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Supersymmetry, Duality and Bound States

Ramzi R. Khuri[1]

Theory Division, CERN, CH-1211, Geneva 23, Switzerland

and

Physics Department, McGill University, Montreal, PQ, H3A 2T8 Canada

P-brane solutions of low-energy string actions have traditionally provided the first evidence for the existence of string dualities, in which fundamental and solitonic p-branes are identified with perturbative and non-perturbative BPS states. In this talk we discuss the composite nature of solutions, which allows for the interpretation of general solutions as bound states or intersections of maximally supersymmetric fundamental constituents. This feature lies at the heart of the recent success of string theory in reproducing the Beckenstein-Hawking black hole entropy formula.

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In the last few years, it has become increasingly clear that p-branes, i.e. $(p+1)$-dimensional extended objects that arise as fundamental or solitonic solutions of string theory, play an important role in establishing the various string dualities and ultimately in the non-perturbative formulation of an underlying theory containing strings and higher membranes.

Under a given duality map, the fundamental p-brane of a given string or higher membrane theory tranforms into a solitonic solution of the dual theory, with the corresponding interchange of singular/non-singular backgrounds, perturbative/non-perturbative states and even spacetime/worldsheet loops (see [1] and references therein).

An interesting example of the implications of duality is that of six-dimensional heterotic string/string duality, which, when reduced to four dimensions, leads to a *duality of dualities*, namely, the interchange of the spacetime strong/weak coupling S-duality with the worldsheet target space T-duality [2]. As a result, the conjectured S-duality follows from the established T-duality provided four-dimensional string/string duality is shown.

More recently, interest has focused on duality between different string theories, in particular, heterotic/type IIA string/string duality in $D = 6$ [3,4]. In this case, one has a more straightforward strong/coupling duality relating the two theories with $g_{IIA} = 1/g_{het}$.

The presence of certain p-branes, however, coupled with the assumption of a given duality, can point to gaps in the formulation of string theory. In particular, it was shown in [5] that, in the context of heterotic/type duality, aside from the usual interchange of fundamental and solitonic strings, there exist membrane solutions which are non-singular (solitonic) in the heterotic theory but singular in the type IIA theory. Such solutions were then interpreted as fundamental membranes in type IIA, since they could neither be ignored as purely singular configurations nor simply counted in the solitonic spectrum of either theory. The authors of [5] then concluded that the formulation of type IIA theory as a theory of strings alone was incomplete. The subsequent discovery of the role of D-branes as carriers of Ramond-Ramond (RR) charge that should be coupled to type II theories [6] (see also [7]) effectively resolved this problem, since the fundamental membrane solution of [5] could now have the interpretation of a D-brane.

A picture then emerges in which fundamental and solitonic p-branes carrying Neveu-Schwarz-Neveu-Schwarz charge correspond to perturbative and non-perturbative BPS states of string theory, while p-branes carrying RR charge correspond to D-brane BPS states which must be coupled to the perturbative spectrum. From both the p-brane and BPS state points of view, the mass of a fundamental solution (state) is independent of the string coupling g, while the mass of a solitonic solution (state) scales as $1/g^2$. By contrast, the mass of a D-brane scales as $1/g$ [8,9]. This intermediate behaviour between fundamental and solitonic states is an inherently stringy departure from the usual $1/g^2$ solitonic scaling found in field theory.

Recent activity has also focused on the existence of an eleven-dimensional theory, the so-called M-theory [4,10–12], whose low-energy limit is eleven-dimensional supergravity and whose construction should lead to the establishment of the various string/string dualities [2,3,4]. While we still do not know exactly what M-theory is, we know that M-theory contains membranes and fivebranes. From the point of view of p-brane solutions, these are represented by a fundamental membrane [13] and a solitonic fivebrane [14].

An example of how p-brane solutions can provide evidence for a duality is the following. In [15], the conjecture was made that the effective theory of heterotic string theory compactified on $K3 \times S^1$ is dual to eleven-dimensional supergravity compactified on a Calabi-Yau threefold. Point-like (electric) solutions are obtained in $D = 5$ by wrapping the membrane from M-theory around two-cycles in the Calabi-Yau space, while string-like (magnetic) solutions in $D = 5$ arise by wrapping the fivebrane around four-cycles. For the specific Calabi-Yau manifold $X_{24}(1, 1, 2, 8, 12)$ with $h_{(1,1)} = 3$ and $h_{(2,1)} = 243$, these point and string solutions/states can be matched with perturbative and non-perturbative solutions/states of heterotic string theory compactified on $K3 \times S^1$ [16].

The heterotic perturbative solutions (or states) include the fundamental string and electrically charged point-like H-monopole [17] and Kaluza-Klein [18] solutions, the latter two being obtained by wrapping the string around S_1. The fundamental string can be identified with one of the three states arising from the M-theory fivebrane, while the electric H-monopole and Kaluza-Klein monopole can be identified with two of the three states arising from the M-theory membrane.

The non-perturbative heterotic solutions/states consist of the point-like solution obtained from the heterotic fivebrane wrapped around $K3 \times S^1$ and the magnetically charged string-like H-monopole and Kaluza-Klein solutions obtained by wrapping the fivebrane around $K3$ only. Here the point-like state can be identified with one of the three states arising from the M-theory membrane, while the magnetic H-monopole and Kaluza-Klein states can be identified with two of the three states arising from the M-theory fivebrane.

The "basic" fundamental and solitonic p-branes preserve half the spacetime supersymmetries, and arise as extremal limits of more general, non-supersymmetric black p-brane solutions of string theory. It turns out, however, that the low-energy supergravity equations of motion possess a feature that allows for the immediate construction of composite solutions from the basic ones. Consider, for example, the double-instanton string solution of heterotic string theory [19]

$$
\begin{aligned}
\phi &= \phi_1 + \phi_2, \\
g_{mn} &= e^{2\phi_1}\delta_{mn} \qquad m,n = 2,3,4,5, \\
g_{ij} &= e^{2\phi_2}\delta_{ij} \qquad i,j = 6,7,8,9, \\
g_{\mu\nu} &= \eta_{\mu\nu} \qquad \mu,\nu = 0,1, \\
H_{mnp} &= \pm 2\epsilon_{mnpq}\partial^q\phi \qquad m,n,p,q = 2,3,4,5, \\
H_{ijk} &= \pm 2\epsilon_{ijkl}\partial^k\phi \qquad i,j,k,l = 6,7,8,9
\end{aligned}
\tag{1}
$$

with $e^{-2\phi_1}\Box_1 e^{2\phi_1} = e^{-2\phi_2}\Box_1 e^{2\phi_2} = 0$, where $\Box_1$ and $\Box_2$ are the Laplacians in the (2345) and (6789) spaces, respectively. For constant chiral spinors $\epsilon = \epsilon_2 \otimes \eta_4 \otimes \eta_4'$, (1) solves the supersymmetry equations with zero background fermi fields. Due to the presence of two independent instantons [17] in the generalized curvature containing H_{MNP} as a torsion, the chiralities of the spinors ϵ_2, η_4 and η_4' are correlated by $(1 \mp \gamma_3)\epsilon_2 = (1 \mp \gamma_5)\eta_4 = (1 \mp \gamma_5)\eta_4' = 0$, so that 1/4 of the spacetime supersymmetries is preserved.

For either $\phi_1 = 0$ or $\phi_2 = 0$ we recover the solitonic fivebrane solution of [20] which preserves 1/2 the spacetime supersymmetries, since only one instanton is present. In this respect, this string is the composite of two independent fivebranes intersecting along the string. This feature can in fact be generalized to arbitrary p-brane solutions [21–24], whereby a given p-brane soliton can be interpreted as the intersection of one or more maximally supersymmetric basic fundamental or solitonic p-branes. From the viewpoint of M-theory, the statement then translates into saying that all p-brane solutions arise as intersections of membrane and fivebranes [21].

Compositeness was also seen in the specific context of string-like solutions of toroidally compactified $D = 4$ heterotic string theory [25], where each solution could be understood in terms of four independent harmonic functions. The existence of these latter solutions also pointed to an interesting interplay between supersymmetry and duality. In particular, the different supersymmetry breaking patterns of the string-like solutions conform to the different large duality groups (containing both S and T duality) of the various compactifications [25].

A related composite picture of p-brane solutions is the bound states picture [26–31]. For the simplest case of extremal black hole solutions in $D = 4$, consider the Einstein-Maxwell scalar action

$$S = \int d^4x \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{-a\phi}F_2^2 \right), \tag{2}$$

where a is an arbitrary parameter. It turns out that for the specific values of $a = \sqrt{3}, 1, 1/\sqrt{3}$ and 0, supersymmetric extreme black holes arising from string compactifications were found. Moreover, from both the spacetime solutions [27] and supersymmetries [28] point of view, these four solutions can be interpreted as bound states of $1, 2, 3$ and 4 distinct $a = \sqrt{3}$ black holes, respectively, the latter corresponding to maximally supersymmetric ($N = 4$ supersymmetry in an $N = 8$, $D = 4$ theory) Kaluza-Klein [18] or H-monopole [17] solutions with flat metric on moduli space [32]. For example, the $a = 0$ Reissner-Nordström black hole arises as a bound state of two T-dual pairs of electric/magnetic $a = \sqrt{3}$ black holes, each pair producing an $a = 1$ black hole. Again, this feature extend quite naturally to arbitrary supersymmetric p-branes in any dimension, as well as to non-extremal, non-supersymmetric p-branes [24].

This compositeness feature lies at the heart of the recent success of string theory in reproducing the Beckenstein-Hawking formula for the entropy of black holes [33–35]. In particular, for a given p-brane arising as a bound state or as intersections of basic constituent p-branes of charge Q_1,Q_2,. . .,Q_n, the area law for the entropy yields

$$S = A/4G = 2\pi\sqrt{Q_1Q_2\ldots Q_n}, \tag{3}$$

which agrees, for large Q_i, with the microscopic entropy formula obtained by counting string states. This was first seen for five-dimensional extremal black holes in [33] and subsequently for four-dimensional extremal black holes in [34]. Analogous results for near-extremal black holes were obtained in [35], which seems to indicate that this sort of factorization is not a property of supersymmetry alone, although it is only for supersymmetric solutions that one can invoke non-renormalization theorems to protect the counting of states in going from the perturbative state-counting picture to the non-perturbative black hole picture.

The question then arises as to whether a similar property holds for arbitrary non-extremal black holes. Such a formula was, in fact, found from the p-brane picture in [35], where, however, it was noted that the corresponding D-brane counting argument was unknown. More recently, it was argued in [36] that the counting arguments relating perturbative states to black holes break down for "fat" black holes, *i.e.* large black holes far from extremality (such as the Schwarzschild black hole). However, this does not in itself rule out the possibility of an analogous, if not identical, compositeness feature in the general case which will once again recover the Beckenstein-Hawking formula and confirm an important success of string theory as a theory of quantum gravity.

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