term, due to the smearing from the proper time resolution, is important for large $\Delta m_q$, the requirements on the proper time resolution for $B_s$ mixing are much more stringent than for $B_d$ mixing. For the more exclusive analyses the decrease in statistics is usually compensated by improvements in purity, mistag and proper time resolution.

## 3 $\Delta m_d$ Results

Figure 2 summarises the latest LEP measurements of $\Delta m_d$. Also show for comparison are measurements from the $\Upsilon(4S)$ (using $\tau_{B_d} = 1.57 \pm 0.05$ ps) and CDF. The names of the analyses are chosen such that the words before (after) the slash refer to the method for tagging the decay (production) state. Here L means high $p_t$ lepton, QJ means jet charge, OPT means optimal tagging, $D^*$ means fully reconstructed $D^{*\pm}$ meson and finally $\pi^* L$ means a new decay tag from DELPHI in which a slow pion from the $D^{*\pm}$ is vertexed with a lepton. For the LEP average, only measurements plotted with a filled circle are used, also no attempt is made to take into account the small statistical and systematic correlations. Figure 3 shows a selection of fits for the $\Delta m_d$ measurements.

## 4 $\Delta m_s$ Results

None of the $B_s$ mixing analyses claim to measure $\Delta m_s$; instead a 95% confidence level exclusion is quoted. The exclusion is determined by finding the minimum in the negative log likelihood distribution as a function of the assumed value of $\Delta m_s$ and moving up from this minimum by a fixed number of likelihood units ($\Delta ln\mathcal{L}^{95}$). Naively, for a 95% CL one would expect $\Delta ln\mathcal{L}^{95} = 1.92$; in practice it is found that, due to the presence of fake minima, the value is larger ($\approx$ 2-3), may be a function of the assumed $\Delta m_s$ and differ from analysis to analysis. A calibration of the 95% CL curve must therefore be performed, at each value of $\Delta m_s$, by studying many fast Monte Carlo experiments.

To estimate the systematic uncertainties, when generating each fast Monte Carlo experiment used to calculate the 95% CL curve, ALEPH simultaneously vary the central values of all the parameters under study within their Gaussian error. The net effect is that the $\Delta ln\mathcal{L}^{95}$ is slightly increased ($\approx 0.3 - 0.5$). OPAL have a different approach; they introduce a Gaussian constraint on each of the systematic parameters and maximise the likelihood with respect to these constrained parameters at each value of $\Delta m_s$. In general the systematics are small and mostly affect the low $\Delta m_s$ region. For the inclusive analyses the most important systematic uncertainty is from $f_s$, the fraction of $B_s$ events produced when the $b$ quarks from the $Z^0$ hadronise. For the $D_s$ based analysis the systematic from $f_s B(B_s \to D_s X)$ dominates.

Figure 4 summarises the 95% CL exclusion regions obtained from all the LEP analyses using this procedure. As the quoted exclusion regions are influenced by statistical fluctuations, the $\Delta m_s$ reach of the different analyses can not be easily compared.

The ALEPH $D_s$L/OPT analysis[2] based on only 277 $D_s$-lepton candidates reconstructed in 6 channels ($\phi\pi^\pm, K^{*0}K^\pm, K^0K^\pm, \phi\ell^-\bar{\nu}, K^{*0}K^{*-}, \phi\pi^-\pi^+\pi^-$),

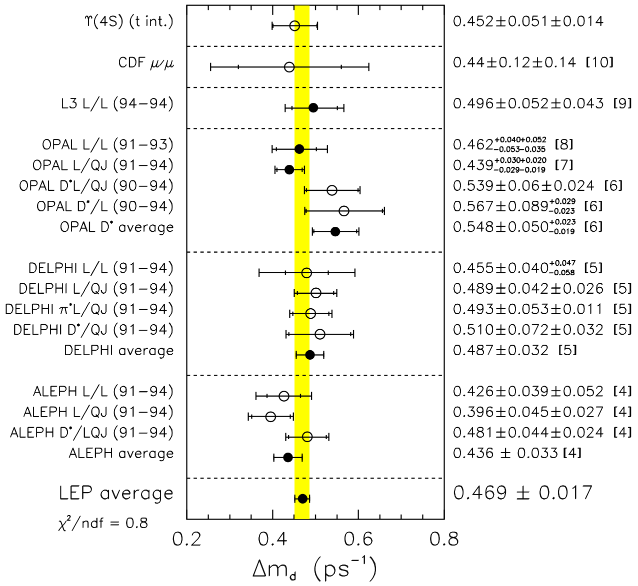

Figure 2: Summary of the LEP $\Delta m_d$ measurements. The number in square brackets refers to the relevant reference. The data sample used is also indicated.

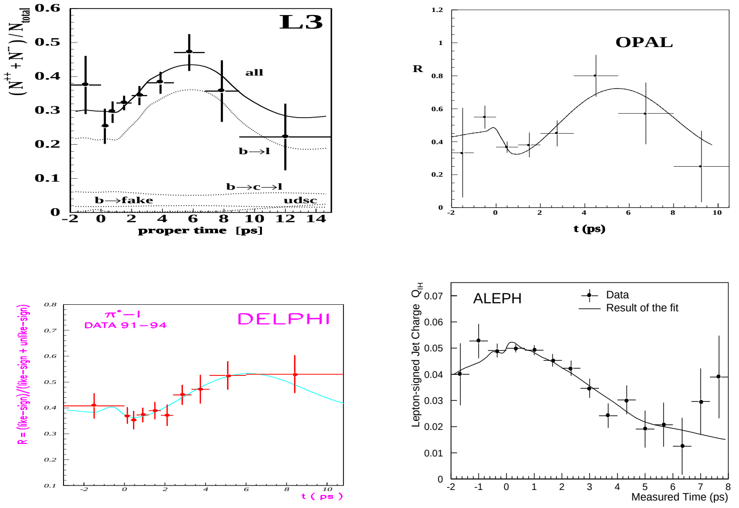

Figure 3: Likelihood fits to the fraction of events tagged as mixed versus proper time for some $\Delta m_d$ analyses: the L3 L/L, the OPAL $D^*$/L, the DELPHI $\pi^*$L/QJ and the ALEPH L/QJ.

yields a limit of 6.6 ps$^{-1}$. The small statistics is compensated by the very high $B_s$ purity ($\approx$ 65%), and the good vertex ($\sigma_\ell \approx 230\ \mu m$) and momentum resolutions ($\sigma_g/g \approx 10\%$).

ALEPH have developed another new analysis[11] (Fig. 5) based on vertexing the $D_s^\pm$ with an oppositely charged hadronic track selected to be likely to come from the $B_s$ vertex ($D_s$H/OPT). Compared to the previous analysis, this approach suffers from additional backgrounds due to $D_s^\pm$ mesons from direct charm production and the misassignment of tracks from the primary vertex to the $B$ vertex.

The ALEPH L/L analysis [12] uses 9710 leptons with a proper time measurement and assuming $f_s = (10.2 \pm 1.6)\%$ excludes $\Delta m_s < 5.6$ ps$^{-1}$. In the ALEPH L/QJ analysis [13], after events used in the L/L analysis are removed to ensure statistical independence, a total of 63131 leptons are tagged using a jet charge based on information from both hemispheres.

All the DELPHI analyses [14] use data up to 1994. As they have not calibrated their 95% CL a conservative value of $\Delta ln\mathcal{L}^{95} = 3$ is used here to define their exclusion regions. They assume $f_s = (10.0 \pm 2.2)\%$. In the DELPHI $D_s$L/QJ analysis, the channel $D_s \rightarrow f_0(980)\pi^+$ is reconstructed in addition to those used by ALEPH. The DELPHI L/L analysis uses 4778 events and provides a limit of 2.2 ps$^{-1}$. The DELPHI L/QJ analysis uses 60381 tagged events and provides a limit of 3.6 ps$^{-1}$.

OPAL have calibrated their 95% CL curves and assume $f_s = (12 \pm 3.6)\%$. The OPAL $D_s$L/QJ analysis [16], which also includes a sample requiring only a $\phi$ from the charm particle, has a log likelihood distribution with a large dip near 0.5 ps$^{-1}$. The OPAL L/QJ analysis [7] uses over 90000 events and provides a limit of 3.1 ps$^{-1}$. Their L/L analysis [8] use data only up to 1993 and gives a limit of 2.2 ps$^{-1}$.

### *4.1 Combination of $\Delta m_s$ Results*

In order to combine the various analyses, naively one would simply add the individual log likelihoods, find the new minimum and apply the same prescription as previously discussed. In this case the calibration of the 95% CL limit curve is rather difficult, as strictly it would require a “grand” Monte Carlo simulation incorporating all the individual analyses. This becomes even more problematic when attempting to combine likelihoods from different experiments. A new procedure to combine mixing results (“amplitude method”), in which the amplitude of the oscillation is fitted as a function of $\Delta m_s$, has been proposed to overcome this problem [3]. As many of the analyses do not provide the necessary information to perform an amplitude method combination, for

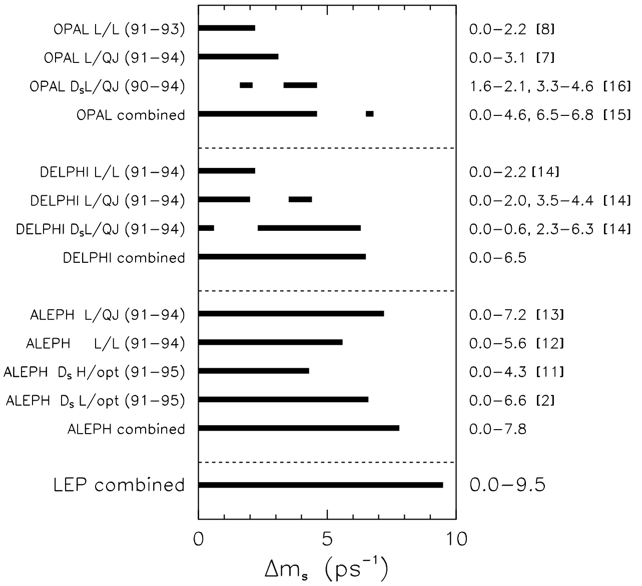

Figure 4: A summary of the 95% CL exclusion regions for $\Delta m_s$. The number in square brackets refers to the relevant reference. The data sample used is also indicated.

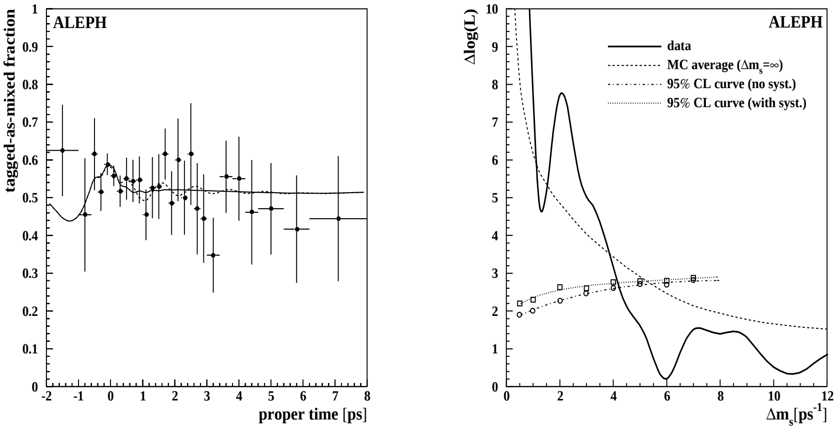

Figure 5: The ALEPH $D_sH$/OPT analysis: (a) Fraction of candidates tagged as mixed as a function of the proper time. The solid curve is the likelihood fit for $\Delta m_s$. (b) The negative log-likelihood with respect to the minimum as a function of $\Delta m_s$. The solid curve shows the data. The dotted (dot-dashed) line is the 95% CL curve with (without) systematics.

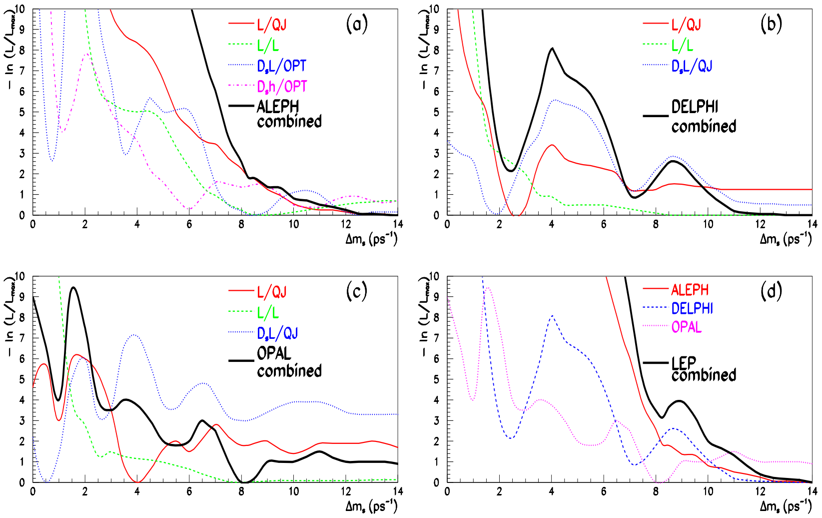

Figure 6: The individual log likelihoods distributions and their sum for the various analyses. (a) ALEPH analyses, (b) DELPHI analyses, (c) OPAL analyses, (d) LEP combined.

this paper, the naive “addition of log likelihood” method is adopted assuming a conservative $\Delta ln\mathcal{L}^{95} = 3$.

The individual log likelihoods and their sum, relative to the minimum, for the ALEPH, DELPHI and OPAL analyses are shown in Figs. 6(a),(b) and (c) respectively. The combined ALEPH/DELPHI/OPAL lower limits are $\Delta m_s > 7.8/6.5/4.6$ ps$^{-1}$. Possible statistical overlap in the DELPHI L/L and L/QJ analysis is ignored. Fig. 6(d) shows the sum of the ALEPH, DELPHI and OPAL combined log likelihoods. Again assuming $\Delta ln\mathcal{L}^{95} = 3$ the combined LEP average is $> 9.5$ ps$^{-1}$. Using the world average $\tau_{B_s} = 1.61 \pm 0.10$ ps [17] this corresponds to $x_s > 15.3$.

## 5 Conclusion

The measurement of $\Delta m_d$ continues to improve; the latest LEP average is $\Delta m_d = 0.469 \pm 0.017$ ps$^{-1}$, i.e. a relative error of 3.6%. This can be compared with the relative theoretical error on the quantity $f^2_{B_d} B_{B_d}$, used in equation 1 to access $|V_{td}|$, of $\approx 40\%$ [1]. In order to reduce the theoretical uncertainties, the emphasis of the mixing analyses is to measure the ratio $\Delta m_s/\Delta m_d$. The latest LEP limit on $\Delta m_s > 9.5$ ps$^{-1}$ gives $\Delta m_s/\Delta m_d > 19.5$ which, via equation 2, implies $|V_{ts}/V_{td}| > 3.8$. In terms of the unitarity triangle this corresponds to

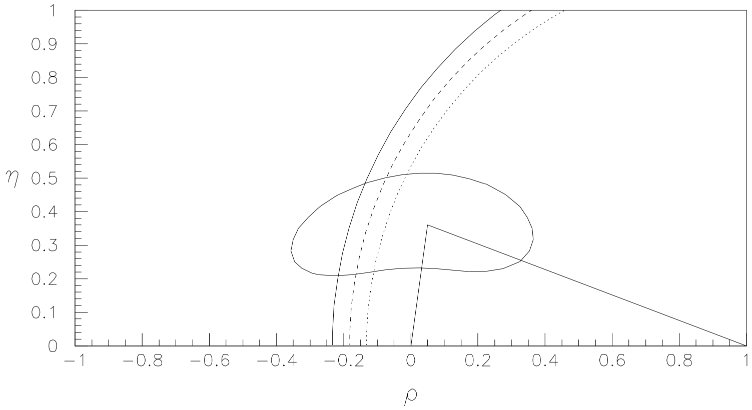

Figure 7: The constraint in the $\rho - \eta$ plane from the LEP bound $\Delta m_s/\Delta m_d > 19.5$. The three curves correspond to $\xi = 1.2$ (solid), $\xi = 1.15$ (dashed) and $\xi = 1.1$ (dots). The region to the left of the curves are excluded. The allowed region for the apex is taken from Ref. [1].

a circle in the $\rho - \eta$ plane which starts to significantly constrain the allowed region for its apex (Fig. 7).

The $B_s$ mixing analyses at LEP are statistics limited. As LEP has recently increased its energy to the $W^+W^-$ threshold, where $B_s$ mesons are no longer copiously produced, future prospects for significantly improved limits or possible observation of $B_s$ mixing at LEP are not bright. Nevertheless, some modest progress can be expected as analyses improve and all experiments incorporate the 1995 data (+20%). The conclusive observation of the elusive $B_s$ oscillation may therefore have to wait for future experiments at Tevatron, HERA and LHC.

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