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Energy Loss to Coaxial Vacuum Chambers in LEP and LHC

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Abstract

In many high-energy storage rings the beam chamber is connected to a separate pump chamber by a metallic wall with many holes or slots which permit passage of the rest gas. In LEP, the pump chamber contains a metallic "negstrip" pump, and thereby becomes a coaxial transmission line. Also in LHC, a coaxial line is formed by the "liner" and the surrounding cold vacuum chamber which it shields from heating by synchrotron radiation. Since the phase velocity of electro-magnetic fields in a coax line is close to light velocity, the fields will be almost in synchronism with the particle beam. This will cause coherent coupling of the fields of the beam and the pump chamber, which may result in a large resistive impedance and could lead to instability, loss of beam energy, and excessive heating of the chamber walls. Here we estimate the rate of field buildup analytically, and in a subsequent report we will compare these results with numerical computations using 3-D computer codes. The results are tested for diagnostic purposes on a "slot coupler" with short and wide holes designed to extract energy efficiently.

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1 Introduction

In many high-energy storage rings the beam chamber is connected to a separate pump chamber by a metallic wall with many holes or slots which permit passage of the rest-gas to be pumped out. In LEP, the pump chamber contains a metallic "negstrip" pump, which forms a coaxial transmission line with the surrounding chamber.

Also in LHC, a coaxial line will be formed by the "liner" and the surrounding cold vacuum chamber which it shields from heating by synchrotron radiation. Since the phase velocity of electro-magnetic fields in a coaxial line is close to light velocity, they will be almost in synchronism with the particle beam. This will cause coherent coupling of the fields of the beam and the pump chamber, which may result in a large resistive impedance and could lead to instability, loss of beam energy, and excessive heating of the chamber walls. Here we estimate the rate of field buildup analytically, and in a subsequent report we will compare these results with numerical computations with a 3-D computer code. The results are tested for diagnostic purposes on a "slot coupler" with short and wide holes, designed to extract energy efficiently.

The calculation of the coupling impedance of holes and slots has been pursued vigorously in recent years, in particular for the vacuum chamber liners of the LHC and the former SSC project. Based on a variety of methods, the results are now usually expressed in terms of the polarizabilities and the susceptibilities of a particular hole shape. For circular or elliptical holes, analytic expressions can be derived, while for other shapes numerical estimates are available. Also the effects of the wall thickness can be included by defining inside and outside polarizability and susceptibility. The effect of a large number of holes has been estimated and it was shown that periodic disposition of the holes can lead to a large, coherent buildup of losses. For the LHC liner, the slots in each section will therefore be disposed irregularly ("randomly"). In the LEP vacuum chamber, the pumping slots are regularly spaced, but grouped in sections with different lengths varying from 3 to almost 12 m.

Here we estimate the rate of field buildup analytically, and in a future report we will compare these results with numerical computations. The results are also tested on a "slot coupler" with short and wide holes, designed to couple efficiently to the beam chamber for diagnostic purposes.

Our model consists of a beam pipe of radius s_1 coupled to a coax of inner radius s_2, outer radius s_3 by a regular array of M elliptical slots of semi-major axis a and semi-minor axis b. The wall thickness of the coupling slot is L and each slot is separated from the previous one by a distance Δ. The geometry is shown in Fig. 1.

The goal of the calculation is to predict the buildup of the TEM mode in the coax which is in synchronism with the beam bunch travelling at $v = c$. We neglect all modes in both the beam pipe (waveguide) and coax which travel with $v \neq c$,

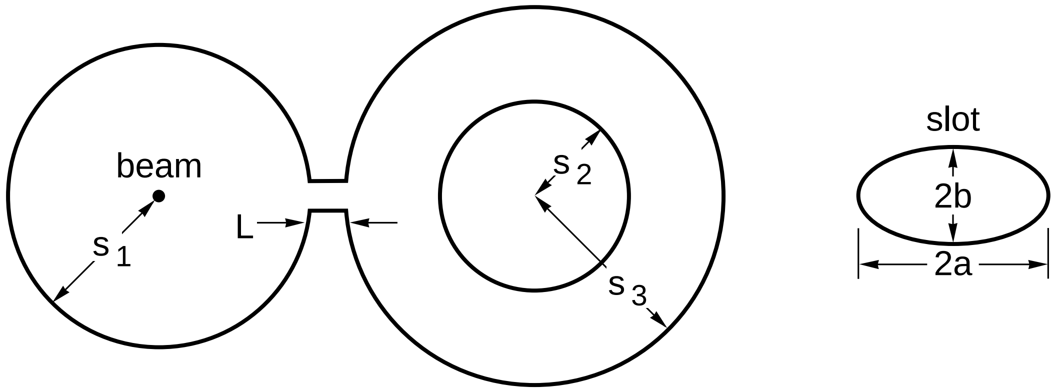

Figure 1: Slots in separating wall between beam and pump chamber in LEP

although we recognise that some of these could build up at particular resonant frequencies. We also consider bunch lengths greater than the slot dimensions, so that the standard electric polarizability and magnetic susceptibility can be used, but we do include a wall of finite thickness. We also neglect attenuation due to wall dissipation in the coax.

2 Source Field in the Waveguide

The source bunch density is assumed to be

$$\tilde{\rho}(x, y, z, t) = Q\delta(x)\delta(y)\tilde{g}(z - ct) \tag{1}$$

where $\tilde{g}(u)$ is the bunch shape. With

$$f(k) = \int du e^{jku}\tilde{g}(u) \tag{2}$$

we can write the charge density in the frequency domain (without a tilde $\sim$) as

$$\rho(x, y, z, k) = Qf(k)\delta(x)\delta(y)e^{-jkz}. \tag{3}$$

To return any quantity to the time domain one simply multiplies by $e^{jkct}dk/2\pi$ and integrates over k.

For a Gaussian bunch of rms length σ, we have

$$\tilde{g}(u) = \frac{1}{\sigma\sqrt{2\pi}}e^{-u^2/2\sigma^2} \tag{4}$$

and thus

$$f(k) = e^{-k^2\sigma^2/2}. \tag{5}$$

In the frequency domain, the waveguide field in the vicinity of a slot is given by

$$E_1 = Z_0 H_1 = \frac{QcZ_0}{2\pi s_1} f(k) e^{-jkz}, \tag{6}$$

where s_1 is the distance from the beam to the coupling slot.

3 Coax Field

We write the longitudinal electric field component in the form

$$E_{2z} = \int dq e^{-jqz} B(q) \frac{F_0(Kr)}{F_0(Ks_3)}, \tag{7}$$

where

$$F_0(u) = Y_0(u) J_0(Ks_2) - J_0(u) Y_0(Ks_2) \tag{8}$$

and where

$$K^2 = k^2 - q^2. \tag{9}$$

Here r is the radial variable with respect to the coax center, s_2 and s_3 are the inner and outer radii of the coax respectively, and the integration contour in the q plane passes below the poles on the negative real axis and above the poles on the positive real axis.

Because the integrand in (7) is not singular at $K = 0$, a coax without holes is only able to propagate a TEM mode, which contains no z-component of the electric field. But the holes will produce a perturbed TEM mode, which will now be obtained by constructing the r-component of the electric field, and then evaluating the contribution from the singularity at $K = 0$.

In the vicinity of the hole at $r = s_3$, the radial electric field corresponding to (7) is

$$-E_{2r} \equiv E_2 = \int dq e^{-jqz} B(q) \frac{jq}{K} \frac{F_0'(Ks_3)}{F_0(Ks_3)}. \tag{10}$$

The perturbed TEM mode corresponds to $q \simeq k$ or $K \simeq 0$. Using

$$Y_0(v) \simeq \frac{2}{\pi}(\ln \frac{v}{2} + \gamma) \text{ as } v \to 0 \tag{11}$$

we can evaluate (10) for $z > 0$ (after a hole) by closing the contour in the lower half plane, thus enclosing the pole at $q = k$. Near $K = 0$ we write

$$\frac{1}{K} \frac{F_0'(Ks_3)}{F_0(Ks_3)} \simeq \frac{1}{(q^2 - k^2) s_3 \ln(s_3/s_2)} \tag{12}$$

so that

$$E_2 = Z_0 H_2 = \frac{\pi B(k)}{s_3 \ln(s_3/s_2)} e^{-jkz}. \tag{13}$$

for the increment to the TEM coax mode travelling in synchronism with the beam when it passes the hole at z.

In order to find $B(q)$, we evaluate the inverse transform of (7) at $r = s_3$, obtaining

$$B(k) = \frac{1}{4\pi^2 s_3} \int_{\text{hole}} dS E_{2z}(s_3, k) e^{jkz}. \tag{14}$$

We now replace the hole geometry shown in Fig. 2 by the symmetric (in the electrostatic potential) and asymmetric geometries shown in Fig. 3. The integral over dS in (14) can then be written in terms of the electrostatic polarizability and magnetic susceptibility (for the corresponding magnetic field decomposition) as

$$\begin{aligned} B(k) &= \frac{jk}{8\pi^2 s_3}\Big[(E_1 - E_2)\chi_s - (E_1 + E_2)\chi_a \\ &- Z_0(H_1 - H_2)\psi_s + Z_0(H_1 + H_2)\psi_a\Big] e^{jkz} \end{aligned} \tag{15}$$

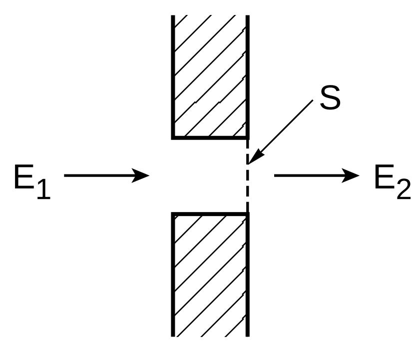

Figure 2: Symmetric geometry for hole

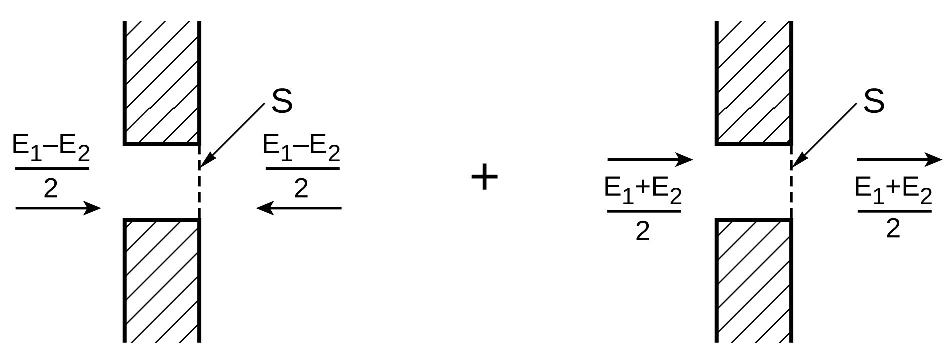

Figure 3: Asymmetric geometry for hole

where the subscripts s, a stand for symmetric and antisymmetric. Using $E_1 = Z_0 H_1$ in (5) and $E_2 = Z_0 H_2$ in (13) we eventually obtain

$$B(k) = \frac{jk}{8\pi^2 s_3}\Big[-E_1(\psi_{\rm out} - \chi_{\rm out}) + E_2(\psi_{\rm in} - \chi_{\rm in})\Big]e^{jkz} \tag{16}$$

where[1]

$$\left.\begin{array}{ll} \chi_{\rm in} = \chi_s + \chi_a & , \quad \chi_{\rm out} = \chi_s - \chi_a \\ \psi_{\rm in} = \psi_s + \psi_a & , \quad \psi_{\rm out} = \psi_s - \psi_a \end{array}\right\}. \tag{17}$$

Using (13) we finally have

$$\delta E_2 = \frac{jk}{8\pi s_2^2(\ln s_3/s_2)}\Big[-E_1\xi_{\rm out} + E_2\xi_{\rm in}\Big], \tag{18}$$

where

$$\xi \equiv \psi - \chi. \tag{19}$$

4 Buildup of TEM Field in the Coax

We can use (18) to determine the buildup of E_2 as the bunch passes each slot. Assuming $E_2 = 0$ before the first slot, we can solve (18) as

$$\frac{E_2}{E_1} = \frac{\xi_{\rm out}}{\xi_{\rm in}}\Big[1 - e^{-jm\beta k\Delta}\Big] , \tag{20}$$

where

$$\beta = \frac{\xi_{\rm in}}{8\pi s_3^2 \Delta \ln(s_3/s_2)}. \tag{21}$$

Obviously, the introduction of damping in the coax will eventually cause the exponential to die out, leaving a constant field amplitude in the coax proportional to the source field.

To see what happens in the time domain we use (7) to write

$$\tilde{E}_2 = \int \frac{dk}{2\pi} e^{jkct} \frac{\xi_{\rm out}}{\xi_{\rm in}} \frac{QcZ_0}{2\pi s_1} f(k) e^{-jkz}(1 - e^{-j\beta kz}), \tag{22}$$

where we write $z = m\Delta$. Using (1), this becomes

$$\tilde{E}_2 = \frac{\xi_{\rm out}}{\xi_{\rm in}} \frac{QcZ_0}{2\pi s_1}\Big[\tilde{g}(z - ct) - \tilde{g}((1+\beta)z - ct)\Big], \tag{23}$$

with the source field in the time domain being

$$\tilde{E}_1 = \frac{QcZ_0}{2\pi s_1}\tilde{g}(z - ct). \tag{24}$$

[1]See R.L. Gluckstern and J.A. Diamond, IEEE Transactions on Microwave Theory and Techniques, **39** No. 2, p. 274 (1991).

Writing $\zeta = z - ct$ as the longitudinal coordinate in the system travelling with the bunch, we find

$$\tilde{E}_2 = \frac{\xi_{\rm out}}{\xi_{\rm in}} \frac{Q_c Z_0}{2\pi s_1} \left[\tilde{g}(\zeta) - \tilde{g}((1+\beta)\zeta + \beta ct)\right] \tag{25}$$

which suggests that the effect of the holes is to generate a pulse receding slowly with velocity $\simeq \beta c$.

We can also calculate the power flow in the coax. In the time domain, this is

$$\int E_2 H_2 2\pi r dr = \frac{Q^2 c^2 Z_0}{(2\pi s_1)^2} \left(\frac{\xi_{\rm out}}{\xi_{\rm in}}\right)^2 \frac{1}{4\pi^2} \int dk f(k) e^{jkct - jkz} (1 - e^{-j\beta kz}) \times \int dk' f(k') e^{jk'ct + jk'z} (1 - e^{j\beta k' z}). \tag{26}$$

Taking $\int_{-\infty}^{\infty} dt$ to obtain the energy leads to

$$\frac{W}{Q^2} = \frac{cZ_0}{(2\pi s_1)^2} \frac{\xi_{\rm out}}{\xi_{\rm in}} \frac{1}{\pi} \int dk f^2(k) (1 - \cos \beta kz). \tag{27}$$

For the Gaussian bunch this becomes

$$\frac{W}{Q^2} = \frac{cZ_0}{2\pi} \left(\frac{\xi_{\rm out}}{\xi_{\rm in}}\right)^2 \left(\frac{s_3}{s_1}\right)^2 \ln\left(\frac{s_3}{s_2}\right) \frac{1}{\sigma\sqrt{\pi}} \left[1 - e^{-(\beta z/2\sigma)^2}\right], \tag{28}$$

corresponding to the expression for the loss factor in the standard literature. For small β the loss factor is then expected to be proportional to the square of the number of slots.

Before presenting some numerical applications, we should emphasise that the analytic estimate in (28) is valid only as long as the wavelength (or bunch length σ) is larger than the slot dimensions (a, b, L, Δ).

If this is *not* the case, one expects resonant behaviour in the coupling slots, and the approximations used in the derivations break down.

5 Numerical Applications

5.1 Slot Coupler

The slots are aligned so that the long dimension is perpendicular to the beam axis. We will use the published tables[2] for elliptical slots to calculate $\xi_{\rm in}$ and $\xi_{\rm out}$ for this example. The slot geometry corresponds to

$$\left.\begin{array}{lll} a = 10\text{mm} & , \; b = 2\text{mm} & , \; L = 2\text{mm} \\ p = \dfrac{a-b}{a+b} = \dfrac{2}{3} & , \; \dfrac{L}{a} = 0.2 & , \; \dfrac{L}{b} = 1.0 \end{array}\right\} \tag{29}$$

[2] B. Radak and R.L. Gluckstern, IEEE Transactions in Microwave Theory and Techniques, **43** No. 1, p. 194 (1995).

We then need $\chi_{\rm in}$, $\chi_{\rm out}$, $\psi_{xx,{\rm in}}$ and $\psi_{xx,{\rm out}}$.

Tables 1, 3[2] allow us to interpolate in p to find

$$\frac{3\chi_{\rm in}}{2\pi ab^2} \simeq 0.78 \ , \ \frac{3\chi_{xx,{\rm out}}}{2\pi a^3} \simeq 0.31 \tag{30}$$

Tables 2,4[2] allow us to interpolate in p to find

$$\ln \frac{\chi_{\rm out}}{\chi_0} \simeq -1.9 \ , \ \ln \frac{\chi_{xx,{\rm out}}}{\chi_{xx,0}} = -0.65 \tag{31}$$

where

$$\frac{3\chi_0}{2\pi ab^2} = \frac{1}{E(1-b^2/a^2)} = \frac{1}{E(.96)} = 0.95 \tag{32}$$

and

$$\frac{3\chi_{xx,0}}{2\pi a^3} = \frac{1-b^2/a^2}{K(1-b^2/a^2)E(1-b^2/a^2)} = \frac{.96}{3.02-1.05} = 0.49. \tag{33}$$

This leads to the values

$$\frac{3\chi_{\rm in}}{2\pi ab^2} \simeq 0.78 \ , \ \frac{3\chi_{xx,{\rm in}}}{2\pi ab^2} = 7.82 \tag{34}$$

$$\frac{3\chi_{\rm out}}{2\pi ab^2} = 0.14 \ , \ \frac{3\chi_{xx,{\rm out}}}{2\pi ab^2} = 6.4. \tag{35}$$

Using $2\pi ab^2/3 = 84$ mm^3, we have

$$\chi_{\rm in} = 66 \ {\rm mm}^3 \ , \ \chi_{xx,{\rm in}} = 660 \ {\rm mm}^3 \tag{36}$$

$$\chi_{\rm out} = 12 \ {\rm mm}^3 \ , \ \chi_{xx,{\rm out}} = 540 \ {\rm mm}^3 \tag{37}$$

$$\psi_{\rm in} = 590 \ {\rm mm}^3 \ , \ \psi_{xx,{\rm in}} = 530 \ {\rm mm}^3. \tag{38}$$

The guide and coax geometry is taken to be

$$s_1 = 16 \ {\rm mm} \ , \ s_2 = 1 \ {\rm mm} \ , \ s_3 = 5 \ {\rm mm} \ , \ \Delta = 10 \ {\rm mm} \ , \ \sigma = 70 \ {\rm mm} \tag{39}$$

and we find from (21) that[3]

$$\beta = 0.0066. \tag{40}$$

According to (28), the asymptotic loss factor is

$$\left.\frac{W}{Q^2}\right|_\infty = 8 \times 10^8 \frac{\rm volts}{\rm coulomb}. \tag{41}$$

Half this value is reached when

$$\frac{\beta z}{2\sigma} = \sqrt{\ln 2} \ {\rm or} \ z \simeq 250\sigma = 18 \ {\rm meters}. \tag{42}$$

[3] This value of β implies that the TEM mode in the coax travels with the reduced velocity $(1-0.0066)c$ because it is loaded by the periodic slots.

5.2 LEP slots

The elliptical slots are here aligned along the beam axis, and the slot geometry is

$$a = 10 \text{ mm} \ , \ b = 5 \text{ mm} \ , \ L = 5 \text{ mm}, \tag{43}$$

where we use $b = 5$ mm instead of $b = 4.5$ mm to allow us to use the entries in Tables, 1, 2, 5, 6[2]. If we ignore any high frequency effects, we find

$$\chi_{\text{in}} = 4.3 \text{ mm}^3 \ , \ \chi_{yy,\text{in}} = 5.1 \text{ mm}^3 \tag{44}$$

$$\chi_{\text{out}} = 0.64 \text{ mm}^3 \ , \ \chi_{yy,\text{out}} = 0.98 \text{ mm}^3 \tag{45}$$

$$\psi_{\text{in}} = 0.8 \text{ mm}^3 \ , \ \psi_{\text{out}} = 0.34 \text{ mm}^3. \tag{46}$$

The guide and coax geometry is taken to be

$$s_1 = 60 \text{ mm} \ , \ s_2 = 15 \text{ mm} \ , \ s_3 = 20 \text{ mm} \ , \ \Delta = 25 \text{ mm} \ , \ \sigma = 10 \text{ mm} \tag{47}$$

and we find from (21) that

$$\beta = 1.6 \times 10^{-6}. \tag{48}$$

According to (28), the asymptotic loss factor is

$$\left.\frac{W}{Q^2}\right|_{\infty} = 5 \times 10^9 \frac{\text{volts}}{\text{coulomb}} \tag{49}$$

Half this value is reached when

$$\frac{\beta z}{2\sigma} = \sqrt{\ln 2} \text{ or } z = 10^6 \sigma = 10^4 \text{ meters}. \tag{50}$$

Obviously, any section of reasonable length will give much lower values of W/Q^2. But the entire calculation for the LEP slots should not be taken at face value because a 10 mm bunch has important high frequency components which are likely to have resonant behaviour. This is being investigated presently with the use of 3-D computer codes, whose results can also be compared with the analytic work presented here in the long wave length region.

6 Conclusions

Energy is lost coherently by a beam coupled to a coaxial transmission line such as formed by the neg-strips in the pump chamber of LEP, or the liner inside the vacuum chamber in LHC. The buildup rate of the loss depends on the size, shape, and number of holes in the separating walls. Here we derive analytical expressions valid in the long wave length limit applicable to long bunches in LHC, while the higher frequency region more important for the short bunches in LEP will have to be treated numerically.