EXPERIMENTAL INVESTIGATION OF SINGLE-BEAM AND BEAM-BEAM SPACE CHARGE EFFECTS

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Summary

In order to improve the performance of the CERN Intersecting Storage Rings (ISR), it has become increasingly important to investigate experimentally the transverse space charge effects of both single-beam and beam-beam origin. The image-dominated, single-beam effects cause, in the first order, closed orbit distortions which have been measured and related to the coherent tune-shift variation across the aperture. Corrections for high precision bumps, used for luminosity measurements, have been calculated. This effect is also important when optimizing the beam-beam count rate. In the second order, these forces cause an incoherent tune-shift which is taken into account when adjusting the tunes across the aperture. The excitation of non-linear resonances by higher order effects has also been observed. The vertical closed orbit distortion due to beam-beam forces is one order of magnitude smaller than the corresponding single-beam effect. Measurements of this distortion have been related to the beam-beam tune-shift. Higher order effects are the strongest source of excitation for vertical non-linear resonances. The horizontal and coupled resonances are excited to a much lesser extent. It has been shown experimentally that by the appropriate adjustment of the beam-beam separations in the intersecting regions, this excitation can be minimized so reducing the decay rate and improving the physics conditions.

Introduction

Space charge effects in the ISR can be conveniently divided into single-beam effects and beam-beam effects. The single-beam effects are image-dominated and can be further sub-divided. Firstly, these image forces affect the motion of the beam as a whole, which can be conveniently studied in terms of a coherent tune-shift, and secondly, the motion of the individual protons is affected, which manifests itself as an incoherent tune-shift. To a lesser extent, the resonance excitation in the machine is also affected. The category of beam-beam effects can be similarly sub-divided. The effects on the closed orbits and tune-shifts are an order of magnitude less than for the single-beam case but the resonance excitation is, by contrast, a very important effect. The influence of space charge has become increasingly important as the operation of the ISR has become steadily more refined and sophisticated.

Single-Beam Coherent Tune-Shifts

The coherent tune governs the closed orbit, as well as the motion of the whole beam when it is disturbed. Thus, changes in the coherent tune directly affect the high precision closed orbit bumps[1] used for luminosity measurements[2]. Conversely, the closed orbit changes make it possible to measure the coherent tune-shift as a function of position with respect to the stack.

The closed orbit distortion can be expressed as a Fourier series in normalized co-ordinates (η, ψ):

$$\eta\ (\psi) = \sum_k F_k\, e^{ik\psi} \quad k = 0, 1, 2, \ldots \infty \qquad (1)$$

where:

- η is the orbit distortion normalized by the square root of the local betatron amplitude,
- ψ is the betatron phase normalized by the tune value,
- F_k is the kth azimuthal, harmonic amplitude of the orbit distortion.

The harmonics can be expressed as a function of the coherent tune, Q, and the corresponding azimuthal harmonics, f_k, of the magnetic imperfections[3].

$$F_k\ (Q) = \left(\frac{Q^2}{Q^2 - k^2}\right) f_k \qquad (2)$$

Providing the amplitude of the kth harmonic can be measured before and after any change in the space charge conditions, the coherent tune-shift ΔQ_c can be calculated from (2).

$$\Delta Q_c = \frac{Q}{2}\left[\frac{(1-E)(Q^2-k^2)}{E\ (Q^2-k^2) - Q^2}\right] \qquad (3)$$

where:

$$E = \frac{F_k\ (Q)}{F_k\ (Q + \Delta Q_c)} \qquad (4)$$

The method is most sensitive for k values close to Q which is 9 or 8 for the ISR.

The closed orbit measurements are made inside a stacked beam by detecting the position of empty scanning buckets in a way analogous to the closed orbit measurement made with single-beam pulses. By adding a thin tail to a dense beam, the empty scanning buckets can be "materialized" across the whole aperture and the variation of coherent tune-shift with position can be found. This has been done with a 10 A beam with a tail of 2.8 A spread across the aperture. Figure 1 gives the spatial variation of the 9th harmonic of the vertical closed orbit distortion with and without this stack. These results are used in the same Figure to give

$$\gamma\ \frac{\Delta Q_{c,vertical}}{\Delta I}$$

where:

- ΔI is the beam current exciting the ΔQ_c
- γ is the total energy normalized by the rest energy.

At the stack centre, $\{\gamma\ \Delta Q_{c,vertical}/\Delta I\}$ has its peak value of -0.15, which agrees well with other measurements made with the ISR[4].

The influence of space charge on half wavelength orbit bumps can now be investigated in some detail. Figure 2 shows a half wavelength closed orbit bump of amplitude F in the normalized co-ordinates (η, ψ). This deformation is expressed by:

$$\begin{aligned} \eta &= F \cos\ (Q\psi) \quad \text{for } |\psi| \leq \pi/2Q \\ \eta &= 0 \qquad\qquad\quad \text{for } |\psi| > \pi/2Q \end{aligned} \qquad (5)$$

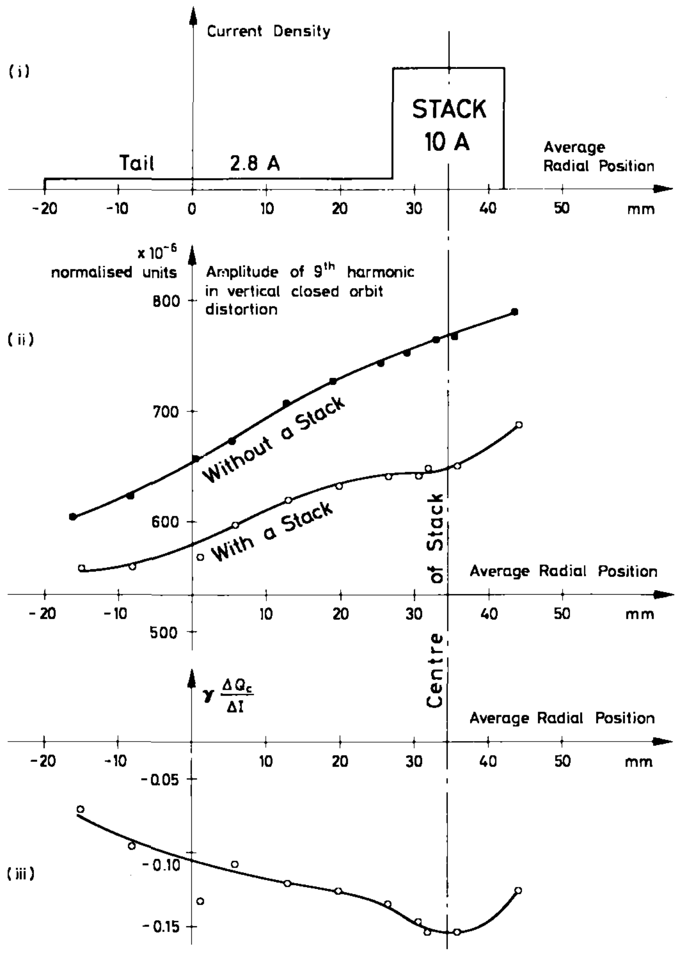

Figure 1. (i) Current Distribution in Stack
(ii) Variation of the 9th Closed Orbit Harmonic across Chamber
(iii) Variation of $(\Delta Q_{c,vertical}/\Delta I)\ \gamma$ across Chamber.

Giving harmonic amplitudes:

$$F_k = \frac{1}{\pi}\left[\frac{\sin\frac{\pi}{2}\left(1-\frac{k}{Q}\right)}{(Q-k)} + \frac{\sin\frac{\pi}{2}\left(1+\frac{k}{Q}\right)}{(Q+k)}\right] \qquad (6)$$

$$k = 1, 2, 3, \ldots \infty$$

Using (2), the effect can be calculated of a coherent tune-shift on the harmonic amplitudes given by (6). The resulting changes constitute a residual orbit distortion around the whole of the machine. Table 1 gives the errors arising in all intersections in the ISR from a single orbit bump in one intersection. These errors are expressed as a percentage of the height of

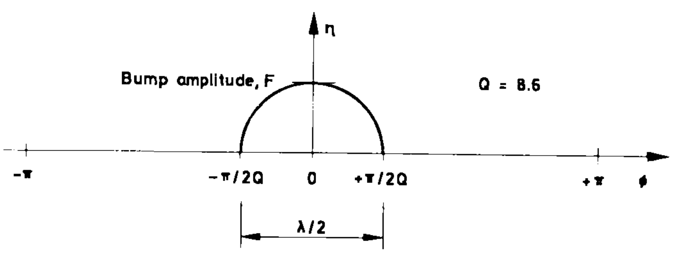

Figure 2. Half Wavelength Closed Orbit Bump in Normalized Co-ordinates.

the bump per ampere of circulating beam current with energy γ. The effect is assumed to be linear for $|\Delta Q_c| < 0.2$. It is common practice in the ISR for luminosity measurements to take place simultaneously in several intersections. As can also be seen from Table 1, the compound errors can amount to a few percent when all bumps are excited with a beam of say 10 A. This does not favourably compare to the wanted accuracy of $\pm$ 0.5 %. By alternating the signs of the bumps in adjacent intersections, these residual effects are greatly reduced, but a more exact compensation is possible.

TABLE 1

Residual Closed Orbit Distortion Introduced by the Image Space Charge Forces on a Single Vertical Orbit Bump

Ring 2	I1*	I2	I3	I4	I5	I6	I7	I8
R	-0.35	1.17	1.56	3.20	3.45	1.29	1.56	-0.27

* Intersection with applied bump.

(The residual distortion caused by space charge (expressed as percentages of the primary bump height) = (R/γ) % per ampere. Although this example uses Intersection 1 in Ring 2 for the primary bump, the table is, in fact, quite general providing the cyclic order of the outer and inner arcs is respected.)

The ISR bumps are excited by two magnets separated by approximately half a wavelength with two more magnets acting as trimmers. Calculations were made to change the excitation of these magnets to compensate the mismatch in the 8th and 9th harmonics arising from the space charge, but this was found to be equivalent to the simpler procedure of calculating the orbit bumps with a tune-shift equal to the coherent one introduced into the computer program by modifying all the magnet gradients.

The stability or reproducibility of the closed orbit is also of great importance. It is usual practice to optimize the beam positions in the intersections and to measure the luminosity using 3 A stacks and then to re-stack high intensity beams. The new beams, however, are no longer optimized owing to the change in space charge conditions. This effect is found by summing over the harmonics in equation (2). The beam shift is not negligible. Consider a beam of 10 A at 22 GeV/c with a closed orbit dominated by a $\pm$ 3 mm peak-to-peak 8th order harmonic and a central non-space charge tune value of 8.6. Using (2) and $\gamma\ \Delta Q_c/\Delta I = -0.15$, an increase of 10 % is found in the 8th harmonic. If the peak falls at an intersection, there will be a shift of 0.3 mm in the orbit position. In addition to a small, and generally negligible reduction in luminosity, the change in the relative beam positions will produce an excitation of non-linear resonances. This is discussed later in this paper.

Single-Beam Incoherent Tune-Shifts

This is perhaps the best known space charge effect[5]. Recently, the dynamic compensation of the incoherent tune shift during stacking has become operational practice in the ISR. This is dealt with in detail in Reference 6 which is also presented at this Conference.

Single-Beam High Order Effects

The higher order moments of the single-beam space charge forces affect the excitation of non-linear resonances, which in turn are known to largely govern the decay rates and the background conditions in the ISR. However, resonance excitation is generally much stronger when the second beam is present. The investigation of these single-beam high order effects has only recently been started in the ISR. The experimental technique used consists of moving empty RF buckets across the machine aperture inside a stack. When the bucket position coincides with the position of a resonance, the surrounding particles are moved into that resonance. Aperture limiting the beam enhances the beam losses, which mark the resonances.

Beam-Beam Interaction

In the 8 intersecting regions of the ISR, the two beams interact electromagnetically. These forces have been studied theoretically for their influence on the tune values and on the excitation of non-linear resonances[7,8,9]. In these References, two extreme cases are generally considered; the case of head-on collisions between the two beams and the case of machines, such as the ISR, with a large crossing angle between the two beams. In the latter case, the two beams only interact over a short distance over which the variations of phases, betatron amplitudes and beam dimensions are negligible. Under these conditions, the electromagnetic forces cancel in the plane of crossing and add in the other plane, which explains why the main beam-beam effects in the ISR are observed in the vertical plane.

Dipole and Quadrupole Beam-Beam Effects

The vertical forces, $F_z(z,s)$ exerted by a beam on a test particle of the other beam is an odd function of the beam separation z and vanishes for z = 0. For the vertical motion, this is equivalent to a horizontal magnetic field. $B_x(z,s)$, the gradient of which imposes a tune-shift on the test particle of:

$$\Delta Q_V = \frac{1}{4\pi}\frac{1}{B\rho}\;\beta_V^{int}\int\left(\frac{\partial B_x(z,s)}{\partial z}\right)_{z=0} ds \quad \text{per intersection,} \tag{7}$$

where:

$B\rho$ is the magnetic rigidity,

β_V^{int} is the vertical betatron amplitude function at the intersection, and

the integral is extended over the whole interaction length, s being the distance along the beam.

For small values of z, the vertical force $F_z(z,s)$ produces a vertical closed orbit distortion given by:

$$\Delta z = \frac{1}{B\rho}\frac{\sqrt{\beta_V\,\beta_V^{int}}}{2\,\sin(\pi Q)}\; z \cos Q\,(\pi+\psi-\psi_{int})\int\left(\frac{\partial B_x(z,s)}{\partial z}\right) ds \tag{8}$$

The integral can be eliminated between the two equations (7) and (8) giving:

$$\Delta z = \frac{2\pi}{\sin(\pi Q_V)}\sqrt{\frac{\beta_V}{\beta_V^{int}}}\;\Delta Q_V\; z\; \cos Q\,(\pi+\psi-\psi_{int}) \tag{9}$$

β_V and the normalized betatron phase ψ are at the point where Δz is measured and β_V^{int} and ψ_{int} are taken at the intersection. Measurements of these effects in the ISR were made using a very sensitive magnetic beam detector. The vertical displacements of a coasting 6 A beam were measured near an intersection when vertically steering the other beam of 11.2 A by an amount z in all intersections at 26 GeV/c. The result $\Delta z/z = -6\cdot 10^{-3}$ corresponds to a tune-shift of $-2.2\cdot 10^{-4}$, which compares favourably with $\Delta Q_V = -2.7\cdot 10^{-4}$ calculated using the theoretical formula[10]:

$$\Delta Q_V = \sqrt{\frac{2}{\pi}}\;\frac{\lambda r_o\,\beta_V^{int}}{\gamma\,\alpha\,\sigma_o} \tag{10}$$

where:

- $\lambda = \frac{N}{2\pi R}$ is the linear particle density
- r_o is the classical proton radius
- α is the crossing angle (14.77°)
- γ is the normalized energy
- σ_o is the r.m.s. half height of the beam (2.0 mm as measured by scraping the beam after the experiment).

In the case of high intensity beams, these vertical orbit distortions can introduce errors in luminosity measurements. For example, with two beams of 11.2 A at 26 GeV/c, the beam separations at the intersecting points would be increased by:

- 1.2 % when bumps of the same sign and amplitude are applied in all intersections simultaneously.
- 0.1 % when these bumps have alternate signs.

The comparison between the two cases gives an additional argument to use alternate bumps in the ISR luminosity calibrations. By scaling these results to the lowest ISR energy and by keeping the same vertical beam dimensions, the error is less than 0.1 % for the usual 3 A beams used for luminosity measurements when alternate bumps are used.

Beam-Beam Excitation of Non-Linear Resonances

In the approximation of a purely vertical resultant of the beam-beam forces and assuming a length of electromagnetic interaction short with respect to the wavelength of the azimuthal harmonic associated with the resonance, only vertical resonances $NQ_V = p$ can be excited, where N is the order of the resonance. The excitation term is the pth harmonic of the azimuthal angle θ of the (N-1)th derivative of the force $F_z(z,s)$. The force $F_z(z,s)$ is an odd function of z as are its even derivatives. The contrary is true for the odd derivatives which are even functions of z, thus giving for the excitation produced by one intersection the curves of Figure 3. The contributions of all intersections have to be added according to their phase (i.e. $p\theta$ or $N\mu_V$ where μ_V is the vertical betatron phase). In a perfect machine (z = 0) and with a superperiodicity of 4, only vertical even order resonances are excited which have p as a multiple of 4. In a practical machine, as soon as z is different in all the intersections, all the vertical resonances occur.

The more sophisticated theories for the beam-beam interaction[9] lead to the same qualitative results: all the resonances $n_1 Q_H + n_2 Q_V = p$ where $(n_1 + n_2 = N)$ are excited but the excitation drops rapidly when n_1 increases from 0 to N. The excitation due to the beam-beam interaction is in general much stronger than that resulting from the magnetic imperfections of the ma-

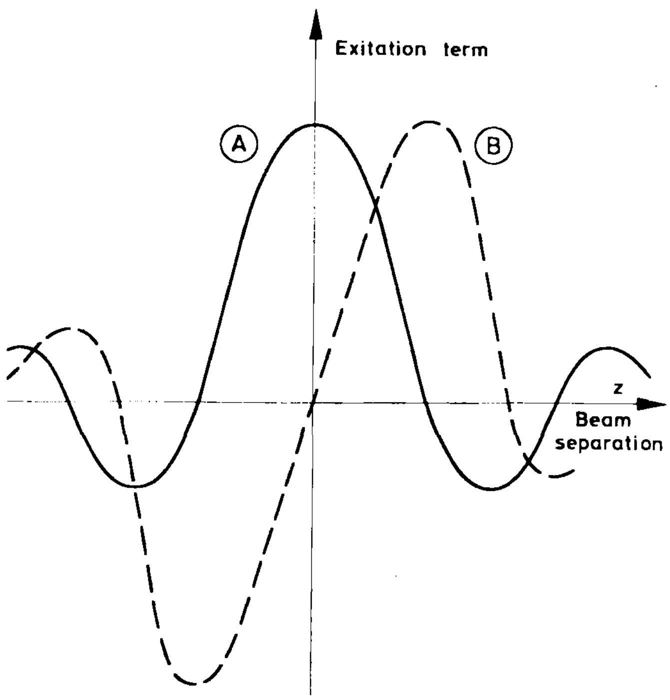

Figure 3. Variation of Resonance Excitation with the Beam Separation 'z' for an even order Resonance (A) or an odd order Resonance (B)

chine and in the ISR it was found to be the main cause of erratic decay rates and high background when using the 5C working line which crosses 5th order resonances (see Figure 4).

The method used to investigate these effects consisted mainly in displacing small beams from the injection orbit to the external orbit and vice versa by RF acceleration, and watching for beam losses when crossing resonances. Figure 4 shows such aperture scans made in Ring 1 in the presence of a stack of 14.5 A in Ring 2. As expected, only the purely vertical resonances $7\ Q_V = 60$, $5\ Q_V = 43$ and $8\ Q_V = 69$ are visible. The losses on $8\ Q_V = 69$ are one order of magnitude smaller than those observed on $5\ Q_V = 43$ which explains why a working line sitting across 8th order resonances rather than 5th order resonances gives improved decay rates and physics conditions[6].

However, as soon as only one resonance predominates in a stack, it is possible to zero the excitation of this particular resonance by suitably adjusting the beam separation in the intersections. This was done in Ring 1 for the resonance $5\ Q_V = 43$ (see Figure 4) by steering the beam in Ring 2 by 0.4 and 0.6 mm in two pairs of intersections, I4/I8 and I2/I6. These intersections were chosen since they are separated by 90^o for the 43rd harmonic in θ. This method of compensation greatly improves the decay rates and the background conditions of stacks. An experiment was carried out using the same 14.5 A beam in Ring 2 and a 12.5 A beam in Ring 1 placed on the working line 5C26 so as to avoid the 7th order resonances (see Figure 4). The vertical aperture of Ring 1 was reduced in order to increase artificially the decay rates and to simulate old physics stacks. When passing from the most unfavourable conditions of resonance excitation to a perfect compensation by vertical steering in Ring 2, the decay rate in Ring 1 was reduced by a factor 15. The required precision for the vertical steering was of the order of 1/10 mm. The gain observed in Ring 2 was smaller (a factor 3). This can be explained, since only two pairs of intersections were used to compensate the defects of 8 intersections. It follows that since the phase-shifts in internal and external arcs of the ISR are different, a compensation scheme which uses only two pairs of intersections cannot satisfy the requirements of both rings simultaneously. This problem can be overcome by distributing the corrections in all the intersections and experiments are continuing in this direction in the ISR.

The importance of these high order beam-beam effects have been recently demonstrated in relation with the acceleration of large intensity beams by phase displacement[10].

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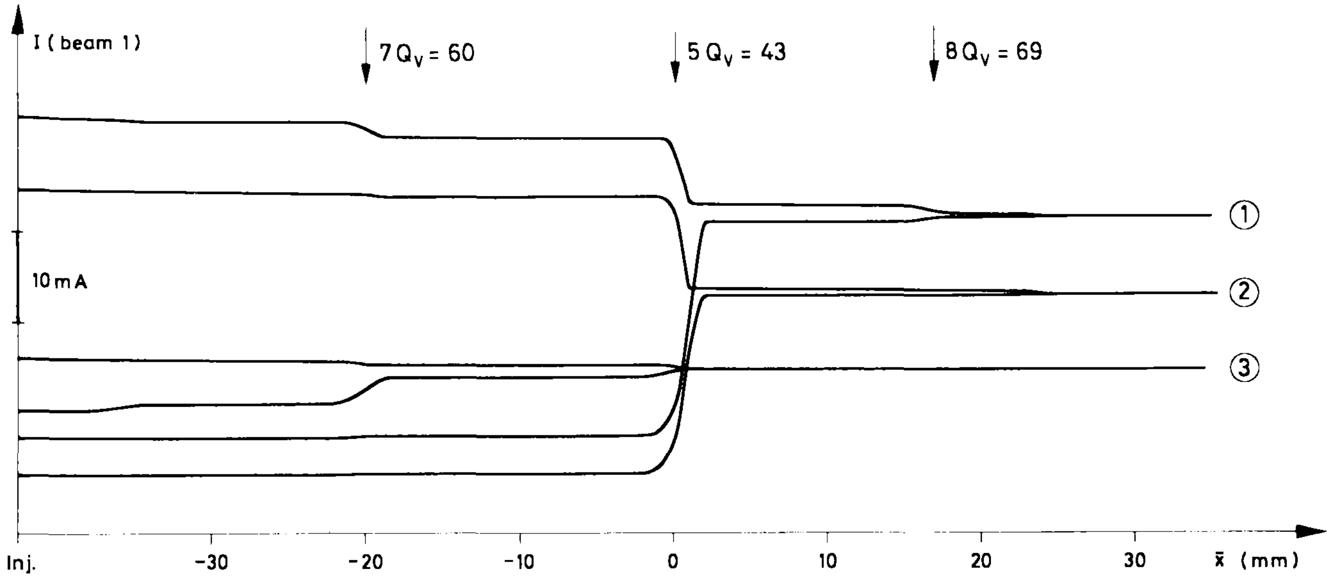

Vertical steering of beam 2 (mm) in the two pairs of intersections

	I 4	I 8	I 2	I 6
Scan 1	0	0	0	0
Scan 2	- 0.4	+ 0.4	- 0.6	+ 0.6
Scan 3	+ 0.4	- 0.4	- 0.6	+ 0.6

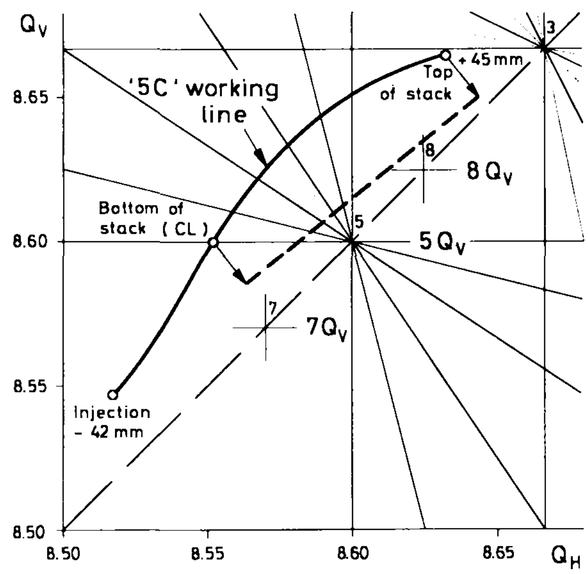

Figure 4. Beam-beam excitation of resonances

Small beams of 75 mA were displaced across the aperture at a speed of 1.6 mm s^{-1}. The tune changes according to the 5C working line shown in the above Figure. Beam losses occur when crossing certain resonances. The broken working line is the 5C line deformed by the incoherent tune shift for a 12.5 A beam at 26 GeV/c.