NON-DESTRUCTIVE DIAGNOSTICS OF COASTING BEAMS WITH SCHOTTKY NOISE

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Summary

The finite number of protons in a circulating beam gives rise to statistical fluctuations in the beam current and beam's centre of gravity. This Schottky noise is used to monitor the distribution of particles in longitudinal momentum as well as to measure the extrema of the Q-values in a stack without any interference with the beam coasting for many hours. It is also instrumental in detecting the growth of betatron amplitudes at particular orbits of a stack, which helps to discern presence and strength of non-linear resonances.

Theory

The statistical noise due to the incoherent motion of particles in a coasting beam can be calculated from the signal of one particle. At the azimuth of a pick-up station the line charge density of the i'th proton is a sequence of delta-functions:

$$\lambda_i(t) = \frac{e}{R} \sum_{\ell=-\infty}^{+\infty} \delta(2\pi\ f_i\ t + \theta_i - 2\pi\ \ell) \qquad (1)$$

where e is its charge, f_i its revolution frequency, θ_i its azimuth at $t = 0$, and $2\pi R$ is the machine circumference. This can be Fourier analysed: in terms of non-negative frequencies:

$$\lambda_i(t) = \frac{e}{2\pi\ R}\left(1 + 2\sum_{n=1}^{\infty} \cos n(2\pi\ f_i\ t + \theta_i)\right) \qquad (2)$$

it is a spectrum of lines with frequency spacing f_i and mean square values

$$2\left(\frac{e}{2\pi\ R}\right)^2$$

We have present particles with their f_i in some range, say f_o to $f_o + \Delta f_o$: then for any one given n the resulting frequencies of (2) are in a band of width $n\Delta f_o$. To avoid confusion we work in the region

$$n < f_o/\Delta f_o$$

so that this band does not overlap those belonging to the neighbouring n-values. In the ISR this means working below $n \sim 3000$, or 1 GHz. The signals contributing to this band are given by the n'th term in (2), summed over all the particles:

$$\sum_i \lambda_{i,n}(t) = \frac{2e}{2\pi\ R} \sum_i \cos n(2\pi\ f_i\ t + \theta_i) \qquad (3)$$

Let the spectrum of revolution frequencies be defined by $N(f_i)\ df_i$, the number present in an interval df_i at f_i. We take an idealised model of a spectrum analyser; when tuned to a frequency f it responds to all signals in a small interval δf , called its resolution, and evaluates their root mean square. The number of particles contributing is therefore

$$N\left(\frac{f}{n}\right)\frac{\delta f}{n} \qquad (4)$$

and the result is

$$\lambda_{rms}\ (f) = \frac{2e}{2\pi\ R}\left\langle\left[\sum_{\delta f} \cos n(2\pi\ f_i\ t + \theta_i)\right]^2\right\rangle^{\frac{1}{2}} \qquad (5a)$$

$$= \frac{\sqrt{2}e}{2\pi\ R}\left[N\left(\frac{f}{n}\right)\frac{\delta f}{n}\right]^{\frac{1}{2}} \qquad (5b)$$

where the summation is restricted to the particles (4). A necessary assumption for going from (5a) to (5b) is that the particles in any group having all the same revolution frequencies f_i must have randomly distributed phases $n\theta_i$, so we are excluding cases where the beam has any coherent disturbance.

This (5b) is effectively Schottky's formula [1] for the statistical fluctuations of a d.c. current. It shows that the spectrum analyser gives out the spectrum of particle revolution frequencies, with a scale factor n in the abscissae and a square root function in the ordinates.

Now consider one particle making transverse betatron oscillations. At the azimuth of the pick-up station it will have a vertical displacement given by

$$z_i(t) = z_o + A_i \cos(2\pi\ Q_i\ f_i\ t + \phi_i) \qquad (6)$$

where z_o is the closed orbit displacement and A_i the particle's betatron amplitude at that azimuth.

A difference pick-up responds to the line dipole density, which we call d. d_i is given by multiplying (1) or (2) by (6), and its expansion in single frequencies is

$$d_i(t) = \frac{e}{2\pi R} \; z_o + A_i \cos(2\pi\, Q_i\, f_i\, t + \phi_i)$$

$$+ 2z_o \sum_{n=1}^{\infty} \cos n(2\pi\, f_i\, t + \theta_i)$$

$$+ A_i \sum_{n=1}^{\infty} \cos(n - Q_i)(2\pi\, f_i\, t + \psi_i) \qquad (7)$$

$$+ A_i \sum_{n=1}^{\infty} \cos(n + Q_i)(2\pi\, f_i\, t + \chi_i)$$

Again it is possible to work in the region of n low enough that the bands belonging to different n values do not overlap, and to look with the spectrum analyser at any of the three n'th terms in (7), summed over all the particles.

We note in passing that information about the closed orbit z_o may be obtainable from the transverse signals in the "longitudinal", nf_i , frequency band. Calculating the root mean square in δf , as before, the $n \mp Q$ bands give

$$d_{rms}\ (f) = \frac{\sqrt{\tfrac{1}{2}}e}{2\pi\ R}\ A(f)\ [N_n(f)\ \delta f]^{\frac{1}{2}}$$

where $N_n(f)$ is the number of particles per unit interval of

$$f = (n \mp Q_i)f_i$$

and $A(f)$ is their *rms* amplitude.

In the general case Q-spread, f_i-spread and amplitude will all affect the spectra and need to be disentangled, but any marked feature like big amplitudes or missing particles occurring at some specific tune Q_k and revolution frequency f_k will show up as a spike or slot at the frequencies $f = (n \mp Q_k)f_k$ for every n.

Application

Figure 1 shows a scan in the frequency domain of the noise picked up from the coasting beam. It is the difference signal derived from two parallel plates located on opposite sides of the beam. The central peaks are enhanced by a resonant transformer which was used to form the difference. The small spikes arise from the longitudinal Schottky noise; they are harmonics of the revolution frequency $f_o \simeq 320$ kHz. The transverse Schottky noise gives the large signals corresponding to Fourier expansion of the incoherent motion into slow and fast transverse waves. Their different spread in frequency is determined by the interplay between Q-spread and spread in revolution frequency. In the case of slow waves, $f = (n - Q)f_o$, the contributions of the two spreads add up if the accelerator is operated above transition and with positive chromaticity. Thus the signals are wider. The opposite holds for the fast waves, $f = (n + Q)f_o$. The Q-value was $\sim$ 8,63 when this scan was made.

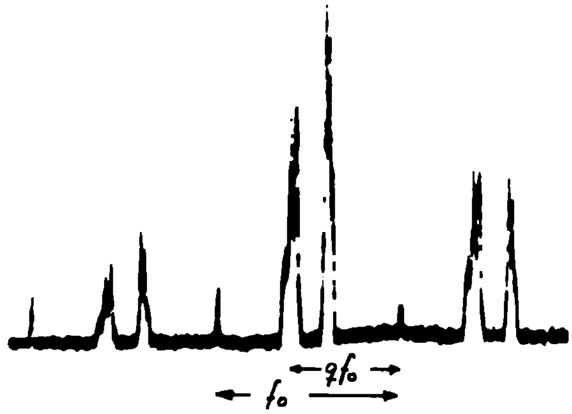


Fig. 1 - Frequency spectrum of the difference signal derived from two plates and induced by the Schottky noise in the beam. Small peaks harmonics of the revolution frequency. Large peaks fast and slow transverse waves.

Longitudinal Schottky scan

Scanning one of the harmonics of the revolution frequency in detail gives the longitudinal Schottky scan. The signal is taken from a sum pick-up, is amplified and displayed in the frequency domain by a HP 141 T frequency analyser. A substantial improvement in signal to noise ratio is achieved by averaging the analogue output of the analyser over many sweeps by means of a 1000 point HP 5480 B signal analyser. The choice of the harmonic is a compromise between conflicting requirements. At a too high frequency the harmonics start to overlap, at a too low harmonic the averaging takes too long because the sweep time of the analyser has to be increased for the same resolution.

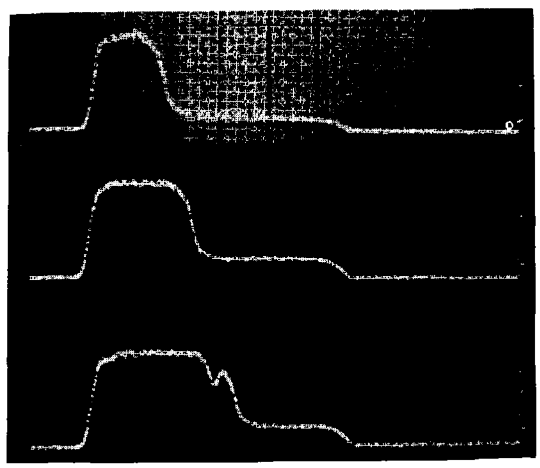
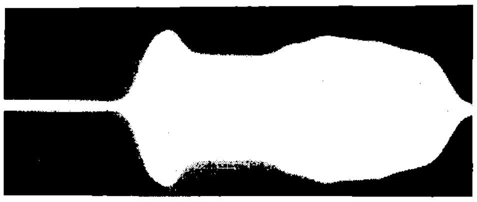


Fig. 2 - Longitudinal Schottky scan taken at different intensity levels during build-up of a stack (10 A, 15 A, 19 A). Average over ~ 2000 samples. $\Delta p/p$ = 6.4 % per hor. Division.

Figure 2 shows a sequence of longitudinal Schottky scans taken during the build-up of a stack. Typically, one averages over 2048 sweeps which takes 1,7 minutes. Figure 3 shows that a longitudinal Schottky scan provides the same information as a scan made by sweeping empty RF buckets through the stack. Moreover, the Schottky scan is not interfering with the beam in any way. This is an important point since the empty bucket scan turned out to be a rather gross perturbation of the coasting beams.



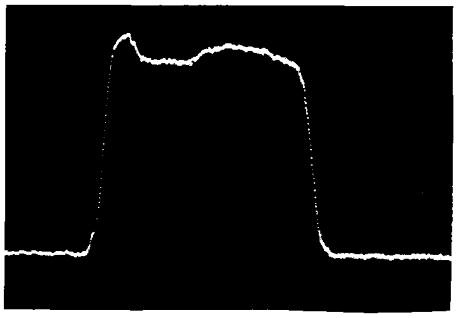


Fig. 3 - Scans of the longitudinal density in a stacked beam.

a) sweeping empty RF buckets through the beam

b) longitudinal Schottky scan

For these reasons the Schottky scan has become the standard technique in the ISR for measuring density in longitudinal momentum space. It is used operationally to monitor the mean momentum and the momentum spread of stacks coasting for many hours. It is also a valuable tool to detect the density variations due to beam loss at particular orbits caused by non-linear resonances, or due to diffusion in momentum space.

Transverse Schottky scan

Vertical Schottky signals in the low frequency range (5 - 20 MHz) are obtained with a resonant difference transformer connected to electrodes above and below the beam. Radial signals are obtained similarly from a pair of lateral electrodes. At higher frequencies (20 MHz - 2 GHz) where the capacitive coupling between beam and electrodes is increased, signals can be obtained with a 50 Ω hybrid transformer, using its difference output. The signals from the resonant transformer are fed to a high impedance low noise amplifier, those from the 50 Ω hybrid to a 50 Ω low noise amplifier. Then the signal is scanned in the frequency domain and suitably averaged as in the case of the longitudinal scan.

The transverse Schottky scans are mainly used to find the maximum and minimum Q-values of a stack. A large Q-spread in the stack is needed to prevent transverse coherent instabilities of the coasting beam. This Q-spread can be just accommodated between lower order non-linear resonances [2)] known to be harmful. Hence, an accurate, "on line" monitoring of the stack extension in the Q-plane is imperative. The usual technique is to display scans of a slow and a fast wave, which are adjacent in frequency, on the same screen by mixing down the fast wave scan. Corresponding points of the stack will appear on opposite sides of each scan. By measuring the real frequency difference Δf between these points one can calculate the non-integral part of the Q-value

$$q = 0{,}5 + (\Delta f/2f_o)$$

where f_o is the revolution frequency in the middle of the stack. Figure 4 shows two such scans. Each of them represents an average over 6000 sweeps of the frequency analyser. The sampling lasts 10 minutes in total. It is obvious from the figure that the ends of the distribution are quite well defined and that the frequency difference can be measured to better than 1 kHz. This means in turn a precision of 10^{-3} in the absolute value of q.

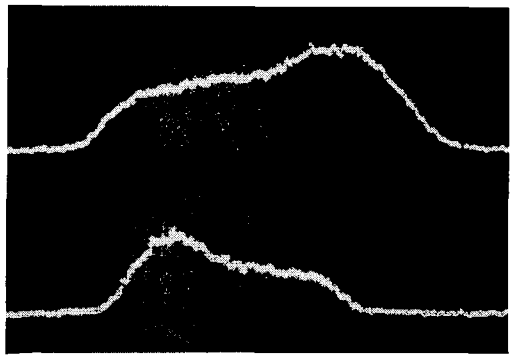


Fig. 4 - Transverse Schottky scan of a 6,5 A beam. Average over ∿ 6000 samples. 2 kHz/hor. Division.

upper trace: scan of slow wave $f = (43 - Q_v)f_o$

lower trace: scan of fast wave $f = (26 + Q_v)f_o$ displaced downwards in frequency by 76 kHz

Since the transverse Schottky scan gives the rms dipole moment as a function of the betatron frequency, one can observe changes in the rms betatron amplitude at a particular Q-value and orbit, provided that no beam loss occurs at this orbit; and the latter can be checked with the longitudinal Schottky scan. Figure 5a gives the vertical Schottky scan of a 15 A beam after stacking. Comparison with Figure 5b, a repetition of the scan after 80 minutes, shows a clear amplitude growth at distinct places. Figure 5c is identical to Figure 5b apart from the higher resolution of the former. The large peaks are due to 8th order resonances whereas 11th order resonances caused the smaller peaks.

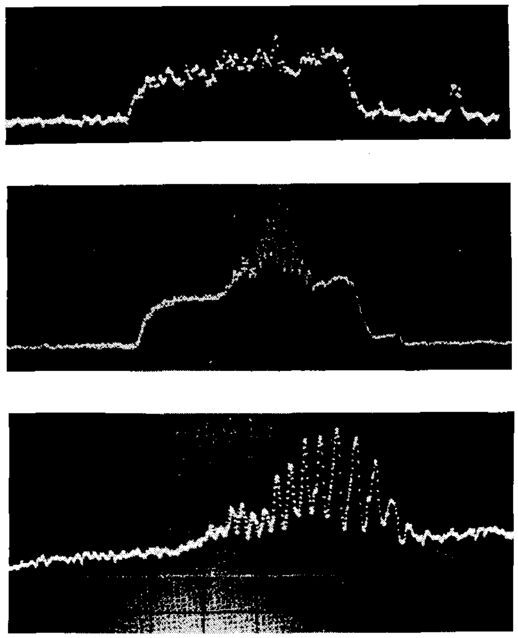


Fig. 5 - Transverse Schottky scan of the slow wave $f = (43 - Q)f_0$ showing amplitude growth due to 8th order and 11th order non-linear resonances.

a) after stacking ΔQ = 0,016/hor. Division

b) 80 min. later ΔQ = 0,016/hor. Division

c) same as b) ΔQ = 0,0063/hor. Division

A further application is the monitoring of the position and extension of slots cut by RF knock-out in the particle distribution. Such a cleaning operation is needed if the beam position is so close to strong resonances that periodical removal is needed of those particles which diffused towards the resonances.

References

1) W. Schottky; Über spontane Stromschwankungen in verschiedenen Electrizitätsleitern. Ann. Physik 57, p. 541 (1918).

2) P. Bryant; Dynamic compensation during stacking of the detuning caused by space charge effects. Contribution to this conference.