

Electroweak phase transition by four dimensional simulations*

F. Csikor ^a Z. Fodor[†] ^b J. Hein ^c J. Heitger ^d A. Jaster ^c I. Montvay ^c

^aInstitute for Theoretical Physics, Eötvös University, H-1088 Budapest, Hungary.

^bTheory Division, CERN, CH-1211 Geneva 23, Switzerland

^cDESY, Notkestr. 85, D-22603 Hamburg, Germany

^dInstitut für Theoretische Physik I, Universität Münster, D-48149 Münster, Germany

The finite temperature phase transition in the SU(2)-Higgs model at a Higgs boson mass $M_H \simeq 35$ GeV is studied in numerical simulations on four dimensional lattices with time-like extensions up to $L_t = 5$. T_c/M_H is extrapolated to the continuum limit and a comparison with the perturbative prediction is made. A one-loop calculation to the coupling anisotropies of the SU(2)-Higgs model on lattices with asymmetric lattice spacings is presented. Our numerical simulations show that the above perturbative result is applicable in the phenomenologically interesting parameter region.

1. Introduction

At high temperatures the electroweak symmetry is restored. Since the baryon violating processes are unsuppressed at high temperatures, the observed baryon asymmetry of the universe has finally been determined at the electroweak phase transition. Due to the bad infrared features of the theory, the perturbative approach predicts $\mathcal{O}(100\%)$ corrections and breaks down in the physically interesting region, $M_H > 65$ GeV [1]. In recent years lattice Monte Carlo simulations have been used in three (e.g. [2]) and four dimensions (e.g. [3]) to clarify the details of the phase transition. Since the bad infrared behaviour is connected with the bosonic sector of the Standard Model, in both cases the fermions are omitted and the SU(2)-Higgs model is studied. The comparison of the two approaches gives not only an unambiguous description of the finite temperature electroweak phase transition, but an understanding of the non-perturbative features of the reduction step, too.

In this talk two selected new results of the four dimensional formulation are discussed. In Sec-

tion 2 the SU(2)-Higgs model at a Higgs boson mass $M_H \simeq 35$ GeV is studied. Our four dimensional, finite temperature lattices have time-like extensions up to $L_t = 5$. We extrapolate T_c/M_H to the continuum limit and compare the result with the perturbative prediction. In Section 3 we present the one-loop calculation to the coupling anisotropies of the SU(2)-Higgs model on lattices with asymmetric lattice spacings. We test the results by numerical simulations and show that the perturbative result is applicable in the phenomenologically interesting parameter region.

The simulations have been performed on the APE (Alenia Quadrics) computers at DESY-IFH and on the CRAY-YMP at HLRZ Jülich.

2. Critical temperature for $M_H \simeq 35$ GeV

The details of the results presented in this Section can be found in [4].

Our studies at Higgs boson masses $\simeq 20, 35, 50$ GeV showed that the perturbative predictions are in quite good agreement with the lattice results for the jump of the order parameter, latent heat and interface tension [1,3,4]. Unfortunately, the non-perturbative results are obtained on lattices with temporal extensions $L_t = 2, 3$ and they have typically $\mathcal{O}(10\%)$ errors, thus the compar-

*talk presented by Z. Fodor

[†]On leave from Institute for Theoretical Physics, Eötvös University, Budapest, Hungary.

ison with continuum perturbative predictions is quite difficult.

The situation is much better for the critical temperature. Its value is $1/L_t$ in lattice units. In order to extract the dimensionless T_c/M_H one needs the transition point at a given L_t and the $T = 0$ mass of the Higgs boson at that point. Both of them can be measured quite precisely.

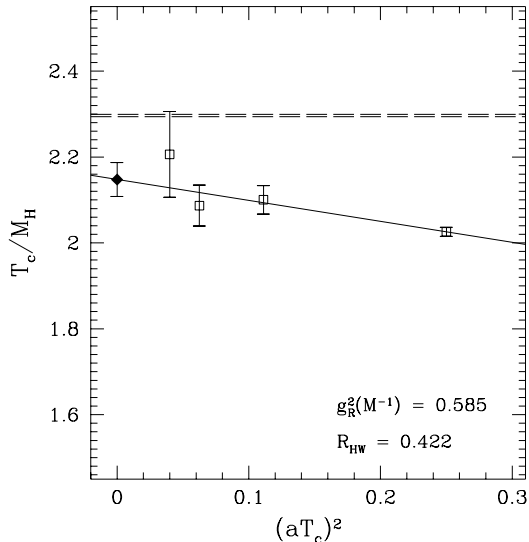


Figure 1. Numerical results for T_c/M_H versus $(aT_c)^2 = L_t^{-2}$. The straight line is the extrapolation to the continuum value shown by the filled symbol. The dashed lines are the perturbative predictions at order g^3 (upper) and g^4 (lower), respectively. R_{HW} gives the mass ratio of the Higgs and W boson masses and g_R^2 is the renormalized gauge coupling.

Fig. 1 shows our results. The non-perturbative masses are determined by a careful extrapolation to infinite volumes. As it can be seen, the T_c/M_H predictions of the one-loop and two-loop perturbation theory almost coincide. The errors of the lattice data are dominated by the uncertainties in the critical hopping parameter. The extrapolated continuum value is $T_c/M_H = 2.147(40)$. The value at $L_t = 2$ is about 5% smaller. This relatively small deviation is better than the expectation based on lattice perturbation theory. The value of T_c/M_H extrapolated to the contin-

uum limit differs by about three standard deviations from the two-loop perturbative result. This is under the assumption that $L_t = 2$ can be included in the extrapolation, which is supported by the good quality of the fit ($\chi^2 \simeq 1$).

3. The SU(2)-Higgs model on asymmetric lattices

The details of the results presented in this Section can be found in [5,6].

For larger M_H (e.g. $M_H = 80$ GeV) the electroweak phase transition gets weaker, the lowest excitations have masses small compared to the temperature, T . From this feature one expects that a finite temperature simulation on isotropic lattice would need several hundred lattice points in the spatial directions even for $L_t = 2$ temporal extension. This difficulty can be solved by using asymmetric lattices, i.e. lattices with different spacings in temporal (a_t) and spatial (a_s) directions. The asymmetry of the lattice spacings is characterized by the asymmetry factor $\xi = a_s/a_t$. The different lattice spacings can be ensured by different coupling strengths, (β_s and β_t for space-space and space-time plaquettes; κ_s and κ_t for space-like and time-like hopping terms) in the action for different directions (c.f. [5]). The anisotropies $\gamma_\beta^2 = \beta_t/\beta_s$ and $\gamma_\kappa^2 = \kappa_t/\kappa_s$ are functions of the asymmetry ξ . On the tree-level the coupling anisotropies are equal to the lattice spacing asymmetry; however, they receive quantum corrections in higher orders of the loop-expansion. On the one-loop level one gets

$$\begin{aligned}\gamma_\beta^2 &= \xi^2 [1 + c_\beta(\xi)g^2 + b_\beta(\xi)\lambda], \\ \gamma_\kappa^2 &= \xi^2 [1 + c_\kappa(\xi)g^2 + b_\kappa(\xi)\lambda],\end{aligned}$$

where g and λ are the bare gauge and scalar coupling, respectively.

In general, the determination of $\gamma_\beta(\xi)$ and $\gamma_\kappa(\xi)$ should be done non-perturbatively. This can be achieved by tuning the coupling strengths requiring that the Higgs- and W-boson correlation lengths in physical units are the same in the different directions.

We have used the above idea in perturbation theory. The one-loop results are summarized on Fig. 2. As it can be seen only $c_\beta(\xi)$ and $c_\kappa(\xi)$

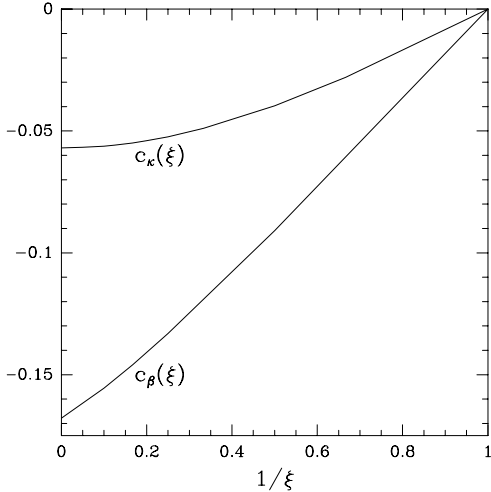


Figure 2. $c_\beta(\xi)$ and $c_\kappa(\xi)$ as functions of $1/\xi$.

are given. The functions $b_\beta(\xi)$ and $b_\kappa(\xi)$ vanish in this order. Our calculation is the extension of [7] to a gauge-Higgs model. Omitting graphs connected with the scalar sector, the result of [7] can be reproduced (the function $c_\beta(\xi)$ of the present paper corresponds to $c_\tau(\xi) - c_\sigma(\xi)$ of ref. [7]).

It is necessary to check the validity of the perturbative result by non-perturbative methods. $M_H \simeq 80$ GeV has been chosen, thus $M_H \simeq M_W$ at zero temperature with typical correlation lengths of 2-4 in lattice units. We have performed simulations with two different sets of anisotropies. First $\gamma_\beta = \gamma_\kappa = 4$ (tree-level). Then $\gamma_\beta = 3.8$, $\gamma_\kappa = 4$. As it can be seen on Fig. 3 in the first case $\xi_W > \xi_H$ ($\xi_i = M_i(\text{space-direction})/M_i(\text{time-direction})$, where i denotes the W or Higgs channel). In the second case $\xi_W < \xi_H$. Rotational invariance is restored if $\xi_W = \xi_H$. Our linear interpolation is shown by the two solid lines. The “matching point” is in complete agreement with the perturbative prediction (full triangle). In addition we have simulated at the perturbatively predicted couplings. At this third point the rotational invariance is restored, thus $\xi_W = \xi_H$ within errorbars.

The non-perturbative features of the theory appear at high temperatures. Since the theory is defined at $T = 0$, where it is weakly interacting, the good agreement between perturbation theory and

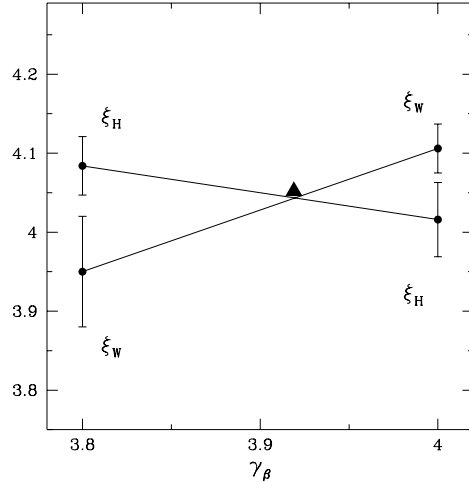


Figure 3. Nonperturbative determination of the anisotropies

non-perturbative results is not surprising. The perturbative predictions open the possibility to study the phase transition for higher Higgs boson masses and to vary the space-like and time-like lattice spacings separately.

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