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Decoupling of a strongly coupled lattice with an application to LHC

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The systematic skew quadrupole field in the LHC superconducting dipole is estimated to be $a^2 = 0.310^{-4}$ at 1 cm. It causes systematic linear coupling resonances to be strongly excited (width up to 0.2 tune unit). With an exact antisymmetry of the optics, LHC is operated close to them. The theory of resonances predicts well the large focusing perturbations observed numerically and allows accurate decoupling with only two families of skew quadrupoles, some being paired. Robustness however favours a solution which does not rely on an accurate knowledge of the perturbation; it is obtained by relaxing the exact antisymmetry to operate the machine with matched tune splits of up to three units.

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DECOUPLING OF A STRONGLY COUPLED LATTICE WITH AN APPLICATION TO LHC

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The systematic skew quadrupole field in the LHC superconducting dipole is estimated to be $a_2 = 0.310^{-4}$ at 1 cm. It causes systematic linear coupling resonances to be strongly excited (width up to 0.2 tune unit). With an exact antisymmetry of the optics, LHC is operated close to them. The theory of resonances predicts well the large focusing perturbations observed numerically and allows accurate decoupling with only two families of skew quadrupoles, some being paired. Robustness however favours a solution which does not rely on an accurate knowledge of the perturbation; it is obtained by relaxing the exact antisymmetry to operate the machine with matched tune splits of up to three units.

1 INTRODUCTION

The large betatron coupling occurring in LHC was found to confuse the decoupling technique in the case of LHC version 2 and raised the question of the theoretical approach to be used to analyse and correct large coupling. The coupling sources, the decoupling scheme and method are presented, followed by the provisions to obtain a lattice robust against variations of the coupling perturbation for instance during the ramp.

2 SOURCES OF COUPLING

The hierarchy of the coupling sources is rather different from other accelerators, due to the super-conducting technology, size of the machine and high energy. The strength of the most important sources are summarized in table 1, where the resonance width [1], in general equal to the closest tune approach, is expressed in tune units. The calculation is made for the nominal working point (LHC versions up to 4.2) where the horizontal and vertical tunes are almost equal ($Q_x - Q_y \approx 0.03$). This working point arises from the antisymmetry of the LHC optics, used so far to minimize the number of quadrupoles circuits in the insertions.

The dominant source of coupling is the systematic a_2 in the dipoles, while the strong CMS solenoid can be neglected already at injection. The so-called systematic a_2 is actually a fabrication tolerance, its expectation value being zero. A systematic misalignment of the sextupole correcting coil in the dipoles by up to a few tenths of millimeter is considered presently possible and could contribute significantly to the resonance width. Random effects are comparable to those observed in LEP and should be easy to correct.

Table 1: *Sources of betatron coupling and resonance width.*

Source	Width
CMS Solenoid, 24 Tm, at injection	0.003
Vertical orbit in sextupoles	0.02
Vertical orbit in random b_3 of dipoles	0.003
Random $a_2 = 1.\ 10^{-4}$ in the dipoles	0.02
Random tilt of quads	0.03
Systematic vertical misalignment of b_3 correctors versus average dipole position	0.08
Systematic $a_2 = .3\ 10^{-4}$ in the dipoles	0.21
TOTAL (at 2 σ)	.38

3 DECOUPLING SCHEME

3.1 LHC version 1

In spite of the strength of the a_2 perturbation, which was then twice larger, the decoupling of LHC version 1 [2] was straightforward. The correction scheme was made of four families of skew quadrupoles. Decoupling was achieved by imposing that the off-diagonal elements of the one-turn linear map vanish. After decoupling, the dynamic aperture was found only insignificantly reduced with respect to a machine without coupling.

3.2 LHC version 2

When the number of cells in the LHC arc was decreased from 25 to 24 (version 2 [5]) to allow a higher filling factor, the decoupling approach described above left large residuals in closest tune approach and β-beating (table 2). They could be reduced painfully by using up to 12 families of skew quadrupoles. The dynamic aperture decreased. Such a pathology had been calculated for RHIC [4]. A similar situation arose during the commissioning of LEP. The resonance compensation could not be achieved in practice and splitting the tunes became necessary. In LHC, such a tune split is inconsistent with the antisymmetry of the optics (antisymmetry about the interaction point and antisymmetry between the two rings at the same azimuth), motivating an analysis of the problem.

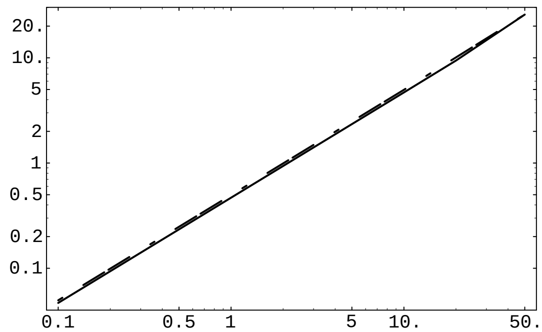

Figure 1: Dependence of the tune split $|Q_I - Q_{II}|$ on a_2 expressed in units of nominal a_2

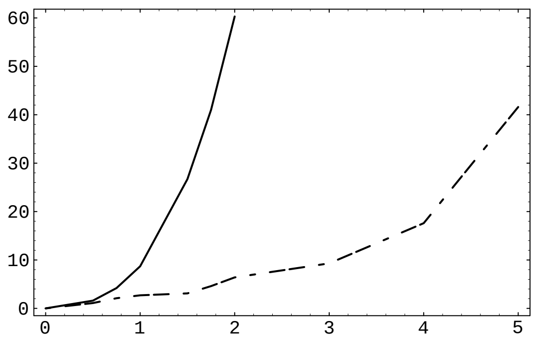

Figure 2: Dependence of β_x (plain) and β_y (dotted) beatings in % on the **compensated** a_2 in LHC version 2 (expressed in units of nominal a_2)

In a first numerical experiment (non-perturbative), the closest tune approach is calculated for an optics perturbed by a_2 and not corrected (fig 1). The dependence of the tune split versus a_2 is found purely linear up to very high values, showing the expected signature of an isolated difference coupling resonance. In a second experiment, the β-beating observed after coupling correction was calculated as a function of a_2 (fig 2). It increases quadratically with the perturbation. This dependence is expected from the focusing effect of the skew quadrupoles when the tilt of the normal modes is significant. The results of both experiments can be interpreted [6] by considering the excitation [1] of the two closest coupling resonance (difference and sum):

$$c^p_{\mp} = \frac{1}{2\pi} \oint \sqrt{\beta_x \beta_y}\, K_s\, e^{i[\mu_x \mp \mu_y - \theta \Delta^p]}\, ds \qquad (1)$$

where K_s is the normalized skew gradient and θ the azimuthal angle (0 to 2π). $\Delta^l = Q_x \mp Q_y - l$ is the distance to the resonance and s the azimuthal position. Before coupling correction, the systematic skew quadrupolar perturbation can only excite significantly the coupling resonances for which the phase term in (1) is stationary. For almost equal betatron tunes, only one difference resonance is excited ($Q_x = Q_y$) and no sum resonance. Such a resonance is expected to yield a tune separation but no significant β-beating. If only the difference coupling reso-

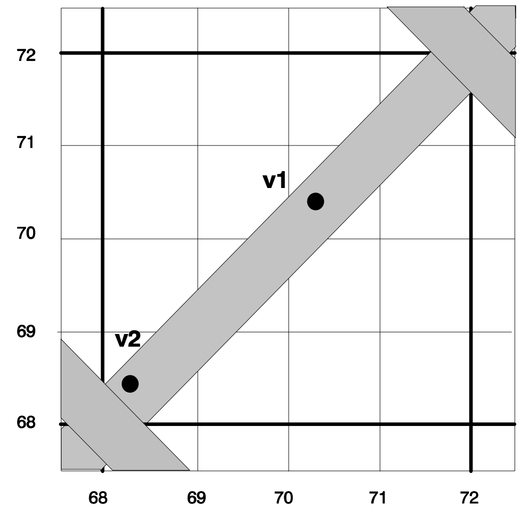

Figure 3: Systematic coupling resonances in LHC in Q_x, Q_y plane

nance is corrected, explicitly or not, the skew quadrupole correctors are liable to excite the sum resonance at the same level as the difference resonance before correction. This occurred in LHC version 2 due to a peculiarity of the LHC optics induced by its antisymmetry (fig 3): although the super-periodicity of the focusing is 4, it becomes 8 for the sum resonance ($Q_x + Q_y$) because the betatron phases advance by the same amount in each cell and each insertion. In LHC version 1 ($Q_x + Q_y \approx 140$), the closest sum resonance is not super-periodic while it is in LHC version 2 ($Q_x + Q_y \approx 17 \times 8$). In the latter case, one expect a second-order contribution to the tune separation and β-beating, as observed in the numerical studies.

3.3 *Correction scheme*

In the light of this analysis, the systematic coupling in LHC should be corrected by compensating the difference coupling resonance with a skew quadrupole scheme which does not excite the sum resonance. To this end, the skew quadrupoles are placed in the arcs, arranged in pairs spaced by a betatron phase shift of $\pi/2$, i.e. one cell in LHC and excited in series. The sum resonance is then not excited eq. (1). To demonstrate the efficiency of the scheme, various solutions were experimented (figure 4 and table 2). With the

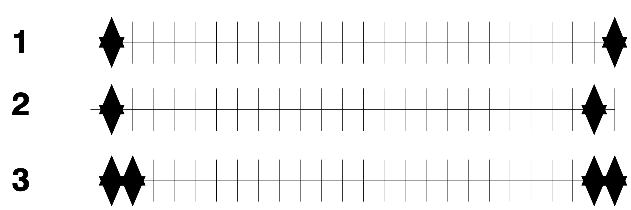

Figure 4: Skew quadrupole schemes in one LHC arc (unit spacing is one cell)

Table 2: *Decoupling results for LHC version 2*

Decoupling scheme	$\|Q_I - Q_{II}\|$	$\Delta\beta_x/\beta_x$	$\Delta\beta_y/\beta_y$
before decoupling	0.496	3.4%	0.3%
1: LHC version 2	0.065	64.%	41.%
2: LHC version 1	0.0027	7.6%	3.6%
3: proposed scheme	0.0026	1.04%	0.96%

Table 3: *Sensitivity of various machines to coupling*

Machine	$\|c\|$ natural	$\|Q_x - Q_y\|$
ISR	0.01	0.025
SPS	0.01	0.005
LEP (tune split of 2)	0.025	0.1
HERA (tune split of 1)	0.06	0.01
LHC version 4	0.4	0.01

proposed scheme, the β-beating disappears and the minimum tune separation becomes small; it may be brought to any arbitrarily smaller value by adding one weak skew quadrupole in quadrature with the first family.

Tracking [7] has shown that this scheme is both necessary and sufficient to prevent a loss of dynamic aperture due to betatron coupling, including a possible increase of the random coupling during the ramp, induced by eddy currents.

4 DYNAMIC EFFECTS

So far, the a_2 perturbation has been assumed to be exactly known. The tune separation for the nominal LHC working point is however only 3% of the width $|c|$ of the coupling resonance (table 3). A very accurate knowledge of the ring integral of a_2, with an accuracy of the order of $2\,10^{-6}$ of the main field component, is required to maintain decoupling. A lattice less sensitive to betatron coupling is therefore needed for a safer operation of LHC.

The coherent contribution of each arc to coupling can be avoided if the horizontal and vertical betatron phase advances are different either in the insertions or in the arc cells. With the exception of an alternating tune split of one unit per arc, the other solutions break the antisymmetry of the nominal optics, used to minimize the number of parameters needed to match the insertions.

The method being explored is nevertheless a fractional tune split in the arcs which disturbs less the machine acceptance. The value for the tune split is deduced from the study of a scenario where the systematic a_2 is assumed to be different for each of four manufacturers and from the inner to the outer channel. Each arc is assumed to be assembled from magnets of the same manufacturer. Each a_2 is drawn from a rectangular distribution. The results over 1000 cases(figure 5) show that the coupling strength can be reduced to a manageable value if the tune split can be chosen between 1 and 3 units. In these conditions, higher-order effects of a_2 on the optics parameters are found negligible.

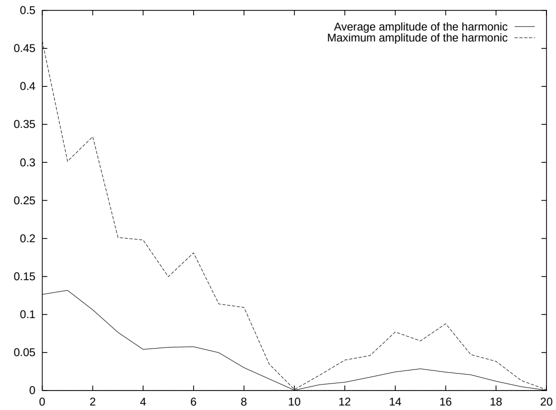

Figure 5: Amplitude of the systematic a_2 versus harmonic number, average and worst case

5 CONCLUSION

Albeit very large, the betatron coupling calculated for LHC is well represented by the excitation of one difference and one sum linear coupling resonance. This model allows the design of a simple scheme mainly based on one family of skew quadrupoles pairs. Their distribution follows the general symmetries of the machine (one pair at each end of each arc). Dynamic aperture studies show that this scheme is both necessary and sufficient. The correction may however not be robust as it relies on the knowledge of the field perturbation to an accuracy of some $2\,10^{-6}$ relative to the main field. A tune split varying between 1 and 3 units, depending on the actual harmonic content of the perturbation, reduces the perturbation to the required level. To achieve it, the insertions are given more flexibility (more quadrupole circuits) and are not constrained to fulfil the exact antisymmetry conditions.

6 REFERENCES

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