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Excited Fermion Contribution to Z^0 Physics at One Loop

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Abstract

We investigate the effects induced by excited leptons at the one-loop level in the observables measured on the Z peak at LEP. Using a general effective Lagrangian approach to describe the couplings of the excited leptons, we compute their contributions to both oblique parameters and Z partial widths. Our results show that the new effects are comparable to the present experimental sensitivity, but they do not lead to a significant improvement on the available constraints on the couplings and masses of these states.

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I. INTRODUCTION

The standard model of electroweak interactions (SM) is not able to give a satisfactory explanation to family repetition and to the complex pattern of the fermion masses. One expects a substantial improvement in the understanding of these problems when considering an underlying fermionic substructure where the usual fermions share some constituents (preons) [1]. In this sense, the SM would be just the low-energy limit of a more fundamental theory, being valid only at energies below the compositeness mass scale Λ.

One of the most unambiguous predictions of the composite models is the existence of an excited lepton state for each known lepton. Unfortunately, we do not yet have a satisfactory model that could reproduce the whole family spectrum. In view of the lack of a unique predictive theory, a model-independent phenomenological analysis of the effects of fermion compositeness seems the most appealing approach. On this ground, we can employ the effective Lagrangian techniques to describe the physics of these excited states below the compositeness scale.

This approach has been employed in several phenomenological studies that analysed the expected signatures of these excited fermions in pp [2,3], e^+e^- [2,4–7], and ep [4,6] collisions at high energies. On the experimental side, several searches for these particles have been carried out, including those at the CERN Large Electron–Positron Collider (LEP) [8] and at HERA [9]. At LEP, the experiments at the Z pole excluded the existence of excited spin–$\frac{1}{2}$ fermions with mass up to 46 GeV from the pair production search ($e^+e^- \to \ell^*\ell^*$), and up to 90 GeV from direct single production ($e^+e^- \to \ell\ell^*$) for a scale of compositeness $\Lambda < 2.5$ TeV [8]. Very recent results from the L3 Collaboration [11], at centre-of-mass energies of 130–140 GeV, determined the lower mass limits at 95% C.L. of 64.7 GeV for the excited electrons, and roughly $\Lambda \geq 1.4$ TeV for $90 \leq M_{e^*} \leq 130$ GeV. The experiments at the DESY ep collider HERA also searched for resonances in the $e\gamma$, νW, and eZ systems [9,10]; however the LEP bounds on excited leptons couplings are about one order of magnitude more stringent in the mass region below the Z mass.

In spite of the failure of all the direct searches for compositeness, we could expect that the next generation of accelerators, working at higher centre-of-mass energies, would be able to obtain a direct evidence of the existence of these composite states. On the other hand, an important source of indirect information about new particles and interactions is the precise measurement of the electroweak parameters done at LEP. Virtual effects of these new states can alter the SM predictions for some of these parameters and the comparison with the experimental data can impose bounds on their masses and couplings.

In this work we investigate the one-loop effects of excited leptons in the observables measured on the Z peak at LEP. Using a general effective Lagrangian approach in terms of dimension six operators to describe the couplings of the excited leptons, we compute their contribution to both oblique and vertex corrections to the electroweak parameters. Our results show that the new effects are comparable to the present experimental sensitivity, but they are only able to constraint very marginally the model parameters beyond the present limits from direct searches.

The outline of the paper is as follows. In section II, we introduce the effective Lagrangian describing the couplings of the excited leptons. Section III contains the relevant analytical expressions for the one-loop corrections induced by the excited leptons. Our results and their respective discussion are given in Section IV. This paper is supplemented with two Appendices. In Appendix A, we list all the relevant Passarino–Veltman functions, and in Appendix B we present our results on the new contribution to the two- and three-point functions in D dimensions.

II. EFFECTIVE INTERACTIONS

In this work, we consider excited fermionic states with spin and isospin $\frac{1}{2}$. We assume that the excited fermions acquire their masses before the $SU(2) \times U(1)$ breaking, so that both left-handed and right-handed states belong to weak isodoublets. We introduce the weak doublets, with hypercharge $Y = -1$, for the usual left-handed fermion (ψ_L) and for the excited fermions (Ψ^*),

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad , \text{ and } \qquad \Psi^* = \begin{pmatrix} N \\ E \end{pmatrix} \ ,$$

The most general dimension-six effective Lagrangian describing the coupling of the excited fermions to the usual fermions, which is $SU(2) \times U(1)$ invariant and CP conserving can be written as [4]

$$\mathcal{L}_{Ff} = -\frac{1}{2\Lambda}\bar{\Psi}^{*}\sigma^{\mu\nu}\left(gf_2\frac{\tau^i}{2}W^i_{\mu\nu} + g'f_1\frac{Y}{2}B_{\mu\nu}\right)\psi_L + \text{ h. c. }, \tag{1}$$

where f_2 and f_1 are weight factors associated to the $SU(2)$ and $U(1)$ coupling constants, with Λ being the compositeness scale, and $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$. We will assume a pure left-handed structure for these couplings in order to comply with the strong bounds coming from the measurement of the anomalous magnetic moment of leptons [12].

In terms of the physical fields, the Lagrangian (1) becomes

$$\mathcal{L}_{Ff} = -\sum_{V=\gamma,Z,W} C_{VFf}\bar{F}\sigma^{\mu\nu}(1-\gamma_5)f\partial_\mu V_\nu - i\sum_{V=\gamma,Z} D_{VFf}\bar{F}\sigma^{\mu\nu}(1-\gamma_5)fW_\mu V_\nu + \text{ h. c. }, \tag{2}$$

where $F = N, E$, and $f = \nu, e$. The non-abelian structure of (1) gives rise to a contact quartic interaction, such as the second term in the l.h.s. of Eq. (2). In this equation, we have omitted terms containing two W bosons, which do not play any role in our calculations. C_{VFf} is the coupling of the vector boson with the different kinds of fermions,

$$\begin{array}{lll} C_{\gamma Ee} = -\frac{e}{4\Lambda}(f_2 + f_1) & , & C_{\gamma N\nu} = \frac{e}{4\Lambda}(f_2 - f_1) \\ C_{ZEe} = -\frac{e}{4\Lambda}(f_2\cot\theta_W - f_1\tan\theta_W) & , & C_{ZN\nu} = \frac{e}{4\Lambda}(f_2\cot\theta_W + f_1\tan\theta_W) \\ C_{WE\nu} = C_{WNe} = \dfrac{e}{2\sqrt{2}\sin\theta_W\Lambda}f_2 \ , & & \end{array} \tag{3}$$

and the quartic interaction coupling constant, D_{VFf}, is given by

$$\begin{aligned} D_{\gamma E\nu} &= -D_{\gamma Ne} = \frac{e^2\sqrt{2}}{4\sin\theta_W\Lambda}f_2 \\ D_{ZE\nu} &= -D_{ZNe} = \frac{e^2\sqrt{2}\cos\theta_W}{4\sin^2\theta_W\Lambda}f_2 \ . \end{aligned} \tag{4}$$

The coupling of gauge bosons to excited leptons can be described by the $SU(2)\times U(1)$ invariant and CP conserving, effective Langragian,

$$\mathcal{L}_{FF} = -\bar{\Psi}^{*}\left[\left(g\frac{\tau^i}{2}\gamma^\mu W^i_\mu + g'\frac{Y}{2}\gamma^\mu B_\mu\right) + \left(\frac{g\kappa_2}{2\Lambda}\frac{\tau^i}{2}\sigma^{\mu\nu}\partial_\mu W^i_\nu + \frac{g'\kappa_1}{2\Lambda}\frac{Y}{2}\sigma^{\mu\nu}\partial_\mu B_\nu\right)\right]\Psi^{*} \tag{5}$$

In terms of the physical fields, this can be written as,

$$\mathcal{L}_{FF} = -\sum_{V=\gamma,Z,W}\bar{F}(A_{VFF}\gamma^\mu V_\mu + K_{VFF}\sigma^{\mu\nu}\partial_\mu V_\nu)F \ . \tag{6}$$

Since we have assumed that the left- and right-handed excited leptons have the same quantum numbers under the standard gauge group, the dimension-four piece in (6) is taken vector-like. A_{VFF} is given by

$$
\begin{aligned}
A_{\gamma EE} &= -e &, \quad A_{\gamma NN} &= 0 \\
A_{ZEE} &= e\frac{(2\sin^2\theta_W - 1)}{2\sin\theta_W\cos\theta_W} &, \quad A_{ZNN} &= \frac{e}{2\sin\theta_W\cos\theta_W} \\
A_{WEN} &= \frac{e}{\sqrt{2}\sin\theta_W}
\end{aligned}
\tag{7}
$$

and K_{VFF} is given by

$$
\begin{aligned}
K_{\gamma EE} &= -\frac{e}{4\Lambda}(\kappa_2 + \kappa_1) &, \quad K_{\gamma NN} &= \frac{e}{4\Lambda}(\kappa_2 - \kappa_1) \\
K_{ZEE} &= -\frac{e}{4\Lambda}(\kappa_2\cot\theta_W - \kappa_1\tan\theta_W) &, \quad K_{ZNN} &= \frac{e}{4\Lambda}(\kappa_2\cot\theta_W + \kappa_1\tan\theta_W) \\
K_{WEN} &= \frac{e}{2\Lambda}\frac{\kappa_2}{\sqrt{2}\sin\theta_W}.
\end{aligned}
\tag{8}
$$

It is important to notice that the phenomenological model for the excited fermions described by the Lagrangians (2) and (6) has been extensively used by several experimental collaborations [8–11] to search for excited states. Therefore the results presented in this paper can be directly compared with the bounds on the excited fermion mass and compositeness scale obtained by these collaborations.

III. ANALYTICAL EXPRESSIONS

In this work we employed the on-shell-renormalization scheme, adopting the conventions of Ref. [13]. We used as inputs the fermion masses, G_F, α, and the Z-boson mass. The electroweak mixing angle is a derived quantity defined through $\sin^2\theta_W = s_W^2 \equiv 1 - M_W^2/M_Z^2$.

As a general procedure to evaluate the virtual contributions of the excited states, with couplings described by (2) and (6), we evaluated the loops in $D = 4-2\epsilon$ dimensions using the dimension regularization method [14] which is a gauge-invariant regularization procedure, and we adopted the unitary gauge to perform the calculations. We identified the poles at $D = 4$ ($\epsilon = 0$) and $D = 2$ ($\epsilon = 1$) with the logarithmic and quadratic dependence on the scale Λ [15]. The finite part of the loop is given by

$$
L_{\text{finite}} = \lim_{\epsilon\to 0}\left[L(\epsilon) - R_0\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + 1\right) - R_1\left(\frac{1}{\epsilon - 1} + 1\right)\right]
$$

where $R_{0(1)}$ are the residues of the poles at $\epsilon = 0(1)$. The final result is written as

$$
L = L_{\text{finite}} + R_0 \log\left(\frac{\Lambda^2}{\mu^2}\right) + R_1\frac{\Lambda^2}{4\pi\mu^2}
$$

In order to compute the loops in D dimensions in terms of the Passarino–Veltman scalar one-loop functions (see Appendix A), we used the Mathematica package FeynCalc [16]. The output of FeynCalc, in the case of the two-point functions, was checked against the results obtained by a direct analytical calculation. Our results for the new contribution to the two- and three-point functions in D dimensions (see below) can be found in Appendix B.

Close to the Z resonance, the physics can be summarized by the effective neutral current

$$J_\mu = \left(\sqrt{2}G_\mu M_Z^2 \rho_f\right)^{1/2} \left[\left(I_3^f - 2Q^f s_W^2 \kappa_f\right)\gamma_\mu - I_3^f \gamma_\mu\gamma_5\right] , \tag{9}$$

where Q^f (I_3^f) is the fermion electric charge (third component of weak isospin), and G_μ is the Fermi coupling constant measured via the muon lifetime. The form factors ρ_f and κ_f have universal contributions, *i.e.* independent of the fermion species, as well as non-universal parts,

$$\rho_f = 1 + \Delta\rho_{\rm univ} + \Delta\rho_{\rm non} , \tag{10}$$

$$\kappa_f = 1 + \Delta\kappa_{\rm univ} + \Delta\kappa_{\rm non} . \tag{11}$$

Excited leptons can affect the physics at the Z pole through their contributions to both universal and non-universal corrections. The universal contributions can be expressed in terms of the unrenormalized vector boson self-energies. Defining the transverse part of vacuum polarization amplitudes between the vector boson $V_1 - V_2$, $\Pi_{\mu\nu}^{V_1V_2}(q^2)$, as

$$\Pi_{\mu\nu}^{V_1V_2}(q^2) \equiv \; g_{\mu\nu}\; \Sigma^{V_1V_2}(q^2)$$

where $V_{1,2} = \gamma$, W, and Z, we can write

$$\begin{aligned}
\Delta\rho_{\rm univ}^{\rm ex}(s) &= -\frac{\Sigma_{\rm ex}^{ZZ}(s) - \Sigma_{\rm ex}^{ZZ}(z)}{s-z} + \frac{\Sigma_{\rm ex}^{ZZ}(z)}{z} - \frac{\Sigma_{\rm ex}^{WW}(0)}{w} + 2\frac{s_W}{c_W}\frac{\Sigma_{\rm ex}^{\gamma Z}(0)}{z} , \\
\Delta\kappa_{\rm univ}^{\rm ex} &= \frac{c_W}{s_W}\frac{\Sigma_{\rm ex}^{\gamma Z}(z)}{z} + \frac{c_W}{s_W}\frac{\Sigma_{\rm ex}^{\gamma Z}(0)}{z} + \frac{c_W^2}{s_W^2}\left[\frac{\Sigma_{\rm ex}^{ZZ}(z)}{z} - \frac{\Sigma_{\rm ex}^{WW}(w)}{w}\right] , \\
\Delta r_{\rm univ}^{\rm ex} &= \Sigma_{\rm ex}^{\prime\gamma\gamma}(0) - \frac{c_W^2}{s_W^2}\left(\frac{\Sigma_{\rm ex}^{ZZ}(z)}{z} - \frac{\Sigma_{\rm ex}^{WW}(w)}{w}\right) + \frac{\Sigma_{\rm ex}^{WW}(0) - \Sigma_{\rm ex}^{WW}(w)}{w} - 2\frac{c_W}{s_W}\frac{\Sigma_{\rm ex}^{\gamma Z}(0)}{z} ,
\end{aligned} \tag{12}$$

where $w(z) = M_{W(Z)}^2$, $s_W(c_W) = \sin(\cos)\theta_W$ and $\Sigma' = d\Sigma/dq^2$.

The diagrams with excited lepton contributions to the self-energies are shown in Fig. 1. The final result for the transverse part of vacuum polarization $\Sigma_{Ff}^{V_1V_2}$ contribution coming from the loop of an excited fermion with mass M and an ordinary massless fermion is

$$
\begin{aligned}
\Sigma^{V_1V_2}_{Ff} = \; & \frac{1}{12\pi^2} C_{V_1Ff} C_{V_2Ff} \left\{ 6q^2\Lambda^2 + q^4 \log\frac{\Lambda^2}{M^2} \right. \\
& -2q^2M^2 - \frac{q^4}{3} + M^2(2M^2-q^2) + (M^2-q^2)(2M^2+q^2) \\
& \left. \times \left[-2 + \left(1 - \frac{M^2}{q^2}\right) \log\left(1 - \frac{q^2}{M^2}\right) \right] \right\}
\end{aligned} \tag{13}
$$

where $V_{1(2)}$ refers to the initial (final) vector boson, and the constants C_{VFf} are defined in (3) for the different vector bosons and fermions.

For the vacuum polarization, $\Sigma^{V_1V_2}_{FF}$, coming from the loop of two excited fermions with mass M, we obtain:

$$
\begin{aligned}
\Sigma^{V_1V_2}_{FF} = \; & \frac{1}{24\pi^2} \frac{q^2}{M^2} \left\{ 6K_{V_1FF}K_{V_2FF}M^2\Lambda^2 + \Big[2A_{V_1FF}A_{V_2FF} \right. \\
& +6(A_{V_1FF}K_{V_2FF} + A_{V_2FF}K_{V_1FF})M + 3K_{V_1FF}K_{V_2FF}\left(\frac{q^2}{3} + 2M^2\right)\Big] M^2 \log\frac{\Lambda^2}{M^2} \\
& +4A_{V_1FF}A_{V_2FF}M^2\left(\frac{1}{3} + \frac{2M^2}{q^2}\right) + 6(A_{V_1FF}K_{V_2FF} + A_{V_2FF}K_{V_1FF})M^3 \\
& +K_{V_1FF}K_{V_2FF}M^2\left(\frac{5q^2}{3} + 4M^2\right) \\
& -2\frac{(4M^2-q^2)^{1/2}}{q} \arctan\left[\frac{q}{(4M^2-q^2)^{1/2}}\right] \Big[2A_{V_1FF}A_{V_2FF}M^2\left(1 + \frac{2M^2}{q^2}\right) \\
& \left. +6(A_{V_1FF}K_{V_2FF} + A_{V_2FF}K_{V_1FF})M^3 + K_{V_1FF}K_{V_2FF}M^2\left(q^2 + 8M^2\right)\Big] \right\}
\end{aligned} \tag{14}
$$

For the purpose of illustration, we derived approximate expressions in the large-M limit. For $R_Q \equiv q^2/M^2 \ll 1$, we obtain

$$
\begin{aligned}
\Sigma^{V_1V_2}_{\rm ex} = \; & \frac{M^2}{12\pi^2} R_Q \left\{ 3\Lambda^2(2C_{V_1Ff}C_{V_2Ff} + K_{V_1FF}K_{V_2FF}) - A_{V_1FF}A_{V_2FF} \right. \\
& -3(A_{V_1FF}K_{V_2FF} + A_{V_2FF}K_{V_1FF})M - 6K_{V_1FF}K_{V_2FF}M^2 - 3C_{V_1Ff}C_{V_2Ff}M^2 \\
& \left. + [A_{V_1FF}A_{V_2FF} + 3(A_{V_1FF}K_{V_2FF} + A_{V_2FF}K_{V_1FF})M + 3K_{V_1FF}K_{V_2FF}M^2] \log\frac{\Lambda^2}{M^2} \right\}
\end{aligned} \tag{15}
$$

We obtain in this approximation the following expressions for the universal corrections,

$$
\begin{aligned}
\Delta\rho(z) = \; & \frac{\alpha}{720 c_W^2 s_W^2 \pi} R_Z \, (c_W^4 + s_W^4) \Big[-24 - 60\,k\,\sqrt{R_L} - 50\,f^2\,R_L - 15\,k^2\,R_L \\
& +60\,f^2\,R_L \log R_L + 15\,k^2\,R_L \log R_L \Big] \\
\Delta\kappa = \; & -\frac{c_W^4}{c_W^4 + s_W^4} \Delta\rho(z) \\
\Delta r = \; & \frac{c_W^2}{c_W^4 + s_W^4} \Delta\rho(z)
\end{aligned} \tag{16}
$$

where $R_Z \equiv M_Z^2/M^2$ and $R_L \equiv M^2/\Lambda^2$. For the sake of simplicity, we have assumed that $f_1 = f_2 = f$ and $k_1 = k_2 = k$.

We should notice that since we are including non-renormalizable dimension six operators the results of the loops are, in principle, quadratically divergent with the scale Λ. However, since we are restricting ourselves to $SU(2) \times U(1)$ gauge invariant operators, the final results for the physical observables are, at most, logarithmically divergent after using the SM counterterms, as can be seen in Eq. (16). Also, it is straightforward to verify that the new physics decouples as the new contributions in Eq. (16) vanish in the limit $R_Z \to 0$ for $R_L \leq 1$.

Corrections to the vertex $Z\bar{f}f$ give rise to non-universal contributions to ρ_f and κ_f. Excited leptons affect these couplings of the Z through the diagrams given in Fig. 2 whose results we parametrize as,

$$-i\frac{e}{2s_W c_W}\left[\gamma_\mu F_{V\text{ex}}^{Zf} - \gamma_\mu\gamma_5 F_{A\text{ex}}^{Zf} - I_3^f\gamma_\mu(1-\gamma_5)\frac{c_W}{s_W}\,\frac{\Sigma_{\text{ex}}^{\gamma Z}(0)}{M_Z^2}\right] , \tag{17}$$

and

$$\Delta\rho_{\text{non}}^{\text{ex}} = \frac{2\,F_{A\text{ex}}^{Zf}(M_Z^2)}{I_3^f} , \tag{18}$$

$$\Delta\kappa_{\text{non}}^{\text{ex}} = -\frac{1}{2s_W^2 Q^f}\left[F_{V\text{ex}}^{Zf} - \frac{I_3^f - 2s_W^2 Q^f}{I_3^f}\; F_{A\text{ex}}^{Zf}(M_Z^2)\right] . \tag{19}$$

There are twelve one-loop Feynman diagrams that involve the contribution of excited fermions to the three-point functions. For each diagram we define $T_i^{V_2}(q^2, M^2, M_V^2)$, $i = 1, \cdots, 12$, where V_2 is the virtual vector boson, with mass M_V, running in the loop. Therefore, we can write the excited lepton contribution to the form factors $F_{V(A)\text{ex}}^{V_1 f}$, for an external vector boson V_1 as,

$$F_{V\text{ex}}^{V_1 f}(q^2) = F_{A\text{ex}}^{V_1 f}(q^2) = \frac{is_W c_W}{e} T_{V_1\to\bar{f}f}(q^2) , \tag{20}$$

with

$$
\begin{aligned}
T_{V_1\to f^+f^-}(q^2) = \; & T_1^{\gamma}(q^2,M^2,0)+T_1^{Z}(q^2,M^2,M_Z^2)+T_1^{W}(q^2,M^2,M_W^2)\\
&+T_2^{\gamma}(q^2,M^2,0)+T_2^{Z}(q^2,M^2,M_Z^2)+T_2^{W}(q^2,M^2,M_W^2)\\
&+T_3^{\gamma}(q^2,M^2,0)+T_3^{Z}(q^2,M^2,M_Z^2)+T_3^{W}(q^2,M^2,M_W^2)\\
&+T_4^{W}(q^2,M^2,M_W^2)\\
&+\frac{1}{2}\left[T_5^{\gamma}(q^2,M^2,0)+T_5^{Z}(q^2,M^2,M_Z^2)+T_5^{W}(q^2,M^2,M_W^2)\right]\\
&+\frac{1}{2}\left[T_6^{\gamma}(q^2,M^2,0)+T_6^{Z}(q^2,M^2,M_Z^2)+T_6^{W}(q^2,M^2,M_W^2)\right]\\
&+T_{11}^{W}(q^2,M^2,M_W^2)+T_{12}^{W}(q^2,M^2,M_W^2)\,.
\end{aligned}
\tag{21}
$$

We have assumed that the ordinary fermions are massless (*i.e.* $m^2 \ll M^2, M_V^2$), and in this limit, $T^{V_2}_{7,8,9,10}(q^2,M^2,M_V^2)=0$. Notice that the external fermions loops (diagrams 5-10 of Fig. 2) only contribute as half, due to the addition of the fermion wave function renormalization counterterms. We also found the relations,

$$
\begin{aligned}
T_2^{V_2}(q^2,M^2,M_V^2) &= T_3^{V_2}(q^2,M^2,M_V^2)\,,\\
T_5^{V_2}(q^2,M^2,M_V^2) &= T_6^{V_2}(q^2,M^2,M_V^2)\,,\\
T_{11}^{V_2}(q^2,M^2,M_V^2) &= T_{12}^{V_2}(q^2,M^2,M_V^2)
\end{aligned}
$$

Our results for $T^{V_2}_{1,2,4,5,11}(q^2,M^2,M_V^2)$, in terms of the Passarino–Veltman scalar one-loop functions, are

$$
\begin{aligned}
T_1^{V_2} = \; & \frac{i}{4\pi^2\,q^2}\,C^2_{V_2Ff}\Bigg\{\Big[A_{V_1FF}\left(2\,M^6-3\,M^4\,M_V{}^2+M_V{}^6+M^2\,M_V{}^2\,q^2+2\,M_V{}^4\,q^2\right)\\
&+K_{V_1FF}\left(2\,M\,M_V{}^4\,q^2-2\,M^3\,M_V{}^2\,q^2\right)\Big]\times\,C_0(0,0,q^2,M^2,M_V^2,M^2)\\
&+A_{V_1FF}\left(-2\,M^4+\,M^2\,M_V{}^2+\,M_V{}^4+\frac{1}{3}\,M^2\,q^2+\frac{2}{9}\,q^4\right)+K_{V_1FF}\left(\,M\,q^4+\,2MM_V^2q^2\right)\\
&-\frac{(4\,M^2-q^2)^{1/2}}{q}\Big[A_{V_1FF}\left(-12M^4+\,6M^2\,M_V^2+\,6\,M_V^4+\,10\,M^2\,Q^2+\,9\,M_V^2\,Q^2-4\,Q^4\right)\\
&+K_{V_1FF}\left(12\,M\,M_V^2\,Q^2-6\,M\,Q^4\right)\Big]\times\arctan\left[\frac{q}{(4\,M^2-q^2)^{1/2}}\right]\\
&+\frac{q^2}{6}\left[A_{V_1FF}\left(18\,M^2+9\,M_V{}^2-4\,q^2\right)-6\,K_{V_1FF}\,M\,q^2\right]\log\frac{\Lambda^2}{M^2}\\
&-\frac{M_V^2}{(M^2-M_V^2)}\left[A_{V_1FF}\left(2\,M^4-M^2\,M_V{}^2-M_V{}^4\right)-2\,K_{V_1FF}\,M\,M_V{}^2\,q^2\right]\log\frac{M^2}{M_V^2}\Bigg\}\\
\simeq \; & \frac{-iM^2}{144\pi^2}\,C^2_{V_2Ff}\Bigg\{A_{V_1FF}(126+117R_V)-R_Q\Big[A_{V_1FF}(64+9R_V)\\
&+K_{V_1FF}\,M(108+18R_V)\Big]+\Big[-A_{V_1FF}\,(108+54R_V)\\
&+R_Q\,(24\,A_{V_1FF}+36\,K_{V_1FF}\,M)\Big]\,\log\frac{\Lambda^2}{M^2}\Bigg\}\,,
\end{aligned}
\tag{22}
$$

where the coupling constants C_{VFf}, A_{VFF}, and K_{VFF} are given by (3), (7), and (8), respectively, and the Passarino–Veltman function $C_0(0,0,q^2,M^2,M_V^2,M^2)$ is given in Appendix A. The approximate expression was obtained for the large-M limit, *i.e.* $R_Q = q^2/M^2 \ll 1$ and $R_V = M_V^2/M^2 \ll 1$.

$$\begin{aligned}
T_2^{V_2} = \; & \frac{-i}{4\pi^2} C_{V_1Ff} C_{V_2Ff} \left(g^a_{V_2} + g^v_{V_2}\right) \Bigg\{ M^2 - 2\,{M_V}^2 - 2\,q^2 - \; q^2 \log\frac{\Lambda^2}{M^2} + 2{M_V}^2 \log\frac{M^2}{M_V^2} \\
& +2{M_V}^2 \left(M^2 - {M_V}^2 - q^2\right) C_0(0,0,q^2,M^2,M_V^2,0) \\
& + \frac{M^2-q^2}{q^2} \left(M^2 - 2\,{M_V}^2 - q^2\right) \log\left(1 - \frac{q^2}{M^2}\right) \Bigg\} \\
\simeq \; & \frac{iM^2}{8\pi^2} C_{V_1Ff} C_{V_2Ff} \left(g^a_{V_2} + g^v_{V_2}\right) R_Q \left(1 + 2R_V \log R_V + 2\log\frac{\Lambda^2}{M^2}\right) \;,
\end{aligned} \tag{23}$$

where g_V^v and g_V^a are the vector and axial coupling of the vector bosons to the usual fermions: for $V=\gamma$, $g_\gamma^v = -e$ and $g_\gamma^a = 0$; for $V=W$, $g_W^v = g_W^a = g/(2\sqrt{2})$; for $V=Z$ and $f=\nu$, $g_Z^v = g_Z^a = g/(4c_W)$; for $V=Z$ and $f=e$, $g_Z^v = g(4s_W^2-1)/(4c_W)$ and $g_Z^a = -g/(4c_W)$.

$$\begin{aligned}
T_4^{V_2} = \; & \frac{i}{144\pi^2\,q^2} C^2_{V_2Ff}\, g_{V_1WW} \Bigg\{ -36\,\Lambda^2\,q^2 + 72\,M^4 - 36\,M^2\,M_V^2 - 36\,M_V^4 - 45\,M^2\,q^2 \\
& +15\,M_V^2\,q^2 + 46\,q^4 + 18\Big(4\,M^6 - 6\,M^4\,M_V^2 + 2\,M_V^6 - M^4\,q^2 + 4\,M^2\,M_V^2\,q^2 \\
& +3\,M_V^4\,q^2 - M^2\,q^4\Big) \times C_0(0,0,q^2,M_V^2,M^2,M_V^2) \\
& -6\frac{(4\,M_V^2 - q^2)^{1/2}}{q} \Big(24\,M^4 - 12\,M^2\,M_V^2 - 12\,M_V^4 - 18\,M^2\,q^2 + 4\,M_V^2\,q^2 \\
& +5\,q^4\Big) \times \arctan\left[\frac{q}{(4\,M_V^2 - q^2)^{1/2}}\right] + 3\,q^2\,(18\,M^2 + 36\,M_V^2 + 5\,q^2) \log\frac{\Lambda^2}{M^2} \\
& + \frac{3}{(M^2 - M_V^2)} \Big(24\,M^6 - 12\,M^4\,M_V^2 - 12\,M^2\,M_V^4 - 18\,M^4\,q^2 - 36\,M_V^4\,q^2 \\
& +5\,M^2\,q^4 - 5\,M_V^2\,q^4\Big) \log\frac{M^2}{M_V^2} \Bigg\} \\
\simeq \; & \frac{iM^2}{288\pi^2} C^2_{V_2Ff}\, g_{V_1WW} \Bigg[-72\frac{\Lambda^2}{M^2} - 18 - 36R_V + R_Q\,(103 + 144R_V + 144R_V \log R_V) \\
& + (108 + 216R_V + 30R_Q) \log\frac{\Lambda^2}{M^2} \Bigg] \;,
\end{aligned} \tag{24}$$

where g_{V_1WW} is the coupling constant of the triple vector boson vertex. For $V_1 = \gamma, Z$ is given by $g_{\gamma WW} = g s_W$ and $g_{ZWW} = g c_W$.

$$
\begin{aligned}
T_5^{V_2} = & \frac{i}{16\left(M^2 - M_V^2\right)\pi^2} C_{V_2Ff}^2 \left(g_{V_1}^a + g_{V_1}^v\right) \Big[14\, M^4 - M^2\, {M_V}^2 - 7\, {M_V}^4 \\
& -6(M^2 - M_V^2)\left(2\, M^2 + {M_V}^2\right) \log \frac{\Lambda^2}{M^2} - 6\frac{M_V^6}{M^2 - M_V^2} \log \frac{M^2}{M_V^2}\Big] \\
\simeq & \frac{iM^2}{16\,\pi^2} C_{V_2Ff}^2 \left(g_{V_1}^a + g_{V_1}^v\right) \left[14 + 13R_V - 6(2+R_V)\log \frac{\Lambda^2}{M^2}\right],
\end{aligned} \tag{25}
$$

and,

$$
\begin{aligned}
T_{11}^{V_2} = & \frac{-i}{32\pi^2} C_{V_2Ff}\, D_{V_1Ff} \Big[4\,\Lambda^2 + 15\, M^2 + 15\, {M_V}^2 - 18\,(M^2 + M_V^2) \log \frac{\Lambda^2}{M^2} \\
& + \frac{18 M_V^4}{M^2 - M_V^2} \log \frac{M^2}{M_V^2}\Big] \\
\simeq & \frac{-iM^2}{32\pi^2} C_{V_2Ff}\, D_{V_1Ff} \left[4\frac{\Lambda^2}{M^2} + 15 + 15R_V - 18(1+R_V)\log \frac{\Lambda^2}{M^2}\right],
\end{aligned} \tag{26}
$$

where D_{VFf} is given in (4).

At first order in R_Q, R_Z and R_W we get the following approximated expression for the form factor $F_{V\mathrm{ex}}^{V_1f}$, which is valid for $V_1 = \gamma, Z$:

$$
\begin{aligned}
F_{V\mathrm{ex}}^{V_1f}(q^2) = & -\frac{M^2 s_W c_W}{288\pi^2 e} R_Q \Big\{128\, A_{V_1F'F'}\, C_{WF'f}^2 + 128\, A_{V_1FF} \left(C_{\gamma Ff}^2 + C_{ZFf}^2\right) \\
& + 72\, C_{V_1F'f'}\, C_{WF'f}\, (g_W^a + g_W^v) + 72\, C_{V_1Ff} \left[C_{\gamma Ff}\,(g_{\gamma f}^a + g_{\gamma f}^v) + C_{ZFf}\,(g_{Zf}^a + g_{Zf}^v)\right] \\
& + 103\, C_{WF'f}^2\, g_{V_1WW} + 216 \left(C_{\gamma Ff}^2\, K_{V_1FF} + C_{ZFf}^2\, K_{V_1FF} + C_{WF'f}^2\, K_{V_1F'F'}\right) M \\
& + \Big[48\, A_{V_1F'F'}\, C_{WF'f}^2 + 48\, A_{V_1FF} \left(C_{\gamma Ff}^2 + C_{ZFf}^2\right) - 144\, C_{V_1F'f'}\, C_{WF'f}\, (g_W^a + g_W^v) \\
& - 144\, C_{V_1Ff} \left[C_{\gamma Ff}\,(g_{\gamma f}^a + g_{\gamma f}^v) + C_{ZFf}(g_{Zf}^a + g_{Zf}^v)\right] - 30\, C_{WF'f}^2\, g_{V_1WW} \\
& + 72 \left(C_{\gamma Ff}^2\, K_{V_1FF} + C_{ZFf}^2\, K_{V_1FF} + C_{WF'f}^2\, K_{V_1F'F'}\right) M\Big] \log R_L \\
& + R_W \Big[18\, A_{V_1F'F'}\, C_{WF'f}^2 + 144\, C_{WF'f}^2\, g_{V_1WW} + 36\, C_{WF'f}^2\, K_{V_1F'F'}\, M \\
& + 144\, C_{WF'f} \left[C_{V_1F'f'}\,(g_W^a + g_W^v) + C_{WF'f}\, g_{V_1WW}\right] \log R_W\Big] \\
& + R_Z \Big[18\, A_{V_1FF}\, C_{ZFf}^2 + 36\, C_{ZFf}^2\, K_{V_1FF}\, M \\
& + 144\, C_{V_1Ff}\, C_{ZFf} \left(g_{Zf}^a + g_{Zf}^v\right) \log R_Z\Big]\Big\}.
\end{aligned} \tag{27}
$$

In order to check the consistency of our calculations, we also analysed the effect of the excited leptons to the $\gamma\bar{f}f$ vertex at zero momentum, which is used as one of the renormalization conditions in the on-shell renormalization scheme. Taking into account the appropriate values for the constants C_{VFf} (3), A_{VFF} (7), and K_{VFF} (8), we verified that the vertex $\gamma\bar{e}e$ cancels at $q^2 = 0$. In the same way, we have also checked that $T_{\gamma\to\bar{\nu}\nu}(q^2 = 0) = 0$ (note that T_4 and T_{11} must change sign for external neutrinos). Therefore our expressions for the new contributions satisfy the appropriate QED Ward identities [17], and leave the fermion electric charge unchanged. In consequence, the vertex corrections are proportional to q^2 as is explicitly shown in the approximate expression (27). Moreover, we also verified that the excited fermions decouple from the vertex correction in the limit of large M.

IV. NUMERICAL RESULTS AND DISCUSSION

The above expressions for the radiative corrections to Z physics due to excited leptons are valid for arbitrary couplings and masses. In order to gain some insight as to which corrections are the most relevant, let us begin our analyses by studying just the oblique corrections, which can also be parametrized in terms of the variables ϵ_1, ϵ_2, and ϵ_3 of Ref. [18]

$$\begin{aligned}
\epsilon_1^{\rm ex} &= \Delta\rho_{\rm univ}^{\rm ex}(z) \\
\epsilon_2^{\rm ex} &= c_W^2\Delta\rho_{\rm univ}^{\rm ex}(z) - 2s_W^2\Delta\kappa_{\rm univ}^{\rm ex} - s_W^2\Delta r_{\rm univ}^{\rm ex} \\
\epsilon_3^{\rm ex} &= c_W^2\Delta\rho_{\rm univ}^{\rm ex}(z) + c_W^2\Delta r_{\rm univ}^{\rm ex} + (c_W^2 - s_W^2)\Delta\kappa_{\rm univ}^{\rm ex}(z)\ .
\end{aligned} \tag{28}$$

Recent global analyses of the LEP, SLD, and low-energy data yield the following values for the oblique parameters [18]

$$\begin{aligned}
\epsilon_1 &= \epsilon_1^{\rm SM} + \epsilon_1^{\rm new} = (5.1 \pm 2.2) \times 10^{-3}\ , \\
\epsilon_2 &= \epsilon_2^{\rm SM} + \epsilon_2^{\rm new} = (-4.1 \pm 4.8) \times 10^{-3}\ , \\
\epsilon_3 &= \epsilon_3^{\rm SM} + \epsilon_3^{\rm new} = (5.1 \pm 2.0) \times 10^{-3}\ ,
\end{aligned} \tag{29}$$

In Table I, we give the attainable values for the new contributions to the ϵ parameters for different values of the excited lepton mass and couplings. As seen from this table, requiring that the new contribution is within the limits allowed by the experimental data (29), we find that the constraints coming from oblique corrections are less restrictive than the available experimental limits. Notice that Λ being the scale of new physics, M must satisfy $M \leq \Lambda$.

As for the vertex corrections, we see in Eq. (20) that the excited leptons alter just the left-handed coupling of the Z. The new contributions to the Z widths, $\Gamma_{ee} \equiv \Gamma(Z \to e^+e^-)$ and $\Gamma_{\rm inv} \equiv 3\ \Gamma(Z \to \bar{\nu}\nu)$, are given by

$$
\begin{aligned}
\Delta\Gamma_{ee} &= \alpha M_Z \frac{(s_W^2 - 1/2)}{3 s_W^2 c_W^2} \times F_V^{Ze}(z) \, , \\
\Delta\Gamma_{\rm inv} &= \alpha M_Z \frac{1}{2 s_W^2 c_W^2} \times F_V^{Z\nu}(z) \, .
\end{aligned}
\tag{30}
$$

The theoretical values for the Z partial width generated by ZFITTER [19], for $m_{\rm top} = 175$ GeV and $M_H = 300$ GeV, are $\Gamma_{ee} = 83.9412$ MeV and $\Gamma_{\rm inv} = 501.482$ MeV. The most recent LEP results [22], assuming lepton universality, are $\Gamma_{\ell\ell}^{\rm LEP}(Z \to \ell^+\ell^-) = 83.93 \pm 0.14$ MeV and for the invisible width $\Gamma_{\rm inv}^{\rm LEP} = 499.9 \pm 2.5$ MeV. Therefore, at 95% C.L., we should have $-0.28 < \Delta\Gamma_{ee} < 0.26$ MeV, and $-6.48 < \Delta\Gamma_{\rm inv} < 3.32$ MeV.

We list in Tables II and III the values of $\Delta\Gamma_{ee}$ and $\Delta\Gamma_{\rm inv}$ attainable in this phenomenological model for some values of the compositeness scale and excited lepton masses, assuming different configurations of the weight factors $f_{1,2}$, and $\kappa_{1,2}$. Our numerical results show that the most restrictive bound on the excited fermion mass and compositeness scale comes from the comparison of $\Delta\Gamma_{ee}$ with the LEP data for this observable.

Let us compare our bounds coming from $\Delta\Gamma_{ee}$ with the ones emerging from the direct search for the excited leptons. First of all, we should point out that the direct search at LEP was just able to reach excited fermion masses up to 130 GeV [11]. On the other hand, the HERA Collaborations [9,10], looking for states that decay into a gauge boson and a usual fermion in the reaction $ep \to f^* X$, can access masses up to 250 GeV.

In Fig. 3, we present the excluded region, at 95% C.L., in the Λ versus M plane imposed by $\Delta\Gamma_{ee}$, for $f_1 = f_2 = \kappa_1 = \kappa_2 = 1$. We have further assumed that $M \leq \Lambda$, leading to the excluded region represented by the shadowed triangle. For comparison, we also present the region excluded by the ZEUS data [10] (below and left of the dashed curve), for $f_1 = f_2 = 1$. Since we have assumed that $BR(e^* \to e\gamma) = 1$, this curve represents an upper limit for the ZEUS bound. As we can see, we were able to exclude just a small region beyond the available limit. We also show our results when we relax the condition of $M \leq \Lambda$. In the latter case, our analysis excludes all excited lepton masses with scales $\Lambda \leq 165$ GeV. It should be realized, however, that relaxing such a condition leads to non-decoupling of the new physics in the large-M limit. Moreover, one can question the use of effective Lagrangians to describe the interactions of particles heavier than the cut-off scale.

In conclusion, we have evaluated the contribution of excited lepton states, up to the one-loop level, to the oblique variables and also to the Z width to leptons. We have compared our results with the precise data on the electroweak observables obtained by the LEP Collaborations in order to extract bounds on some of the free parameters (compositeness scale and excited lepton mass) of the phenomenological model under consideration. We also compared our results with the recent bounds obtained through the direct search for these particles. Our results show that the present precision in the electroweak parameters attained by LEP is very marginally able to constrain the parameters Λ and M beyond the present limits from direct searches.

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APPENDIX A: SCALAR ONE-LOOP INTEGRALS

The relevant Passarino–Veltman functions are [20],

$$
\begin{aligned}
A_0(m_0^2) &= -i(16\pi^2)\mu^{4-D}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2-m_0^2}\ ,\\
B_0(p_1^2,m_0^2,m_1^2) &= -i(16\pi^2)\mu^{4-D}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-m_0^2)[(k+p_1)^2-m_1^2]}\ ,\\
C_0(p_1^2,p_{21}^2,p_2^2,m_0^2,m_1^2,m_2^2) &= -i(16\pi^2)\mu^{4-D}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-m_0^2)[(k+p_1)^2-m_1^2][(k+p_2)^2-m_2^2]}
\end{aligned}
\tag{A1}
$$

where $p_{21}=p_2-p_1$

The scalar function A_0 can be written as [21],

$$
A_0(m_0^2)=m_0^2\left(\Delta-\log\frac{m_0^2}{\mu^2}+1\right)+2\frac{\mu^2}{\pi(D-2)}
\tag{A2}
$$

where we have kept the pole at $D=2$, and

$$
\Delta=\frac{2}{4-D}-\gamma_E+\log 4\pi
\tag{A3}
$$

where γ_E is Euler's constant.

The B_0 and C_0 functions can be written in terms of integrals over Feynman parameters as

$$
B_0(p_1^2,m_0^2,m_1^2)=\Delta-\int_0^1 dx\log\left[\frac{x^2p_1^2-x(p_1^2+m_0^2-m_1^2)+m_0^2-i\epsilon}{\mu^2}\right]
\tag{A4}
$$

and

$$
\begin{aligned}
C_0(p_1^2,p_{21}^2,p_2^2,m_0^2,m_1^2,m_2^2) = -\int_0^1 dx\int_0^x dy\Big[&p_{21}^2x^2+p_1^2y^2+(p_2^2-p_1^2-p_{21}^2)xy\\
&+(m_1^2-m_2^2-p_{21}^2)x+(m_0^2-m_1^2+p_{21}^2-p_2^2)y+m_2^2-i\epsilon\Big]^{-1}
\end{aligned}
\tag{A5}
$$

The function B_0, for some cases of interest, are

$$
\begin{aligned}
B_0(0,0,M^2) &= \Delta+1-\log\left(\frac{M^2}{\mu^2}\right)\ ,\\
B_0(0,M^2,M_V^2) &= \Delta+1-\frac{(M^2+M_V^2)}{2(M^2-M_V^2)}\log\left(\frac{M^2}{M_V^2}\right)-\log\left(\frac{MM_V}{\mu^2}\right)\ ,\\
B_0(q^2,0,M^2) &= \Delta+2-\left(1-\frac{M^2}{q^2}\right)\log\left(1-\frac{q^2}{M^2}\right)-\log\left(\frac{M^2}{\mu^2}\right)\ ,\\
B_0(q^2,M^2,M^2) &= \Delta+2-2\frac{(4M^2-q^2)^{1/2}}{q}\arctan\left[\frac{q}{(4M^2-q^2)^{1/2}}\right]-\log\frac{M^2}{\mu^2}
\end{aligned}
\tag{A6}
$$

In the contribution $T_{11}(q^2, M^2, M_V{}^2)$ (B7), we have the function

$$B_1(0, M^2, M_V{}^2) = -\frac{\Delta}{2} - \frac{1}{4} - \frac{M^2}{2(M^2 - M_V^2)} + \frac{M^4}{2(M^2 - M_V^2)^2} \log\left(\frac{M^2}{M_V^2}\right) + \log\left(\frac{M_V^2}{\mu^2}\right) \tag{A7}$$

The functions C_0, for some cases of interest, are

$$\begin{aligned}
C_0(0,0,0,M^2,0,M^2) = \quad & -\frac{1}{M^2}\ , \\
C_0(0,0,0,M^2,M_V^2,0) = \quad & -\frac{1}{(M^2 - M_V^2)} \log\left(\frac{M^2}{M_V^2}\right)\ , \\
C_0(0,0,0,M^2,M_V^2,M^2) = \quad & -\frac{1}{(M^2 - M_V^2)^2} \left\{ M^2 - M_V^2 \left[1 + \log\left(\frac{M^2}{M_V^2}\right)\right]\right\}\ , \\
C_0(0,0,q^2,M^2,M_V^2,0) = \quad & \frac{1}{q^2}\left[\log\left(\frac{M^2 - q^2}{M_V^2}\right)\log\left(\frac{M^2-q^2}{M_V^2} - 1\right)\right. \\
& -\log\left(\frac{M^2}{M_V^2} - 1\right)\log\left(\frac{M^2}{M_V^2}\right) \\
& \left. + i\pi\log\left(1 - \frac{q^2}{M^2}\right) - \mathrm{Li}_2\left(\frac{M^2}{M_V^2}\right) + \mathrm{Li}_2\left(\frac{M^2-q^2}{M_V^2}\right)\right]\ , \\
C_0(0,0,q^2,M^2,M_V^2,M^2) = \quad & \frac{1}{q^2}\left\{-2\pi\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2(M^2-M_V^2)-q^2}\right]\right. \\
& + 4\arctan\left[\frac{(4M^2-q^2)^{1/2}}{q}\right]\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2(M^2-M_V^2)-q^2}\right] \\
& - \log\left(\frac{M^2}{M_V^2}\right)\log\left[\frac{(M^2-M_V^2)^2}{(M^2-M_V^2)^2 + M_V^2 q^2}\right] \\
& - \mathrm{Li}_2\left(\frac{M^2 q^2}{(M^2-M_V^2)^2 + M_V^2 q^2}\right) + \mathrm{Li}_2\left(\frac{M_V^2 q^2}{(M^2-M_V^2)^2 + M_V^2 q^2}\right) \\
& + \mathrm{Li}_2\left(\frac{\xi^*}{2(M^2-M_V^2)-\xi}\right) - \mathrm{Li}_2\left(\frac{-\xi^*}{2(M^2-M_V^2)-\xi^*}\right) \\
& \left. + \mathrm{Li}_2\left(\frac{\xi}{2(M^2-M_V^2)-\xi^*}\right) - \mathrm{Li}_2\left(\frac{-\xi}{2(M^2-M_V^2)-\xi}\right)\right\}\ , \\
C_0(0,0,q^2,M^2,0,M^2) = \quad & \frac{1}{q^2}\left\{-2\pi\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2M^2-q^2}\right] - \mathrm{Li}_2\left(\frac{M^2 q^2}{M^4}\right)\right. \\
& + 4\arctan\left[\frac{(4M^2-q^2)^{1/2}}{q}\right]\arctan\left[\frac{q(4M^2-q^2)^{1/2}}{2M^2-q^2}\right] \\
& + \mathrm{Li}_2\left(\frac{\xi^*}{2M^2-\xi}\right) - \mathrm{Li}_2\left(\frac{-\xi^*}{2M^2-\xi^*}\right) + \mathrm{Li}_2\left(\frac{\xi}{2M^2-\xi^*}\right) \\
& \left. - \mathrm{Li}_2\left(\frac{-\xi}{2M^2-\xi}\right)\right\}\ ,
\end{aligned} \tag{A8}$$

where $\xi = q^2 + iq(4M^2 - q^2)^{1/2}$, and $\mathrm{Li}_2(x)$ is the dilogarithm or Spencer function, defined as

$$\mathrm{Li}_2(x) = -\int_0^1 \frac{dt}{t}\log(1-xt)\ ,$$

APPENDIX B: RESULTS IN D DIMENSIONS FOR TWO- AND THREE-POINT FUNCTIONS

The result in D dimensions for the transverse part of vacuum polarization $\Sigma_{Ff}^{V_1V_2}$ contribution coming from the loop of an excited fermion with mass M and a usual fermion with mass m (see Fig. 1) is

$$\begin{aligned}\Sigma_{Ff}^{V_1V_2} = & -\frac{1}{4(D-1)\pi^2}C_{V_1Ff}C_{V_2Ff}\Big\{2q^2A_0(m^2)\\ & +(M^2-m^2)\left[q^2-(D-2)(M^2-m^2)\right]B_0(0,m^2,M^2)\\ & +\Big[(D-2)(M^2-m^2)^2-(D-3)q^2(M^2+m^2)-q^4\Big]B_0(q^2,m^2,M^2)\Big\}\end{aligned} \tag{B1}$$

The definition of the A_0 and B_0 functions is given in the Appendix A.

For the vacuum polarization ($\Sigma_{FF}^{V_1V_2}$) contribution coming from the loop of two excited fermions with mass M, we obtain

$$\begin{aligned}\Sigma_{FF}^{V_1V_2} = & -\frac{1}{8(D-1)\pi^2}\Big\{2\left[(D-2)A_{V_1FF}A_{V_2FF}+K_{V_1FF}K_{V_2FF}q^2\right]A_0(M^2)\\ & -\Big[A_{V_1FF}A_{V_2FF}[4M^2+(D-2)q^2]+2(D-1)q^2M(A_{V_1FF}K_{V_2FF}+A_{V_2FF}K_{V_1FF})\\ & +K_{V_1FF}K_{V_2FF}q^2[q^2+4(D-2)M^2]\Big]B_0(q^2,M^2,M^2)\Big\}\end{aligned} \tag{B2}$$

For the three-point functions in D dimensions the results read:,

$$
\begin{aligned}
T_1^{V_2} = \; & \frac{i}{4\,(1-D)\,\pi^2}\, A_{V_1FF}\, C^2_{V_2Ff}\,(2-D)^2\, A_0(M^2) \\
& -\frac{i}{4\,(2-D)\;\pi^2\, q^2}\, C^2_{V_2Ff}\,\Big[A_{V_1FF}\,\Big(4\,M^4 - 2\,D\,M^4 - 6\,M^2\,{M_V}^2 + 2\,D\,M^2\,{M_V}^2 \\
& +2\,{M_V}^4 - 16\,M^2\,q^2 + 8\,D\,M^2\,q^2 - D^2\,M^2\,q^2 + 4\,D\,{M_V}^2\,q^2 - D^2\,{M_V}^2\,q^2\Big) \\
& -K_{V_1FF}\,\Big(32\,M^3\,q^2 - 16\,D\,M^3\,q^2 + 2\,D^2\,M^3\,q^2 - 12\,M\,{M_V}^2\,q^2 + 2\,D\,M\,{M_V}^2\,q^2\Big)\Big] \\
& \times B_0(0, M^2, {M_V}^2) \\
& +\frac{i}{8\,(1-D)\;(2-D)\;\pi^2\, q^2}\, C^2_{V_2Ff}\,\Big[A_{V_1FF}\,\Big(8\,M^4 - 12\,D\,M^4 + 4\,D^2\,M^4 - 12\,M^2\,{M_V}^2 \\
& +16\,D\,M^2\,{M_V}^2 - 4\,D^2\,M^2\,{M_V}^2 + 4\,{M_V}^4 - 4\,D\,{M_V}^4 - 28\,M^2\,q^2 + 50\,D\,M^2\,q^2 \\
& -20\,D^2\,M^2\,q^2 + 2\,D^3\,M^2\,q^2 + 14\,{M_V}^2\,q^2 - 16\,D\,{M_V}^2\,q^2 + 2\,D^2\,{M_V}^2\,q^2 + 8\,q^4 \\
& -12\,D\,q^4 + 3\,D^2\,q^4\Big) + K_{V_1FF}\,\Big(96\,D\,M^3\,q^2 - 64\,M^3\,q^2 - 36\,D^2\,M^3\,q^2 + 4\,D^3 M^3\,q^2 \\
& +24\,M\,{M_V}^2\,q^2 - 28\,D\,M\,{M_V}^2\,q^2 + 4\,D^2\,M\,{M_V}^2\,q^2 + 20\,M\,q^4 \\
& -26\,D\,M\,q^4 + 6\,D^2\,M\,q^4\Big)\Big] \times\; B_0(q^2, M^2, M^2) \\
& -\frac{i}{4\,(2-D)\;\pi^2\, q^2}\, C^2_{V_2Ff}\,\Big[A_{V_1FF}\,\Big(4\,M^6 - 2\,D\,M^6 - 10\,M^4\,{M_V}^2 + 4\,D\,M^4\,{M_V}^2 \\
& +8\,M^2\,{M_V}^4 - 2\,D\,M^2\,{M_V}^4 - 2\,{M_V}^6 - 16\,M^4\,q^2 + 8\,D\,M^4\,q^2 - D^2\,M^4\,q^2 \\
& +26\,M^2\,{M_V}^2\,q^2 - 11\,D\,M^2\,{M_V}^2\,q^2 + D^2\,M^2\,{M_V}^2\,q^2 \\
& -8\,{M_V}^4\,q^2 + D\,{M_V}^4\,q^2 - 8\,{M_V}^2\,q^4 + 2\,D\,{M_V}^2\,q^4\Big) \\
& +K_{V_1FF}\,\Big(44\,M^3\,{M_V}^2\,q^2 - 32\,M^5\,q^2 + 16\,D\,M^5\,q^2 - 2\,D^2\,M^5\,q^2 - 18\,D\,M^3\,{M_V}^2\,q^2 \\
& +2\,D^2\,M^3\,{M_V}^2\,q^2 - 12\,M\,{M_V}^4\,q^2 + 2\,D\,K_{V_1FF}\,M\,{M_V}^4\,q^2 \\
& -16\,M\,{M_V}^2\,q^4 + 4\,D\,M\,{M_V}^2\,q^4\Big)\Big] \times\; C_0(0, 0, q^2, M^2, {M_V}^2, M^2)\;,
\end{aligned}
\tag{B3}
$$

$$
\begin{aligned}
T_2^{V_2} = \; & \frac{-i}{4\pi^2}\, C_{V_1Ff}\, C_{V_2Ff}\, (4-D)\, \left(g^a_{V_2}+g^v_{V_2}\right)\, M^2\, B_0(0,0,M^2) \\
& +\frac{i}{4\,(2-D)\,\pi^2} C_{V_1Ff}\, C_{V_2Ff}\, (10-6D+D^2)\, \left(g^a_{V_2}+g^v_{V_2}\right)\, {M_V}^2\, B_0(0,0,{M_V}^2) \\
& -\frac{i}{4\,(2-D)\,\pi^2}\, C_{V_1Ff}\, C_{V_2Ff}\, \left(g^a_{V_2}+g^v_{V_2}\right)\, \Big(2\,M^2+6\,{M_V}^2 \\
& -6\,D\,{M_V}^2+D^2\,{M_V}^2\Big)\times\; B_0(0,M^2,{M_V}^2) \\
& +\frac{i}{4\,(2-D)\,\pi^2}\, C_{V_1Ff}\, C_{V_2Ff}\, \left(g^a_{V_2}+g^v_{V_2}\right)\, \Big(10\,M^2-6\,D\,M^2+D^2\,M^2 \\
& -4\,{M_V}^2-2\,q^2\Big)\times\; B_0(q^2,0,M^2) \\
& +\frac{i}{4\,(2-D)\,\pi^2}\, C_{V_1Ff}\, C_{V_2Ff}\, \left(g^a_{V_2}+g^v_{V_2}\right)\, {M_V}^2\, \Big(12\,M^2-6\,D\,M^2+D^2\,M^2 \\
& -4\,{M_V}^2-4\,q^2\Big)\times\; C_0(0,0,q^2,M^2,{M_V}^2,0)\;,
\end{aligned}
\tag{B4}
$$

$$
\begin{aligned}
T_4^{V_2} = \; & \frac{i}{4\pi^2}\, C^2_{V_2Ff}\, g_{V_1WW}\, A_0({M_V}^2) \\
& -\frac{i}{4\,(1-D)\,(2-D)\,\pi^2\,q^2}\, C^2_{V_2Ff}\, g_{V_1WW}\, \Big(4\,M^4-6\,D\,M^4+2\,D^2\,M^4-6\,M^2\,{M_V}^2 \\
& +8\,D\,M^2\,{M_V}^2-2\,D^2\,M^2\,{M_V}^2+2\,{M_V}^4-2\,D\,{M_V}^4-7\,M^2\,q^2+13\,D\,M^2\,q^2 \\
& -4\,D^2\,M^2\,q^2+3\,{M_V}^2\,q^2-3\,D\,{M_V}^2\,q^2\Big) \\
& \times B_0(0,M^2,{M_V}^2) \\
& +\frac{i}{8\,(1-D)\,(2-D)\,\pi^2\,q^2}\, C^2_{V_2Ff}\, g_{V_1WW}\, \Big(8\,M^4-12\,D\,M^4+4\,D^2\,M^4-12\,M^2\,{M_V}^2 \\
& +16\,D\,M^2\,{M_V}^2-4\,D^2\,M^2\,{M_V}^2+4\,{M_V}^4-4\,D\,{M_V}^4-18\,M^2\,q^2+24\,D\,M^2\,q^2 \\
& -6\,D^2\,M^2\,q^2+20\,{M_V}^2\,q^2-20\,D\,{M_V}^2\,q^2+4\,D^2\,{M_V}^2\,q^2-7\,q^4+7\,D\,q^4-D^2\,q^4\Big) \\
& \times B_0(q^2,{M_V}^2,{M_V}^2) \\
& -\frac{i}{4\,(1-D)\,\pi^2}\, C^2_{V_2Ff}\, g_{V_1WW}\, \left(M^2-{M_V}^2\right)\, B_1(0,M^2,{M_V}^2) \\
& +\frac{i}{4\,(2-D)\,\pi^2\,q^2}\, C^2_{V_2Ff}\, g_{V_1WW}\, \Big(4\,M^6-2\,D\,M^6-10\,M^4\,{M_V}^2+4\,D\,M^4\,{M_V}^2 \\
& +8\,M^2\,{M_V}^4-2\,D\,M^2\,{M_V}^4-2\,{M_V}^6-7\,M^4\,q^2+2\,D\,M^4\,q^2+12\,M^2\,{M_V}^2\,q^2 \\
& -4\,D\,M^2\,{M_V}^2\,q^2-3\,{M_V}^4\,q^2-7\,M^2\,q^4+2\,D\,M^2\,q^4\Big) \\
& \times C_0(0,0,q^2,{M_V}^2,M^2,{M_V}^2)\;,
\end{aligned}
\tag{B5}
$$

$$
\begin{aligned}
T_5^{V_2} = \; & \frac{i}{4\pi^2} C_{V_2Ff}^2 \, (2-D) \left(g_{V_1}^a + g_{V_1}^v\right) \, A_0({M_V}^2) \\
& + \frac{i}{8m^2\,\pi^2} C_{V_2Ff}^2 \, \left(g_{V_1}^a + g_{V_1}^v\right) \left(M^2 - {M_V}^2\right) \left(2\,m^2 - D\,m^2 + 2\,M^2 - D\,M^2 - {M_V}^2\right) \\
& \times B_0(0, M^2, {M_V}^2) \\
& - \frac{i}{8m^2\,\pi^2} C_{V_2Ff}^2 \, \left(g_{V_1}^a + g_{V_1}^v\right) \left(2\,m^4 - D\,m^4 - 4\,m^2\,M^2 + 2\,D\,m^2\,M^2 + 2\,M^4\right. \\
& \left. - D\,M^4 - 3\,m^2\,{M_V}^2 + D\,m^2\,{M_V}^2 - 3\,M^2\,{M_V}^2 + D\,M^2\,{M_V}^2 + {M_V}^4\right) \\
& \times B_0(m^2, M^2, {M_V}^2) \, ,
\end{aligned} \tag{B6}
$$

Note that we have kept here the light fermion mass m. The limit of $m \to 0$ is made at the end.

$$
\begin{aligned}
T_{11}^{V_2} = \; & \frac{i}{8\,(1-D)\,\pi^2} C_{V_2Ff} \, (5 - 6D + 2D^2) \, D_{V_1Ff} \, A_0({M_V}^2) \\
& + \frac{i}{4\,(1-D)\,\pi^2} C_{V_2Ff} \, (3 - 3\,D + D^2) \, D_{V_1Ff} \, M^2 \, B_0(0, M^2, {M_V}^2) \\
& + \frac{i}{8\,(1-D)\,\pi^2} C_{V_2Ff} \, D_{V_1Ff} \left(M^2 - {M_V}^2\right) B_1(0, M^2, {M_V}^2) \, .
\end{aligned} \tag{B7}
$$

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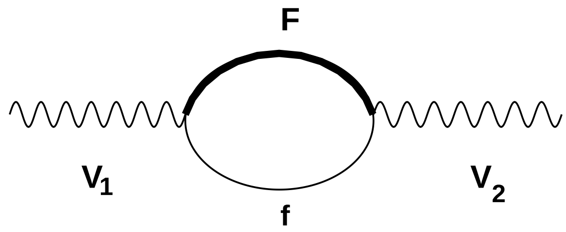
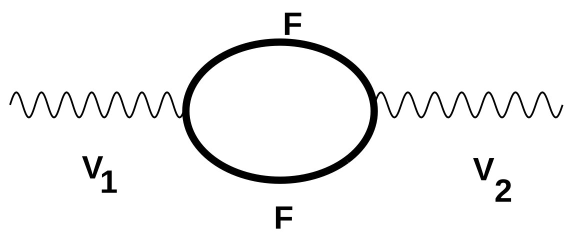
FIGURES



(1)



(2)

FIG. 1. Feynman diagrams leading to contribution of the excited leptons to the two-point functions

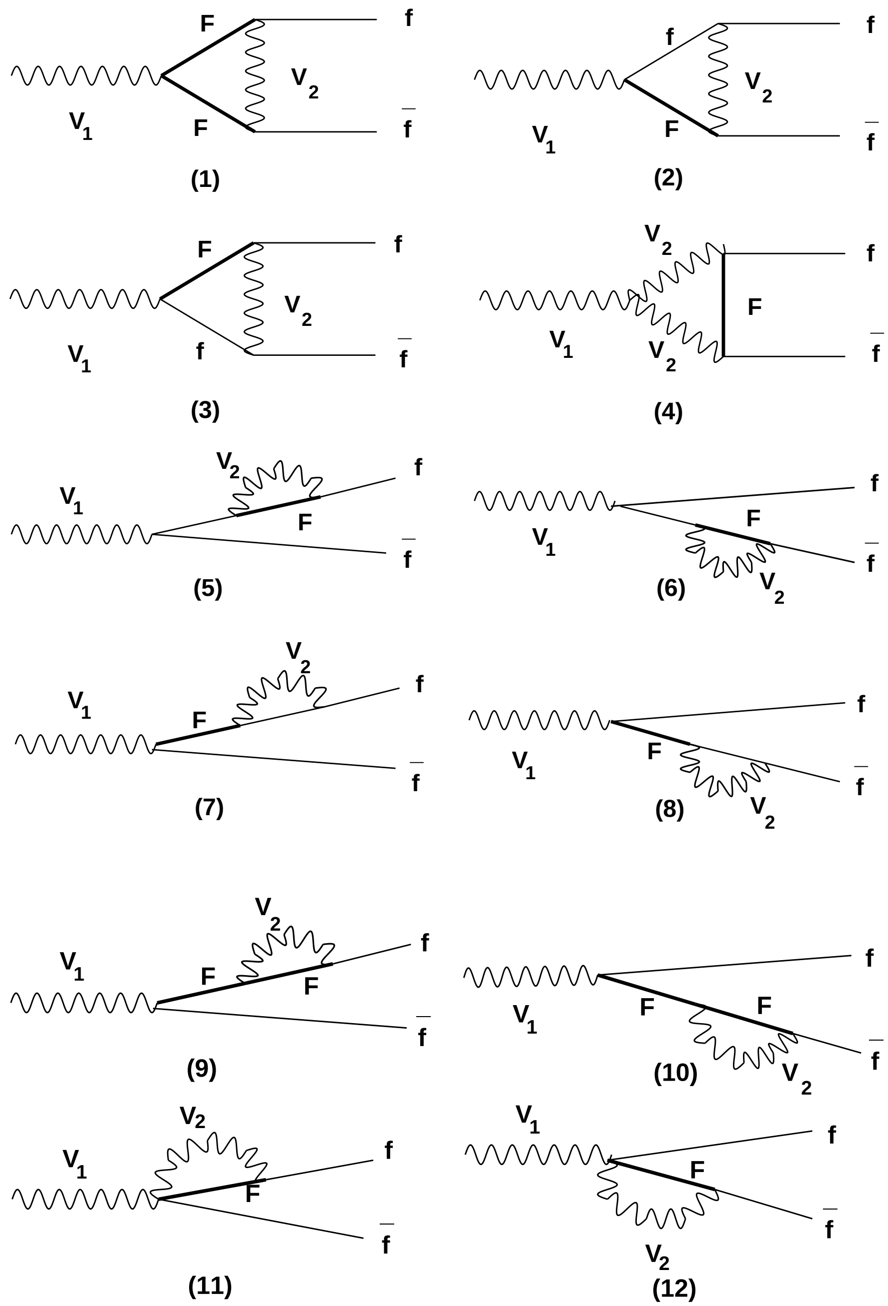


FIG. 2. The contribution of the excited leptons to the three-point functions

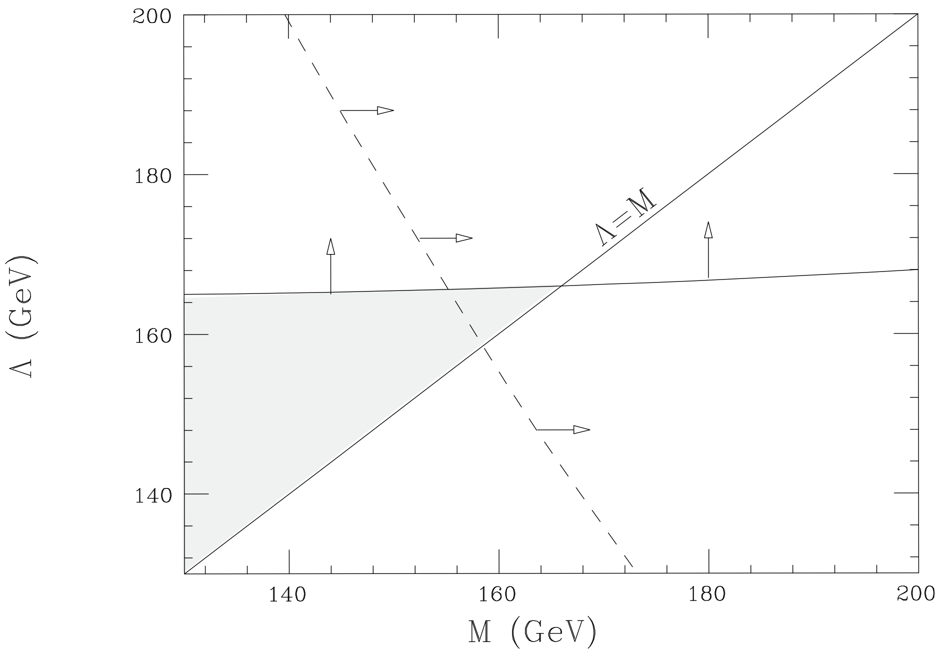


FIG. 3. Excluded regions in the Λ versus M plane from the bounds on $\Delta\Gamma_{ee}$ (shadowed area), and from ZEUS data (below and left of the dashed curve), at 95% C.L.

M (GeV)	Λ (GeV)			
	300	500	1000	2000
	$(f_1, f_2, \kappa_1, \kappa_2) = (1, 1, 1, 1)$			
300	0.18 , 0.17 , 0.21	0.13 , 0.12 , 0.16	0.076 , 0.069 , 0.09	0.048 , 0.044 , 0.057
500	—	0.063 , 0.058 , 0.075	0.04 , 0.037 , 0.048	0.023 , 0.021 , 0.028
1000	—	—	0.016 , 0.014 , 0.019	0.01 , 0.0092 , 0.012
2000	—	—	—	0.0039 , 0.0035 , 0.0046
	$(f_1, f_2, \kappa_1, \kappa_2) = (1, -1, 1, -1)$			
300	0.052 , 0.037 , 0.085	0.055 , 0.044 , 0.079	0.037 , 0.031 , 0.051	0.029 , 0.025 , 0.038
500	—	0.017 , 0.012 , 0.029	0.017 , 0.014 , 0.025	0.012 , 0.01 , 0.016
1000	—	—	0.0041 , 0.0028 , 0.0071	0.0043 , 0.0035 , 0.0062
2000	—	—	—	0.001 , 0.000069 , 0.0018
	$(f_1, f_2, \kappa_1, \kappa_2) = (1, 0, 1, 0)$			
300	0.041 , 0.026 , 0.075	0.037 , 0.026 , 0.062	0.032 , 0.026 , 0.047	0.03 , 0.026 , 0.039
500	—	0.014 , 0.0092 , 0.027	0.013 , 0.0092 , 0.02	0.011 , 0.0092 , 0.016
1000	—	—	0.0036 , 0.0023 , 0.0066	0.0031 , 0.0023 , 0.0051
2000	—	—	—	0.00089 , 0.00057 , 0.0016
	$(f_1, f_2, \kappa_1, \kappa_2) = (0, 1, 0, 1)$			
300	0.17 , 0.17 , 0.17	0.12 , 0.12 , 0.13	0.072 , 0.069 , 0.077	0.046 , 0.044 , 0.052
500	—	0.059 , 0.058 , 0.06	0.038 , 0.037 , 0.04	0.022 , 0.021 , 0.024
1000	—	—	0.014 , 0.014 , 0.015	0.0094 , 0.0092 , 0.0099
2000	—	—	—	0.0036 , 0.0035 , 0.0037

TABLE I. $-\epsilon^{ex}_{1,2,3} \times 10^3$ for $M, \Lambda = 300, 500, 1000, 2000$ GeV and for different configurations of the weight factors

M (GeV)	Λ (GeV)			
	300	500	1000	2000
	$(f_1, f_2, \kappa_1, \kappa_2) = (1,1,1,1)$			
300	0.096	0.042	0.014	0.0045
500	—	0.037	0.012	0.0039
1000	—	—	0.0097	0.0031
2000	—	—	—	0.0025
	$(f_1, f_2, \kappa_1, \kappa_2) = (1,-1,1,-1)$			
300	0.074	0.030	0.0091	0.0027
500	—	0.028	0.0084	0.0025
1000	—	—	0.0073	0.0022
2000	—	—	—	0.0018
	$(f_1, f_2, \kappa_1, \kappa_2) = (1,0,1,0)$			
300	−0.0010	0.00038	0.00040	0.00018
500	—	−0.00043	0.00016	0.00012
1000	—	—	−0.00011	0.000038
2000	—	—	—	−0.000029
	$(f_1, f_2, \kappa_1, \kappa_2) = (0,1,0,1)$			
300	0.087	0.036	0.011	0.0034
500	—	0.033	0.010	0.0031
1000	—	—	0.0087	0.0026
2000	—	—	—	0.0022

TABLE II. $\Delta\Gamma(Z \to e^+e^-)$ in MeV, for $M, \Lambda = 300, 500, 1000, 2000$ GeV and for different configurations of the weight factors

M (GeV)	Λ (GeV)			
	300	500	1000	2000
	$(f_1, f_2, \kappa_1, \kappa_2) = (1, 1, 1, 1)$			
300	0.17	0.090	0.031	0.0096
500	—	0.069	0.026	0.0086
1000	—	—	0.018	0.0068
2000	—	—	—	0.0047
	$(f_1, f_2, \kappa_1, \kappa_2) = (1, -1, 1, -1)$			
300	0.15	0.19	0.086	0.031
500	—	0.067	0.061	0.025
1000	—	—	0.019	0.016
2000	—	—	—	0.0050
	$(f_1, f_2, \kappa_1, \kappa_2) = (1, 0, 1, 0)$			
300	-0.043	-0.013	-0.0025	-0.00049
500	—	-0.016	-0.0030	-0.00060
1000	—	—	-0.0040	-0.00077
2000	—	—	—	-0.0010
	$(f_1, f_2, \kappa_1, \kappa_2) = (0, 1, 0, 1)$			
300	0.34	0.17	0.062	0.021
500	—	0.13	0.050	0.017
1000	—	—	0.035	0.013
2000	—	—	—	0.0089

TABLE III. $\Delta\Gamma(Z \to \text{invisible})$ in MeV, for $M, \Lambda = 300, 500, 1000, 2000$ GeV and for different configurations of the weight factors