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SINGULARITY AND EXIT PROBLEMS

IN TWO-DIMENSIONAL STRING COSMOLOGY

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Abstract

A broad class of two-dimensional loop-corrected dilaton gravity models exhibit cosmological solutions that interpolate between the string perturbative vacuum and a background with asymptotically flat metric and linearly growing dilaton. The curvature singularities of the corresponding tree-level solutions are smoothed out, but no branch-change occurs. Thus, even in the presence of a non-perturbative potential, the system is not attracted by physically interesting fixed points with constant dilaton, and the exit problem of string cosmology persists.

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The scale factor duality of the string effective action [1] has recently motivated the study of a class of cosmological models, in which the Universe starts evolving from the string perturbative vacuum through an initial, pre-big bang phase [2, 3] having "dual" kinematic properties with respect to those of standard cosmology. This initial phase is characterized by an accelerated growth of the curvature and of the string coupling, so that the transition to the post-big bang decelerated evolution is expected to occur in the region of high curvature and/or strong coupling. Such a transition cannot be consistently described in the context of the lowest-order string effective action [4] (unless one adopts a radical quantum cosmology approach, in which the transition is described as a scattering process between asymptotic $|\mathrm{in}\rangle$ and $|\mathrm{out}\rangle$ states in minisuperspace [5], thus neglecting details of the transition region). And in fact, available examples of non-singular backgrounds, describing a smooth transition between pre- and post-big bang configurations, make use of a two-loop non-local dilaton potential [2, 6], or are formulated as exact conformal field theories [7], which automatically take all higher-derivative corrections into account.

The importance of loop corrections for implementing non-singular string cosmology models has been recently emphasized by Easther and Maeda [8]. By extending previous work on one-loop superstring cosmology [9], they have found non-singular four-dimensional solutions that interpolate smoothly between an intial string phase and a final era of the Friedmann-Robertson-Walker type, with constant dilaton and decreasing curvature. Similar results have been recently obtained also by Rey [10], working in the context of the so-called CGHS model of two-dimensional dilaton gravity [11], with the one-loop trace anomaly term supplemented by a local, covariant counterterm in order to preserve a useful classical symmetry [12, 13]. By exploiting such a symmetry to generalize the classical solutions, it has been shown [10] that the curvature and dilaton singularities of the tree-level pre-big bang background are regularized by the quantum one-loop corrections. However, in that example a limited number of conformal scalar fields ($N < 24$) must be assumed, corresponding to a negative contribution to the one-loop anomaly term. This is known to lead to gravitational instabilities, and to the emission of negative-energy Hawking radiation [13]. Also, the curvature is bounded but the dilaton keeps growing, asymptotically, so that higher-loop corrections cannot be neglected.

In this paper we present a different class of solutions of the CGHS model, in which the curvature singularity of the tree-level description is smoothed out by the one-loop terms without spoiling the physical requirement $N > 24$. The dilaton still evolves monotonically but, unlike in the case discussed in [10], the semiclassical back reaction of the produced gravitational radiation grows in time and may become of the same order as the one-loop terms, irrespective of the initial density. However, without some mechanism implementing a change of branch of the solution, the growth of the dilaton can neither be stopped by this back reaction nor by the effects of a non-perturbative dilaton potential.

We start considering the one-loop effective action for a two-dimensional model of dilaton gravity, coupled to N conformal matter fields f_i,

$$S = \int d^2x\sqrt{-g}\left[-e^{-\phi}\left(R + (\nabla\phi)^2 + \Lambda\right) + \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2 + \frac{k}{2}\left(R\nabla^{-2}R + \epsilon\phi R\right)\right]. \qquad (1)$$

Notations: $k = (N-24)/24$, ∇ is the covariant gradient operator, $g_{00} = +1$, and $R_{\mu\nu\alpha}{}^{\beta} = \partial_\mu \Gamma_{\nu\alpha}{}^{\beta} - \ldots$. With our conventions, the dilaton is related to the effective string coupling g_s by $e^\phi = g_s^2$. The first contribution proportional to k in eq. (1) corresponds to the usual trace anomaly, the second one (parametrized by ϵ) is a local counterterm that one is free to add to the definition of the model. The case $\epsilon = 0$ reproduces the original CGHS model [11], the case $\epsilon = 1$ is the conformal-invariant model considered in [12, 13].

We are looking for exact cosmological solutions of the above action with $\Lambda = f_i = 0$ (as in [10]), by keeping for the moment both k and ϵ arbitrary (homogeneous cosmological solutions with non-vanishing Λ, $f = f(t)$ and $\epsilon = 1$ have already been discussed in [14]). We shall work in the cosmic time gauge, the most appropriate to cosmological applications. To this aim we parametrize the two-dimensional metric in terms of the scale factor $a(t)$ and of the lapse function $\mathcal{N}(t)$ as

$$ds^2 = \mathcal{N}(t)dt^2 - a^2(t)dx^2, \qquad a = e^\beta, \qquad \beta = \beta(t). \qquad (2)$$

The action, modulo total derivatives, can thus be rewritten as

$$S = \int \frac{dx^2}{\mathcal{N}} e^\beta \left[-\left(e^{-\phi}\right)^{\cdot}(2\dot\beta - \dot\phi) + k\dot\beta(\epsilon\dot\phi - 2\dot\beta)\right], \qquad (3)$$

where a dot denotes differentiation with respect to cosmic time t.

We shall vary the action with respect to $\mathcal{N}$ and β, fixing the gauge to $\mathcal{N} = 1$. The first variation gives the Hamiltonian constraint:

$$\left(e^{-\phi}\right)^{\cdot}(2\dot\beta - \dot\phi) = k(\epsilon\dot\phi\dot\beta - 2\dot\beta^2). \qquad (4)$$

The second leads to the spatial components of the gravi-dilaton tensor equations, which can be integrated immediately to give

$$\frac{d}{dt}\left(-e^{-\phi}+\frac{k}{2}\epsilon\phi-2k\beta\right)=\frac{e^{-\beta}}{t_0} \tag{5}$$

(t_0 is an integration constant). By eliminating $\left(e^{-\phi}\right)^{\cdot}$ and $\dot\beta$ through eq. (5), the constraint (4) reduces to

$$\dot\phi^2\left[e^{-2\phi}+k(\epsilon-2)e^{-\phi}+\frac{k^2\epsilon^2}{4}\right]=\frac{e^{-2\beta}}{t_0^2}. \tag{6}$$

The square root of this equation, combined with eq. (5), then leads to the system of coupled first-order equations

$$\begin{aligned}
2k\dot\beta &= -\frac{e^{-\beta}}{t_0}\pm\frac{e^{-\beta}}{t_0}\left(e^{-\phi}+\frac{k\epsilon}{2}\right)\left[e^{-2\phi}+k(\epsilon-2)e^{-\phi}+\frac{k^2\epsilon^2}{4}\right]^{-1/2},\\
\dot\phi &= \pm\frac{e^{-\beta}}{t_0}\left[e^{-2\phi}+k(\epsilon-2)e^{-\phi}+\frac{k^2\epsilon^2}{4}\right]^{-1/2},
\end{aligned} \tag{7}$$

from which, setting $e^\phi=g_s^2$,

$$\frac{d\beta}{dg_s^2}=\frac{1}{2kg_s^4}\left(1+\frac{k}{2}\epsilon g_s^2\mp\sqrt{1+k(\epsilon-2)g_s^2+\frac{k^2\epsilon^2}{4}g_s^4}\right). \tag{8}$$

For $\epsilon=1$ we now easily recover the two branches of the exact solution presented in [10], characterized, respectively, by $\beta=\ln g_s$ and by $k\beta=-g_s^{-1}$, namely $2\beta=\phi$ and $k\beta e^{\phi/2}=-1$. Singularities are avoided, in this solution, only for $k<0$, as can easily be seen from eqs. (7) by noting that, for $\epsilon=1$ and $k>0$, both $\dot\phi$ and $\dot\beta$ diverge at $g_s^2=2/k$.

The case $\epsilon=1$, however, is only the particular limiting case of the condition $\epsilon\ge1$, under which eq. (8) provides real solutions for $\beta(g_s)$. For $\epsilon>1$, we obtain from (8) the general integral

$$\begin{aligned}
\beta(g_s^2) &= \beta_0-\frac{1}{2kg_s^2}\left(1\mp\sqrt{1+k(\epsilon-2)g_s^2+\frac{k^2\epsilon^2}{4}g_s^4}\right)+\frac{\epsilon}{4}\ln g_s^2\mp\\
&\mp\frac{|k|\epsilon}{4k}\sinh^{-1}\left[\frac{k^2\epsilon^2g_s^2/2+k(\epsilon-2)}{2|k|\sqrt{\epsilon-1}}\right]\pm\frac{\epsilon-2}{4}\sinh^{-1}\left[\frac{k(\epsilon-2)g_s^2+2}{2|k|g_s^2\sqrt{\epsilon-1}}\right],
\end{aligned} \tag{9}$$

where β_0 is an integration constant. We shall consider, in this paper, the physical case $k>0$ (i.e. $N>24$), and we shall concentrate on the upper branch of the solution, the one that reduces asymptotically, for $t\to-\infty$, to the tree-level, superinflationary pre-big bang solution [2] $a \sim (-t)^{-1}$, $\phi \sim -2\ln(-t)$ (in the other branch $\dot{\phi}$ diverges as $t \to -\infty$). For positive values k can be absorbed into g_s^2 (with a redefinition of the integration constant β_0); in addition, for $\epsilon > 1$, the dilaton is a monotonic function of cosmic time (see eq. (7)). The general, upper branch solution with $k > 0, \epsilon > 1$ can thus be given as a function of the monotonic coupling parameter $g_s^2(t) = \exp[\phi(t)]$ as:

$$a(g_s) = e^{\beta} = e^{\beta_0} \left| \frac{g_s^2}{\epsilon(r + g_s^2) + \epsilon - 2} \right|^{\epsilon/4} \left| \frac{2r + 2 + (\epsilon - 2)g_s^2}{g_s^2} \right|^{(\epsilon-2)/4} \exp\left(\frac{r-1}{2g_s^2}\right), \tag{10}$$

$$\dot{\phi}(g_s) = \frac{g_s^2}{at_0 r} \quad , \qquad 2\dot{\beta} = \frac{1}{at_0 r}\left(1 + \frac{\epsilon}{2}g_s^2 - r\right), \tag{11}$$

where

$$r(g_s) = \sqrt{1 + (\epsilon - 2)g_s^2 + \frac{\epsilon^2}{4}g_s^4} \tag{12}$$

and $g_s(t)$ is given implicitly by

$$\frac{t}{t_0} = \int \frac{dg_s^2}{g_s^4} r(g_s)\; a(g_s). \tag{13}$$

In this branch, the evolution of the scale factor is monotonic, as $d\beta/dg_s^2 > 0$ (see eq. (8). Asymptotically, at $t \to -\infty$ and $g_s \to 0$, we find from eqs. (10) and (13) that $a_-(g_s) \sim g_s$ and $g_s^2 = e^{\phi} \sim (-t)^{-2}$. At $t \to +\infty$, $g_s \to \infty$, the scale factor approaches a constant, $a_+(g_s) \sim$ const, and $g_s^2 = e^{\phi} \sim e^{ct}$, $c =$ const. The above solution thus describes, for any $\epsilon > 1$, a smooth transition between the two-dimensional version of the well-known [2] dilaton-dominated, pre-big bang inflationary evolution $a \sim (-t)^{-1}$, $\phi \sim -2\ln(-t)$, and a final configuration characterized asymptotically by flat space-time and linearly growing dilaton [15]. The transition occurs without singularities in $\dot{\beta}$ and $\dot{\phi}$ for all ϕ ranging from $-\infty$ to $+\infty$, as can easily be checked from eq. (7) and from the fact that $f(g_s)$ has no real zeros. The scalar curvature $R = -2(\ddot{\beta} + \dot{\beta}^2)$ is everywhere bounded, approaches zero at $t \to \pm\infty$, and reaches a maximum around the transition region $g_s \sim 1$. The plot of $a(\phi)$, $\dot{\phi}(\phi)$, $\dot{\beta}(\phi)$ and $R(\phi)$ is shown in Fig. 1 for the particular case $\epsilon = 2$.

The above class of exact solutions is not completely satisfactory as an example of regular cosmological backgrounds, however, because the dilaton keeps growing as $t \to +\infty$ ($\phi \sim t$), thus reaching asymptotically a regime in which the perturbative approximation breaks down (as in the solution presented in [10]), and the effective action becomes dominated by higher-loop corrections. Unlike in the four-dimensional solutions discussed in [8], there is no way to obtain from eqs. (10)–(13) a coupling parameter e^{ϕ} which remains bounded at all times.

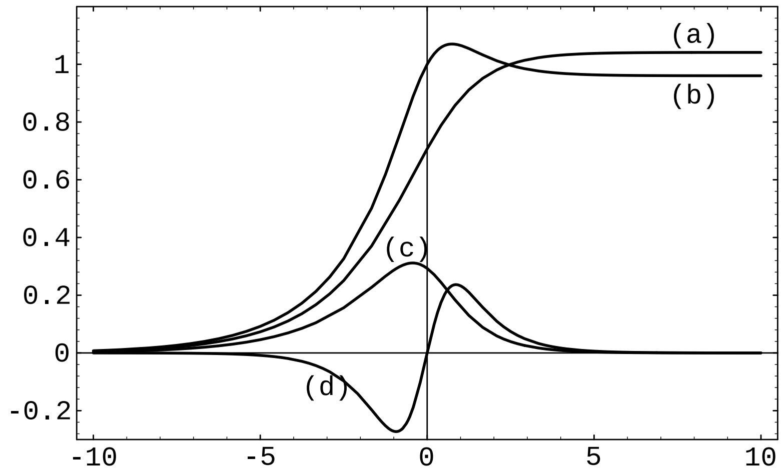

Fig. 1. *Plot versus $\phi = 2\ln g_s$ of (a) the scale factor a, (b) the dilaton growing rate $\dot{\phi}$, (c) the expansion rate $\dot{\beta}$, and (d) the scalar curvature $R = -2(\ddot{\beta} + \dot{\beta}^2)$, for the solution (10), (11), in the particular case $\epsilon = 2$. We have chosen t_0 equal to the fundamental string length λ_s, and we have normalized a in such a way that $\dot{\phi} = 1$, in string units, when $\phi = 0$.*

In the context of a realistic cosmological model, however, we should take into account the effect of a non-perturbative dilaton potential $V(\phi)$. Such a potential, typically required by supersymmetry-breaking models, is known to go very rapidly to zero at small coupling, $V(\phi) \sim \exp(-g_s^{-2})$ for $\phi \to -\infty$, while it tends to grow with a complicated, in general non-monotonic behaviour in the opposite, large-coupling limit [16]. Any potential $V(\phi)$ that grows enough at large ϕ can thus dominate the one-loop contributions to the background energy density, $ke^{\phi}\dot{\beta}\dot{\phi}$, which stay constant at large ϕ, and might suppress the asymptotic growth of the dilaton (different examples of regular two-dimensional backgrounds, without loop terms but with an appropriate potential, have been discussed in [17] in the context of models implementing the limiting curvature hypothesis [18]).

In order to discuss this possibility we add to the action (3) a potential $V(\phi)$,

$$S = \int \frac{dx^2}{\mathcal{N}} e^{\beta} \left\{ e^{-\phi} \left[\dot{\phi}(2\dot{\beta} - \dot{\phi}) - \mathcal{N}^2 V(\phi) \right] + k\dot{\beta}(\epsilon\dot{\phi} - 2\dot{\beta}) \right\}. \tag{14}$$

The variation with respect to $\mathcal{N}, \beta$ and ϕ provides the equations (in the gauge $\mathcal{N} = 1$)

$$\dot{\phi}^2 - 2\dot{\beta}\dot{\phi} - V = ke^{\phi}\dot{\beta}(\epsilon\dot{\phi} - 2\dot{\beta}), \tag{15}$$

$$\left(\ddot{\phi} + \dot{\beta}\dot{\phi}\right)\left(1 + k\frac{\epsilon}{2}e^{\phi}\right) - 2ke^{\phi}\left(\ddot{\beta} + \dot{\beta}^2\right) - \dot{\phi}^2 + V = 0, \tag{16}$$

$$\ddot{\phi} = \left(\ddot{\beta} + \dot{\beta}^2\right)\left(1 + k\frac{\epsilon}{2}e^{\phi}\right) + \frac{1}{2}\dot{\phi}^2 - \dot{\beta}\dot{\phi} + \frac{1}{2}(V' - V), \tag{17}$$

where $V' = \partial V/\partial\phi$. These equations can be exactly solved by the particular (de Sitter-like) configuration with constant dilaton, $\phi = \phi_0 =$ const, and constant curvature, $\dot{\beta} = H_0 =$ const, provided the potential satisfies, at $\phi = \phi_0$,

$$V_0 = 2kH_0^2 e^{\phi_0}\ , \qquad V_0' = -V_0\left(\frac{e^{-\phi_0}}{k} + \frac{\epsilon - 2}{2}\right) \tag{18}$$

(the second condition is required to satisfy the dilaton equation (17)). It is interesting to note that ϕ_0 is not necessarily an extremum of $V(\phi)$, if $V_0 \neq 0$. The background energy density, in this context, may thus become vacuum-dominated even if the scalar field is not at the minimum of the potential, and this may have interesting implications for the solution of the cosmological constant problem.

Unfortunately, however, such a frozen configuration $\{\phi_0, H_0\}$ is not a stable fixed point towards which the background can be attracted, if we consider the branch of the solution that includes the phase of pre-big bang evolution from the perturbative vacuum. In fact, by taking the square root of eq. (15), and eliminating $\ddot{\phi}$ in eq. (16) through eq. (17), we find that in the presence of a potential the two branches are defined by the equations:

$$\begin{aligned}
\dot{\phi} &= \dot{\beta}\left(1 + \frac{\epsilon}{2}g_s^2\right) \pm \sqrt{\dot{\beta}^2 r^2 + V}, & (19)\\
\ddot{\beta} &= -\dot{\beta}^2 + \frac{1}{2r^2}\left[\left(\dot{\phi}^2 - V\right)\left(1 - \frac{\epsilon}{2}g_s^2\right) - V'\left(1 + \frac{\epsilon}{2}g_s^2\right)\right] & (20)
\end{aligned}$$

(again, we have absorbed k into g_s^2). The pre-big bang branch, which reduces to the solution (10)–(13) for small enough g_s (when $V(\phi)$ becomes negligible), corresponds to the upper sign in eq. (19). Both branches are satisfied by the constant dilaton and constant curvature solution (18), with $H_0 < 0$ for the upper branch, and $H_0 > 0$ for the lower one. By perturbing eqs. (19), (20) around such a configuration, to first order in $\delta\phi$ and $\delta\dot{\beta}$, we can easily compute the 2×2 matrix $\mathcal{M}$ characterizing the small oscillations of the background, such that

$$\begin{pmatrix}\delta\dot{\phi}\\ \delta\ddot{\beta}\end{pmatrix} = \mathcal{M}\begin{pmatrix}\delta\phi\\ \delta\dot{\beta}\end{pmatrix}, \qquad \dot{\beta} = H_0, \quad \dot{\phi}_0 = 0 = \ddot{\beta}_0. \tag{21}$$

A necessary condition for the stability of the point $\{\phi_0, H_0\}$ is the presence of a negative real part in both the eigenvalues of $\mathcal{M}$, namely Tr $\mathcal{M} < 0$. We find that, for the two branches,

$$\text{Tr}\ \mathcal{M} = \pm|H_0|, \tag{22}$$

so that $\phi = \phi_0$, $\dot\beta = H_0$ cannot be an attractor for a gravi-dilaton background, emerging from the small coupling regime and described by the upper branch of the solution for which Tr $\mathcal{M} > 0$. This conclusion can be easily checked by a numeric integration of eqs. (19), (20), for any shape of the dilaton potential.

As another possibility of stopping the dilaton growth, even without a dilaton potential, we recall that the transition between two phases with different asymptotic vacua, in general, leads to the parametric amplification of the initial vacuum fluctuations, and to the production of radiation [19]. In our two-dimensional class of backgrounds, the contribution of the radiation energy density (ρ_r) to the space-time curvature, $e^\phi \rho_r$, tends to grow exponentially in time since $e^\phi \rho_r \sim e^\phi a^{-2} \to e^\phi$ for $t \to +\infty$. Consequently, for $\epsilon > 1$, the radiation contribution will eventually become of the same order as that of the one-loop terms, whose contribution to the curvature, $ke^\phi \dot\beta\dot\phi$, tends to a constant for $t \to +\infty$. This is to be contrasted with the $\epsilon = 1$ case [10], where $e^\phi \rho_r$ goes to a constant, asymptotically, and remains negligible.

If we take into account the back reaction of this radiation produced semiclassically, we must add the term $e^\phi \rho_r$ to the right hand-side of eq. (15), while the remaining equations (16) and (17) are left unchanged because of the particular form of the radiation equation of state, $p = \rho$, in two dimensions. In the absence of a potential the two branches are then simply defined by the equations

$$\dot\phi = \dot\beta\left(1 + \frac{\epsilon}{2} g_s^2\right) \pm \sqrt{\dot\beta^2 r^2 + \rho_0 e^{\phi - 2\beta}}, \tag{23}$$

$$\ddot\beta = -\dot\beta^2 + \frac{\dot\phi^2}{2r^2}\left(1 - \frac{\epsilon}{2} g_s^2\right), \tag{24}$$

where ρ_0 is a free parameter. Thanks to the one-loop terms that cancel the radiation contribution, these equations admit the particular vacuum solution with constant dilaton and globally flat space-time,

$$\dot\phi = 0, \qquad \ddot\beta + \dot\beta^2 = 0, \qquad \rho_r \equiv \rho_0 e^{-2\beta} = 2k\dot\beta^2, \tag{25}$$

describing a linear contraction in the upper (pre-big bang) branch, $\dot\beta < 0, a \sim (-t), t < 0$, and linear expansion in the other branch, $\dot\beta > 0, a \sim t, t > 0$. However, as in the previous case with the dilaton potential, this solution is unstable in the upper branch (as $\delta\ddot\beta = \pm 2|\dot\beta|\delta\dot\beta$), and the particular solution (25) cannot be approached by a background that evolves from the pre-big bang configuration without branch changing.

In conclusion, combining these with other [2], [6]–[10] examples of smooth transitions for a typical string cosmology scenario, the following picture seems to emerge. In the solutions of the low-energy string effective action the growth of the curvature is unbounded, and prevents a continuous evolution from the initial accelerated phase to the present decelerated regime. At small coupling, higher-order α' corrections, typically due to finite-size effects and weighted by the inverse of the string tension, can "flatten" the growth of the curvature, leading eventually to a constant-curvature, de Sitter-like evolution. At higher coupling, quantum loop corrections seems to be able to induce a "bounce" of the curvature, implementing the transition to the decelerated, decreasing curvature regime. This transition is accompanied, in general, by radiation production, whose semiclassical back reaction may be expected to become important, and eventually dominate ("reheating"). The possible residual growth of the string coupling, however, can be permanently stopped by the radiation back reaction, or by the effects of non-perturbative self-interactions, only in the phase described by the decelerated branch with Tr $\mathcal{M} < 0$. A true "graceful exit" from the string to the standard cosmological phase thus seems to require a change of branch of the solution, possibly implemented by the contribution of higher derivative terms, which are absent in the example discussed here.

Much work is still needed, of course, to clarify all the details of this cosmological scenario. A full four-dimensional analysis, in particular, should complement this two-dimensional discussion. These preliminary results provide useful hints for future investigations and for implementing, in a string theory context, a consistent description of the Universe which starts evolving from the flat and cold perturbative vacuum, and ends up in the present matter-dominated state, with all the relic aspects of the big bang explosion.

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