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OHSTPY-HEP-T-96-014

A Global χ^2 Analysis of Electroweak Data (including Fermion Masses and Mixing Angles) in SO(10) SUSY GUTs

Tomáš Blažek,[a] Marcela Carena,[b] Stuart Raby,[a*] and Carlos Wagner[b]

[a] Department of Physics, The Ohio State University
174 W. 18th Ave., Columbus, OH 43210

[b]Theory Division, CERN
Geneva, Switzerland

arXiv:hep-ph/9608273v1 8 Aug 1996

We present a global χ^2 analysis of electroweak data, including fermion masses and mixing angles, in SO(10) SUSY GUTs. Just as precision electroweak data is used to test the Standard Model, the well determined Standard Model parameters are the precision electroweak data for testing theories beyond the Standard Model. In this talk we use the latest experimentally measured values for these parameters. We study several models discussed in the literature. One of these models provides an excellent fit to the low energy data with $\chi^2 \sim 1$ for 3 degrees of freedom. We present our predictions for a few selected points in parameter space.

1. Introduction

There are many theories of fermion masses and they are all different. Some of these theories assume a grand unified symmetry, while others only require the Standard Model gauge symmetry. Some contain low energy supersymmetry, while others do not and some include U(1) family symmetries to restrict Yukawa matrices; others contain non-abelian family symmetries. Many theories of fermion masses make predictions only for a restricted subset of low energy observables. For example, some test bottom-tau unification, the Georgi-Jarlskog relation $\frac{m_s}{m_d} = \frac{1}{9}\frac{m_\mu}{m_e}$, or the Harvey-Ramond-Reiss relation $V_{cb} = \sqrt{\frac{m_c}{m_t}}$. One thing is certain, all of these theories **cannot** describe Nature. Nevertheless, many are consistent with the low energy data to within an *order of magnitude*.

There are many who believe that, with our present knowledge of field theory, this is the best we can do. We think not. We are encouraged in our belief by the following guiding principle: that the simple observed pattern of fermion masses and mixing angles is not due to some *random* dynamics at an effective cut-off scale M but is instead evidence that a small set of fermion mass operators are dominant. This leads to the fundamental hypothesis of our work –

1. that a few effective operators at an effective cut-off scale M (where $M = M_{Planck}$ or M_{string}) (or at a GUT scale M_G) dominate the quantitative behavior of fermion masses and mixing angles; and

2. that a more fundamental theory, incorporating Planck or stringy dynamics, will generate an effective field theory below M including these dominant operators.

With this hypothesis in mind, we are encouraged to find this effective field theory. However, in order to make progress, it is clear that *order of magnitude* comparisons with the data are insufficient. Moreover, since the predictions of any simple theory are correlated, a *significant test* of any theory requires a *global fit to all the low energy data*. This work makes the first attempt to bring the tests of theories of fermion masses into conformity with the accuracy of the low energy

*Talk presented by S. Raby at the Fourth International Conference on Supersymmetry (SUSY96), College Park, MD, May 29,1996.

data.

This analysis would not have been feasible ten years ago. The top quark mass and the CKM angle V_{ub} had not been measured; while V_{cb} and the weak mixing angle $sin^2\theta_W$ were measured, but with large error bars. Today, as a result of both experimental and theoretical progress, all but one of the 18 parameters of the Standard Model are known to much better accuracy [the Higgs mass has yet to be measured]. These parameters are the precision electroweak data for testing theories of fermion masses.

In this talk we present a global χ^2 analysis of precision electroweak data, *including fermion masses and mixing angles*, within the context of several theories of fermion masses based on SO(10) SUSY GUTs. Of course, the analysis we describe can be applied to any *predictive* theory.

Why look at SO(10) SUSY GUTs?

- We use SUSY GUTs since they give the simplest explanation for the experimental observation that the three gauge couplings appear to meet at a scale of order 10^{16} GeV[1]. Moreover, *the grand unified symmetry gaurantees that this result is insensitive to radiative corrections.*

- We use SO(10) since it provides the simplest explanation for the observed family structure of the light fermions[2].

2. Low energy observables – experimental values

Our χ^2 function includes 20 low energy observables. In addition we have incorporated the experimental bounds on sparticle masses into the code as a penalty in χ^2; added if one of these bounds is violated. This gaurantees that we remain in the experimentally allowed regions of parameter space. Let us now discuss the experimental observables and their errors.

We have

- 6 parameters associated with the Standard Model gauge and electroweak symmetry breaking sectors — α_{EM}, $\sin^2\theta_W(M_Z)$, M_W, M_Z, ρ_{new}, and $\alpha_s(M_Z)$;

- 13 parameters associated with fermion masses and mixing angles — M_t, $m_b(m_b)$, $(M_b - M_c)$, m_s, m_d/m_s, Q, M_τ, M_μ, M_e, V_{us}, V_{cb}, V_{ub}/V_{cb}, and J; and

- the branching ratio for $b \rightarrow s\gamma$

where ρ_{new} includes the added contribution to the electroweak ρ parameter from physics beyond the SM, Q is the Kaplan-Manohar-Leutwyler ellipse parameter[3] relating u, d, and s quark masses and J is the Jarlskog CP violating invariant[4].

The experimental values for each observable are given in table 1 with the associated experimental or theoretical error σ. *Note, σ is taken to be either the experimental error or 1/2%, whichever is larger.* This is because our numerical calculations contain theoretical uncertainties from round-off errors alone which are of order 1/2%.[2] Thus the listed errors for M_Z, M_W, α_{EM}, M_τ, M_μ, M_e are dominated by our theoretical uncertainties. Note, also, mass parameters denoted with a capital letter M are defined as pole masses, while m_b, m_s, m_d, m_u are defined as the $\overline{MS}$ running masses at the scale m_b for the bottom quark or 1 GeV for the three light quarks.

The Standard Model values for $\sin^2\theta_W(M_Z)$ or J are not in table 1. Instead of $\sin^2\theta_W(M_Z)$ we use the Fermi constant G_μ, while instead of J we use the Bag constant $\hat{B}_K$.

The Fermi constant receives small corrections from SUSY, while $\sin^2\theta_W(M_Z)$, on the other hand, is more sensitive to the SUSY spectrum[5]. As a consequence, the value of $\sin^2\theta_W$ varies by a percent or more in different regions of soft SUSY breaking parameter space. In our calculations we include only the leading logarithmic corrections when evaluating $\sin^2\theta_W(M_Z)$ and we neglect the SUSY box and vertex corrections to G_μ.

Similarly, we test the CP violating parameter J by comparing to the experimental value of $\epsilon_K = (2.26 \pm 0.02) \times 10^{-3}$. The largest theoretical uncertainty however comes in the value of the QCD bag constant $\hat{B}_K$. We thus give the value for $\hat{B}_K$ as obtained from recent lattice calculations[6]. We then compare this to a theoretical

[2] For $\sin^2\theta_W$ we take σ to be 1%. This is to account for the additional error resulting from our neglecting SUSY box and vertex corrections.

value of $\hat{B}_K$ defined as that value needed to agree with ϵ_K for a given set of fermion masses and mixing angles, assuming only the SM box diagrams.

The experimental value for ρ_{new} is obtained from Langacker's combined fits to the precision electroweak data, presented at SUSY96[7].

For fermion masses and mixing angles we use those combinations of parameters which are known to have the least theoretical and/or experimental uncertainties. For example, while the bottom and top quark masses are known reasonably well, the charm quark mass is not known as accurately. On the other hand, heavy quark effective theory relates the mass difference $M_b - M_c$ between the bottom and charm quark *pole masses* to 5% accuracy[8]. We thus use this relation, instead of the charm quark mass itself, to test the theory. M_b and M_c are calculated from the $\overline{MS}$ running masses, $m_b(m_b)$ and $m_c(m_c)$ using two loop QCD threshold corrections.

Similarly, among the three light quarks there is one good relation which severely constrains any theory. This is the Kaplan-Manohar-Leutwyler ellipse given by

$$1 = \frac{1}{Q^2}\frac{m_s^2}{m_d^2} + \frac{m_u^2}{m_d^2} \qquad (1)$$

or

$$Q = \frac{\frac{m_s}{m_d}}{\sqrt{1 - \frac{m_u^2}{m_d^2}}} \qquad (2)$$

where Q is the ellipse parameter. The experimental value for Q is obtained from a weighted average of lattice results and chiral Lagrangian analysis, with important contributions from the violation of Dashen's theorem[9]. Note that Q^2 is free of $O(m_q)$ corrections, but m_d/m_s is not. Hence σ is so much smaller for Q^2 than for m_d/m_s.

The remaining parameters are more or less self evident. We just remark that the central value for V_{cb}, as well as the error bars, has steadily decreased in the last 5 years, making it a very significant constraint. In addition, the value for V_{ub}/V_{cb} changed dramatically in 1992. It changed from approximately 0.15 ± 0.05 to its present value 0.08 ± 0.02, where the errors were and continue to be dominated by theoretical model dependence. Clearly the systematic uncertainties were large but are now *hopefully* under control.

3. Low energy observables – computed values

In our analysis we consider the minimal supersymmetric standard model defined at a GUT scale with, in all cases (but one), tree level GUT boundary conditions on gauge couplings and Yukawa matrices. In this one case, we include an arbitrary parameter, ϵ_3, which parametrizes the one loop threshold correction to gauge coupling unification.[3] We also include 7 soft SUSY breaking parameters — an overall scalar mass m_0 for squarks and sleptons, a common gaugino mass $M_{1/2}$, and the parameters A_0, B and μ. In addition we have allowed for non-universal Higgs masses, m_{H_u} and m_{H_d}.[4] Thus the number of arbitrary parameters in the effective GUT includes the 3 gauge parameters, 7 soft SUSY breaking parameters and n_y Yukawa parameters. The number n_y and the form of the Yukawa matrices are model dependent. The effective theory between M_G and M_Z is the MSSM. We use two loop SUSY renormalization group equations [RGE] (in a $\overline{DR}$ renormalization scheme) for dimensionless parameters and one loop RGE for dimensionful parameters from M_G to M_Z. However we have

[3] ϵ_3 is calculable in any complete SUSY GUT. It is also constrained somewhat by the bounds on the nucleon lifetime[10].

[4] Note that if the messenger scale of SUSY breaking is M_{Planck} then our analysis is not completely self-consistent. In any complete SUSY GUT defined up to an effective cut-off scale $M > M_G$, the interactions above M_G will renormalize the soft breaking parameters. This will, in general, split the degeneracy of squark and slepton masses at M_G even if they are degenerate at M. On the other hand, bounds on flavor changing neutral current processes, severely constrain the magnitude of possible splitting. Thus these corrections must be small. In addition, in theories where SUSY breaking is mediated by gauge exchanges with a messenger scale below (but near) M_G, the present analysis is expected to apply unchanged. Since in this case squarks and sleptons will be nearly degenerate at the messenger scale. The Higgs mases, on the other hand, are probably dominated by new interactions which also generate a μ term. It is thus plausible to expect the Higgs masses to be split and independent of squark and slepton masses. The parameter A_0 could also be universal at the messenger scale.

checked that the corrections to our results obtained by using two loop RGE for dimensionful parameters are insignificant.

At M_Z we include one loop corrections to the W and Z masses. Thus the W and Z masses are given by the formulae:

$$M_W^2 = \frac{1}{2} g_2^2 v^2 + \delta_W^2 \tag{3}$$

$$M_Z^2 = \frac{1}{2} (\frac{3}{5} g_1^2 + g_2^2) v^2 + \delta_Z^2 \tag{4}$$

where δ_W, δ_Z are the one loop corrections to the pole masses and g_1, g_2 are $\overline{DR}$ gauge couplings evaluated at M_Z in the MSSM.

The Higgs vacuum expectation value v is an implicit function of soft SUSY breaking parameters, gauge and Yukawa couplings. It is determined by self consistently demanding minimization of the tree level Higgs potential. The actual value for v is fixed by minimizing χ^2.

To compare with the $\overline{DR}$ value of $\sin^2\theta_W(M_Z)$ in the MSSM we use the relation

$$\sin^2\theta_W = \frac{\frac{3}{5}\alpha_1}{\frac{3}{5}\alpha_1 + \alpha_2} \tag{5}$$

evaluated at M_Z.

$\alpha_s(M_Z) \equiv \alpha_3(M_Z)$ is evaluated in the $\overline{MS}$ scheme. One loop threshold corrections are included in these parameters in order to correctly account for states with mass greater than M_Z. Finally for α_{EM} we first evaluate $\alpha(M_Z) = \alpha_2(M_Z)\sin^2\theta_W(M_Z)$ in the $\overline{MS}$ scheme and then renormalize $\alpha(M_Z)$ to zero momentum. In both cases the $\overline{DR}$ values of $\alpha_i,\ i = 1,2,3$ are first changed to $\overline{MS}$ values using the threshold conditions

$$(1/\alpha_i(M_Z))|_{\overline{MS}} = (1/\alpha_i(M_Z))|_{\overline{DR}} + C_i \tag{6}$$

with $C_i = \frac{C_2(G_i)}{12\pi}$ and $C_2(G_i) = N$ for SU(N) or $= 0$ for U(1).

For fermion masses, we have at tree level

$$m_f = \lambda_f \frac{v}{\sqrt{2}} f(\beta) \tag{7}$$

where λ_f are the 3×3 Yukawa matrices at M_Z and $f(\beta) = sin\beta(cos\beta)$ for up quarks (down quarks and charged leptons). In addition, at M_Z we include the leading ($O(\tan\beta)$) one loop threshold corrections to fermion masses and mixing angles[11].

Finally, we compute the theoretical value for $\hat{B}_K$ needed to fit ϵ_K and also the branching ratio for $b \to s\gamma$[12].

We then form a χ^2 function including the 20 low energy observables — 6 in the gauge sector (M_W, M_Z, α_{EM}, $\sin^2(\theta_W)$, α_s, ρ), 13 in the fermion mass sector (9 charged fermion masses and 4 quark mixing angles) and the branching ratio for $b \to s\gamma$.[5] This χ^2 function is minimized self-consistently by iteratively varying the GUT parameters with m_0, $M_{1/2}$, and μ fixed. The procedure takes two steps. First the Higgs mass parameters are varied until the tree level Higgs potential is minimized, for a fixed trial value for v and $\tan\beta$. Then m_{H_u} and m_{H_d} along with the rest of the parameters are varied using the Minuit routine from the CERN library to minimize the χ^2 function.

In addition to the 20 observables given in table 6 we have to deal with the experimental lower bounds on sparticle masses. We require that the contribution of the lightest neutralino (if below the LEP1 threshold) to the invisible Z partial width is bounded by $\Gamma(Z \to \text{ invisibles}) = (-1.5 \pm 2.7)$ MeV, as measured at LEP1. We also demand that the lightest chargino be heavier than 65 GeV. Finally we require the pseudoscalar Higgs, A, to have mass greater than 30 GeV.[6] In order to take these bounds into account (within Minuit) we have added a large penalty to the χ^2 function whenever one of these bounds is exceeded.

4. Model 4 of ADHRS

In this work we have analyzed several models of fermion masses. We have studied model 4 of ADHRS[13]. In this model $n_y = 5$. The model is defined by the following 4 operators defined in the effective theory at M_G. $A_1,\ A_2$ and $\tilde{A}$ are adjoint

[5] The χ^2 function also includes the conditions for extremizing the Higgs potential.

[6] Note these bounds are on the tree level running masses. The state A (along with the other physical Higgs states) receives significant corrections to its tree level mass in the regime of large $\tan\beta$ in which we work.

scalars with vacuum expectation values in the B-L, Y or X (SU(5) invariant) directions of SO(10), respectively.

$$\begin{aligned}\mathcal{O}_{33} &= 16_3\ 10_1\ 16_3 \\ \mathcal{O}_{23} &= 16_2\ \frac{A_2}{\tilde{A}}\ 10_1\ \frac{A_1}{\tilde{A}}\ 16_3 \\ \mathcal{O}_{12} &= 16_1\ (\frac{\tilde{A}}{\mathcal{S}_M})^3\ 10_1\ (\frac{\tilde{A}}{\mathcal{S}_M})^3\ 16_2\end{aligned} \quad (8)$$

There are six possible choices for the 22 operator; all give the same $0:1:3$ Clebsch relation between up quarks, down quarks and charged leptons responsible for the Georgi-Jarlskog relation[15].

The resulting Yukawa matrices at M_G are given by –

$$Y_u = \begin{pmatrix} 0 & C & 0 \\ C & 0 & -\frac{1}{3}B \\ 0 & -\frac{4}{3}B & A \end{pmatrix}$$

$$Y_d = \begin{pmatrix} 0 & -27C & 0 \\ -27C & Ee^{i\phi} & \frac{1}{9}B \\ 0 & -\frac{2}{9}B & A \end{pmatrix}$$

$$Y_e = \begin{pmatrix} 0 & -27C & 0 \\ -27C & 3Ee^{i\phi} & B \\ 0 & 2B & A \end{pmatrix}$$

The best fits give $\chi^2 = 13$ for 5 degrees of freedom. Note that the regions with lowest χ^2 contain significant (of order (4 - 6)%) one loop SUSY threshold corrections to fermion masses and mixing angles. Thus these corrections are necessary to improve the agreement of the model with the data.

All low energy observables are fit to within their 2σ bounds. However the fits for 4 observables, ($V_{cb} = 0.0451$, $V_{ub}/V_{cb} = 0.0468$, $\hat{B}_K = 0.957$ and $Q^{-2} = 0.00172$) lie in the $(1-2)\sigma$ region. Note that Minuit typically tries to equalize the contribution of all the observables to χ^2. We now argue that these results are true predictions of the theory.

Consider the first three parameters. It was shown by Hall and Rasin[14] that the relation

$$\frac{V_{ub}}{V_{cb}} = \sqrt{\frac{\lambda_u}{\lambda_c}} \quad (9)$$

holds for any fermion mass texture in which the 11, 13 and 31 elements of the mass matrices are zero and perturbative diagonalization is permitted. Note λ_u, λ_c are the up and charm quark Yukawa couplings evaluated at a common renormalization scale. A typical value for the right-hand side of the equation is 0.05 which is too small for the left-hand side by more than 20%.

We now show that the fits for V_{ub}/V_{cb}, V_{cb} and $\hat{B}_K$ are correlated. Consider the formula for ϵ_K given by

$$\epsilon_K \approx (V_{cd}\,\frac{V_{ub}}{V_{cb}}\,V_{cb}^2 sin(\xi))\ \hat{B}_K \times (\text{one loop factors}) \quad (10)$$

where the first factor is just the Jarlskog parameter, J defined by the expression –

$$\begin{aligned} J &= Im(V_{ud}V_{ub}^* V_{cb} V_{cd}^*) \\ &= |V_{cd}||V_{ub}/V_{cb}||V_{cb}|^2\ \sin\xi \end{aligned} \quad (11)$$

where ξ is the CP violating phase. We see that if V_{ub}/V_{cb} is small, then V_{cb} and $\hat{B}_K$ must be increased to compensate. As a consequence, V_{cb} and $\hat{B}_K$ are both too large. Addition of 13 and 31 mass terms can accomodate larger values of V_{ub}/V_{cb} and thus lower values for V_{cb} and $\hat{B}_K$.

Now consider the ellipse parameter Q. This parameter is strongly controlled by the Georgi-Jarlskog relation[15]

$$\frac{m_s}{m_d} \approx \frac{1}{9}\frac{m_\mu}{m_e} \quad (12)$$

which is satisfied by model 4. This is an important zeroth order relation to try to satisfy. However unless there are small calculable corrections to this relation, it leads to values of $m_s/m_d \sim 25$ and thus values of Q which are too large. Note, that introducing 13 and 31 terms in the down quark and charged lepton mass matrices can, in principle, perturb the zeroth order Georgi-Jarlskog relation.

Thus the disagreement between model 4 and the data seems significant. It is unlikely that it can be fixed with the inclusion of small threshold corrections to the Yukawa relations at M_G. Before one adds new operators, however, it is worthwhile to consider the possibility that perhaps one of the experimental measurements is wrong. It is interesting to ask whether the agreement with the data can be significantly improved by removing one contribution to the χ^2 function; essentially discarding one piece of data[7]. In order to check this possibility we have removed several observables from the analysis (one at a time), to see if this significantly improves the fit. We find no significant improvements with this procedure.

On the other hand, we have found that we can indeed improve the results by adding one operator contributing to the 13 and 31 elements of the Yukawa matrices. We discuss this possibility in the next section. The additional terms are obtained by adding one new effective mass operator. Of course there are many possible 13 operators. In this work we have not performed a search over all possible 13 operators. Instead we study two 13 operators which are motivated by two complete SO(10) extensions of model 4.

5. Model 4(a,b,c) of LR

In this section we analyze two models derived from complete SO(10) SUSY GUTs discussed recently by Lucas and S.R.[10]. The models were constructed as simple extensions of models 4 of ADHRS. The label (a,b,c) refers to the different possible 22 operators which give identical Clebsch relations for the 22 element of the Yukawa matrices. However in the extension to a complete SUSY GUT these different operators lead to inequivalent theories. The different theories are defined by the inequivalent U(1) quantum numbers of the states. When one demands "naturalness", i.e. includes all terms in the superspace potential consistent with the symmetries of the theory one finds an additional 13 operator for models 4(a,b,c) given by –

$$\mathcal{O}_{13} = \tag{13}$$

$$\begin{array}{ll} (a) & 16_1\,(\frac{\tilde{A}}{\mathcal{S}_M})^3\,10_1\,(\frac{\tilde{A}A_2}{\mathcal{S}_M{}^2})\,16_3 \\ (b) & 0 \\ (c) & 16_1\,(\frac{\tilde{A}}{\mathcal{S}_M})^3\,10_1\,(\frac{A_2}{\mathcal{S}_M})\,16_3 \end{array}$$

[7]We thank M. Barnett for bringing this idea to our attention

One of these models (model 4b) is identical to model 4 of ADHRS when states with mass greater than M_G are integrated out. The results for this model are thus identical to those presented in the previous section; hence we will not discuss it further. The other two models include one new effective operator at the GUT scale. The addition of this 13 operator introduces two new real parameters in the Yukawa matrices at M_G; thus we have $n_y = 7$.

Models 4a and 4c differ only by the 13 operator. For model 4c the resulting Yukawa matrices are given by –

$$Y_u = \begin{pmatrix} 0 & C & -\frac{4}{3}De^{i\delta} \\ C & 0 & -\frac{1}{3}B \\ \frac{1}{3}De^{i\delta} & -\frac{4}{3}B & A \end{pmatrix}$$

$$Y_d = \begin{pmatrix} 0 & -27C & \frac{2}{3}De^{i\delta} \\ -27C & Ee^{i\phi} & \frac{1}{9}B \\ -9De^{i\delta} & -\frac{2}{9}B & A \end{pmatrix}$$

$$Y_e = \begin{pmatrix} 0 & -27C & -54De^{i\delta} \\ -27C & 3Ee^{i\phi} & B \\ -De^{i\delta} & 2B & A \end{pmatrix}.$$

We find that model 4a gives a best fit $\chi^2 \sim 4$ for 3 dof, while model 4c gives $\chi^2 \leq 1$ for 3 dof. For model 4c, the preferred regions of soft SUSY breaking parameter space for $\mu = 80$ GeV correspond to $M_{1/2} > 220$ GeV with $m_0 > 300$ GeV. The lower bound on m_0 varies slowly with $M_{1/2}$. In addition as μ increases the lower bound on m_0 increases significantly. Finally the lower bound on $M_{1/2}$ is determined by LEP limits on chargino/neutralino masses.

Note that the one loop SUSY threshold corrections for fermion masses scale as

$$\frac{\mu M_{1/2}}{m_0^2} \text{ or } \frac{\mu A_0}{m_0^2}. \tag{14}$$

We find that, in this case, the best fit tends to minimize these corrections, although for $\chi^2 \sim 1$ these are still of order (4 - 6)%. As a result, in the large region with $\chi^2 < 1$ and the SUSY corrections to fermion masses negligible, the effective number of degrees of freedom is actually larger than 3, since in this region there are 7 parameters in the Yukawa matrices determining the 13 low energy observables in the fermion mass sector.

In table 2, we give the computed values for the low energy observables in model 4c and the partial contributions to χ^2 for each observable for a particular point in soft SUSY breaking parameter space. We note that the best fit value for α_s is always on the low side of 0.12.

In table 3, we also include values for the SUSY spectra and the CP violating angles α, β and γ, measurable in neutral B decays. We note that the values for α, β and γ do not significantly change over soft SUSY breaking parameter space. Hence the predictions for these CP violating angles are robust; they are not significantly dependent on the values for the soft SUSY breaking parameters and thus provide a powerful test of the theory.

6. Conclusions

In this talk we have presented a global χ^2 analysis of the low energy data (including fermion masses and mixing angles) within the context of a class of predictive SO(10) supersymmetric GUTs. One of these models in fact provides an excellent fit to the low energy data. Moveover since our global analysis can be applied to any predictive theory, it provides a means of comparing the quality of the fits for different models. By applying such an analysis to all theories of fermion masses, we may hope to find the effective theory of Nature, valid below the Planck (or string) scale.

This research was supported in part by the U.S. Department of Energy contract DE-ER-01545-681. T.B. would like to thank Piotr Chankowski for many useful discussions.

REFERENCES

1. S. Dimopoulos, S. Raby and F. Wilczek, *Phys. Rev.* **D24** 1681 (1981); S. Dimopoulos and H. Georgi, *Nucl. Phys.***B 193** 150 (1981); L. Ibanez and G.G. Ross, *Phys. Lett.* **105B** 439 (1981); M. B. Einhorn and D. R. T. Jones, *Nucl. Phys.* **B196** 475 (1982); W. J. Marciano and G. Senjanovic, *Phys. Rev.* **D 25** 3092 (1982). For recent analyses, see P. Langacker and N. Polonsky, *Phys. Rev.***D47** 4028 (1993); ibid., **D49** 1454 (1994); M. Carena, S. Pokorski, C.E.M. Wagner, *Nucl. Phys.***B406** 59 (1993).
2. H. Georgi, Particles and Fields, Proceedings of the APS Div. of Particles and Fields, ed C. Carlson (1974); H. Fritzsch and P. Minkovski, *Ann. Phys.* **93** 193 (1975).
3. D. Kaplan and A. Manohar, Phys. Rev. Lett. **56** 2004 (1986); H. Leutwyler, Nucl. Phys. **B337** 108 (1990).
4. C. Jarlskog, Phys. Rev. Lett. **55** 1039 (1985).
5. P.H. Chankowski, Z. Pluciennik and S. Pokorski, Nucl. Phys. **B439**, 23 (1995).
6. R. Gupta, D. Daniel, G.W. Kilcup, A. Patel, and S.R. Sharpe, Phys. Rev. **D47**, 5113 (1993); G. Kilcup, Phys. Rev. Lett. **71**, 1677 (1993); S. Sharpe, Nucl. Phys. B (Proc. Suppl.) **34** 403 (1994).
7. See P. Langacker this conference.
8. M. Shifman, N.G. Uraltsev, and A. Vainshtein, Phys. Rev. **D51**, 2217 (1995); N.G. Uraltsev, Int. J. Mod. Phys. **A11**, 515 (1996); P. Ball, M. Beneke, and V.M. Braun, Phys. Rev. **D52**, 3929 (1995).
9. J.F. Donoghue, B.R. Holstein and D. Wyler, Phys. Rev. **D47**, 2089 (1993); J. Bijnens, Phys. Lett. **B306**, 343 (1993). Summarized in H. Leutwyler, Cern preprint CERN-TH/96-44, hep-ph/9602366 (1996).
10. V. Lucas and S. Raby, "GUT Scale Threshold Corrections in a Complete Supersymmetric SO(10) Model: $\alpha_s(M_Z)$ vs. Proton Lifetime," OSU Preprint OHSTPY-HEP-T-95-028 (1995); e-Print Archive: hep-ph/9601303; to be published in Phys. Rev. D.
11. T. Blazek, S. Raby and S. Pokorski, Phys. Rev. **D52**, 4151 (1995).
12. S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. **B353**, 591 (1991); S. Bertolini, F. Borzumati and A. Masiero, in B Decays, edited by S. Stone, World Scientific (Singapore) (1991).
13. G. Anderson, S. Dimopoulos, L.J. Hall, S. Raby and G. Starkman, *Phys. Rev.***D49** 3660 (1994).
14. L.J. Hall and A. Rasin, Phys. Lett. **B315**,164 (1993).
15. H. Georgi and C. Jarlskog, Phys. Lett. **86B**,

Observable	Central value	σ
M_Z	91.188	0.46
M_W	80.385	0.40
α_{EM}^{-1}	137.04	0.69
G_μ	0.11664×10^{-4}	0.0012×10^{-4}
$\alpha_s(M_Z)$	0.118	0.006
ρ_{new}	-0.9×10^{-3}	2.9×10^{-3}
M_t	174.46	8.1
$m_b(m_b)$	4.26	0.11
$M_b - M_c$	3.4	0.2
m_s	180	50
m_d/m_s	0.05	0.015
Q^{-2}	0.00203	0.00020
M_τ	1.777	0.0089
M_μ	105.66	0.53
M_e	0.5110	0.0026
V_{us}	0.2205	0.0026
V_{cb}	0.040	0.003
V_{ub}/V_{cb}	0.08	0.02
$\hat{B}_K$	0.8	0.1
$B(b \to s\gamma)$	0.232×10^{-3}	0.092×10^{-3}

Table 1
Experimental observables

297 (1979).

Observable	Computed value	Contribution to χ^2
M_Z	91.15	< 0.1
M_W	80.38	< 0.1
α_{EM}^{-1}	137.0	< 0.1
G_μ	0.1165×10^{-4}	< 0.1
$\alpha_s(M_Z)$	0.1130	0.69
ρ_{new}	0.1806×10^{-3}	0.14
M_t	175.0	< 0.1
$m_b(m_b)$	4.292	< 0.1
$M_b - M_c$	3.468	0.12
m_s	183.7	< 0.1
m_d/m_s	0.0496	< 0.1
Q^{-2}	0.002046	< 0.1
M_τ	1.776	< 0.1
M_μ	105.7	< 0.1
M_e	0.5110	< 0.1
V_{us}	0.2205	< 0.1
V_{cb}	0.04074	< 0.1
V_{ub}/V_{cb}	0.07395	< 0.1
$\hat{B}_K$	0.8109	< 0.1
$B(b \to s\gamma)$	0.2444×10^{-3}	< 0.1

Table 2
Results for Model 4c for $\mu = 80$ $M_{1/2} = 280$ and $m_0 = 400$.

Observable	Predictions			
$\sin 2\alpha$	0.95			
$\sin 2\beta$	0.51			
$\sin 2\gamma$	-0.66			
$sin^2\theta_W$	0.2340			
gluino	723			
charginos	70	260		
neutralinos	52	95	130	260
up squarks	470	612	746	769
down squarks	509	550	748	773
charged sleptons	75	339	422	448
sneutrinos	329		440	441

Table 3
More Results for Model 4c. For squark and slepton masses, the first two columns are for the third family which are significantly split, while the third and fourth columns are mean values for the nearly degenerate states of the second and first families, respectively.