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Strongly Coupled QED*

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ABSTRACT

A short review of some of the most relevant contributions to non-perturbative QED is done. Since a Gaussian behaviour of QED à la $\lambda\phi^4$ has been ruled out by the numerical data, I analyse the other two most reliable scenarios, i.e. triviality à la Nambu-Jona Lasinio and non-Gaussian critical behaviour. I give a suggestive theoretical argument against a Gaussian behaviour of QED à la Nambu-Jona Lasinio, and show how the numerical data for the susceptibility at the critical point of QED support this result.

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1. Introduction

Quantum Electro-Dynamics (QED) is the most standard and popular example of a non-asymptotically free gauge theory. Due to the peculiar form of its β renormalization group function, as obtained in perturbation theory, this model suffers from the Landau pole problem, which means that the renormalized coupling increases with the energy and diverges at some finite energy scale. The only way to construct a consistent theory in the infinite cut-off limit is to put the renormalized coupling equal to zero and the model is therefore trivial. In spite of that, QED is also the most successful quantum field theory from a phenomenological point of view. In fact, at energy scales much lower than the cut-off scale, QED describes perfectly the electromagnetic interactions. There is no doubt that QED incorporates very well the main features of electromagnetic interactions between electrons and photons in Nature.

There are several possible issues to the triviality problem of perturbative QED:

i. Non-perturbative contributions to the β function, which would be negligible at small energies, become relevant at large energy scales and change the structure of the β function. An ultraviolet stable or non-Gaussian fixed point could appear in this way as driving to a non-trivial interacting theory in the infinite cut-off limit. This issue includes also the possibility that a summation of all the perturbative series be able to change the structure of the β function. Therefore "non-perturbative contributions" should be understood in a wide sense.

ii. At large energy scales, and in the context of Unification Models for gauge interactions, other physical interactions between particles become relevant; these new interactions are responsible for the necessary changes in the β function in order to get a non-trivial model.

iii. The real world is only described by effective theories, which are interacting at small energy scales compared with the cut-off scale but are trivial if the cut-off is sent to infinity.

It could be that, after all, the perturbative Landau pole problem remains, even taking into account all "non-perturbative" contributions to the β function. However, and independently of this, it is very interesting and stimulating, at least from a theoretical point of view, to understand if non-asymptotically free gauge theories can be consistently constructed against the standard prejudice, and to look for the possible existence of non Gaussian fixed points in QED. This is however a hard business since, contrary to what happens in asymptotically free gauge theories, perturbative solutions of the renormalization group equations cannot be used as a guide in these investigations.

Another source of interest in non-perturbative QED comes from the composite models for the Higgs sector of the Standard Model. In fact it has been argued that a technigauge model able to strongly couple fermions and to produce massive fermion-antifermion bound states at large energy and with large anomalous dimensions (not asymptotically free!) could be on the basis of the dynamical mass generation mechanism.

These are the main reasons that motivated people 12 years ago [1] to start investigations on non-perturbative QED and to search for the existence of a non-Gaussian fixed point in this model. An important amount of work in this direction has been done in these 12 years. References [1-11] are some of the most relevant contributions to the field.

The existence of a continuous chiral transition at finite inverse gauge coupling β, separating a strongly coupled phase where chiral symmetry is broken from a weak coupling phase where the symmetry is realized, was found in [1] for the non-compact model in the quenched approximation. Another very interesting and important contribution to the field was the work by Leung, Love and Bardeen [3]: using the continuum formulation and solving the Schwinger-Dyson equations for the fermion self-energy in the quenched ladder approximation, they found large anomalous dimensions near the critical point for the composite operator $\bar{\psi}\psi$; this implies a scale dimension of 2 for $\bar{\psi}\psi$ and strongly suggests that four-fermion interactions become renormalizable outside perturbation theory. We should therefore include four-fermion interactions in a full analysis of non perturbative QED.

Reference [4] was the first numerical simulation of the compact model with dynamical fermions whereas in [5] it was shown that the compact model undergoes a first order chiral phase transition and therefore a continuum limit cannot be defined for this model. This point deserves further investigations since, as recently found [12], a continuous chiral transition appears in the compact model after the inclusion of four fermion interactions.

Reference [6] contains the results of the first numerical simulation of non-compact QED with dynamical fermions. In [7] it was shown that the chiral condensate follows a power law behaviour near the critical point, as opposed to the essential singularity behaviour or Miransky scaling [2].

The DESY-Jülich group claimed in [8] that they were able to fit the data for the chiral condensate with a logarithmically improved scalar mean field equation of state. This was a five-parameter fit, one of them being the critical coupling. From the results of these fits they concluded that QED has a trivial continuum limit. Triviality would be described by a Gaussian fixed point, the critical behaviour around it being similar to the one of the $\lambda\phi^4$ model. In reference [9], two completely independent and uncorrelated calculations were done using different numerical approaches and different operators. The extracted critical couplings, which were in perfect agreement, did not support however the logarithmically improved scalar mean field fit of the DESY-Jülich group. The data of [9] were compatible, as discussed below, with a Gaussian behaviour of QED à la Nambu-Jona Lasinio as well as with a pure power law equation of state.

Actually, the high degree of disagreement between the numerical estimations of the critical coupling in [8] and [9] is surprising, a disagreement that in our more recent calculations in the 14^4 lattice is of about 100 standard deviations [13]. Since the numerical data for the chiral condensate in both calculations were in good agreement, the only source of disagreement must be in the analysis of these data, i.e. in the use of different fitting equations in [8] and [9]. The DESY-Jülich group has recently published new results on the logarithmically improved scalar mean field fits of QED with higher statistics [14]. In this paper the $\frac{\chi^2}{d.o.f.}$ of the fits is also shown. The best reported value is $\frac{\chi^2}{d.o.f.} = 7.63$ for around 70 degrees of freedom. These numbers give an extremely low confidence level for these fits and rule out a Gaussian behaviour of QED à la $\lambda\phi^4$. Indeed, as suggested by Horowitz a few years ago [10], if the continuum limit of QED is trivial, triviality would manifest itself as in the Nambu-Jona Lasinio model rather than as in the $\lambda\phi^4$ model.

It is simple to understand this statement if we take it into account that QED in the infinite gauge coupling limit and Nambu-Jona Lasinio in the infinite four-fermion coupling limit are equivalent. Both models have composite scalars and pseudo-scalars in the spectrum and, as suggested by Kocić and Kogut, and corroborated in a numerical simulation of the Nambu-Jona Lasinio model with discrete Z_2 symmetry [10], triviality in a theory with composite scalars seems not to manifest itself in the same way as in a theory with fundamental scalars. This result can be shown for the Nambu-Jona Lasinio model in a very simple and elegant way by means of the large N expansion.

Reference [11] contains the results of an analysis of the gauged Nambu-Jona Lasinio model, i.e. QED plus four-fermion interactions with continuous chiral symmetry. A mean field approximation for the fermion field was used in [11], whereas fluctuations of the gauge field were taken into account outside of approximations. The finding of a continuous phase transition line of critical points in the gauge-coupling–four-fermion-coupling plane, with critical indices taking non-mean field values, at least near the critical point of QED [11], was very suggestive. Notwithstanding that we would like to understand the nature of the continuum limit of strongly coupled QED outside of approximations.

I will develop in the next section a suggestive argument against a Gaussian behaviour of QED à la Nambu-Jona Lasinio. In section 3 I will discuss some recent results of the Frascati-Zaragoza collaboration on the chiral equation of state of QED at criticality, and the last section will be devoted to the conclusions. Even if I am not going to discuss it here, I would like to state that there are other interesting approaches for the pure gauge compact model based on a suppression of monopole contributions [15, 16], the results being appealing enough to deserve further investigations.

2. The Effective Action

As discussed in the previous section, a Gaussian behaviour of QED as in a logarithmically improved scalar mean field theory has been ruled out [9, 13], also by the very low confidence level of the fits reported in [14]. This notwithstanding, a Gaussian behaviour of QED à la Nambu-Jona Lasinio seems to be compatible with the data and it is a difficult task to distinguish this behaviour from a non-Gaussian behaviour. Indeed in both cases the effective δ exponent which measures the response of the system to a external symmetry breaking field at the critical point is less than 3. The effective magnetic exponent β_m, which controls the behaviour of the chiral condensate near the critical point, is larger than its mean field value of 0.5 in both cases; the data for the susceptibility exponent γ point clearly to the mean field value $\gamma = 1$ [17]. To differentiate between logarithmic violations to mean field scaling and a pure power law scaling is actually very difficult (see next section). This is the reason why, before discussing the numerical results, I would like to give an argument based on theoretical grounds, strongly suggesting that QED and Nambu-Jona Lasinio are in different universality classes. This argument is based on the definition of an effective action for the Nambu-Jona Lasinio model and on its comparison with the fermion effective action of QED at fixed energy density.

As we have shown in the past [9, 17], the critical behaviour of QED is controlled by a singularity of the fermion effective action, which holds at a critical value of the gauge energy density E around 1. I will now show how we can define an effective action for the Nambu-Jona Lasinio model, the strong four-fermion coupling expansion of which has as first contribution the

QED effective action. Before that, let me review the main features of the construction of the fermion effective action in QED.

The lattice action of non-compact QED with staggered fermions is

$$S = \frac{\beta}{2}\sum_{n,\mu<\nu} F^2_{\mu\nu}(n) + \frac{1}{2}\sum_{n,\mu}\eta_\mu(n)(\bar\psi_n U_\mu(n)\psi_{n+\mu} - \bar\psi_{n+\mu}U^+_\mu(n)\psi_n) + m\sum_n \bar\psi_n\psi_n. \tag{1}$$

where $\beta = 1/g^2$, $F_{\mu\nu}$ is the naive regularization of the continuum e.m. tensor, $\eta_\mu(n)$ are the Kogut-Susskind phases and $U_\mu(n)$ are the compact link connections. In the chiral limit this action is invariant under the continuous chiral $U(1)$ transformations

$$\psi_n \;\rightarrow\; \psi_n\, e^{i\alpha(-1)^{n_1+\ldots+n_d}} \qquad \bar\psi_n \;\rightarrow\; \bar\psi_n\, e^{i\alpha(-1)^{n_1+\ldots+n_d}}. \tag{2}$$

Action (1) can be written in a compact form as

$$S = \frac{\beta}{2}\sum_{n,\mu<\nu} F^2_{\mu\nu}(n) \;+\; \bar\psi\Delta(m, U_\mu(n))\psi \tag{3}$$

with the following form of the Dirac operator

$$\Delta = mI + i\Lambda(U_\mu(n)) \tag{4}$$

$\Lambda(U_\mu(n))$ being a Hermitian matrix. After integrating out the fermion degrees of freedom the partition function is

$$Z = \int [dA_\mu(n)] e^{-\frac{\beta}{2}\sum_{n,\mu<\nu} F^2_{\mu\nu}(n)} \det\Delta(m, U_\mu(n)) \tag{5}$$

In order to construct the fermion effective action at fixed gauge energy density we introduce in the previous expression a trivial identity in the following way [18]:

$$Z = \int [dA_\mu(n)] dE \delta\left(\frac{1}{12V}\sum F^2_{\mu\nu}(n) - E\right) e^{-\frac{\beta}{2}\sum_{n,\mu<\nu} F^2_{\mu\nu}(n)} det\Delta \tag{6}$$

where V in (6) is the lattice volume. If we now introduce the density of states $N(E)$ at fixed gauge energy density as

$$N(E) = \int [dA_\mu(n)]\delta\left(\frac{1}{12V}\sum F^2_{\mu\nu}(n) - E\right), \tag{7}$$

the partition function (6) can be written as

$$Z = \int dEN(E)e^{-V6\beta E}\langle\,\det\Delta\rangle_E, \tag{8}$$

with the following definition for the mean value of the fermion determinant at fixed E:

$$\langle\,\det\Delta\rangle_E = \frac{\int[dA_\mu(n)]\det\Delta\delta(\frac{1}{12V}\sum F_{\mu\nu}^2(n)-E)}{\int[dA_\mu(n)]\delta(\frac{1}{12V}\sum F_{\mu\nu}^2(n)-E)}. \tag{9}$$

Contrary to what happens in the compact case, the partition function (6), (8) is divergent due to gauge invariance and to the use of non-compact variables. These divergences, which are avoidable by fixing the gauge, can also be regularized, and they cancel when taking vacuum expectation values.

Since the pure gauge action is a quadratic form of the gauge fields, the density of states $N(E)$ can be analytically computed [9]. At large lattice volumes it behaves like

$$N(E) = CE^{V\frac{3}{2}}, \tag{10}$$

where C is a divergent constant that contains the divergences due to the non-compact integration, and which cancel when computing mean values.

Now taking into account the fact that the fermion determinant is positive-definite, we can introduce the normalized effective action $S_{eff}^{QED}(E,m)$ as

$$S_{eff}^{QED}(E,m) = -\frac{1}{V}\log\langle\,\det\Delta\rangle_E, \tag{11}$$

which takes a finite value in the thermodynamical limit. Expressions (8), (10) and (11) allow us to write the partition function as a simple one-dimensional integral over the gauge energy density:

$$Z = \int dEe^{V\frac{3}{2}logE-V6\beta E-VS_{eff}^{QED}(E,m)}. \tag{12}$$

The only unknown function in the previous expression is the effective action $S_{eff}^{QED}(E,m)$. It contains all the information on the critical behaviour of this model.

The thermodynamics of this model can be exactly solved in the infinite volume limit by the saddle point technique [9]. The mean plaquette energy for instance will be given by the solution of the saddle point equation, which maximizes the integrand in (12).

Let me now consider the chiral limit. In this limit, the model has a continuous phase transition at some finite value of the gauge coupling, in perfect analogy with magnetic systems. Since first-order phase transitions are excluded at small numbers of flavours [19, 20] it is obvious that if the effective action $S_{eff}^{QED}(E)$ is a regular function of E the solutions of the saddle point equations will also be regular. No singularities in the plaquette energy or specific heat will be found at any finite β. Therefore in order to get a continuous phase transition at finite β we need a singularity in the effective action $S_{eff}^{QED}(E)$ at some finite value of E. This was just what we found by numerical computation of $S_{eff}^{QED}(E)$ [9].

The investigation of the main features of this singularity by numerical methods is rather difficult. However, what is important for the argument I am going to develop here is that this singularity controls the critical behaviour of the model and determines its critical exponents.

The next step in this argument is to define an effective action for the Nambu-Jona Lasinio model and to compare it with the QED effective action. The lattice action for the Nambu-Jona Lasinio model with continuous chiral symmetry and in the chiral limit is

$$S_{NJL} = \frac{1}{2}\sum_{n,\mu}\eta_\mu(n)(\bar{\psi}_n\psi_{n+\mu} - \bar{\psi}_{n+\mu}\psi_n) \; - \; G\sum_{n,\mu}\bar{\psi}_n\psi_n\bar{\psi}_{n+\mu}\psi_{n+\mu} \tag{13}$$

where G is the four-fermion coupling. This action is also invariant under the continuous chiral transformations (2).

We can bilinearize the previous action with the help of an auxiliary compact $U(1)$ vector field as follows

$$\begin{aligned} S_{NJL} &= \frac{1}{2}\sum_{n,\mu}\eta_\mu(n)(\bar{\psi}_n\psi_{n+\mu} - \bar{\psi}_{n+\mu}\psi_n) \\ &+ G^{\frac{1}{2}}\sum_{n,\mu}\eta_\mu(n)(\bar{\psi}_n e^{i\theta_\mu^F(n)}\psi_{n+\mu} - \bar{\psi}_{n+\mu}e^{-i\theta_\mu^F(n)}\psi_n). \end{aligned} \tag{14}$$

$\theta_\mu^F(n)$ are compact phases taking values in the $(-\pi, \pi)$ interval. Action (14) can be written in a compact form as

$$S_{NJL} = \bar{\psi}\Delta_{NJL}(\theta_\mu^F)\psi \tag{15}$$

with the following expression for the Dirac operator:

$$\Delta_{NJL} = \Delta_0 \; + \; (4G)^{\frac{1}{2}}\Delta(\theta_\mu^F). \tag{16}$$

Δ_0 is the Dirac operator for a free fermion theory, whereas $\Delta(\theta_\mu^F)$ is the QED Dirac operator with the gauge field A_μ replaced by the auxiliary vector field θ_μ^F.

The partition function is the path integral over the auxiliary vector field configurations of the determinant of the Dirac operator

$$Z_{NJL} = \int [d\theta^F] \det \Delta_{NJL}(\theta_\mu^F). \tag{17}$$

Since the fermion determinant in (17) is a periodic function of the compact variables θ_μ^F, we can also use non-compact variables in the definition of Z_{NJL}. The divergences that will appear from the use of non-compact degrees of freedom can be regularized; they cancel when computing vacuum expectation values, as in non-compact QED.

The next steps are the same as were used before in the definition of the QED effective action. First we define the energy density of the auxiliary vector field θ_μ^F just as if it were a gauge field with a "e.m. tensor":

$$F_{\mu\nu}^F(n) = \theta_\mu^F(n) \ + \ \theta_\nu^F(n+\mu) \ - \ \theta_\mu^F(n+\nu) \ - \ \theta_\nu^F(n). \tag{18}$$

Inserting now in the partition function (17) a δ function

$$\delta\left(\frac{1}{12V}\sum_{n,\mu<\nu} F_{\mu\nu}^2(n) - E\right)$$

and an integral over the energy density E of the auxiliary vector field, we get

$$Z_{NJL} = \int dE e^{V\frac{3}{2}\log E - VS_{eff}^{NJL}(E,G)} \tag{19}$$

with

$$S_{eff}^{NJL}(E,G) = -\frac{1}{V}\log \langle \det \Delta_{NJL}\rangle_E, \tag{20}$$

and where the definition of the mean value $\langle\,\rangle_E$ over configurations at fixed energy density of the vector field θ_μ^F is the same as given in (9).

Again in this case the thermodynamics of the system can be exactly solved in the infinite volume limit by the saddle point technique. For instance, the mean value of the four-fermion term in the action (13) will be given by

$$\langle \bar\psi_n\psi_n\bar\psi_{n+\mu}\psi_{n+\mu}, \rangle = \frac{\partial S_{eff}^{NJL}(E,G)}{\partial G} \tag{21}$$

evaluated at the solution $E = E_0(G)$ of the saddle point equation for each fixed value of the four-fermion coupling G.

Doing now a cumulant expansion of $\langle \det \Delta_{NJL}, \rangle_E$ and inserting in it the expression (16) for the Nambu-Jona Lasinio Dirac operator, we get a strong four-fermion coupling expansion for the Nambu-Jona Lasinio effective action

$$S_{eff}^{NJL}(E,G) = S_{eff}^{QED}(E) \ + \ S_1(E,G). \tag{23}$$

The first contribution to this expansion $S_{eff}^{QED}(E)$ is just the effective action of QED, whereas $S_1(E,G)$ in (23) vanishes in the $G = \infty$ limit.

We have now at hand everything we need to finish the argument. As discussed before, the QED effective action has a singularity that holds at a critical value of the energy E around 1 [9, 17] and which controls the critical behaviour of the model. Since the QED effective action is the first contribution to the Nambu-Jona Lasinio effective action (23) in the strong four fermion coupling expansion of it, the QED singularity will also be a singularity of the Nambu-Jona Lasinio effective action. Hence, if by running the four-fermion coupling G the solution of the saddle point equations $E_0(G)$ for this model reaches the critical energy of QED, the critical behaviour of the Nambu-Jona Lasinio model will be controlled also by the same singularity that controls the critical behaviour in QED. Both models will be in the same universality class even if the behaviour far from the critical point could be very different.

What breaks the previous argument is the assumption that the singularity of QED is accessible to the Nambu-Jona Lasinio model. In fact since there is no kinetic term for the auxiliary vector field in action (14) and since the Nambu-Jona Lasinio effective action is bounded, the solution of the saddle point equation is $E_0(G) = \infty$ for every value of the four-fermion coupling G. The singularity of QED is not accessible to the Nambu-Jona Lasinio model. The critical behaviour of the last model must be controlled by a singularity in the four-fermion coupling G of its effective action at $E = \infty$.

The argument I have developed here has a close resemblance with real space renormalization group based arguments. Even if it does not constitute a rigorous proof, it strongly suggests that both models are in different universality classes.

3. The QED Chiral Equation of State at Criticality

In this section I will discuss some recent results on the behaviour of the chiral order parameter at the critical point of QED [13]. The main ingredients of this calculation are the use of the Microcanonical Fermion Average approach (MFA) to simulate dynamical fermions [18], and the computation of the chiral transverse susceptibility in the Coulomb phase and in the chiral limit, without the help of arbitrary mass extrapolations.

In the MFA approach the massless Dirac operator is exactly diagonalized with the help of a modified Lanczos algorithm. In contrast to what happens with other standard algorithms such as Hybrid Monte Carlo, computations in the chiral limit are therefore allowed in this approach. We only need to take care that the commutation of the chiral and thermodynamical limits can be done for the operator we are interested in. In the broken phase, characterized by a non-symmetric vacuum under chiral transformations, this commutation will give wrong results if applied to operators which are not invariant under chiral transformations. However the symmetry of the vacuum under chiral transformations allows in principle such a commutation in the Coulomb phase. This result can be rigorously demonstrated, for instance for the transverse susceptibility. The only ingredient in this demonstration is the fact that the susceptibility in the Coulomb phase and in the chiral limit is always finite, which implies that the spectral density of eigenvalues of the Dirac operator $\rho(\lambda)$ near the origin behaves like λ^p, with $p > 1$. Using this result, it is simple to show that the transverse susceptibility is a continuous function of the bare fermion mass m at $m = 0$.

The computation of the chiral susceptibility in the Coulomb phase and in the chiral limit allowed us to get very precise measurements of the critical couplings [13, 17]. This is a crucial point since, as discussed in [9], the numerical determination of critical indices is very sensitive to the value of the critical coupling. As I mentioned in the introduction, the numerical estimations of the critical coupling by the Illinois group and the Frascati-Zaragoza group [9] were in very good agreement even if numerical approaches and techniques used in these calculations were very different. I would like to mention here that we have four independent determinations of the critical coupling based on the analysis of the fermion effective action [9], probability distribution function of the chiral order parameter [21], Lee-Yang zeros of the partition function and chiral susceptibilities [13, 17]. The results of the four calculations were in perfect agreement, but the values extracted from the computation of the chiral susceptibility are the best from a statistical point of view. The reason for that was the observation that chiral susceptibility follows an almost perfect linear behaviour as a function of the gauge energy density in the Coulomb phase, even relatively far from the critical point. The very high confidence level of the linear fits for this operator are responsible for the very precise measurements of the critical couplings (see Table I).

Using the values for the critical couplings reported in Table I, fits of the data for the chiral order parameter at the critical point were done with the two most reliable equations of state. As previously stated, a Gaussian behaviour of strongly coupled QED à la $\lambda\Phi^4$ has been ruled out by the numerical data [9, 13, 14]. I also discussed how, from theoretical prejudices, we expect that, if QED is trivial, triviality would manifest itself as in the Nambu-Jona Lasinio model. Even if I have given in the previous section a strong argument against a Gaussian behaviour of strongly coupled QED à la Nambu-Jona Lasinio, I will show here the results of the fits of the chiral order parameter with a Nambu-Jona Lasinio equation of state and with a pure power law, the last corresponding to a non-Gaussian behaviour of the model.

The Nambu-Jona Lasinio-like equation of state at the critical point is parametrized as follows

$$A\langle\,\bar{\psi}\psi\,\rangle^3 \log\left(\frac{1}{\langle\,\bar{\psi}\psi\,\rangle}\right) + C\langle\,\bar{\psi}\psi\,\rangle^3 = m. \tag{24}$$

The pure cubic term in this equation fixes the scale of the logarithmic violations to scaling. The power of the logarithm is set equal to 1, as follows from the $\frac{1}{N}$ expansion of the Nambu-Jona Lasinio model [22].

The pure power law equation of state has the following simple form

$$B\langle\,\bar{\psi}\psi\,\rangle^\delta = m. \tag{25}$$

The fits of Figs. 1 and 2 have been done in the mass range $0.01 < m < 0.08$ and 14^4 lattice. In both Figs. we plot the left-hand side of the corresponding equation of state against the right-hand side. Very high quality fits are obtained in the two cases, as follows from the results reported in Table II, where I show the $\frac{\chi^2}{d.o.f.}$ of the fits in $10^4, 12^4$ and 14^4 lattices and for a number of flavours running from 1 to 8. The errors in this table take into account the error in the determination of the critical coupling (see Table I).

The conclusion of this analysis is that the data for the chiral condensate at the critical point are not enough sensitive to distinguish between the two possibilities. Both equations of state, with the corresponding fitted parameters, are actually very similar in the mass region explored and even at much smaller masses.

The next step in complexity of the numerical analysis is to test the behaviour of the derivative of the order parameter, i.e. the longitudinal susceptibility. We expect this operator, being a higher order derivative of the free energy, to be more sensitive to different equations of state.

In the case of a Nambu-Jona Lasinio-like equation of state we get for the longitudinal susceptibility χ_L the following equation

$$3m - \langle \bar{\psi}\psi \rangle \chi^{-1} = A \langle \bar{\psi}\psi \rangle^3 \tag{26}$$

whereas for the pure power law the equation is

$$3m - \langle \bar{\psi}\psi \rangle \chi^{-1} = (3-\delta) B \langle \bar{\psi}\psi \rangle^{\delta}. \tag{27}$$

Equations (26) and (27) have two remarkable things. First, the logarithmic violations to scaling of equation (24) disappear in the corresponding equation for the susceptibility (26). This is very welcome from a numerical point of view since it is very difficult to distinguish $\delta = 3$ with logarithmic violations driving to a $\delta_{eff} < 3$ from $\delta < 3$. Second, if we do a log–log plot of $\log(3m - \langle \bar{\psi}\psi \rangle \chi^{-1})$ against $\log \langle \bar{\psi}\psi \rangle$ we expect a linear behaviour in both cases (26), (27). The only difference is the slope of the linear fit, which should be 3 in the Gaussian case and less than 3 for the non-Gaussian one.

Figure 3 contains the results for the log–log plot in the 12^4 lattice. All the plotted points in this Fig. are in the mass range $(0.015 - -0.035)$. The straight line in this Fig. is a linear fit. From the slope of this fit we get that the mean field-like equation of state (Nambu-Jona Lasinio) is also ruled out since the extracted value for the critical index is $\delta = 1.98(16)$.

These results, which are very appealing, unfortunately need to be improved in larger lattices before they can be taken as a "proof" of the existence of a non-Gaussian fixed point in strongly coupled QED. We have not enough statistics to check the previous results in the 14^4 lattice. This notwithstanding, the result $\delta = 1.98(16)$, which is very similar to the quenched value reported in [23], is far enough from the Gaussian $\delta = 3$ value to consider it as a strong indication of a non-Gaussian behaviour.

To finish this section I would like to comment on some features of Fig. 3. We have found deviations from the linear behaviour of the function plotted in this Fig. in both the left-hand and right-hand corners. The right-hand corner corresponds to very small bare fermion masses, and we expect deviations from scaling in this region due to finite-size effects. This is the region we should explore on larger lattices in order to see if the scaling window enlarges to smaller fermion masses.

The physical origin for the deviations from scaling in the left-hand corner is different. In this case we have large bare fermion masses and it is most likely that deviations from scaling come from subleading contributions to the dominant scaling in equations (25), (27).

Another surprising feature, which follows from the analysis of the results plotted in Fig. 3, is the value $\delta = 1.98(16)$. This value is quite different from the value $\delta = 2.81$ obtained for the same lattice size from the fits of the chiral order parameter with the pure power law equation (25). To understand this discrepancy we should take into account that the fits of equation (25) were done in a much larger mass interval than the one used in Fig. 3. As stated before, the chiral order parameter is not very sensitive to the different equations of state. Playing with the two free parameters of equation (25), we can of course reproduce the data for the chiral condensate in the mass interval $(0.015 - -0.035)$ and $\delta \sim 2$. The susceptibility fits, as expected, are much more sensitive to the form of the equation of state, even if the mass interval were these fits work well is significantly reduced.

4. Summary

I have done in the Introduction of this paper a short review of some of the most relevant contributions to the investigation of non-perturbative QED [1–11]. The relevance of four-fermion interactions in this kind of investigations [3] has been emphasized. Unfortunately, and due to technical difficulties, results for the full model with four-fermion interactions are available only in the mean field approximation for the fermion field [11]. These results are however very exciting, since a non-Gaussian behaviour appears in this approximation [11].

A Gaussian behaviour of QED à la $\lambda\phi^4$ has been ruled out by the results of the Illinois and Frascati-Zaragoza groups [9, 13] as well as by the extremely low confidence level of the logarithmically improved scalar mean field fits of the DESY-Jülich group [14]. These results, beside theoretical prejudices based on the equivalence between QED and Nambu-Jona Lasinio models in the strong coupling limit, tell us that the most reliable realization of triviality in QED, assuming it is trivial, is à la Nambu-Jona Lasinio.

I have given in section 2 a suggestive theoretical argument against a Gaussian behaviour of QED à la Nambu-Jona Lasinio. The argument is based on the definition of an effective action for the last model and on its comparison with the fermion effective action of QED at fixed energy density. I have shown how the singularity in the QED effective action, which controls the critical behaviour of this model, is not accessible to the Nambu-Jona Lasinio model and therefore both models are most likely in different universality classes.

The previous expectations are corroborated by the results of the fits of the longitudinal susceptibility at the critical point of QED reported in section 3. The value of the critical index $\delta = 1.98(16)$ extracted from these fits is quite far from its mean field value $\delta = 3$. The most reliable scenario is therefore that in which strongly coupled QED has a non-Gaussian fixed point and, provided that hyperscaling is verified, a non-trivial continuum limit.

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Figure captions

Figure 1. $0.97\langle\,\bar\psi\psi\,\rangle^3\log(\frac{1}{\langle\,\bar\psi\psi\,\rangle})+2.12\langle\,\bar\psi\psi\,\rangle^3$ against m in the 14^4 lattice. The solid line is a fit with a straight line of slope equal to 1 and crossing the origin.

Figure 2. $2.39\langle\,\bar\psi\psi\,\rangle^{2.73}$ against m in the 14^4 lattice. The solid line is a fit with a straight line of slope equal to 1 and crossing the origin.

Figure 3. $\log\,(3m-\langle\,\bar\psi\psi\,\rangle\chi^{-1})$ against $\log\,\langle\,\bar\psi\psi\,\rangle$ at the critical point of the 12^4 lattice. The solid line is a linear fit.

Table captions

Table I Critical couplings extracted from the susceptibilities in $10^4, 12^4$ and 14^4 lattices at $N_f = 1, 2, 3, 4, 8$.

Table II Results for the constant B and the critical index δ (pure power law equation (25)) and for the two constants A and C (mean field equation (24)) at lattice sizes $10, 12, 14$ and $N_f = 1, 2, 3, 4, 6, 8$. All the fits were done in the mass interval $(0.01 - -0.08)$.

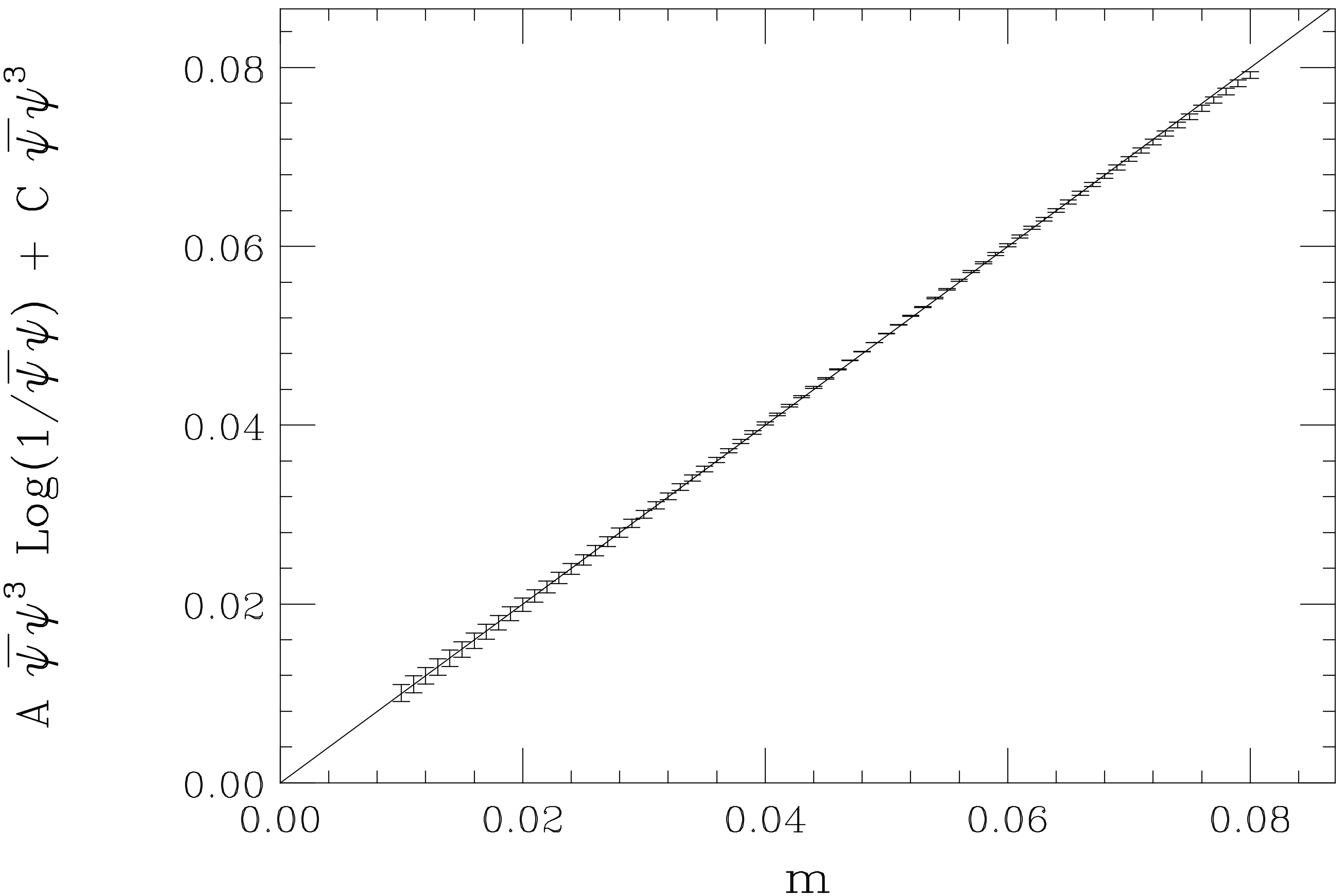

A $\bar{\psi}\psi^3$ Log$(1/\bar{\psi}\psi)$ + C $\bar{\psi}\psi^3$
0.00
0.02
0.04
0.06
0.08
0.00
0.02
0.04
0.06
0.08
m

Fig. 1

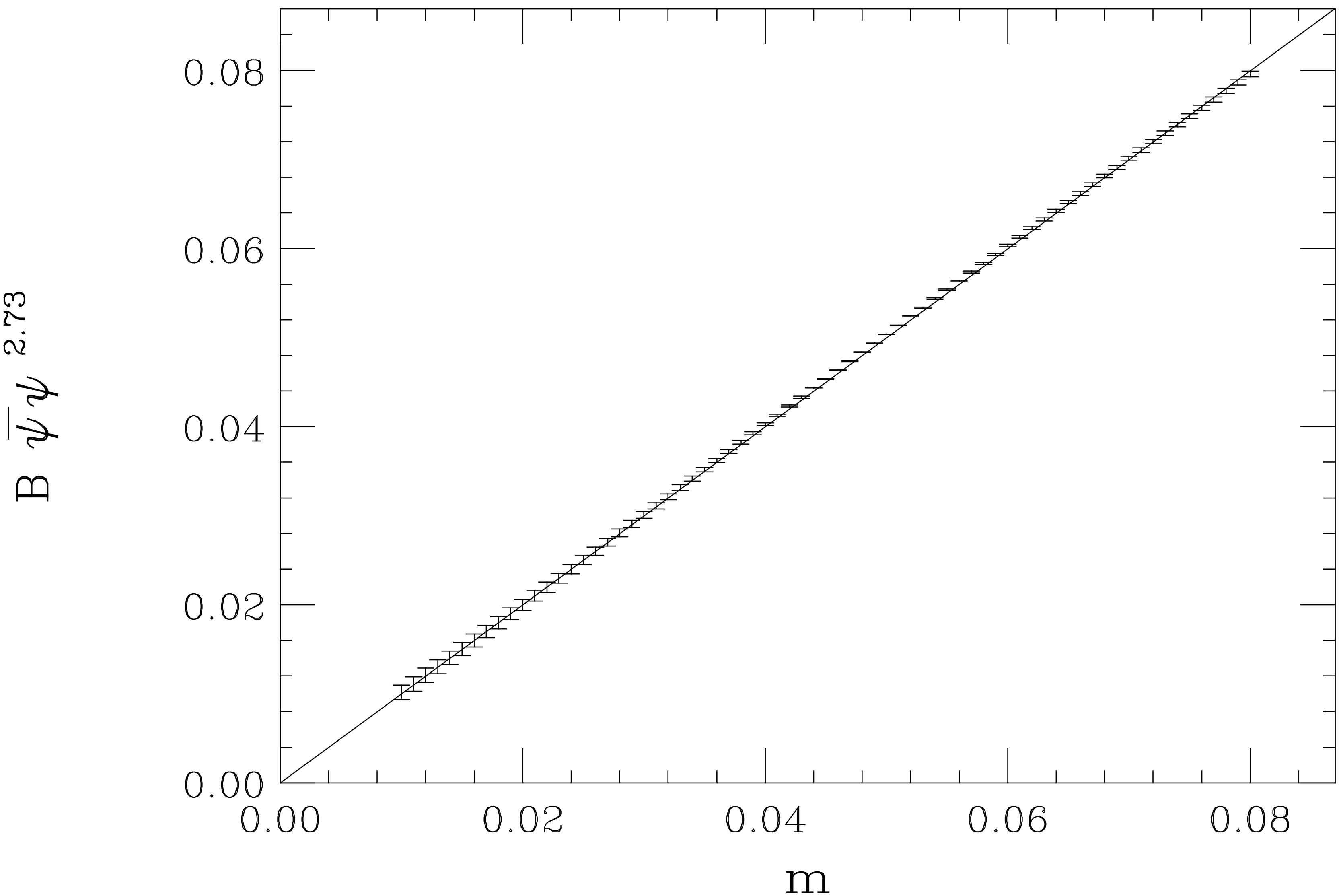
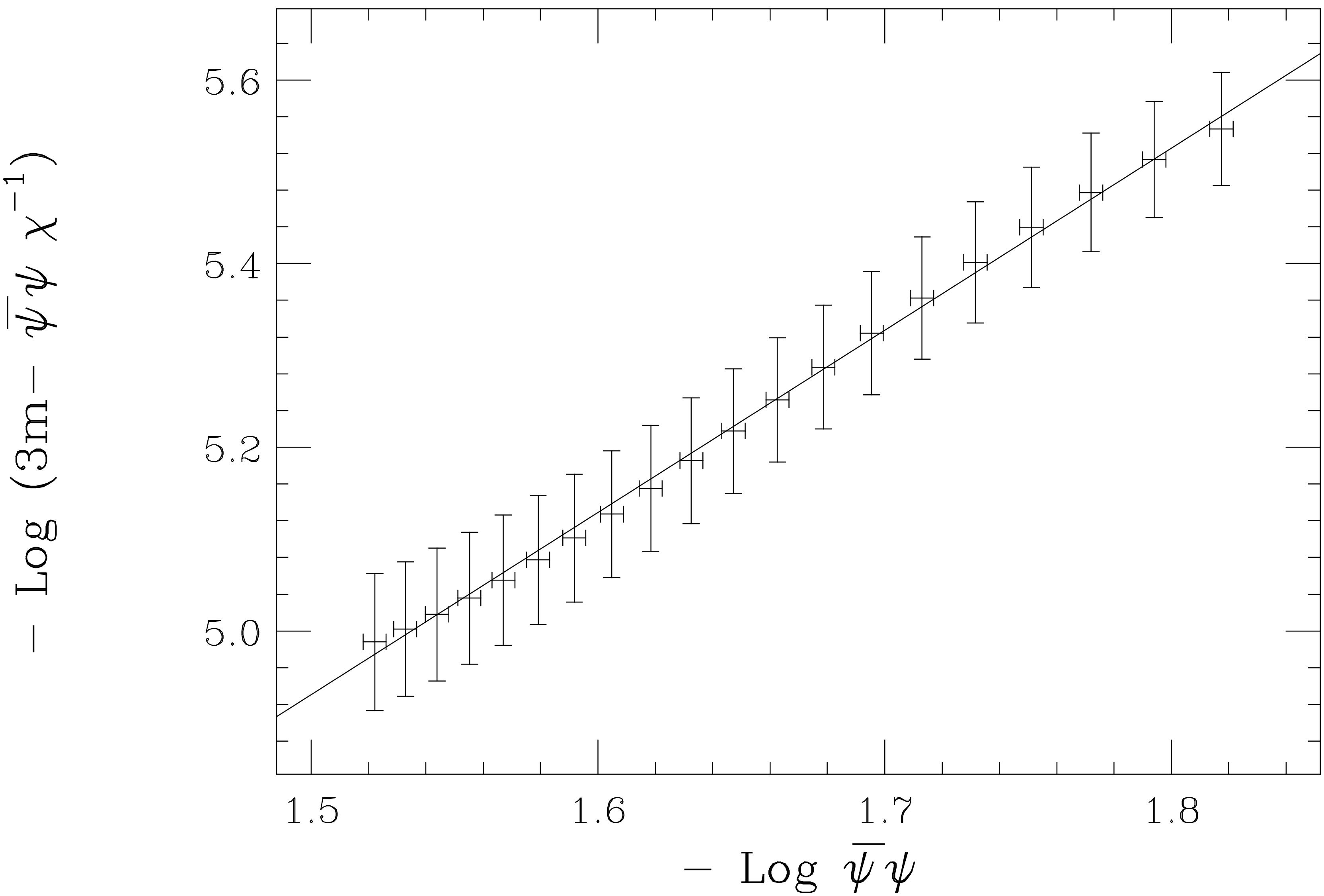


Fig. 2



5.6
5.4
5.2
5.0
1.5
1.6
1.7
1.8
$-\ \mathrm{Log}\ (3\mathrm{m} - \bar{\psi}\psi\ \chi^{-1})$
$-\ \mathrm{Log}\ \bar{\psi}\psi$

Fig. 3

L	n_f	β_c
10	1	0.2332(4)
	2	0.2230(3)
	3	0.2127(2)
	4	0.2025(2)
	6	0.1820(2)
	8	0.1616(2)
12	1	0.2356(3)
	2	0.2255(2)
	3	0.2153(2)
	4	0.2051(2)
	6	0.1847(2)
	8	0.1644(2)
14	1	0.2391(3)
	2	0.2289(3)
	3	0.2187(2)
	4	0.2086(2)
	6	0.1882(2)
	8	0.1679(2)

Table I

L	n_f	B	δ	χ^2/dof	A	C	χ^2/dof
10	1	2.68(11)	2.93(4)	0.667	0.20(11)	2.67(13)	0.624
	2	2.51(13)	2.86(5)	0.279	0.42(15)	2.44(17)	0.253
	3	2.52(13)	2.86(5)	0.259	0.43(15)	2.46(16)	0.234
	4	2.53(13)	2.86(5)	0.339	0.43(15)	2.47(16)	0.306
	6	2.52(12)	2.86(6)	0.475	0.44(14)	2.45(16)	0.427
	8	2.50(11)	2.85(6)	0.586	0.47(14)	2.42(15)	0.522
12	1	2.47(6)	2.82(2)	0.555	0.58(7)	2.36(8)	0.520
	2	2.47(4)	2.82(3)	0.359	0.62(7)	2.35(6)	0.332
	3	2.48(2)	2.81(2)	0.406	0.62(5)	2.36(4)	0.376
	4	2.481(4)	2.814(8)	0.427	0.62(4)	2.36(1)	0.397
	6	2.48(3)	2.815(2)	0.429	0.612(7)	2.37(5)	0.400
	8	2.49(5)	2.815(6)	0.426	0.61(2)	2.37(7)	0.398
14	1	2.32(10)	2.72(4)	0.133	1.03(14)	2.02(17)	0.148
	2	2.38(11)	2.73(4)	0.279	0.98(15)	2.10(19)	0.270
	3	2.39(12)	2.73(4)	0.725	0.97(15)	2.1(2)	0.674
	4	2.39(12)	2.73(4)	1.01	0.97(15)	2.1(2)	0.928
	6	2.39(12)	2.73(4)	1.14	0.97(15)	2.1(2)	1.05
	8	2.39(12)	2.73(4)	1.15	0.97(15)	2.1(2)	1.06

Table II