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Chiral Symmetry Breaking in QCD: A Variational Approach

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Abstract

We develop a “variational mass” expansion approach, recently introduced in the Gross-Neveu model, to evaluate some of the order parameters of chiral symmetry breakdown in QCD. The method relies on a reorganization of the usual perturbation theory with the addition of an *arbitrary* quark mass m , whose non-perturbative behaviour is inferred partly from renormalization group properties, and from analytic continuation in m properties. The resulting ansatz can be optimized, and in the chiral limit $m \rightarrow 0$ we estimate the dynamical contribution to the “constituent” masses of the light quarks $M_{u,d,s}$; the pion decay constant F_π and the quark condensate $\langle \bar{q}q \rangle$.

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1 Introduction

A still challenging question in strong interaction physics is the derivation of the low-energy properties of the spectrum from “QCD first principle”, due to our limited present skill with non-perturbative physics. At very low energy, where the ordinary perturbation theory cannot be applied, Chiral Perturbation Theory [1] and Extended Nambu–Jona-Lasinio models [2, 3]¹ give a consistent framework in terms of a set of parameters that have to be fixed from the data; yet the bridge between those effective parameters and the basic QCD degrees of freedom remains largely unsolved. Although lattice QCD simulations recently made definite progress [5] in that direction, the consistent treatment of dynamical unquenched quarks and the chiral symmetry remains a serious problem.

In this paper, we investigate a new, semi-analytical method, to explore *how far* the basic QCD Lagrangian can provide, in a self-consistent way, non-zero dynamical quark masses, quark condensates, and pion decay constant, in the limit of vanishing Lagrangian (current) quark masses. Such a qualitative picture of chiral symmetry breakdown (CSB) can be made more quantitative by applying a new “variational mass” approach, recently developed within the framework of the anharmonic oscillator [6], and in the Gross–Neveu (GN) model [7, 8]. The starting point is very similar to the ideas developed a long time ago and implemented in various different forms in refs.[9], [10]. There, it was advocated that the convergence of conventional perturbation theory may be improved by a variational procedure in which the separation of the action into “free” and “interaction” parts is made to depend on some set of auxiliary parameters. The results obtained by expanding to finite order in this redefined perturbation series are optimal in regions of the space of auxiliary parameters where they are least sensitive to these parameters. Recently there appeared strong evidence that this optimized perturbation theory may indeed lead to a rigorously convergent series of approximations even in strong coupling cases [11]. An essential novelty, however, in [6]–[8] and the present paper, is that our construction combines in a specific manner the renormalization group (RG) invariance with the properties of an analytically continued, arbitrary mass parameter m . This, at least in a certain approximation to be motivated, allows us to reach *infinite* order of the variational-perturbative expansion, and therefore presumably optimal, provided it converges. Our main results are a set of non-perturbative ansatzs for the relevant CSB quantities, as functions of the variational mass m , which can be studied for extrema and optimized. Quite essentially, our construction also provides a simple and consistent treatment of the renormalization, reconciling the variational approach with the inherent infinities of quantum field theory and the RG properties.

Before proceeding, let us note that there exists a quite radically different attitude towards CSB in QCD, advocating that the responsible mechanism is most probably the non-perturbative effects due to the *instanton* vacuum [12], or even more directly related to confinement [13]. However, even if the instanton picture of CSB is on general grounds well motivated, and many fruitful ideas have been developed in that context², as far as we are aware there is at present no sufficiently rigorous or compelling evidence for

¹For a recent complete review see [4].

²See e.g. ref. [14] for a review and original references.

it, derived from “first principle”. In any event, it is certainly of interest to investigate quantitatively the “non-instantonic” contribution to CSB, and we hope that our method is a more consistent step in that direction.

2 Dynamical quark masses

In what follows we only consider the $SU(n_f)_L \times SU(n_f)_R$ part of the chiral symmetry, realized by the QCD Lagrangian in the absence of quark mass terms, and for $n_f = 2$ or $n_f = 3$ as physically relevant applications.

Following the treatment of the anharmonic oscillator [6] and its generalization to the GN model [7, 8], let us consider the following modification of the usual QCD Lagrangian,

$$L_{QCD} \rightarrow L_{QCD}^{free}(g_0 = 0, m_0 = 0) - m_0 \sum_{i=1}^{n_f} \bar{q}_i q_i + L_{QCD}^{int}(g_0^2 \rightarrow x g_0^2) + x m_0 \sum_{i=1}^{n_f} \bar{q}_i q_i, \quad (1)$$

where L_{QCD}^{int} designates the ordinary QCD interaction terms, and x is a convenient “interpolating” expansion parameter. This formally is equivalent to substituting everywhere in the bare Lagrangian,

$$m_0 \rightarrow m_0 (1 - x); \quad g_0^2 \rightarrow g_0^2 x, \quad (2)$$

and therefore in any perturbative (bare) quantity as well, calculated in terms of m_0 and g_0^2 . Since the original massless QCD Lagrangian is recovered for $x \rightarrow 1$, m_0 is to be considered as an *arbitrary* mass parameter after substitution (2). One expects to optimize physical quantities with respect to m_0 at different, possibly arbitrary orders of the expansion parameter x , eventually approaching a stable limit, i.e. *flattest* with respect to m_0 , at sufficiently high order in x .

However, before accessing any physical quantity of interest for such an optimization, the theory should be renormalized, and there is an unavoidable mismatch between the expansion in x , as introduced above, and the ordinary perturbative expansion as dictated by the mass and coupling counterterms. Moreover, it is easy to see that at any finite order in the x expansion, one always recovers a trivial result in the limit $m \rightarrow 0$ (equivalently $x \rightarrow 1$), which is the limit in which to identify non-zero order parameters of CSB.

These problems can be circumvented by advocating a specific ansatz, which resums the (reorganized) perturbation series in x and is such that the limit $x \rightarrow 1$ no longer gives a trivial zero mass gap. As was shown in detail in ref. [8], the ansatz for the dynamical mass is most easily derived by following the steps³:

i) Consider first the general solution for the running mass, given as

$$m(\mu') = m(\mu) \exp \left\{ - \int_{g(\mu)}^{g(\mu')} dg \frac{\gamma_m(g)}{\beta(g)} \right\} \quad (3)$$

in terms of the effective coupling $g(\mu)$, whose RG evolution is given as $\mu dg(\mu)/d\mu \equiv \beta(g)$, and $\gamma_m(g) \equiv -(\mu/m)d(m(\mu))/d\mu$. Solving (3) imposing the “fixed point” boundary condition:

$$M \equiv m(M), \quad (4)$$

³See also ref. [15] for a detailed derivation in the QCD context.

at two-loop RG order we obtain, after some algebra (we use the normalization $\beta(g) = -b_0 g^3 - b_1 g^5 - \dots$, $\gamma_m(g) = \gamma_0 g^2 + \gamma_1 g^4 + \dots$):

$$M_2 = \bar{m} f^{-\frac{\gamma_0}{2b_0}} \left[\frac{1 + \frac{b_1}{b_0} \bar{g}^2 f^{-1}}{1 + \frac{b_1}{b_0} \bar{g}^2} \right]^{-\frac{\gamma_1}{2b_1} + \frac{\gamma_0}{2b_0}}, \quad (5)$$

where $\bar{m} \equiv m(\bar{\mu})$, $\bar{g} \equiv g(\bar{\mu})$ ($\bar{\mu} \equiv \mu \sqrt{4\pi} e^{-\gamma_E/2}$), and $f \equiv \bar{g}^2/g^2(M_2)$ satisfies

$$f = 1 + 2b_0 \bar{g}^2 \ln \frac{M_2}{\bar{\mu}} + \frac{b_1}{b_0} \bar{g}^2 \ln \left[\frac{1 + \frac{b_1}{b_0} \bar{g}^2 f^{-1}}{1 + \frac{b_1}{b_0} \bar{g}^2} f \right]; \quad (6)$$

(note in (5) and (6) the recursivity in both f and M_2). The necessary non-logarithmic perturbative corrections to those pure RG results are then consistently included as

$$M_2^P \equiv M_2 \left(1 + \frac{2}{3} \gamma_0 \frac{\bar{g}^2}{f} + \frac{K}{(4\pi^2)^2} \frac{\bar{g}^4}{f^2} + \mathcal{O}(g^6) \right), \quad (7)$$

where the complicated two-loop coefficient K was calculated exactly in ref. [16]. Equation. (7) defines the (infrared-convergent, gauge-invariant) pole mass [17] M_2^P , in terms of the \overline{MS} mass at two-loop order, and can be shown [15] to resum the leading (LL) and next-to-leading logarithmic (NLL) dependence in \bar{m} to all orders.

ii) Perform in expressions (5), (6), (7) the substitution $\bar{m} \rightarrow \bar{m}v$, and integrate the resulting expression, denoted by $M_2^P(v)$, according to

$$\frac{1}{2i\pi} \oint \frac{dv}{v} e^v M_2^P(v), \quad (8)$$

where the contour is around the negative real v axis.

In [8] it was shown that the previous steps correspond (up to a specific renormalization scheme (RS) change, allowed on general grounds from RG properties) to a resummation of the x series as generated from the substitution (2)⁴. Moreover this is in fact the only way of rendering compatible the above x expansion and the ordinary perturbative one, thus obtaining finite results. Actually the resummation coincides with the exact result in the large- N limit of the GN model. Now, since the summation can be formally extended to arbitrary RG orders [8], including consistently as many arbitrary perturbative correction terms as known in a given theory, in the QCD case we make the assumption that it gives an adequate “trial ansatz”, to be subsequently optimized in a way to be specified next. After appropriate rescaling of the basic parameters, \bar{g} and \bar{m} , by introducing the RG-invariant basic scale $\Lambda_{\overline{MS}}$ [18] (at two-loop order), and the convenient scale-invariant dimensionless “mass” parameter

$$m'' \equiv \left(\frac{\bar{m}}{\Lambda_{\overline{MS}}} \right) 2^C [2b_0 \bar{g}^2]^{-\gamma_0/(2b_0)} \left[1 + \frac{b_1}{b_0} \bar{g}^2 \right]^{\gamma_0/(2b_0) - \gamma_1/(2b_1)}, \quad (9)$$

we end up with the following dynamical mass ansatz:

$$\frac{M_2^P(m'')}{\Lambda_{\overline{MS}}} = \frac{2^{-C}}{2i\pi} \oint dy \frac{e^{y/m''}}{F^A(y)[C + F(y)]^B} \left(1 + \frac{\mathcal{M}_1}{F(y)} + \frac{\mathcal{M}_2}{F^2(y)} \right), \quad (10)$$

⁴ v is related to the original expansion parameter x as $x = 1 - v/q$, q being the order of the expansion.

where $y \equiv m''v$, and F is defined as

$$F(y) \equiv \ln[y] - A \ln[F(y)] - (B - C) \ln[C + F(y)], \quad (11)$$

with $A = \gamma_1/(2b_1)$, $B = \gamma_0/(2b_0) - \gamma_1/(2b_1)$, $C = b_1/(2b_0^2)$, in terms of the RG coefficients [19]. Finally the perturbative corrections in (10) are simply given as $\mathcal{M}_1 = (2/3)(\gamma_0/2b_0)$ and $\mathcal{M}_2 = K/(2b_0)^2$.

Observe in fact that, were we in a simplified QCD world, where there would be *no* non-logarithmic perturbative contributions (i.e. such that $\mathcal{M}_1 = \mathcal{M}_2 = \dots = 0$ in (10)), the latter ansatz would then resum exactly the x variational expansion. In that case, (10) would have a very simple behaviour near the origin $m'' \rightarrow 0$. Indeed, it is easy to see that (11) admits an expansion $F(y) \simeq C^{(B-C)/A} y^{1/A}$ for $y \rightarrow 0$, which immediately implies that (10) would give a simple pole at $y \rightarrow 0$, with a residue giving $M_2 = (2C)^{-C} \Lambda_{\overline{MS}}$. Moreover one can always choose an appropriate renormalization scheme in which b_2 and γ_2 are set to zero, as well as all higher order coefficients, so that there are no other corrections to the simple above relation.

Now, in the realistic world, \mathcal{M}_1 , \mathcal{M}_2 , etc can presumably not be neglected. We can nevertheless expand (10) near $m'' \rightarrow 0$ for any known non-zero \mathcal{M}_i , using

$$\frac{1}{2i\pi} \oint dy e^{y/m''} y^\alpha = \frac{(m'')^{1+\alpha}}{\Gamma[-\alpha]}, \quad (12)$$

and the resulting Laurent expansion in $(m'')^{1/A}$ may be analysed for extrema and optimized at different, in principle arbitrary $(m'')^{1/A}$ orders. An important point, however, is that the perturbative corrections do depend on the RS choice, as is well known. Since the pure RG behaviour in (10) already gives the order of magnitude, $M \simeq \text{const} \times \Lambda_{\overline{MS}}$, we can hope that a perturbative but optimized treatment of the remaining corrections is justified. In other words we shall perform an ‘‘optimized perturbation’’ with respect to m'' around the non-trivial fixed point of the RG solution.

To take into account this RS freedom, we first introduce in (10) an arbitrary scale parameter a , from $\bar{\mu} \rightarrow a \bar{\mu}$. Accordingly the perturbative coefficients \mathcal{M}_i in (10) take a logarithmic dependence in a , simply fixed order by order from (5)–(7) and the requirement that (10) differs from the original \overline{MS} expression only by higher order terms. The a -dependence will eventually exhibit a non-trivial extrema structure and we shall also optimize the result with respect to a^5 . Actually there are other possible changes of renormalization prescriptions affecting expression (10) in addition to the a dependence, which may be taken into account as well. More precisely, the second coefficient of $\gamma_m(g)$, γ_1 , do depend on the RS choice, even in MS schemes [20]. As it turns out, this additional RS freedom is very welcome in our case: in fact, the previous picture is invalidated, due to the occurrence of extra branch cuts in the y plane at $Re[y_{cut}] > 0$, as given by the zeros of dy/dF from (11) (in addition to the original cut on the negative real y axis). This prevents using the expansion near the origin, eq. (12), since it would lead to ambiguities

⁵This procedure indeed gave very good results [8] in the GN model, where in particular for low values of N the optimal values found, a_{opt} , are quite different from 1.

of $\mathcal{O}(\exp(\text{Re}[y]/m''))$ for $m'' \rightarrow 0^6$. The specific contour around the negative real axis was suggested by the known properties of the large N limit of the GN model, and it is not surprising if the analytic structure is more complicated in QCD. However, the nice point is that the actual position of those cuts do depend on the RS, via $A(\gamma_1)$ in (11). Defining $\gamma'_1 \equiv \gamma_1 + \Delta\gamma_1$, we can choose $\text{Re}[y_{cut}] \simeq 0$ for $\Delta\gamma_1 \simeq 0.00437$ (0.00267) for $n_f = 2$ ($n_f = 3$), respectively. We therefore consider [15] the general RS change

$$m' = \bar{m} (1 + B_1 \bar{g}^2 + B_2 \bar{g}^4); \quad g'^2 = \bar{g}^2 (1 + A_1 \bar{g}^2 + A_2 \bar{g}^4) \quad (13)$$

(implying $\Delta\gamma_1 = 2b_0 B_1 - \gamma_0 A_1$), and optimize with respect to this new arbitrariness⁷.

However one soon realizes that our extension of the ‘‘principle of minimal sensitivity’’ (PMS) [9] defines a rather complicated optimization problem. Fortunately, we can study this problem within some approximations, which we believe are legitimate. Since the ansatz (10) (with the above RS change understood, to make it consistent) would indeed be optimal with respect to m'' for *vanishing* perturbative non-logarithmic corrections, $\mathcal{M}_i = 0$, we shall assume that the expansion for small m'' is as close as possible to an optimum, and define the $m'' \rightarrow 0$ limit by some relatively crude but standard approximation, avoiding numerical optimization with respect to m'' . The approximation we are looking for is not unique: given (10), one could construct different approximations leading to a finite limit for $m'' \rightarrow 0$ [8]. Here we shall only demonstrate the feasibility of our program in the simplest possible realization. In fact, since we shall anyhow optimize with respect to the RS dependence we assume that it largely takes into account this non-uniqueness due to higher order uncertainties.

Padé approximants are known to greatly improve perturbative results [22] and often have the effect of smoothing the RS dependence. We thus take a simple Padé approximant which by construction restitutes a simple pole for $F \rightarrow 0$ (i.e $m'' \rightarrow 0$) in (10), and gives

$$M^{Pad\acute{e}}(a, \Delta\gamma_1, B_1, m'' \rightarrow 0) = \Lambda_{\overline{MS}} (2C)^{-C} a \exp\left\{\frac{A_1}{2b_0}\right\} \left[1 - \frac{\mathcal{M}_1^2(a, \Delta\gamma_1, B_1)}{\mathcal{M}_2(a, \Delta\gamma_1, B_1)}\right] \quad (14)$$

We have performed a rather systematic study of the possible extrema of (14) for arbitrary a , B_1 (with $\Delta\gamma_1$ fixed so that the extra cuts start at $\text{Re}[y] \simeq 0$). We obtain the flattest such extrema for $a \simeq 2.1$, $B_1 \simeq 0.1$, which leads to the result

$$M_{opt}^{Pad\acute{e}}(m'' \rightarrow 0) \simeq 2.97 \Lambda_{\overline{MS}}(2) \quad (15)$$

for $n_f = 2$. Similarly, we obtain $M_{opt}^{Pad\acute{e}}(m'' \rightarrow 0) \simeq 2.85 \Lambda_{\overline{MS}}(3)$ for $n_f = 3$. Note that these values of the dynamical quark masses, if they are to be consistent with the expected range [4] of $M_{dyn} \simeq 300\text{-}400$ GeV, call for relatively low $\Lambda_{\overline{MS}} \simeq 100\text{-}150$ GeV, which is indeed supported by our results in the next section.

⁶ The origin of those singularities is rather similar to the ambiguities related to renormalons [21]. An essential difference, however, is that the present singularities occur in the analytic continuation of a mass parameter rather than a coupling constant, and it is possible to move those singularities away by an appropriate RS change, as we discuss next. See ref. [15] for an extended discussion.

⁷ We also impose a further RS choice, $b'_2 = 0$, $\gamma'_2 = 0$, which fixes A_2 , B_2 in (13) and guarantees that our two-loop convention for $\Lambda_{\overline{MS}}$ remains unaffected. Note, however, that (13) implies $\Lambda_{\overline{MS}} \rightarrow \Lambda_{\overline{MS}} \exp\left\{\frac{A_1}{2b_0}\right\} \equiv \Lambda'$. In what follows we express the results in terms of the original $\Lambda_{\overline{MS}}$.

3 Composite operators and F_π

We shall now generalize the ansatz (10) for the pion decay constant F_π . The main idea is to do perturbation theory around the same RG evolution solution with the non-trivial fixed point, as specified by the function F in (11), with perturbative correction terms obviously specific to F_π . A definition of F_π suiting all our purposes is [1, 24]

$$i \int d^4q e^{iq \cdot x} \langle 0 | T A_\mu^i(x) A_\nu^k(0) | 0 \rangle = \delta^{ik} g_{\mu\nu} F_\pi^2 + \mathcal{O}(p_\mu p_\nu) \quad (16)$$

where the axial vector current $A_\mu^i \equiv (\bar{q} \gamma_\mu \gamma_5 \lambda^i q) / 2$ (the λ^i 's are Gell-Mann $SU(3)$ matrices or Pauli matrices for $n_f = 3$, $n_f = 2$, respectively). Note that according to (16), F_π is to be considered as an order parameter of CSB [23].

The perturbative expansion of (16) for $m \neq 0$ is available to the three-loop order, as it can be easily extracted from the very similar contributions to the electroweak ρ -parameter, calculated at two loops in [25] and three loops in [26].

The appropriate generalization of (10) for F_π now takes the form

$$\frac{F_\pi^2}{\Lambda_{\overline{MS}}^2} = (2b_0) \frac{2^{-2C} a^2}{2i\pi} \oint \frac{dy}{y} y^2 e^{y/m''} \frac{1}{F^{2A-1} [C + F]^{2B}} \times \delta_\pi \left(1 + \frac{\alpha_\pi(a)}{F} + \frac{\beta_\pi(a)}{F^2} + \dots \right) \quad (17)$$

in terms of $F(y)$ defined by eq. (11) and where δ_π , $\alpha_\pi(1)$ and $\beta_\pi(1)$, whose complicated expressions will be given elsewhere [15], are fixed by matching the perturbative \overline{MS} expansion in a way to be specified next. Formula (17) necessitates some comments: apart from the obvious changes in the powers of F , y , etc, dictated by dimensional analysis, note that the perturbative expansion of the (composite operator) $\langle A_\mu A_\nu \rangle$ in (16) starts at one-loop, but zero g^2 order. This leads to the extra $2b_0 F$ factor in (17), corresponding to an expansion starting at $\mathcal{O}(1/g^2)$ ⁸. Another difference is that the perturbative expansion of (16) is ambiguous due to remaining divergences after mass and coupling constant renormalization. Accordingly it necessitates additional subtractions which, within our framework, are nothing but the usual renormalization procedure for a composite operator, which is (perturbatively) well-defined [20]. The only consequence is that, after a consistent treatment of the subtracted terms (i.e respecting RG invariance), the unambiguous determination of the $1/F^n$ perturbative terms in (17) necessitates the knowledge of the $(n + 1)$ order of the ordinary perturbative expansion. The nice thing, however, is that the subtracted terms only affect the values of α_π and β_π , but not the *form* of the ansatz (17), as soon as the order of the variational-perturbative expansion is larger than 1 [8]. The consistency of our formalism is checked by noting that the re-expansion of (17) do reproduce correctly the LL and NLL dependence in \bar{m} of the perturbative expansion of the composite operator to all orders.

The analyticity range with respect to $\Delta\gamma_1$, discussed in section 2, remains valid for (17) as well, since the branch cuts are determined by the very same relation (11). We can thus proceed to a numerical optimization with respect to the RS dependence, along the

⁸ The $\mathcal{O}(1/g^2)$ first-order term cancels anyhow after the necessary subtraction discussed below.

same line as the mass case in section 2. Using an appropriate Padé approximant form to define the $F \rightarrow 0$ ($m'' \rightarrow 0$) limit, we obtain the optimal values as

$$F_{\pi, \text{opt}}^{\text{Padé}}(m'' \rightarrow 0) \simeq 0.55 \Lambda_{\overline{MS}}(2) \quad (0.59 \Lambda_{\overline{MS}}(3)), \quad (18)$$

for $n_f = 2$ (3). With $F_\pi \simeq 92$ MeV, this gives $\Lambda_{\overline{MS}} \simeq 157$ (168) MeV, for $n_f = 3$ (2).

4 $\langle \bar{q}q \rangle$ ansatz

As is well known [20, 4], $\langle \bar{q}q \rangle$ is not RG-invariant, while $m\langle \bar{q}q \rangle$ is; this is thus the relevant quantity to consider for applying our RG-invariant construction. A straightforward generalization of the derivation in section 3 leads to the ansatz

$$\frac{\bar{m}\langle \bar{q}q \rangle}{\Lambda_{\overline{MS}}^4} = (2b_0) \frac{2^{-4C} a^4}{2i\pi} \oint \frac{dy}{y} \frac{e^{y/m''} y^4}{(F)^{4A-1} [C + F]^{4B}} \delta_{\langle \bar{q}q \rangle} \left(1 + \frac{\alpha_{\langle \bar{q}q \rangle}(a)}{F(y)} \right) \quad (19)$$

where again the coefficients $\delta_{\langle \bar{q}q \rangle}$ and $\alpha_{\langle \bar{q}q \rangle}(1)$ are obtained from matching the ordinary perturbative expansion after a subtraction, and will be given explicitly elsewhere [15]. The perturbative expansion, known up to two-loop order [27, 15] implies that one only knows unambiguously the first order correction $\mathcal{O}(1/F)$ in (19), as previously discussed. Apart from that, (19) has all the expected properties (RG invariance, resumming LL and NLL dependence etc), but a clear inconvenience is that $\langle \bar{q}q \rangle$ cannot be directly accessed, being screened by tiny explicit symmetry breaking effects due to $m \neq 0$. This is of course a well-known problem, not specific to our construction.

However, it is not clear how to consistently include explicit symmetry breaking effects within our framework. As amply discussed, in (19) m'' is an arbitrary parameter, destined to reach its chiral limit $m'' \rightarrow 0$. Accordingly, $\bar{m} \rightarrow 0$ for $m'' \rightarrow 0$, so that one presumably expects only to recover a trivial result, $\bar{m}\langle \bar{q}q \rangle \rightarrow 0$ for $m'' \rightarrow 0$. This is actually the case: although (19) potentially gives a non-trivial result in the chiral limit, namely the simple pole residue ($\equiv 2b_0(2C)^{-C} \delta_{\langle \bar{q}q \rangle} \alpha_{\langle \bar{q}q \rangle}(a)$, upon neglecting unknown higher-order purely perturbative corrections), when we require extrema of this expression with respect to RS changes, using for the $m'' \rightarrow 0$ limit a Padé approximant similar to the one for F_π , we do *not* find non-zero extrema. Such a result is not conclusive regarding the actual value of $\langle \bar{q}q \rangle(\bar{\mu})$, but it may be considered a consistency cross-check of our formalism.

On the other hand, we should mention that our basic expression (19) *does* possess non-trivial extrema for some $m''_{\text{opt}} \neq 0$. These we however refrain from interpreting in a more quantitative way since, within our framework, we cannot give to $m'' \times \Lambda_{\overline{MS}}$ the meaning of a true, explicit quark mass (whose input we in principle need in order to extract a $\langle \bar{q}q \rangle$ value from (19)). At least, it strongly indicates that it should be possible to extract $\langle \bar{q}q \rangle$ in the chiral limit, by introducing in a consistent way a small explicit symmetry-breaking mass, $-m_{0, \text{exp}} \bar{q}_i q_i$, to the basic Lagrangian (1).

5 Summary

In this paper we have shown that the variational expansion in arbitrary m'' , as developed in the context of the GN model [8], can be formally extended to the QCD case, apart

from the complication due to the presence of extra singularities, which can be however removed by appropriate RS change. As a result we obtain in the chiral limit non-trivial relationships between $\Lambda_{\overline{MS}}$ and the dynamical masses and order parameters, F_π , $\bar{m}\langle\bar{q}q\rangle$. The resulting expressions in a generalized RS have been numerically optimized, using a well-motivated Padé approximant form, due to the complexity of the full optimization problem. The optimal values obtained for M_q and F_π are quite encouraging, while for $\langle\bar{q}q\rangle$ they are quantitatively not conclusive, due to the inherent screening by an explicit mass term of this quantity, in the limit $m \rightarrow 0$. A possible extension to include consistently explicit breaking mass terms in our formalism is explored in ref. [15].

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