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COMBINED-FUNCTION ELECTRON STORAGE RINGS

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A combined-function lattice permits reduction of the RF power required for compensation of synchrotron radiation, compared to a separated-function machine of the same circumference. At the same time, the required beam aperture is slightly smaller, and the damping aperture is much larger. Damping of radial betatron oscillations is achieved by making the defocusing magnets slightly stronger than the focusing ones.

Introduction

All strong-focusing electron storage rings so far have been built with separate-function lattices. This offers not only the advantage of increased flexibility (independent adjustment of the working point, and hence of the beam size), but also provides damping of the horizontal betatron oscillations which is absent in an isomagnetic combined-function lattice.

At higher beam energies, the energy loss due to synchrotron radiation is very large, and the required RF power becomes the dominant part of operational costs. The energy loss per turn is proportional to the field in the bending magnets, and for a given machine circumference it would be desirable to fill as much as possible of it with bending magnets in order to keep their field low. If combined-function magnets could be used, the need for additional space for quadrupoles could be avoided. By providing the magnets with pole-face windings, the space required for correction lenses could also be gained. The total saving in radiation energy would be typically of the order of 10 - 20%, depending on the ratio of quadrupole and correction lens length to the total circumference. Although this may appear small, in a typical 20 GeV machine it may amount to over 1 MW of RF power, and to even more in network power consumption. Therefore, this will be a rather important argument in these times of increasing power costs and shrinking energy reserves.

Damping of the horizontal betatron oscillation can be provided by making the defocusing magnets stronger than the focusing ones, as was proposed by Robinson [1] a long time ago. The adjustment of the beam size could be done by wiggler magnets in straight sections. Small variations of the working point can be obtained with the pole-face windings, so that the reduced flexibility seems less severe.

An additional advantage of the combined function lattice is its wide damping aperture - i.e. the horizontal width of average radii which give damping for all modes of oscillations. In fact, it can easily be made larger than the physical aperture. In a separate-function machine, the damping aperture is usually very small, and can only be increased by making the quadrupoles longer, which leaves even less space for the bending magnets. Just how much damping aperture is really needed, is hard to specify; however, all operations requiring beam handling, such as chromaticity measurements, become easier the larger it is.

Finally, a combined-function machine will need fewer large power supplies, and no bus-bars to feed quadrupoles, which may be an appreciable saving in construction costs. The pole-face windings require only low power, and may even be needed in a separate function machine to correct field distortions if the magnetic field at injection energy is low.

The synchrotron radiation integrals in a combined-function lattice

For the calculation of momentum compaction, energy loss, and damping in electron storage rings we need the integrals I_1 to I_5.[2,3] They can be evaluated as sums over all bending magnets.

In order to simplify the calculations, we assume that the lattice consists entirely of alternating F and D magnets, with no straight sections between them (the extension to a FODO lattice is straightforward). The two most interesting cases are the one with magnets of equal length and the one with magnets of equal profile. The requirement of equal phase-shift in both the horizontal and vertical planes then yields the focusing strengths and the lengths of the magnets respectively.

For a machine consisting of N periods of focusing (subscript F) and defocusing (subscript D) half-cells of length L, the circumference C is given by

$$N(L_F + L_D) = C \tag{1}$$

and the total bending angle (bending radius ρ)

$$N\left(\frac{L_F}{\rho_F} + \frac{L_D}{\rho_D}\right) = 2\pi \tag{2}$$

We will express our results as function of the phase-shift per cell μ. The tune Q then is given by

$$Q = \frac{N\mu}{2\pi} \tag{3}$$

Damping of the oscillations in all three planes is assured if $I_4 = 0$. (Then the partition numbers are $J_h = J_v = 1$, $J_\varepsilon = 2$ as in a separate-function lattice.)

$$I_4 = N\left[(1 - 2n_F)\frac{L_F}{\rho_F^2}\langle\eta_F\rangle + (1 - 2n_D)\frac{L_D}{\rho_D^2}\langle\eta_D\rangle\right] \tag{4}$$

where $\langle\eta\rangle$ is the average dispersion. The field-index n is given by

$$n_{F,D} = \mp (K\rho^2)_{F,D} \tag{5}$$

for rectangular magnets, where $K = |B'/B\rho|$ is the focusing strength. Since $K\rho^2 >> 1$, we get to a very good approximation

$$I_4 = 2N\left[\frac{L_F K_F}{\rho_F}\langle\eta_F\rangle - \frac{L_D K_D}{\rho_D}\langle\eta_D\rangle\right] \tag{6}$$

If we call for short

$$\left.\begin{aligned} r &= \frac{\rho_F}{\rho_D} = \frac{1+\varepsilon}{1-\varepsilon} \\ p &= \sqrt{\frac{K_F}{K_D}} = \frac{1-\kappa}{1+\kappa} \\ q &= \frac{L_F}{L_D} = \frac{1+\lambda}{1-\lambda} \end{aligned}\right\} \tag{7}$$

the condition for $I_4 = 0$ becomes (see Table I for $\langle\eta\rangle$)

$$(r - 1)\,\frac{1 + p^2 r}{1 + r^2/q}\;\frac{s_F\, S_D/x_F}{p\, s_F\, C_D - c_F\, S_D} = 1 \qquad (8)$$

where

$$\left.\begin{array}{ll} s_F = \sin x_F & S_D = \mathrm{Sinh}\, x_D \\ c_F = \cos x_F & C_D = \mathrm{Cosh}\, x_D \\ x_F = \frac{L_F}{2}\sqrt{K_F} & x_D = \frac{L_D}{2}\sqrt{K_D} \end{array}\right\} \qquad (9)$$

It is immediately obvious that this condition cannot be fulfilled for r = 1, i.e. for equal bending radii in F and D magnets ("isomagnetic machine"). The D magnets have to be stronger than the F magnets (r > 1) since all the other terms on the LHS of Eq. (8) are positive.

For magnets of equal length (q = 1) the condition of equal phase-shift in both planes yields also p = 1, i.e. the focusing strengths have to be the same in F and D magnets. In this case the condition for damping simplifies considerably to give

$$\frac{r^2 - 1}{r^2 + 1}\;\frac{s\, S/x}{sC - cS} = 1 \qquad (10)$$

with

$$s = \sin x,\; c = \cos x,\; S = \mathrm{Sinh}\, x,\; C = \mathrm{Cosh}\, x,\; x = \frac{L}{2}\sqrt{K} \qquad (11)$$

The phase-shift per cell is then given by

$$\cos\mu = \cos 2x\, \mathrm{Cosh}\, 2x \qquad (12)$$

If μ is not too large ($\lesssim \pi/2$), we find approximately

$$\mu = \frac{4x^2}{\sqrt{3}} \qquad (13)$$

and the condition for damping becomes approximately

$$\varepsilon = \frac{\mu}{4\sqrt{3}} \qquad (14)$$

Although the derivation is a bit more complicated, the same approximation is found for the case of equal profile parameter magnets ($r = q = 1/p^2$). The exact evaluation has been done by computer and is shown in Fig. 1. All synchrotron integrals can now be evaluated[4] and are shown in Fig. 2 together with their approximations as ratios to the isomagnetic values.

$$\frac{I_1}{I_1^{iso}} = 1 - \frac{\mu^2}{24}, \quad I_1^{iso} \approx \frac{C}{Q^2}\left(1 + \frac{3\mu^2}{40}\right) \qquad (15)$$

$$\frac{I_2}{I_2^{iso}} = 1 + \frac{\mu^2}{48}, \quad I_2^{iso} = \frac{4\pi^2}{C} \qquad (16)$$

$$\frac{I_3}{I_3^{iso}} = 1 + \frac{\mu^2}{16}, \quad I_3^{iso} = \frac{8\pi^3}{C^2} \qquad (17)$$

$$\frac{I_5}{I_5^{iso}} = 1 + \frac{\mu^2}{48}, \quad I_5^{iso} \approx \frac{4\pi^2}{CQ^3}\left(1 + \frac{3\mu^2}{40}\right) \qquad (18)$$

Damping aperture and profile parameter

Because the field increases with radius in the F magnets - and decreases in the D magnets - the damping of the horizontal betatron oscillations disappears at some positive momentum deviation. The damping of the phase oscillation disappears at about twice that value on the negative side.

We estimate that deviation as about half the value where the fields become equal, which is given by

$$B_F\left(1 + K_F\,\rho_F\,\langle\eta_F\rangle\,\frac{\Delta p}{p}\right) = B_D\left(1 - K_D\,\rho_D\,\langle\eta_D\rangle\,\frac{\Delta p}{p}\right) \qquad (19)$$

or

$$\left(\frac{\Delta p}{p}\right)_{damp} = \frac{1}{2}\,\frac{B_D - B_F}{B_F\, K_F\, \rho_F\, \langle\eta_F\rangle + B_D\, K_D\, \rho_D\, \langle\eta_D\rangle} \qquad (20)$$

Substituting for the average dispersion, we find with the damping condition

$$\left(\frac{\Delta p}{p}\right)_{damp} = \frac{q}{2}\left(\frac{r - 1}{r + q}\right)^2 \qquad (21)$$

For equal length magnets (p = q = 1) we then get

$$\left(\frac{\Delta p}{p}\right)_{damp} = \frac{1}{2}\left(\frac{r - 1}{r + 1}\right)^2 \approx \frac{\mu^2}{96} \qquad (22)$$

and a slightly larger value is obtained for equal profile parameter (q = r).

For a typical machine[4] the damping aperture is well above 10σ in both cases for $\mu = \pi/2$, and thus practically fills the vacuum chamber. In separated function machines, on the other hand, it is difficult to get damping apertures of more than a few σ at high energies if one is not willing to waste space for very long and weak quadrupoles.

Although the field gradients required for focusing are actually quite weak, the very low dipole field of the magnets has the consequence of large profile parameters in a combined-function lattice.

For equal length magnets, the profile parameter is slightly larger in the weaker F magnets. We find generally

$$\left(\frac{n}{\rho}\right)_F = K_F\,\rho_F = \frac{4\pi\sqrt{3}\,Q^2}{\mu\, C}\;\frac{1 - \varepsilon\,\lambda}{(1 + \lambda)(1 - \varepsilon)} \qquad (23)$$

or, for equal length magnets

$$\left(\frac{n}{\rho}\right)_F = \frac{8\sqrt{3}\,Q^2}{\mu\, C\left(1 - \frac{\mu}{4\sqrt{3}}\right)} \qquad (24)$$

For equal profile paramters

$$\left(\frac{n}{\rho}\right)_F = \left(\frac{n}{\rho}\right)_D = \frac{8\pi\sqrt{3}\,Q^2}{\mu\, C} \qquad (25)$$

which is also the isomagnetic value.

If n/ρ turns out to be uncomfortably large, one can reduce it by reducing Q. However, with decreasing Q the required aperture increases. In practice, a reasonable compromise has to be found[4] or special magnets have to be considered.

Conclusions

A combined-function lattice for electron storage rings permits a reduction of the RF power required for compensation of the synchrotron radiation losses over a machine with a separated function lattice of the same circumference. Due to the absence of separate quadrupoles, more of the circumference can be filled with bending magnets with correspondingly lower fields.

The synchrotron radiation losses will be slightly larger than those of a combined-function, isomagnetic machine of equal circumference, but still well below that of a separate-function machine. The horizontal and vertical aperture requirements are slightly smaller than for a separate-function machine of equal phase-shift per cell, as the beta-functions remain smaller. Another advantage of the combined-function lattice is the large damping aperture, which can easily be made to exceed the physical aperture of the vacuum chamber.

Damping of the horizontal betatron oscillations is obtained by making the bending field of the defocusing magnets stronger than that of the focusing ones. Two alternatives are investigated in detail: the first one has magnets of equal length and focusing strength, and the second one has equal profile parameters. There appears to be no strong advantage of one scheme over the other.

The main disadvantage of the machine is the reduced flexibility in the choice of a working point. However, beam size control can be performed with a wiggler magnet, and small adjustments of the working point can be made with pole-face windings. At present, it appears that the combined-function lattice is a strong competitor for the more conventional separate-function lattice for electron storage rings.

References

1. K. Robinson; Phys. Rev. 111, 373 (1958)
2. M. Sands; SLAC-Report 121 (1970)
3. R. Helm et al.; IEEE-NS20, 900 (1970)
4. A. Hofmann, B. Zotter; CERN/ISR-GS/76-28 (unpublished)

Table I - Comparison of lattice functions

Q = tune, R = average radius, $L = \mu R/2Q$ half-cell length, $K = (4 \cdot x \cdot Q/\mu R)^2$ focusing strength ($B'/\rho B$)

$\varepsilon = (B_D - B_F)/(B_D + B_F)$, $D = \sin x \operatorname{Cosh} x - \cos x \operatorname{Sinh} x$, $x = L\sqrt{K}/2$ solution of $\cos 2x \operatorname{Cosh} 2x = \cos\mu$

	Combined-function FD lattice (equal length magnets)			Separate-function FODO lattice		
	μ phase-shift/cell	$\mu = \pi/3$	$\mu = \pi/2$	short-lens approx.	$\mu = \pi/3$	$\mu = \pi/2$
$\beta_{max} \cdot \frac{Q}{R}$	$\mu \frac{\sin 2x \operatorname{Cosh} 2x + \operatorname{Sinh} 2x}{4x \sin\mu}$	1.685	2.405	$\frac{\mu}{\sin\mu}\left(1 + \sin\frac{\mu}{2}\right)$	1.814	2.682
$\beta_{min} \cdot \frac{Q}{R}$	$\mu \frac{\cos 2x \operatorname{Sinh} 2x + \sin 2x}{4x \sin\mu}$	0.643	0.5	$\frac{\mu}{\sin\mu}\left(1 - \sin\frac{\mu}{2}\right)$	0.605	0.460
$\eta_{max} \cdot \frac{Q^2}{R}$	$\left(\frac{\mu}{4x}\right)^2 \left(\frac{2\operatorname{Sinh} x}{D} + 1 - \varepsilon\right)$	1.305	1.597	$\left(\frac{\mu/2}{\sin\mu/2}\right)^2 \left(1 + \frac{1}{2}\sin\frac{\mu}{2}\right)$	1.371	1.670
$\eta_{min} \cdot \frac{Q^2}{R}$	$\left(\frac{\mu}{4x}\right)^2 \left(\frac{2\sin x}{D} - 1 - \varepsilon\right)$	0.832	0.847	$\left(\frac{\mu/2}{\sin\mu/2}\right)^2 \left(1 - \frac{1}{2}\sin\frac{\mu}{2}\right)$	0.822	0.798
$\langle\eta\rangle_{F,D} \cdot \frac{Q^2}{R}$	$\left(\frac{\mu}{4x}\right)^2 \left(\frac{2\sin x \operatorname{Sinh} x}{x D} \pm 1 - \varepsilon\right)$	1.245 / 0.930	1.463 / 0.963	$\left(\frac{\mu/2}{\sin\mu/2}\right)^2 \left(1 - \frac{\sin^2\mu/2}{12}\right)$	1.074	1.182

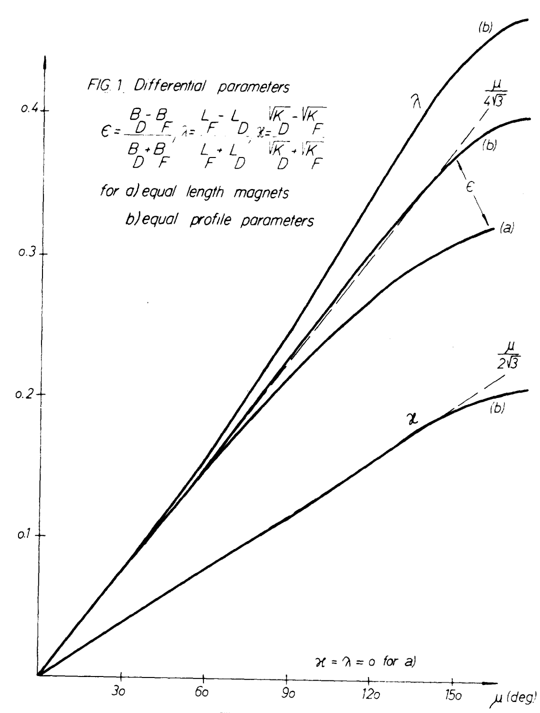

Figure 1.

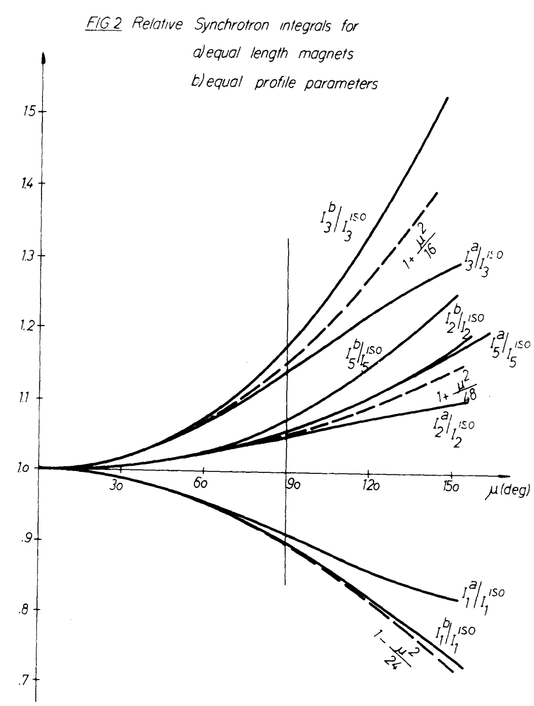

Figure 2.