

# MEASUREMENTS OF THE TUNE VARIATIONS INDUCED BY NON-LINEARITIES IN LEPTON MACHINES

R. Bartolini, M. Giovannozzi, W. Scandale, A. Verdier CERN, Geneva, Switzerland  
 C. Pellegrini, P. Tran UCLA  
 E. Todesco, INFN-Bologna  
 J. Corbett, M. Cornacchia SSRL

## Abstract

The precise measurement of the betatron tune as a function of the oscillation amplitude provides a useful information on non-linear beam dynamics. In lepton accelerators, this measurement is made difficult due to various damping mechanisms. To counteract this, we propose to use algorithms that provide a precise measurement of the tune in a small number of turns. We apply these procedures in LEP at injection and collision energy, as well as in SPEAR at injection energy. Collections of experimental data, and a first comparison with the results of model-based simulations are discussed.

## 1 INTRODUCTION

Precise algorithms for the tune measurement were recently investigated [1, 2, 3] and applied both to tracking simulations and to experimental data [4, 5]. The possibility of a precise determination of the tune with a small number of turns opens the perspective of measuring the instantaneous tune in presence of ripples or detuning with amplitude in lepton machines, at least in conditions where the damping time is not too fast (i.e.  $\geq 100$  turns). Several sets of data were collected from the LEP and SPEAR storage rings in operating conditions useful for the measure of the betatron tune. In the LEP case the data were collected during a machine experiment performed in 1993, devoted to the measurement of the dynamic aperture, both at injection and collision energies. The goal of that experiment was to observe the beam stability at large values of the vertical action. Therefore, a vertical deflection of several millimeters was applied to the beam, and the subsequent horizontal and vertical beam positions were collected and stored for 1024 consecutive turn. In the SPEAR case the data were collected during the study of the effect of a family of four octupoles for the Landau damping of coupled bunch motion at high current [5] and their effectiveness in the new lattice NOQ3 [6] which is currently in operation. Both sets of data allow to measure the betatron tunes as a function of the vertical action. This will provide informations on the non-linear spurious fields present along the circumference of the machine. In the available data the oscillations are damped by various damping mechanisms, like radiation and head-tail effect. This provides a mean to explore various regions of the phase-space and therefore to measure the tune for different values of the actions. On the other hand, this fact reduces the number of oscillations corresponding to a constant value of the action and thus affect-

ing the precision of the tune measurement. The algorithms of Ref. [2, 3] provide precise values of the tunes already with a small number of turns. Using them, we could find a procedure to deduce the detuning with the amplitude with sufficient precision.

## 2 MACHINE CONDITIONS

**LEP** During the data collection, LEP was set under the conditions described in [7], with the G05P46hv2 lattice (90/60 arc cells). The pretzel was turned off. ALEPH was not running. Its coupling correction had been subtracted. The r.m.s. closed orbit distortions were about 0.6 mm in vertical and 0.7 mm in horizontal. Four electron bunches of  $92\mu A$  were circulating in the machine.  $Q_s$  was 0.065. The fractional tunes were  $Q_y = 0.2021$  and  $Q_x = 0.2885$ . The emittance and damping wigglers were on. Under these conditions the r.m.s. energy spread is  $1.50 \times 10^{-3}$  and the horizontal emittance is 35 nm (MAD calculations). The beam was kicked in the vertical plane by means of the dump kicker. The latter was calibrated from the analysis of the amplitude of the kicked beam, since there was a discrepancy between this calibration and the kick angle computed from the voltage.

**SPEAR** In the SPEAR operating condition referring to the collected data, the energy was 2.3 GeV in the injection configuration of the NOQ3 lattice. The residual coupling was less than 1% and the chromaticity was adjusted to be slightly positive 0.05 vertical and 0.02 horizontal in normalized units. The current was 4.3 mA, in a single bunch. The tunes were  $Q_y = 5.302$  and  $Q_x = 7.156$ .

## 3 ANALYSIS OF DATA

The analysis of the data is based on the high precision algorithms for the tune measurement [2, 3]. Due to linear and non-linear coupling, the vertical motion induces small but observable oscillations in the horizontal plane. The horizontal and the vertical tunes are computed using sets of  $N$  consecutive values of the beam position and applying to them the interpolated FFT algorithm. The value of  $N$  results from a compromise between the precision required and the damping time of the signal. In our case we choose  $N = 32$  for LEP data and  $N = 128$  for SPEAR data. The calculation of the tunes and of the action is iterated over successive windows of  $N$  turns. In each iteration the time window moves by one or more turns until the whole set of useful data is

scanned. In order to obtain the betatron frequency, the available set of  $N$  data is filtered by a Hanning window. After this filtering, the frequency spectrum is evaluated with a FFT algorithm, and the leading frequency is identified. The tune is finally computed with the following interpolating formula:

$$\nu_{\text{FHan}} = \frac{k}{N} + \frac{1}{2\pi} \arcsin \left[ \Psi \sin \frac{2\pi}{N} \right], \quad (1)$$

where  $\Psi$  is given by

$$\Psi = A \left( |\phi(\nu_k)|, |\phi(\nu_{k+1})|, \cos \frac{2\pi}{N} \right) \quad (2)$$

and the function  $A$  is given by

$$A(a, b, c) = \frac{-(a + bc)(a - b) + b\sqrt{\Delta}}{a^2 + b^2 + 2abc} \quad (3)$$

where

$$\Delta = c^2(a + b)^2 - 2ab(2c^2 - c - 1). \quad (4)$$

We assume that  $\nu_k$  is the leading frequency in the FFT spectrum  $\phi(\nu_i)$  and that the spectral amplitude  $\phi(\nu_{k+1})$  is larger than  $\phi(\nu_{k-1})$ . If the last hypothesis is not true, one has to use  $\phi(\nu_{k-1})$  instead of  $\phi(\nu_{k+1})$  in Eq. (1). In absence of noise the frequency error scales with the number of turns  $N$  as:

$$|\epsilon_{\text{Fint}}| \leq \frac{C_{\text{FHan}}}{N^4}. \quad (5)$$

where  $C_{\text{FHan}}$  is a scaling constant smaller than 1.

**LEP data** We analysed in details the data of the runs 1940 and 1941. In the run 1940 the energy was 20 GeV and the voltage of the vertical kicker was 1.0 kV. In the run 1941 the energy was 46 GeV and the voltage of the vertical kicker of 1.8 kV. The amplitude was measured at two places : PU.QL4B.L1 where  $\beta_x$  is 11.6 m and  $\beta_y$  is 142.6 m, and PU.QL17.L7 where  $\beta_x$  is 112.6 m and  $\beta_y$  is 20.16 m (theoretical values computed with MAD). At injection energy (20 GeV) the persistence of the oscillations was  $\approx 200$  turns as shown in Fig (1), whilst at collision energy (46 GeV) it was only  $\approx 60$  turns. A great care is necessary to identify the frequency of the horizontal mode of oscillation in the induced horizontal signal shown in Fig. 2. The vertical action is evaluated by averaging the peak oscillation over the same set of 32 turns, using the previously established calibration factor and the theoretical values of the  $\beta$  functions. The analysis is stopped when the oscillation amplitude is of the order of 0.1 mm (digital resolution of the 8-bit ADC used in the position monitors as mentioned above). Due to the digital noise of about 0.1 mm amplitude, the scaling law of the frequency error is in fact approximately proportional to  $1/N^2$ , see Ref. [2]. Therefore the tune error in a 32 turns measurement will be of the order of  $10^{-3}$ . The tunes estimated from this analysis and applied to the data of the run No. 1940 are shown in Fig (3). The interpolating

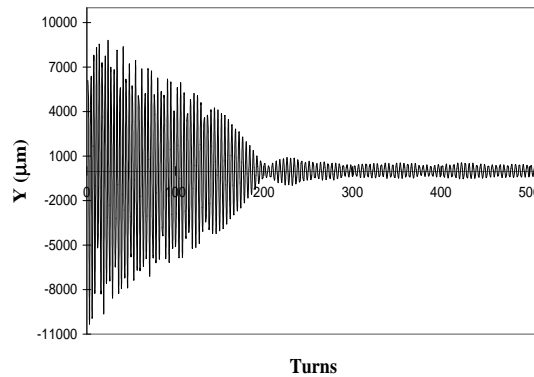


Figure 1: Vertical amplitude as from measurement in run 1940 of LEP at 20 GeV.

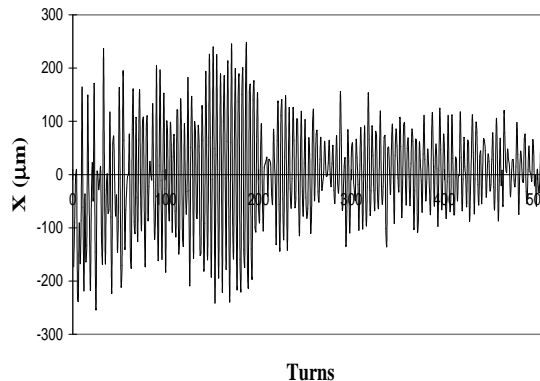


Figure 2: Horizontal amplitude as from measurement in run 1940 of LEP at 20 GeV.

curves used there assume a linear dependence of the tunes on the action. The extrapolated values of the tunes at zero amplitude, are not in a good agreement with the measured values. This could be due to a badly estimated coupling in the machine. The slope of the interpolating lines is compared with the theoretical expectations in the next section.

**SPEAR data** We analysed the data of the run called 27. They were collected after a vertical kick of 12 kV, from two PU's: BPM1S2 where  $\beta_x$  is 20.6 m and  $\beta_y$  is

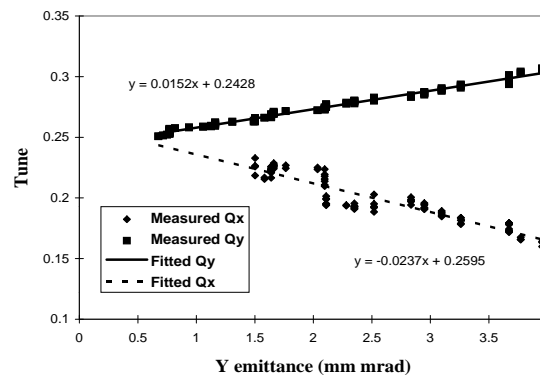


Figure 3: Detuning curve for the LEP machine at 20 GeV obtained using data measured at PU.QL4B.L1

11.8 m and BPMWIS1 where  $\beta_x$  is 4.9 m and  $\beta_y$  is 48.9 m. This allowed to reconstruct the evolution of the Courant-Snyder coordinate. An analogous analysis was performed using windows of 128 data as the damping time is approximately 700 turns. The results of these analysis are reported in Figs. (4) and (5).

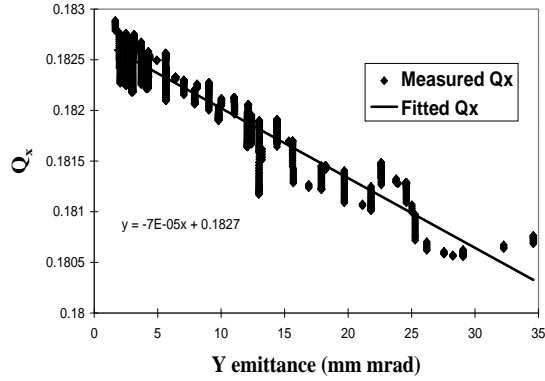


Figure 4: Vertical detuning curve for the SPEAR machine at 2.3 GeV

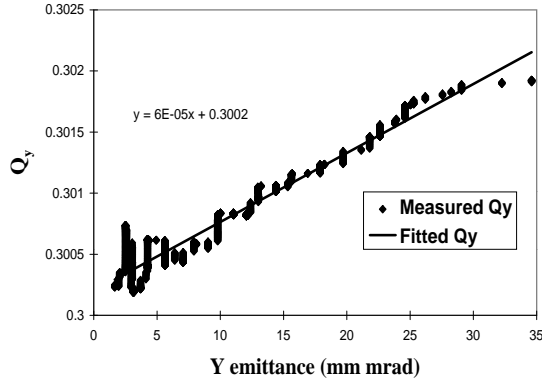


Figure 5: Horizontal detuning curve for the SPEAR machine at 2.3 GeV

#### 4 COMPARISON WITH THE MODELS

The detuning with amplitude has been obtained from the estimation of the sextupole and octupole components in the LEP dipoles at 46 GeV [8] and 20 GeV [9]. The anharmonicities have been computed with MAD (normal form analysis in 4D or 6D). At 46 GeV they are:

$$\frac{\partial Q_x}{\partial E_x} = 7.9 \times 10^3, \frac{\partial Q_y}{\partial E_y} = +3.0 \times 10^4, \frac{\partial Q_x}{\partial E_y} = -4.4 \times 10^4$$

The tune-shifts associated with the emittance estimated above is:

$$\Delta Q_x = -0.11 \pm 0.03, \Delta Q_y = +0.07 \pm 0.02$$

At 20 GeV, with both sextupolar and octupolar field errors in the dipoles, we have:

$$\frac{\partial Q_x}{\partial E_x} = 3.2 \times 10^3, \frac{\partial Q_y}{\partial E_y} = +1.44 \times 10^4, \frac{\partial Q_x}{\partial E_y} = -2.46 \times 10^4$$

The tune-shifts associated with the emittance estimated above is then :

$$\Delta Q_x = -0.11 \pm 0.016, \Delta Q_y = +0.034 \pm 0.02$$

Only with sextupolar field errors in the dipoles, we have:

$$\frac{\partial Q_x}{\partial E_x} = 10.8 \times 10^3, \frac{\partial Q_y}{\partial E_y} = +8.1 \times 10^3, \frac{\partial Q_x}{\partial E_y} = -1.7 \times 10^4$$

Finally, without any field error, i.e. only the chromaticity sextupoles are in the machine, we have:

$$\frac{\partial Q_x}{\partial E_x} = -6.4 \times 10^3, \frac{\partial Q_y}{\partial E_y} = +8.1 \times 10^3, \frac{\partial Q_x}{\partial E_y} = -1.7 \times 10^4$$

Let's now compare these theoretical values with the experimental data. As the measurements were done with vertical kicks, we have only measurements for the quantities

$$\frac{\partial Q_x}{\partial E_y}, \frac{\partial Q_y}{\partial E_y}$$

The comparison gives quite contradictory answers. At 20 GeV there is an excellent agreement of the measurements with the computation from the LEP model that contains parasitic sextupoles and octupoles in the dipoles. In fact the measurements of the errors [9] were done in the dipoles of the first octant and in the simulations these measured errors were put in all LEP dipoles. The agreement of the measured and computed detuning with amplitude confirm the hypothesis that the field errors are the same in all LEP dipoles. On the other hand it also suggests that the octupole components in the quadrupoles of the straight sections are negligible compared with those of the dipoles. At 46 GeV the agreement is lacking. In our opinion the discrepancy at high energy depends on the poor quality of the data available. We tried to deduce informations on the detuning by extracting 32 values of the positions from a set of about 60 significant values. Obviously the damping time was too small for our study.

For the SPEAR machine, we found a qualitative agreement with previous comparison of the total vertical detuning. A more refined analysis is in progress.

#### 5 REFERENCES

- [1] E.Asseo, CERN PS Note (LEA) 85-3 (1985).
- [2] R. Bartolini et al., CERN SL Note (AP) 95-84 (1995) and Part. Acc. in press.
- [3] R. Bartolini et al., these proceedings.
- [4] R. Bartolini et al., SL-MD Note 207, (1996).
- [5] P.Tran et al., Proceedings of PAC Conference, Dallas, (1995).
- [6] H.D. Nuhn SLAC-PUB-6457, (1994).
- [7] L. Ducimetière et al., SL-MD Note 113, (1994).
- [8] A. Verdier, SL-MD Note 117, (1994).
- [9] A. Verdier, SL-MD Note 110, (1993).