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Aspects of Type I – Type II – Heterotic Triality in Four Dimensions*

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Abstract

We discuss the equivalence between Type I, Type II and Heterotic $N = 2$ superstring theories in four dimensions. We study the effective field theory of Type I models obtained by orientifold reductions of Type IIB compactifications on $K_3 \times T^2$. We show that the perturbative prepotential is determined by the one-loop corrections to the Planck mass and is associated to an index. As is the case for threshold corrections to gauge couplings, this renormalization is entirely due to $N = 2$ BPS states that originate from $D = 6$ massless string modes. We apply our result to the so-called *S-T-U* model which admits simultaneous Type II and Heterotic descriptions, and show that all three prepotentials agree in the appropriate limits as expected from the superstring triality conjecture.

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1. Introduction

There has been some very convincing evidence accumulated so far for the equivalence of theories which were believed in the past to describe truly different types of superstrings. Type I, Type II and Heterotic theories seem merely to provide complementary descriptions of a more complicated theory of fundamental interactions, and the larger framework of superstring dualities now includes also M-theory and F-theory descriptions. In order to reach various points on the web of connected models, it is often convenient to start from ten dimensions and to descend to lower dimensions by compactifying these well-known theories. The equivalence of various superstring compactifications can then be understood as a consequence of a few fundamental dualities originating from higher dimensions [1].

Among the four-dimensional models, the most familiar examples of dual pairs are based on Type II and Heterotic constructions [2] whose equivalence originates from the well-established six-dimensional duality between Type IIA compactified on K_3 and Heterotic compactified on T^4 [3]. These models have $N = 4$ or $N = 2$ spacetime supersymmetry in $D = 4$ and their equivalence has been checked in many ways, including some highly non-trivial quantitative comparisons of the respective low-energy effective actions [2, 4, 5]. As a generic feature, the string coupling of the Heterotic side is mapped under such duality to a "geometric" modulus on the Type II side.

Type I theory remained a wild card in duality conjectures until quite recently Polchinski and Witten presented several arguments for the equivalence of Type I and Heterotic theories in ten dimensions [6]. Although in $D = 10$ this is a strong-weak coupling duality, it turns out that upon appropriate compactification to $D = 4$ one obtains $N = 2$ supersymmetric dual pairs with the Heterotic gauge coupling mapped to a Type I gauge coupling in a way that some weakly coupled regions overlap on both sides. This work is focused on Type I – Heterotic duality in $D = 4$. We discuss the mapping of special coordinates of the special Kähler manifold describing the massless vector multiplet sector. We compute the perturbative prepotential on the Type I side for a general class of $K_3 \times T^2$ orientifold reductions of the underlying Type IIB theory. We obtain a general expression which involves $N = 2$ BPS states only. It agrees with the appropriate limit of the corresponding expression in the Heterotic theory. We apply this result to a specific example, which admits all three Type I, Type II and Heterotic descriptions. The agreement of all three prepotentials provides here a convincing evidence for a true superstring triality in $D = 4$.

The paper is organized as follows. In section 2, we review Type I – Heterotic duality in $D = 6$ [7, 8, 9] and recall some basic features of the effective field theory describing Type I orientifold reductions of Type IIB theory [10, 11, 7] that are dual to Heterotic K_3 compactifications. In section 3, we discuss the tree-level effective actions of four-dimensional models obtained by toroidal compactifications of six-dimensional Type I models. We identify the universal vector moduli S, T and U (in Heterotic notation) and describe the duality mapping of special coordinates. In section 4 we discuss quantum corrections, explaining what kind of useful information can be extracted from purely perturbative Type I computations. We discuss the problem of determination of the one-loop prepotential. In the Heterotic theory one can extract it from the universal part of threshold corrections to gauge couplings [12, 13, 14]. In Type I theory, the one-loop threshold corrections have recently been analyzed in ref.[15]; however it is not possible to extract from them the Kähler metric. The reason is that unlike the Heterotic case, the Planck mass receives non-vanishing corrections which force redefinitions of special coordinates at the one-loop level. As a result, the universal part of threshold corrections is absorbed into the tree-level gauge coupling. However the Kähler metric, hence also the perturbative prepotential, can be extracted from the one-loop Planck mass. In section 5, we present the one-loop computation of the Planck mass which allows a determination of the Kähler metric in Type I theory. The metric, hence also the prepotential are completely determined by the BPS spectrum of the theory. Some technical details and the expressions for various propagators used in the computation are given in the Appendix. In section 6, we consider a specific orientifold model with the massless spectrum consisting of 3 vector multiplets and 244 hypermultiplets [8, 9]. This is the so-called S-T-U model which admits simultaneous Type II and Heterotic descriptions [2]; the exact form of its prepotential has been determined before by using Type II – Heterotic duality [5]. We apply the general formula of section 5 to determine the one-loop Kähler metric. The result reproduces the $T \to i\infty$ limit of the Heterotic case [13], as expected from duality. We summarize our results in section 7.

2. Type I Effective Field Theory in Six Dimensions

In this work, we will consider four-dimensional $N = 2$ Type I superstrings obtained by toroidal compactifications of $D = 6$, $N = 1$ models. The latter can be constructed as orientifold compactifications of Type IIB theories [11, 16]. In this section we review some basic features of the effective field theory in $D = 6$, in connection with Type I - Heterotic duality [7].

Anomaly cancellation constrains the massless spectrum to satisfy

$$n_H - n_V = 244 - 29(n_T - 1) \ , \tag{2.1}$$

where n_H, n_V and n_T are the numbers of hyper, vector and tensor multiplets, respectively. Since we are interested in theories dual to Heterotic compactifications, we restrict our discussion to $n_T = 1$. These theories contain one two-index antisymmetric tensor field whose self-dual part belongs to the tensor multiplet while its anti-self-dual part belongs to the gravitational multiplet. In Type I theory the antisymmetric tensor arises from the Ramond-Ramond (R-R) sector.

The scalar component of the tensor multiplet is related to the K_3 volume and determines the gauge coupling constants. In fact, a standard dimensional reduction from $D = 10$ to $D = 6$ gives

$$\mathcal{L}^{(6)} = -e^{-2\phi_6}\left\{\frac{1}{2}R^{(6)} + 2\left(\frac{\partial\omega}{\omega}\right)^2 - 2(\partial\phi_6)^2\right\} - \frac{1}{4}e^{-\phi_6}\omega^2 F^2 - \frac{1}{16}\omega^4(dB-\Omega)^2 + \dots \tag{2.2}$$

Here, ω^4 is the volume of K_3, $R^{(6)}$ is the scalar curvature, ϕ_6 is the six-dimensional dilaton, F is the gauge field strength, dB is the antisymmetric tensor field strength and Ω is the gauge Chern-Simons term. It is now easy to see that in the Einstein frame the factor $e^{-\phi_6}$ drops from the gauge kinetic terms, so that the gauge coupling constant becomes $1/\omega$. The scalar ω belongs to the tensor multiplet while the string coupling e^{ϕ_6} belongs to a hypermultiplet.

The gauge couplings of eq.(2.2), obtained by a simple dimensional reduction, are not the most general ones. First of all, in orientifold constructions, there appear additional gauge bosons associated to open strings with end-points fixed on 5-branes [16]. The corresponding gauge kinetic terms are

$$-\frac{1}{4}e^{-\phi_6}\omega^{-2}F'^2 - \frac{1}{16}B\wedge F'\wedge F' + \ldots \tag{2.3}$$

By going again to the Einstein frame, one sees that the coupling constant becomes ω for these (primed) gauge fields. In the most general case the gauge couplings are given by linear combinations [10]:

$$\frac{1}{g_i^2} = v_i\omega^2 + v_i'\omega^{-2} \tag{2.4}$$

where v_i and v_i' are constants. In models involving non-vanishing constants of both types, *i.e.* $v_iv_j' \neq 0$ for some gauge group generators i, j, an additional complication arises because the effective field theory action given by the sum of eqs. (2.2) and (2.3) is not consistent with supersymmetry [17]. The last terms of eqs. (2.2) and (2.3) are Wess-Zumino terms which cancel gauge anomalies in analogy with the Green-Schwarz mechanism in $D = 10$. We will come back to this problem later, after compactifying to $D = 4$.

Type I – Heterotic duality originates from $D = 10$, where both theories are believed to be equivalent after inverting the string couplings and rescaling the Regge slope α' [6]:

$$\phi_{10}^I = -\phi_{10}^H \qquad\qquad \alpha_I' = e^{\phi_{10}^H}\alpha_H' \tag{2.5}$$

where I and H refer to Type I and Heterotic, respectively. Using the relation between six-and ten-dimensional dilatons, $e^{-2\phi_6} = e^{-2\phi_{10}}\omega^4$, one finds

$$e^{-2\phi_6^I} = \omega_H^4 \qquad\qquad \omega_I^4 = e^{-2\phi_6^H} \tag{2.6}$$

which implies that the six-dimensional theories become equivalent after interchanging the square of the string coupling with the inverse of the K_3 volume, $e^{2\phi_6} \leftrightarrow \omega^{-4}$. As a result, the interactions of tensor and gauge multiplets are purely classical on the Type I side, since the string coupling belongs to a hypermultiplet. On the other hand, the hypermultiplet sector of the Heterotic side does not receive any quantum corrections since the string coupling there belongs to a tensor multiplet.

3. Tree-Level Effective Field Theory in Four Dimensions

After toroidal compactification to $D = 4$ one obtains $N = 2$ superstring models with the massless spectrum consisting of the supergravity multiplet, $n_V + 3$ vector multiplets and n_H hypermultiplets. The three additional vector bosons and the graviphoton arise from the metric and from the R-R antisymmetric tensor. The way scalar particles fit into supermultiplets is more subtle, therefore we discuss them now in some detail.

In addition to the scalar components of the n_V vector multiplets which are open string states, and the scalar of the tensor multiplet, there are also 5 scalars which appear upon compactification to $D = 4$. Three of them come from the torus metric G_{IJ} and two from the antisymmetric tensor: the axion dual to $D = 4$ components $B_{\mu\nu}$, and the internal component B_{IJ}. Note that the usual NS-NS (Neveu-Schwarz) antisymmetric tensor is eliminated by the orientifold projection. Straightforward dimensional reduction of the $D = 6$ effective action (2.2), (2.3) yields

$$\begin{aligned}
\mathcal{L}^{(4)} \;\; = \;\; & -e^{-2\phi_4}\left\{\frac{1}{2}R^{(4)} + 2\left(\frac{\partial\omega}{\omega}\right)^2 - 2(\partial\phi_4)^2 - \frac{\partial U\partial\bar{U}}{(U-\bar{U})^2} + \frac{1}{4}\left(\frac{\partial\sqrt{G}}{\sqrt{G}}\right)^2\right\} \\
& -\frac{1}{4}e^{-\phi_4}G^{1/4}\omega^2F^2 - \frac{1}{4}e^{-\phi_4}G^{1/4}\omega^{-2}F'^2 + \dots
\end{aligned} \tag{3.1}$$

where for the moment we kept only the terms which are relevant for the identification of the supermultiplets. The four-dimensional dilaton (defined as the string coupling constant in $D=4$) is given by $e^{-2\phi_4}=e^{-2\phi_6}\sqrt{G}$. In eq.(3.1), U is the usual complex modulus which determines the complex structure of the torus: $U=(G_{45}+i\sqrt{G})/G_{44}$.

Guided by the form of gauge coupling constants, we define two complex fields S and S', with the imaginary parts given by

$$S_2=e^{-\phi_4}G^{1/4}\omega^2 \qquad\qquad S'_2=e^{-\phi_4}G^{1/4}\omega^{-2}\ . \tag{3.2}$$

The real parts S_1 and S'_1 are defined as the scalar dual to $B_{\mu\nu}$ and B_{45}, respectively. In terms of these fields, eq.(3.1), transformed into the Einstein frame and supplemented by the dimensionally reduced kinetic term $(dB)^2$ of eq.(2.2), reads

$$\begin{aligned}\mathcal{L}^{(4)} \quad &= \quad -\frac{1}{2}R^{(4)}+\frac{\partial U\partial\bar{U}}{(U-\bar{U})^2}-\frac{(\partial S_1)^2}{4S_2^2}-\frac{(\partial S'_1)^2}{4S'^2_2}-\frac{(\partial S_2)^2}{4S_2^2}-\frac{(\partial S'_2)^2}{4S'^2_2}\\ &\quad -\frac{1}{2}(\partial\phi_6)^2-\frac{1}{4}S_2F^2-\frac{1}{4}S'_2F'^2+\ldots\end{aligned} \tag{3.3}$$

The complex scalars S, S' and U belong to vector multiplets while the six-dimensional dilaton ϕ_6 remains in a hypermultiplet. Equation (3.3) shows that in the absence of open string vector multiplets, the three universal scalars S, S' and U parameterize a $[SU(1,1)]^3$ manifold, with the corresponding prepotential $F=SS'U$.

Type I theory exhibits two continuous Peccei-Quinn symmetries associated to S and S' axion shifts which remain valid to all orders of perturbation theory since the corresponding axions originate from the R-R sector. In terms of the independent scalars appearing in eq.(3.3), the four-dimensional string coupling is a combination of fields belonging to hyper and vector multiplets:

$$e^{-2\phi_4}=e^{-\phi_6}(S_2S'_2)^{1/2}\ . \tag{3.4}$$

This means that both hyper and vector multiplet sectors can in principle receive quantum corrections in four-dimensional Type I theory, once $e^{-2\phi_4}$ combines with appropriate factors to form a hyper or a vector multiplet component.

We can now use Type I – Heterotic relations (2.6) in $D = 6$ to deduce the duality mapping in four dimensions:

$$S_I = S_H \qquad\qquad S'_I = T_H \qquad\qquad U_I = U_H\,. \tag{3.5}$$

Here, $S_H = \alpha + ie^{-2\phi_4^H}$, where α is the axion dual to $B_{\mu\nu}$ and ϕ_4^H is the Heterotic dilaton; $T_H \equiv T = B_{45} + i\sqrt{G}$ is the usual Kähler-class modulus of the 2-torus. This means that at the exact level the two theories are equivalent upon identification of S' with T and of the hypermultiplet scalar $e^{-2\phi_6}$ with the K_3 volume, according to eq.(2.6).

In order to see how the prepotential depends on the additional n_V open string vector multiplets, we first discuss the simplest case of vectors obtained by dimensional reduction from $D = 10$. Starting from the Lagrangian (2.2) one obtains:

$$\begin{aligned}\mathcal{L}^{(4)} \;=\; & -\frac{1}{2}R^{(4)} + \frac{\partial S\partial\bar{S}}{(S-\bar{S})^2} + \frac{\partial U\partial\bar{U}}{(U-\bar{U})^2} - \frac{(\partial S'_2)^2}{4S'^2_2} - \frac{(\partial S'_1 + \frac{1}{2}\sum_i a_4^i \overleftrightarrow{\partial} a_5^i)^2}{4S'^2_2} \\ & + \textstyle\sum_i \displaystyle\frac{|U\partial a_4^i - \partial a_5^i|^2}{(S'-\bar{S}')(U-\bar{U})} + \ldots\end{aligned} \tag{3.6}$$

where a_4, a_5 are the scalars arising from the compact components of six-dimensional vector fields. These Lagrangian terms can be derived from the $N = 2$ prepotential

$$F^{(0)} = S(S'U - \tfrac{1}{2}\textstyle\sum_i A_i^2) \tag{3.7}$$

where the special coordinates of gauge fields are defined by [18]

$$A_i = a_4^i U - a_5^i\;, \tag{3.8}$$

and S' is redefined as

$$S' = S'|_{A=0} + \tfrac{1}{2}\textstyle\sum_i a_4^i A_i\;. \tag{3.9}$$

The duality transformation (3.5) maps A_i into perturbative gauge multiplets on the Heterotic side.

Instead of the vectors A_i coming from the ten-dimensional gauge group $[SO(32)]$, consider now the vector multiplets related to 5-branes discussed in section 2. Their $D = 6$

kinetic terms are of the form (2.3), and lead to the same $D=4$ effective Lagrangian and prepotential as in eqs.(3.6) and (3.7), with the replacement of A by A' [defined as in eq.(3.8)] and with the interchange $S \leftrightarrow S'$. On the Heterotic side, these vector multiplets have a non-perturbative origin, and the corresponding gauge couplings are determined by the T modulus instead of the Heterotic dilaton S (modulo exponentially suppressed instanton corrections).

A more complicated situation arises in the simultaneous presence of A- and A'-type of fields, or in the presence of vector multiplets with gauge couplings involving non-vanishing v and v' as in eq.(2.4). If one naively starts from the combined action (2.2)+(2.3) and goes down to $D=4$, one finds that in terms of the redefined complex fields

$$S' = S'|_{A=0} + \textstyle\sum_i \frac{v_i}{2} a_4^i A_i \qquad\qquad S = S|_{A=0} + \sum_i \frac{v'_i}{2} a_4^i A_i \ , \tag{3.10}$$

the scalar kinetic terms are determined by the Kähler potential

$$\begin{aligned} K &= -\ln\{(S'-\bar{S}')(U-\bar{U}) - \textstyle\sum_i \frac{v_i}{2}(A_i-\bar{A}_i)^2\} - \ln\{(S-\bar{S})(U-\bar{U}) - \sum_i \frac{v'_i}{2}(A_i-\bar{A}_i)^2\} \\ &\qquad + \ln(U-\bar{U}) \\ &= -\ln\{(S-\bar{S})(S'-\bar{S}')(U-\bar{U}) - \tfrac{1}{2}\textstyle\sum_i [v_i(S-\bar{S}) + v'_i(S'-\bar{S}')](A_i-\bar{A}_i)^2 \\ &\qquad + \frac{1}{(U-\bar{U})}[\textstyle\sum_i \frac{v_i}{2}(A_i-\bar{A}_i)^2][\sum_j \frac{v'_j}{2}(A_j-\bar{A}_j)^2]\} \ . \end{aligned} \tag{3.11}$$

The corresponding scalar manifold, although Kähler, is not of the special type, which is a consequence of the fact that the six-dimensional action was not consistent with supersymmetry, as already mentioned before. This six-dimensional anomaly disappears in lower dimensions, where “anomalous” terms are canceled by local counterterms. It is not difficult to realize that the role of the counterterms is to cancel the last term in eq.(3.11), so that the Kähler manifold becomes special, as required by $N=2$ supersymmetry. The corresponding prepotential is

$$F^{(0)} = SS'U - \tfrac{1}{2}\textstyle\sum_i (v_i S + v'_i S') A_i^2 \ . \tag{3.12}$$

The above result agrees with the analysis of $D = 5$ compactification of the same theory [17]. Moreover, it can be verified directly at the string level in various examples.

Note that in the case when $v_i v_i' < 0$ for some i, the corresponding gauge kinetic term may vanish for finite values of S and S'. This singularity is inherited from the corresponding term in $D = 6$ and is related to the appearance of tensionless strings [7]. On the Heterotic side, for perturbative gauge fields, v is the Kac-Moody level while a non-zero v' may arise from one-loop threshold corrections in the $T \to i\infty$ limit.

4. Type I – Heterotic Duality and Quantum Corrections

In this section we discuss perturbative corrections to the prepotential on the Type I side. We want to understand how duality can be tested by comparing prepotentials, and eventually what information can be extracted from purely perturbative Type I computations. The two Peccei-Quinn symmetries dictate the following form of Type I prepotential:

$$F(S, S', U, A) = F^{(0)} + f_I(U, A) + \text{non-perturbative corrections}, \tag{4.1}$$

where $F^{(0)}$ is the tree-level prepotential (3.12) and $f_I(U, A)$ is the one-loop correction. Type I non-perturbative terms include instanton terms which are suppressed in the large S_2 and/or S_2' limit. Although f_I cannot depend on S and S' in a continuous way, its form may be different in the regions $S_2 > S_2'$ and $S_2 < S_2'$, *i.e.* for large and small K_3 volumes, *c.f.* eq.(3.2). In models which are invariant under "T-duality" ($\omega \to 1/\omega$) one obtains the same result in the two regions.

On the Heterotic side, there is only one perturbative Peccei-Quinn symmetry (associated to S), therefore the analogous expression is

$$F(S, T, U, A) = F^{(0)} + f_H(T, U, A) + \text{non-perturbative corrections}, \tag{4.2}$$

where we used the duality relation (3.5) which maps S' into T. Type I – Heterotic duality implies that

$$\lim_{T_2\to\infty} f_H = f_I|_{S_2>S_2'} . \tag{4.3}$$

As indicated above, this relation between perturbative prepotentials is valid only for $S_2 > S_2'$, since in the perturbative expansion of the Heterotic theory the large S limit is taken first. The other region, $S_2 < S_2'$, can only be reached non-perturbatively from the Heterotic side, therefore Type I perturbation theory can be *a priori* useful in studying the corresponding region $T_2 > S_2 \to \infty$. The two regions can be related, though, by $\omega \to 1/\omega$ duality which corresponds to non-perturbative $S \leftrightarrow T$ exchange. Note that if a given model admits also a Type II description, the full prepotential $F(S,T,U,A)$ can be computed exactly at the classical level on the Type II side.

Let us consider now a class of models based on orientifold reductions of Type IIB theory compactified on the K_3 orbifold T^4/Z_2 [11, 16]. In $D=6$ these models have one tensor multiplet and a maximal gauge group $U(16)\times U(16)'$. The two group factors are associated to open strings with Neumann-Neumann (N-N) and Dirichlet-Dirichlet (D-D) boundary conditions, respectively. In addition, there are massless hypermultiplets in the representations $2\times[(\mathbf{120},\mathbf{1})+(\mathbf{1},\mathbf{120})]$, $1\times(\mathbf{16},\mathbf{16})$, and 20 singlets. The $U(16)\times U(16)'$ model has an $\omega\to 1/\omega$ duality which interchanges the two $U(16)$ group factors. After compactifying on T^2 one obtains a $D=4$ model with the tree-level prepotential given by a special case of eq.(3.12):

$$F^{(0)} = SS'U - \tfrac{1}{2}\textstyle\sum_i (SA_i^2 + S'A_i'^2) \ , \tag{4.4}$$

where A and A' refer now to $U(16)$ and $U(16)'$ gauge multiplets, respectively. Note that, from the Heterotic point of view, $U(16)'$ has a purely non-perturbative origin.

In order to determine perturbative corrections to the prepotential in Type I theory, one could in principle follow the method applied on the Heterotic side, by extracting the one-loop Kähler potential $K^{(1)}$ from the universal (gauge group-independent) part of threshold corrections to gauge couplings [12, 13]. In fact, the one-loop threshold corrections have been recently studied in the Coulomb phase of the $U(16)\times U(16)'$ model [15]. They depend on U and Wilson-line moduli only, which is consistent with the general form of perturbative expansion (4.1). Without losing generality, we can focus on the $SU(16)$ subgroup originating from N-N boundary conditions. At zero Wilson lines, the corresponding gauge coupling takes the form:

$$\frac{4\pi^2}{g^2}=\frac{\pi}{2}S_2+\Delta \tag{4.5}$$

where S_2 is the tree-level contribution. The threshold correction is[1]

$$\Delta=6\int_0^\infty\frac{dt}{t}Z(t), \tag{4.6}$$

where

$$Z(t)=\sum_{p\in\Gamma_2}e^{-\pi t|p|^2/2} \tag{4.7}$$

is the partition function of the two-dimensional torus lattice Γ_2, with momenta restricted to Kaluza-Klein modes:

$$p=\frac{m_1+m_2\bar U}{\sqrt{2U_2}\,G^{1/4}} \tag{4.8}$$

with integer m_1 and m_2. Due to this restriction $SL(2,Z)_T$ symmetry is lost while $SL(2,Z)_U$ remains as a perturbative symmetry. The integral (4.6) has a logarithmic infrared divergence at $t\to\infty$, which reproduces the correct low-energy running of the gauge coupling with the beta function coefficient $b=6$.[2]

In the Heterotic theory, as mentioned before, the one-loop Kähler metric can be extracted from threshold corrections by using the relation [12]

$$\partial_U\partial_{\bar U}\Delta=-\frac{b}{(U-\bar U)^2}+4\pi^2K^{(1)}_{U\bar U}\ . \tag{4.9}$$

Using the identity

$$\partial_U\partial_{\bar U}e^{-\pi t|p|^2/2}=-\frac{1}{(U-\bar U)^2}\partial_t t^2\partial_t e^{-\pi t|p|^2/2}\ , \tag{4.10}$$

[1]Here we use the standard field theory normalization of gauge couplings which amounts to multiplying the result of ref.[15] by a factor of 2.

[2]The apparent ultraviolet divergence in eq.(4.6) disappears when the expression is appropriately cut off [15]. The potential divergence is anyway U-independent, and thus does not affect our discussion here.

which follows from eq.(4.8), we obtain after differentiating eq.(4.6):

$$\begin{aligned}\partial_U\partial_{\bar U}\Delta &= -\frac{6}{(U-\bar U)^2}\int_0^\infty dt\partial_t^2[tZ(t)]\\ &= -\frac{6}{(U-\bar U)^2}\partial_t\left(t\sum_{p\in\Gamma_2}e^{-\pi t|p|^2/2}\right)\Bigg|_0^\infty = -\frac{6}{(U-\bar U)^2}.\end{aligned} \tag{4.11}$$

The final result comes from the boundary term at $t=\infty$; the boundary at $t=0$ does not contribute as one can see easily by performing a double Poisson resummation in m_1 and m_2, cf. eq.(4.8). This result coincides with the first term of eq.(4.9). Does this mean that $K^{(1)}_{U\bar U}=0$?

The answer turns out to be no. In the Heterotic case, the above procedure relied on the fact that there are no one-loop corrections to the Planck mass [19]. In contrast, we will see that such corrections do appear in Type I theory. As a result, the Type I S field as defined in eq.(3.2) and below requires a redefinition at the one-loop level in order to remain an $N=2$ special coordinate. Indeed, assuming that the Einstein term receives a one-loop correction δ, so that the coefficient of $R^{(4)}$ in eq.(3.1) is

$$-\frac{1}{2}(e^{-2\phi_4}+\delta)R^{(4)}\ , \tag{4.12}$$

one has to redefine the dilaton $e^{-\phi_4}\to e^{-\phi_4}+\frac{1}{2}\delta e^{\phi_4}$ (to the leading order). The gauge coupling S_2 of eq.(3.2) is then redefined as $S_2\to S_2+\sqrt{G}\delta/(2S_2')$. As a consequence, the gauge couplings (4.5) receive a universal correction which upon using the relation (4.9) translates to

$$K^{(1)}_{U\bar U}=\frac{1}{16\pi S_2'}\sqrt{G}\,\partial_U\partial_{\bar U}\delta\ . \tag{4.13}$$

The above equation is also valid in the presence of Wilson lines. The momentum lattice Γ_2 is then shifted in a way described in [15]. Depending on the sector, one has $\Gamma_2\to A_i+\Gamma_2$ or $A_i+A_j+\Gamma_2$. The A_i's are defined in eq.(3.8) and the shifted lattice $A_i+\Gamma_2$ is defined with momenta as in eq.(4.8) with the numerator replaced by $m_1+m_2\bar U+\bar A_i$. It is now easy to verify that eq.(4.10) remains valid, hence also eq.(4.13). In the next section we compute the Planck mass correction δ.

Before concluding this section, we would like to make a few comments concerning the gauge group dependent part of threshold corrections in Type I string theories. Integrating eq.(4.11) and using $SL(2,Z)_U$ symmetry one obtains $\Delta = -b\ln[U_2|\eta(U)|^4] + \text{const}$, where η is the Dedekind eta-function. This result is valid for any gauge group factor and gives the U-modulus dependence of the one-loop gauge couplings in open string models, including the case of S'-dependent tree-level couplings (3.12) with $v' \neq 0$. The coefficient v' can be determined by anomaly cancellation in $D = 6$ and was shown [20] to be related to the four-dimensional $N = 2$ beta-function, $v'_i - v'_j = (b_i - b_j)/6$. This is consistent with duality since Type I theory reproduces the familiar result of Heterotic string models for the group dependent part of threshold corrections [21], $\Delta_i - \Delta_j = -(b_i - b_j)\ln[U_2T_2|\eta(U)\eta(T)|^4]$, in the limit $T = S' \to i\infty$.

5. One-loop Correction to Planck Mass

In order to extract the one-loop correction to Newton's constant, we consider an amplitude with two external graviton insertions

$$\partial_U \sum_{\substack{\text{one-loop}\\ \text{surfaces}}} \ll V_h(p_1,\varepsilon^1)V_h(p_2,\varepsilon^2) \gg \, = \, -\frac{1}{4}\varepsilon^1_{\mu\nu}\varepsilon^2_{\lambda\rho}\eta^{\mu\lambda}p_1^\rho p_2^\nu \,\partial_U\delta + \mathcal{O}(p^4), \tag{5.1}$$

where $\ll\gg$ stands for the path integral over world-sheets of given topology, $\epsilon^{1,2}$ are the polarization tensors and

$$V_h(p,\varepsilon) = 8\int d^2z\ \varepsilon_{\mu\nu} : (\bar\partial x^\nu + \frac{1}{2}\tilde\psi^\nu p\cdot\tilde\psi)(\partial x^\mu - \frac{1}{2}\psi^\mu p\cdot\psi)e^{ip\cdot x} : \tag{5.2}$$

is the graviton vertex operator in the zero-ghost picture. Here, x^μ are the space-time coordinates, ψ^μ ($\tilde\psi^\mu$) are their left- (right-) moving fermionic superpartners and $2d^2z \equiv dzd\bar z$. The one-loop surfaces of type-I theory are the torus ($\mathcal{T}$), annulus ($\mathcal{A}$), Möbius strip ($\mathcal{M}$) and Klein bottle ($\mathcal{K}$). Strictly-speaking the amplitude (5.1) vanishes on shell due to momentum conservation and the transversality conditions. A correct procedure is to start with the three-point amplitude between two gravitons and a U-modulus, which are on-shell but have complex momenta. Extracting the desired kinematic structure from this amplitude gives the same result as the amplitude (5.1), if we blindly ignore the fact that in this latter $p_1^\mu \varepsilon^2_{\mu\nu}$ should vanish [12].

In calculating the left-hand side of eq. (5.1) one must contract at least half of the fermions, or else the spin-structure summation gives zero. These contractions supply the desired powers of momenta, so we may set $p = 0$ elsewhere to find

$$\begin{aligned}\partial_U \delta \ &= \ -16 \sum_{\sigma=\mathcal{A},\mathcal{M},\mathcal{K}} \int_0^\infty \frac{dt}{t}(2\pi^2 t)^{-2} \partial_U Z(t) \int d^2z d^2w \frac{1}{2} \sum_{s=2,3,4} (-)^s \frac{\theta_s^2}{\eta^6} Z^{int}_{s,\sigma} \times \\ &\times \Big\{ \langle \partial x(z) \partial x(w) \rangle_\sigma \langle \tilde\psi(\bar z) \tilde\psi(\bar w) \rangle^2_{\sigma,\bar s} - \langle \partial x(z) \bar\partial x(\bar w) \rangle_\sigma \langle \tilde\psi(\bar z) \psi(w) \rangle^2_{\sigma,s} + c.c. \Big\} \quad (5.3)\end{aligned}$$

Here θ_s^2/η^6 is the oscillator contribution of bosonic and fermionic coordinates of the non-compact space plus two-torus; $(-)^s$ is the usual sign of spin-structure summation which for the desired kinematic structure can be restricted to the even ones; the factor $(2\pi^2 t)^{-2}$ comes from the integration over space-time momenta; $Z(t)$ is the sum over torus momenta which carries all U-dependence and, in the absence of Wilson lines, is given by eq.(4.7); finally $Z^{int}_{s,\sigma}$ is the contribution of the internal $N = 4$ superconformal theory describing the K_3 compactification to six dimensions, including for the annulus and Möbius, the multiplicity of Chan-Patton states. Notice that we have omitted the torus diagram in the above expression: this vanishes, as we will argue below, consistently with the fact that the Einstein term is not renormalized in $N = 2$ heterotic models.

The bosonic and fermionic propagators on $\mathcal{A}, \mathcal{M}, \mathcal{K}$ can be obtained from those on the torus by the method of images [23]. This is described in detail in the appendix. Using the fermionic propagators (A.9) one can put the spin-structure summation in the form

$$\sum_{s=2,3,4} (-)^s \frac{\theta_s^2(0)}{\eta^6} Z^{int}_s \times \frac{1}{4} \frac{\theta_s^2(v) {\theta'_1}^2(0)}{\theta_s^2(0)\theta_1^2(v)} = \pi^2 Z^{int}_{s=1} \qquad (5.4)$$

We have here used the fact that the partition function of the internal superconformal theory depends on spin structure only through the characters of the associated level-one

SU(2) Kac Moody algebra, so that the entire sum collapses by the Riemann θ-identity to an index [21]. This index is a trace over open-string Ramond or closed-string Ramond-Ramond states, weighted with the fermion-parity operator $(-)^{F_{int}}$. It implies that only massless six-dimensional states, that give rise to N=2 BPS multiplets in four dimensions, contribute to the amplitude, as is also the case for threshold corrections to the gauge couplings [22, 15]. Notice also the similarity of this result to the analogous expressions for the one loop corrections to gauge couplings and Kähler metric in the heterotic string [21, 12, 14].

To complete the calculation we must still perform the z- and w-integrals of the bosonic correlators of eq.(5.3). The corresponding calculation for the torus diagram would give zero for the following reason: fermions can only contract when they are both holomorphic or antiholomorphic, and $\langle \partial_z x \partial_w x \rangle$ is the derivative of a periodic function and thus vanishes, when integrated over the entire torus. This argument does not go through for the other three one-loop surfaces, which are obtained by modding out covering tori with an appropriate Z_2 involution I_σ. This is explained in the appendix, where we also derive the expressions

$$\langle \partial x(z) \partial x(w) \rangle_\sigma \;=\; \partial_z \partial_w P_B(z,w;\tau) + \frac{\pi}{4\tau_2} \tag{5.5}$$

$$\langle \partial x(z) \bar\partial x(\bar w) \rangle_\sigma \;=\; \partial_z \partial_{\bar w} P_B(z, I_\sigma(w);\tau) - \frac{\pi}{4\tau_2} \tag{5.6}$$

where P_B is the bosonic propagator on the covering torus, with modular parameter $\tau = it/2$, $1/2 + it/2$, $2it$ for the surfaces $\sigma = \mathcal{A}$, $\mathcal{M}$, $\mathcal{K}$, respectively. Now using the fact that for a function f that is periodic on the covering torus

$$\int_\sigma \partial_w f(w) - \partial_{\bar w} f(I_\sigma(w)) = \int_{\mathcal{T}} \partial_w f(w) = 0 \ , \tag{5.7}$$

we can easily perform the integrals of the bosonic propagators in eq. (5.3) with the result

$$\int d^2 z \int d^2 w \, \left\{ \langle \partial x(z) \partial x(w) \rangle_\sigma - \langle \partial x(z) \bar\partial x(\bar w) \rangle_\sigma + c.c. \right\} \;=\; \tag{5.8}$$

$$= \frac{\pi}{4}\tau_2 \quad = \quad \begin{cases} \pi t/8 \text{ for } \sigma = \mathcal{A}, \mathcal{M} \\ \pi t/2 \text{ for } \sigma = \mathcal{K} \end{cases}$$

Putting together all results, we arrive at our final expression for the one-loop renormalization of Newton's constant

$$\partial_U \delta = -\frac{1}{2\pi}\Big(\frac{1}{2}Z^{int}_{1,\mathcal{A}} + \frac{1}{2}Z^{int}_{1,\mathcal{M}} + 2Z^{int}_{1,\mathcal{K}}\Big)\int_0^\infty \frac{dt}{t^2}\partial_U Z(t) \ , \tag{5.9}$$

The index discussed previously counts hypermultiplets minus the graviton and vector multiplets in four dimensions [21, 14]. The relative factor of four between surfaces with and without a boundary, accounts for the fact that while an open-string hypermultiplet has four Ramond states, a closed-string hypermultiplet contains only a single Ramond-Ramond state [22, 15]. The final result takes thus the form

$$\partial_U \delta = -\frac{2}{\pi}\int_0^\infty \frac{dt}{t^2}\partial_U \left\{ \sum_{\text{BPS hypermultiplets}} e^{-\pi t M^2/2} - \sum_{\text{BPS vector multiplets}} e^{-\pi t M^2/2} \right\} \ , \tag{5.10}$$

where the masses in this expression originate from momentum in the internal two-torus. This expression is similar to the general formula for the one-loop $N = 2$ prepotential in the Heterotic case [14].

6. Example of String Triality

In this section we discuss one specific model which has simultaneous Type I, Heterotic and Type II descriptions. On the Type I side it originates from a six-dimensional model with one tensor multiplet and a completely broken gauge group. Anomaly cancellation (2.1) constraints such a model to contain 244 hypermultiplets. This model can be obtained from the class of orientifold constructions discussed in section 4, which are based on the K_3 orbifold T^4/Z_2. It belongs to a subclass which has a perturbative Heterotic description either as $SO(32)$ or $E_8 \times E_8$, compactified on $K_3 \times T^2$ with instanton numbers $(12, 12)$ [8]. The 8 five-branes are then located half at each fixed point of the orbifold. The D-D gauge group is broken down to $U(1)^{16}$, which furthermore obtain mass by coupling to sixteen hypermultiplets coming from the closed-string twisted sector [22]. The maximal gauge group in $D = 6$ is thus $SU(16) \times U(1)$ and the hypermultiplet spectrum consists of two antisymmetric **120**'s from the N-N sector, 16 fundamental **16**'s from the D-N sector and 4 singlets from the closed string sector. It is easy to see that this N-N gauge group can be broken completely by scalar vacuum expectation values, leaving exactly 244 hypermultiplets. Four of those come from the closed string sector, while 240 remain from the open strings.

Upon toroidal compactification to $D = 4$ one finds also the 3 universal vector multiplets S, S' and U. A Heterotic – Type II dual pair with the same massless spectrum has been considered before in refs.[2, 5]. On the Type II side, it corresponds to a IIA compactification on the Calabi-Yau threefold $WP_{1,1,2,8,12}(24)$ with Hodge numbers $h_{(1,1)} = 3$ and $h_{(1,2)} = 243$. The first indication that this pair is also equivalent to the above Type I construction comes from its perturbative $SL(2,Z)_U$ symmetry as well as from the $S \leftrightarrow S'$ symmetry which is the remnant of the six-dimensional $\omega \to 1/\omega$ duality. The latter is mapped to $S \leftrightarrow T$ exchange which was found to be an exact symmetry of the Calabi-Yau compactification [5]. In order to make a quantitative comparison, we will first use the formulae derived in section 5 to determine the one-loop correction to the Type I prepotential.

Applying eq.(5.10) in the case under consideration, one finds

$$\partial_U \delta = -\frac{2}{\pi} \int_0^\infty \frac{dt}{t^2} \times 240 \partial_U Z(t), \tag{6.1}$$

where the torus partition function $Z(t)$ is given in eq.(4.7). We can now extract the one-loop Kähler metric by applying $\partial_{\bar{U}}$ to eq.(5.9) and, using eq.(6.1) and the identity (4.10), to obtain

$$\partial_U \partial_{\bar{U}} \delta = -\frac{120}{\pi} \frac{1}{U_2^2} \int_0^\infty \frac{dt}{t^2} \partial_t t^2 \partial_t Z(t)\ . \tag{6.2}$$

After the change of variables $t = 1/l$ and double Poisson resummation in the T^2 partition function (4.7) one finds

$$\partial_U \partial_{\bar U} \delta = -\frac{480}{\pi} \frac{\sqrt{G}}{U_2^2} \int_0^\infty dl\, l \partial_l l^2 \partial_l \sum_{n_1,n_2}{}' e^{\frac{-4\pi\sqrt{G}}{U_2}|n_1+n_2\bar U|^2 l} = -\frac{240}{\pi^3\sqrt{G}} \sum_{n_1,n_2}{}' \frac{1}{|n_1+n_2U|^4} \qquad (6.3)$$

with the sum running over all integers except for $n_1 = n_2 = 0$. Substituting this result to eq.(4.13) we obtain the following one-loop correction to the Kähler metric:

$$K^{(1)}_{U\bar U} = -\frac{15}{\pi^4} \frac{1}{S_2'} \sum_{n_1,n_2}{}' \frac{1}{|n_1 U + n_2|^4} \; . \qquad (6.4)$$

We now turn to the Heterotic side of the model. The one-loop prepotential has been determined in ref.[13]. In the limit $T_2 \to \infty$, its third derivative reads

$$\partial_U^3 f_H(U, T_2 \to \infty) = -\frac{[\partial_U j(U)]^2}{\pi^2 j(U)[j(U) - j(i)]} = 4E_4(U) \qquad (6.5)$$

where $j(U)$ is the $SL(2,Z)$ modular function with a simple pole at infinity while the weight-4 lattice function

$$E_4(U) = \frac{45}{\pi^4} \sum_{n_1,n_2}{}' \frac{1}{(n_1+n_2U)^4} \; . \qquad (6.6)$$

Using the standard $N = 2$ formulae one finds that the one-loop corrected Kähler metric is

$$K_{U\bar U} = -\frac{1}{(U-\bar U)^2} + \frac{1}{S_2} K^{(1)}_{U\bar U} \qquad (6.7)$$

with $K^{(1)}_{U\bar U}$ given by the same expression as in eq.(6.4) with S_2' replaced by T_2. We see that the Type I result (6.4) corresponds to the $T \to i\infty$ limit of the Heterotic case, as expected from duality.

7. Summary

In this work, we studied the general features of the effective field theory describing $N = 2$ compactifications of Type I superstrings. A particular role is played by two dilaton-like fields associated to continuous Peccei-Quinn symmetries which remain unbroken in perturbation theory. Under Type I – Heterotic duality one of them is mapped to the Heterotic dilaton S and the other to the T modulus.

The one-loop computations presented in sections 5 and 6 provide a strong test of Type I – Heterotic duality conjecture for a class of $N = 2$ models based on $K_3 \times T^2$ compactifications. Weakly coupled Type I theory is recovered in the weakly coupled regime of the Heterotic theory in the limit of large Kähler modulus T (T^2 volume), provided that the K_3 volume of Type I compactification is large ($\omega^4 > 1$). When the K_3 volume is small, Type I perturbation theory probes a non-perturbative region in Heterotic theory. On the other hand, space-time non-perturbative effects in Type I theory which are exponentially suppressed at large T are mapped to world-sheet instantons on the Heterotic side. The most interesting conclusion of this work is the fact that the type-I prepotential is determined by the renormalization of Newton's constant and it is related to an index. It should be straightforward to extend these results to other type-I models.

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Appendix

We present here the derivation of various propagators which we use in the calculation of the one-loop amplitude (5.1, 5.3) on the annulus ($\mathcal{A}$), Möbius strip ($\mathcal{M}$) and Klein bottle ($\mathcal{K}$). These surfaces can be defined as quotients of tori under different involutions (see fig.1)

$$I_{\mathcal{A}}(z) = I_{\mathcal{M}}(z) = 1 - \bar{z} \ , \qquad\qquad I_{\mathcal{K}}(z) = 1 - \bar{z} + \tau/2 \ , \tag{A.1}$$

where $\tau = \tau_1 + i\tau_2$ is the modular parameter of the defining torus. The fundamental cells of the involutions can be chosen as follows:

$$\mathcal{A}: \quad z \in [0, 1/2] \times [0, \tau_2] \qquad \mathcal{M}: \quad z \in [1/2, 1] \times [0, \tau_2] \qquad \mathcal{K}: \quad z \in [0, 1] \times [0, \tau_2/2] \ .$$

Actually in section 5 we use the periodicity properties to make the integration region for the Möbius strip identical to the one for the annulus. The open string boundaries, corresponding to the loci of fixed points, are drawn as thick lines in fig.1. There are no fixed points for the Klein bottle representing the evolution and orientation flip of a closed string. Notice also that the three covering tori are characterized by different modular parameters: $\tau = it/2,\ \ 1/2 + it/2,\ \ 2it$ for the surfaces $\sigma = \mathcal{A},\ \mathcal{M},\ \mathcal{K}$, respectively.

The bosonic correlators can be expressed in terms of the propagator on the torus $\mathcal{T}$

$$\langle x(z)x(w)\rangle_{\mathcal{T}} = -\frac{1}{4}\ln\left|\frac{\theta_1(z-w|\tau)}{\theta_1'(0|\tau)}\right|^2 + \frac{\pi(z_2-w_2)^2}{2\tau_2} \equiv P_B(z,w) \ , \tag{A.2}$$

by symmetrizing under the corresponding involutions:

$$\begin{aligned} \langle x(z)x(w)\rangle_{\sigma} &= \frac{1}{2}\Big[P_B(z,w) + P_B(z, I_\sigma(w)) + P_B(I_\sigma(z), w) + P_B(I_\sigma(z), I_\sigma(w))\Big] \\ &= P_B(z,w) + P_B(z, I_\sigma(w)) \ . \end{aligned} \tag{A.3}$$

We follow throughout the conventions of Green, Schwarz and Witten [24] and we set $\alpha' = 1/2$. The above expressions must be supplemented with the usual holomorphic-regularization prescription, which ensures that left- and right-movers communicate only

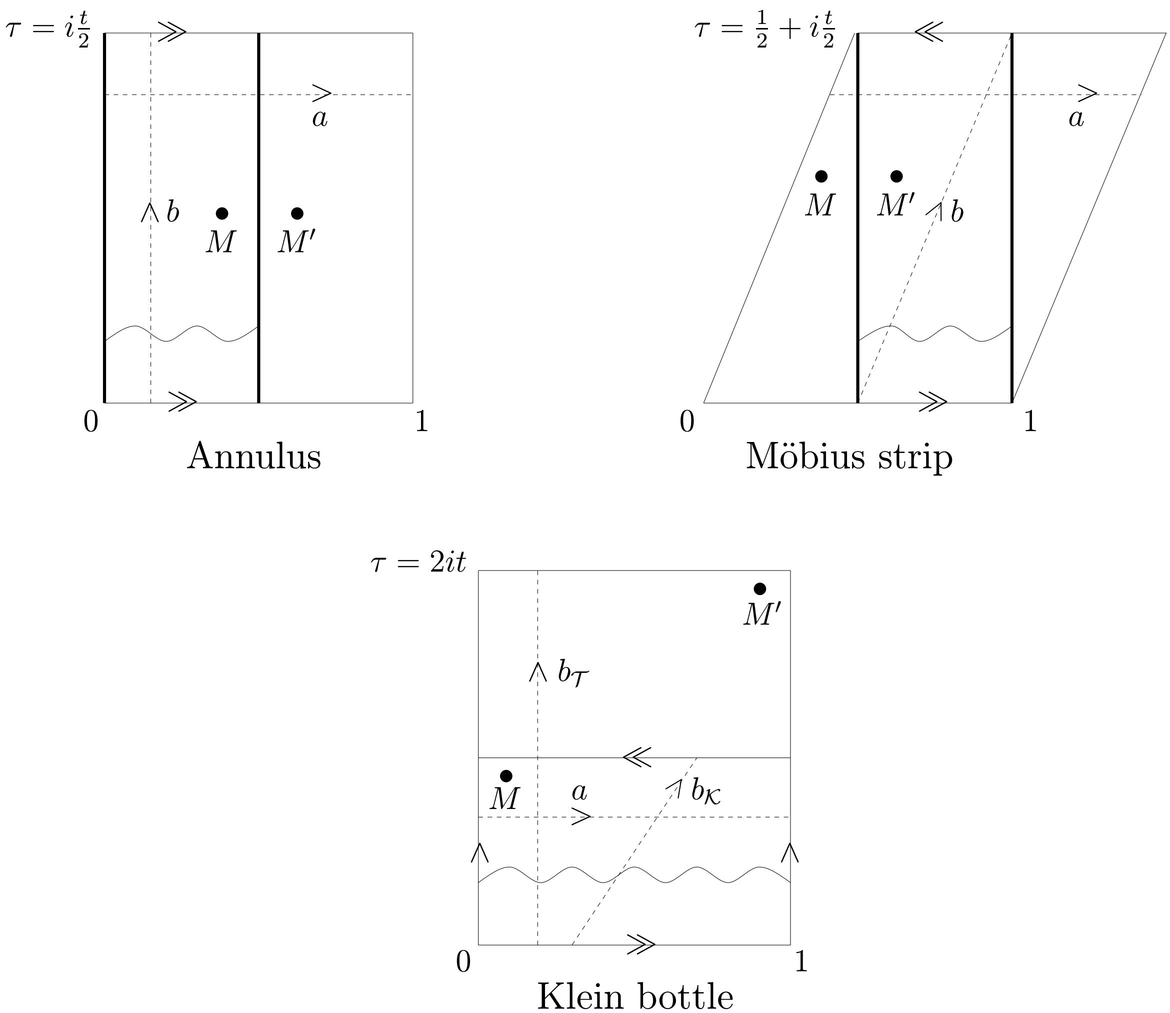

Figure 1: Covering tori and fundamental cells for the three one-loop surfaces $\sigma = \mathcal{A}$, $\mathcal{M}$, $\mathcal{K}$. The cycles are represented by dashed lines. The points M' are images of M under the appropriate involutions. The loci of fixed points drawn in thick are open-string boundaries.

through their zero modes. For the torus this implies that

$$\langle \partial x(z) \partial x(w) \rangle_{\mathcal{T}} = \partial_z \partial_w P_B(z,w;\tau) \ , \quad \text{but} \quad \langle \partial x(z) \bar{\partial} x(w) \rangle_{\mathcal{T}} = -\frac{\pi}{4\tau_2} \tag{A.4}$$

For the other three surfaces we have likewise

$$\langle \partial x(z) \partial x(w) \rangle_{\sigma} = \partial_z \partial_w P_B(z,w;\tau) + \frac{\pi}{4\tau_2} = -\frac{1}{4} \partial_z \partial_w \ln \left| \frac{\theta_1(z-w|\tau)}{\theta_1'(0|\tau)} \right|^2 + \frac{\pi}{2\tau_2} \tag{A.5}$$

and

$$\langle \partial x(z) \bar{\partial} x(\bar{w}) \rangle_{\sigma} = \partial_z \partial_{\bar{w}} P_B(z, I_\sigma(w);\tau) - \frac{\pi}{4\tau_2} = -\frac{1}{4} \partial_z \partial_{\bar{w}} \ln \left| \frac{\theta_1(z - I_\sigma(w)|\tau)}{\theta_1'(0|\tau)} \right|^2 - \frac{\pi}{2\tau_2} \ . \tag{A.6}$$

As a check one can verify that these propagators have the correct short distance singularity and periodicity properties on each surface. Furthermore the normal derivatives on the boundaries vanish, consistently with our choice of Neumann boundary conditions for the non-compact space-time coordinates.

We now turn to fermionic correlators. For 2-dimensional Majorana spinors

$$\Psi(z,\bar{z}) = \begin{pmatrix} \psi(z) \\ \tilde{\psi}(\bar{z}) \end{pmatrix} \tag{A.7}$$

the propagator on the torus reads

$$\langle \Psi(z,\bar{z}) \Psi^T(w,\bar{w}) \rangle_{\mathcal{T}} = P_F(s;z,w) \left(\frac{1+\gamma^3}{2} \right) + \bar{P}_F(\bar{s};\bar{z},\bar{w}) \left(\frac{1-\gamma^3}{2} \right) \ , \tag{A.8}$$

where $\gamma^3 = \text{diag}(1,-1)$, s and $\bar{s}$ are the even spin structures of the left and right components,

$$P_F(s;z,w) \equiv \langle \psi(z) \psi(w) \rangle_{\mathcal{T}}^s = \frac{i}{2} \frac{\theta_s(z-w|\tau)}{\theta_1(z-w|\tau)} \frac{\theta_1'(0|\tau)}{\theta_s(0|\tau)} \ . \tag{A.9}$$

and θ_s $(s = 2,3,4)$ are the even theta functions. The propagators on the other surfaces can be determined again by the method of images [23]. The left and right components of fermions have the same spin structure on all covering tori, except for the Möbius strip for which the three even spin structures are $(s,\bar{s}) = (2,2),(3,4)$ and $(4,3)$. One way to understand this subtlety is by noting that for the Möbius strip $\tau = \frac{1}{2} + \frac{it}{2}$ so that $\bar{\theta}_3 = \theta_4$.

The Z_2 involutions exchange left- with right-moving fermions up to a subtle choice of signs. One consistent choice is

$$\mathcal{I}^2_\sigma = \mathcal{I}^3_\sigma = \mathcal{I}^4_\sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \sigma = \mathcal{A},\, \mathcal{M}. \tag{A.10}$$

$$\mathcal{I}^2_\mathcal{K} = \mathcal{I}^3_\mathcal{K} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} , \qquad \mathcal{I}^4_\mathcal{K} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{A.11}$$

Symmetrizing the torus propagator under these involutions one finds

$$\begin{aligned} \langle \psi(z)\psi(w)\rangle_\sigma &= P_F(s; z, w) \\ \langle \psi(z)\tilde{\psi}(\bar{w})\rangle_\sigma &= P_F(s; z, I_\sigma(w)) \\ \langle \tilde{\psi}(\bar{z})\tilde{\psi}(\bar{w})\rangle_\sigma &= \bar{P}_F(\bar{s}; \bar{z}, \bar{w}). \end{aligned} \tag{A.12}$$

The reader can check that these propagators have the correct pole structure and periodicity properties.

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