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Twelve-Dimensional Aspects of Four-Dimensional N=1 Type I Vacua

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Abstract

Four-dimensional supergravity theories are reinterpreted in a 12-dimensional F-theory framework. The $O(8)$ symmetry of $N = 8$ supergravity is related to a reduction of F-theory on T_8, with the seventy scalars formally associated, by $O(8)$ triality, to a fully compactified four-form A_4. For the $N = 1$ type I model recently obtained from the type IIB string on the Z orbifold, we identify the Kähler manifold of the untwisted scalars in the unoriented closed sector with the generalized Siegel upper-half plane $Sp(8, R)/(SU(4) \times U(1))$. The $SU(4)$ factor reflects the holonomy group of Calabi-Yau fourfolds.

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1 Introduction

Non-trivial backgrounds for the scalar fields of type IIB supergravity have provided a geometrical setting [1, 2] for some peculiar six-dimensional (6D) string vacua [3] previously derived as open descendants [4] of type IIB K3 compactifications. The resulting models differ markedly from conventional K3 reductions, since their massless spectra contain different numbers of (anti)self-dual tensors that take part in a generalized Green-Schwarz mechanism [5, 6]. These peculiar features find an elegant rationale in the compactification of a putative 12D F-theory [1] on elliptically fibered Calabi-Yau (CY) threefolds [2], a construction that generalizes previous work on supergravity vacua with scalar backgrounds [7] by taking into account subtle global issues. Many of the recent developments can be traced to some remarkable properties of the scalar manifolds of supergravity theories that have been known since the late seventies [8]. These non-compact spaces admit a natural action of generalized dualities, and their negative curvatures allow for non-trivial scalar backgrounds compatibly with supersymmetry and vanishing vacuum energy.

In this letter we consider the possibility of endowing 4D supergravities with a 12D interpretation in the spirit of F-theory. This relates the internal part of the 12D Lorentz group to the $O(8)$ symmetry of $N = 8$ supergravity [9, 10], only an $O(7)$ subgroup of which is manifest in the 11D interpretation [11]. In CY compactifications of 10D superstrings, an $SU(3)$ subgroup of the internal $O(6)$ is identified with the holonomy of the CY threefold. In a similar fashion, in the decomposition of $O(8)$ into $O(2) \times O(6)$, $SO(8)$ triality allows a natural interpretation of the $O(6) \sim SU(4)$ subgroup as the holonomy of a CY fourfold. The chiral type I vacua recently constructed in [12] offer a nice setting to elicit this correspondence. Just as the "parent" type IIB string on the Z orbifold [13] may be regarded as a singular limit of a compactification on a CY threefold, these $N = 1$ models may be regarded as singular limits of F-theory compactifications on CY fourfolds, some examples of which have been recently discussed in [14]. In the following we give evidence that the Kähler manifold of the untwisted scalars in the projected theory is $Sp(8, R)/(SU(4) \times U(1))$, precisely in the spirit of this correspondence.

2 The Scalar Manifold

Let us begin by reviewing the results in [12], where open descendants of the Z orbifold limit of a CY threefold have been constructed. The unoriented closed-string sector is obtained combining the torus amplitude $\mathcal{T}$ for the type IIB string on the Z orbifold [13] with a Klein-bottle projection $\mathcal{K}$. The original massless closed spectrum containing the $N = 2$ supergravity multiplet, $(9 + 1)$ hypermultiplets from the untwisted sector and 27 additional hypermultiplets from the twisted sectors is thus truncated to $N = 1$ supergravity coupled to $(1+9+27)$ chiral multiplets. The tadpole cancellation conditions require the introduction of open-string states with a Chan-Paton gauge group $SO(8) \times SU(12) \times U(1)$, and the resulting massless spectrum includes three generations of chiral multiplets in the $(8, 12^*)$ and $(1, 66)$ representations. The $U(1)$ factor is anomalous, and a generalized Green-Schwarz mechanism involving R-R axions gives a mass of the order of the string scale to the corresponding gauge boson [15]. The $SO(8)$ and $SU(12)$ factors have opposite beta functions, and the latter is strongly coupled in the IR. Along classically flat directions, the Chan-Paton group is generically broken to $U(1)^4 \times SU(2)$. The massless untwisted sector of the closed-string spectrum is encoded in the following terms of the torus and Klein-bottle partition functions:

$$\begin{aligned} \mathcal{T}_u &= \frac{1}{2}\left\{ |V_2 - S_2 - C_2|^2 + \sum_{I=1}^{9}\Big(\big[\, (O_2 - S_2)(\bar{O}_2 - \bar{C}_2)\,\big]_{(I)} + h.c. \Big) \right\} \\ \mathcal{K}_u &= \frac{1}{2}\,(V_2 - S_2 - C_2) \quad , \end{aligned} \tag{2.1}$$

where O_2, V_2, S_2 and C_2 are level-one characters of the transverse Lorentz group $O(2)$. The untwisted sector of the "parent" type IIB model includes the 20 NS-NS fields $(\phi, b_{\mu\nu}, b_{i\bar{j}}, g_{i\bar{j}})$ and the 20 R-R fields $(\phi', b'_{\mu\nu}, b'_{i\bar{j}}, A_{\mu\nu i\bar{j}})$. These fill the universal hypermultiplet, as well as 9 additional hypermultiplets corresponding to the second cohomology group of untwisted $(1, 1)$ forms, and parametrize the quaternionic manifold $E_{6(+2)}/(SU(2) \times SU(6))$. This manifold was obtained by c-map [16] from the special Kähler manifold $SU(3, 3)/(SU(3) \times SU(3) \times U(1))$ of the heterotic string on the Z orbifold [17]. In NS-NS sector, the open descendant retains the dilaton and a 9-dimensional real slice of the complex Kähler cone corresponding to

$$Im(J_{i\bar{j}}) \;=\; Re(g_{i\bar{j}}) + Im(b_{i\bar{j}}) \quad . \tag{2.2}$$

In the R-R sector, one is left with a mixture of ϕ' and $b'_{\mu\nu}$, as well as with mixtures of the other R-R fields. Though somewhat surprising, this result, clearly encoded in the vacuum amplitudes of eq. (2.1), is also supported by the explicit study of tree-level amplitudes, as well as by some rather compelling duality arguments. The 20 scalar fields parametrize a space K_I that, on general supersymmetry grounds, is a Kähler manifold embedded in $E_{6(+2)}/(SU(2) \times SU(6))$ and is expected to have a number of properties:

(a) K_I should be an *irreducible* hermitian symmetric space G/H [18] of complex dimension 10. This is motivated by previous experience with conventional orbifold compactifications, where the reduced holonomy leads to scalar manifolds that are symmetric spaces [24]. Moreover, simple scaling arguments [12] suggest complete mixings between the type I dilaton and the other scalars. This marked difference with respect to the heterotic string reflects the T duality variance of perturbative open-string vacua.

(b) $SU(2) \times SU(6) \supset H \supset O(3) \times O(3)$ and $E_6 \supset G \supset O(3,3) \times R^+$, since both the linearly and the non-linearly realized symmetries of the scalar manifold should be a subset of those of the "parent" type IIB string and should include those of $O(3,3)/(O(3) \times O(3))$, the unconventional real slice of the Kähler cone chosen by the world-sheet orbifold to accommodate the NS-NS fluctuations. This should be contrasted with the classical CY moduli space of closed strings, that would accommodate solely $g_{i\bar{j}}$.

(c) The complex scalars should be in a representation that reduces to the $(3,3) + (1,1)$ under the decomposition of H to the $O(3) \times O(3)$ subgroup. This corresponds to the real slice of the Kähler cone discussed above together with the NS-NS dilaton, since the NS-NS antisymmetric tensor $b_{\mu\nu}$ is projected out of the unoriented closed spectrum.

These conditions uniquely select $Sp(8,R)/(SU(4)\times U(1))$. It should be appreciated that this truncation is vastly different from the heterotic one. The simplicity of the type I model of [12] reflects itself in the linearly realized symmetry, $H=SU(4)\times U(1)$, with an $SU(4)$ factor strongly suggestive of a 12D interpretation.

3 Reduction of $N=8$ Supergravity

The $N=8$ theory includes 70 scalar fields that parametrize the coset $E_{7(+7)}/SU(8)$ [9]. The whole field content has a natural decomposition in terms of $N=2$ supermultiplets, as follows from the branching of $SU(8)$ in $SU(2)\times SU(6)$,

$$\left\{2,2\left(\frac{3}{2}\right),1\right\}+6\times\left\{\frac{3}{2},2(1),\frac{1}{2}\right\}+15\times\left\{1,2\left(\frac{1}{2}\right),2(0)\right\}+20\times\left\{\frac{1}{2},2(0)\right\} \quad . \tag{3.1}$$

The $N=8$ spectrum thus contains 15 $N=2$ vector multiplets and 10 $N=2$ hypermultiplets. Therefore, any truncation of the $N=8$ theory must have $n_V\leq 15$ and $n_H\leq 10$. The untwisted sector of the Z orbifold is the "maximal" consistent reduction, since $n_H=10$ in type IIB (and thus $n_V=9$ in type IIA). The Z_3-projection in the untwisted sector of the type IIB string amounts to retaining only Z_3-singlets in the decomposition of $SU(8)$ into $SU(3)$ induced by $8\rightarrow 4+4\rightarrow 3+1+3+1$. Moreover, in the decomposition (3.1) the hypermultiplets belong to the antisymmetric three-tensor representation of $SU(6)$, that has zero Z_3 triality. Further reduction according to $6\rightarrow 3+3$ preserves this property and, as a result, all the $N=2$ hypermultiplets in (3.1) survive. Conversely, all the 15 vector multiplets are projected out, since they have nonzero triality. The original 28 vector fields belong to the antisymmetric two-tensor of $SU(8)$, and decompose according to $28\rightarrow 6+3+1+3\times(3+3^*)$. Therefore, only the graviphoton survives the Z_3 projection. For the sake of comparison, let us note that in the type IIA massless spectrum the decomposition $8\rightarrow 4+4^*\rightarrow 3+1+3^*+1$ implies $28\rightarrow 8+1+3\times(3)+3\times(3^*)+1$. As a result, there are 9 Z_3-singlet vectors (together with their $N=2$ superpartners) and the graviphoton.

The truncation that leads to the $N=1$ heterotic string follows from the intermediate branching of $SU(8)$ in $SU(4)\times SU(4)$. Accordingly, the $N=8$ supergravity is to be decomposed in terms of $N = 4$ supermultiplets:

$$\left\{2, 4\left(\frac{3}{2}\right), 6(1), 4\left(\frac{1}{2}\right), 2(0)\right\} + 4\times\left\{\frac{3}{2}, 4(1), (6+1)\left(\frac{1}{2}\right), 8(0)\right\} + 6\times\left\{1, 4\left(\frac{1}{2}\right), 6(0)\right\} \quad , \tag{3.2}$$

where the first and last terms correspond to the $N = 1$ supergravity multiplet in $D = 10$, while the middle term accounts for the additional 10D gravitino multiplet. The 36 scalars in the vector multiplets parametrize the coset $O(6,6)/(O(6)\times O(6))$, while the two scalars of the supergravity multiplet parametrize $SU(1,1)/U(1)$ [8]. Keeping Z_3-singlets in the $N = 4$ theory according to $6 \to 3 + 3^*$ results in 10 chiral multiplets, one from the supergravity multiplet and 9 from the vector multiplets. The latter parametrize the special Kähler manifold $SU(3,3)/(SU(3) \times SU(3) \times U(1))$, and the appearance of $SU(3)$ reflects the holonomy of CY threefolds.

Returning to the open descendant of the type IIB string in [12], let us consider the decomposition $8_s \to 6 + 1_+ + 1_-$ of the spinor of $O(8)$ in representations of $O(2) \times O(6)$ induced by the branching $8_v \to 4+4^*$ of tangent vectors on manifolds of $SU(4)$ holonomy. As already noted, the $N = 8$ vector fields belong to the antisymmetric two-tensor of $O(8)$ that decomposes according to $[8\times 8]_A = 15+6_+ +6_- +1$, giving precisely one $O(6)$ singlet, the graviphoton. This field, however, is projected out of the unoriented closed spectrum together with one of the $O(6)$-singlet gravitini. On the other hand, the 40 scalars in the $(2,20)$ of $O(2) \times O(6)$ transform as a 10_+ and a 10^*_-, and the projection retains only the 10_+ in the unoriented spectrum. These states belong to the symmetric two-tensor of $SU(4)$, or equivalently to the self-dual antisymmetric three-tensor of $O(6)$, consistently with a scalar manifold $Sp(8,R)/(SU(4) \times U(1))$.

The generalized Siegel upper-half plane $Sp(2n,R)/U(n)$, familiar from the theory of Riemann surfaces [22], may be seen as the open submanifold $\mathcal{S}_n$ of complex symmetric $n\times n$ matrices Ω, with $Im\Omega > 0$ [18]. It is the natural extension of the type IIB (ϕ, ϕ') system that lives in the upper-half plane. In this case, the counterpart of the $SL(2,Z)$ group of U dualities [20] is $Sp(8,Z)$, that clearly combines S and T transformations. As already remarked, this is consistent with the properties of perturbative type I vacua, that lack both T and S duality invariances. On the other hand, one would expect manifest invariance under dualities involving only complex structure deformations [21] that, however, are lacking in the model of [12]. The $Sp(2n, R)$ group of real matrices

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{3.3}$$

such that $A^T C = C^T A$, $B^T D = D^T B$ and $A^T D - C^T B = 1$ acts projectively on Ω:

$$\Omega \to (A\Omega + B)(C\Omega + D)^{-1} \quad . \tag{3.4}$$

In particular, the continuous Peccei-Quinn symmetry of the R-R fields corresponds to the triangular subgroup

$$S_{PQ} = \begin{pmatrix} 1 & B \\ 0 & 1 \end{pmatrix} \quad , \tag{3.5}$$

with $B = B^T$. It is natural to expect that (world-sheet and space-time) instanton effects induce non-trivial monodromies with respect to $Sp(2n, Z)$, so that the moduli space is actually $\mathcal{S}_n / Sp(2n, Z)$, a generalization of the fundamental domain of the moduli space of elliptic curves. Note that $\mathcal{S}_n$ has a natural Kähler metric with Kähler potential

$$K = - \log \det Im\Omega \quad . \tag{3.6}$$

In the model of [12], a non-perturbatively generated superpotential would be a modular form of $Sp(8, Z)$. More generally, for a Kähler potential transforming as $K \to K - \Lambda_\Gamma - \bar{\Lambda}_\Gamma$ under the action of a monodromy group Γ, the superpotential would be a Γ-modular form transforming as $W \to e^{\Lambda_\Gamma}\ W$ [23].

The analysis may be extended to the untwisted sectors of other abelian orbifolds. For the "parent" type IIB string, the special Kähler manifolds of the vector multiplets and the quaternionic manifolds of the hypermultiplets have been discussed in [24]. Open descendants typically associate to type IIB vector multiplets type I chiral multiplets, with the same scalar field content, and in these cases the special Kähler manifold is unaffected by the projection. On the other hand, the hypermultiplets are truncated to chiral multiplets, and the number of scalar fields is correspondingly halved. For instance, for the $Z_2 \times Z_2$ orbifold the correspondence between the scalar manifolds is

$$\frac{O(4,4)}{O(4) \times O(4)} \to \frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)} \quad . \tag{3.7}$$

4 F-Theory Interpretation

The compactification of F-theory on the simplest CY fourfold, the T_8 torus, gives $N = 8$ supergravity in $D = 4$. If T_8 is equipped with a complex structure, its fourth cohomology group decomposes as

$$H^4 = H^{(4,0)} + H^{(0,4)} + H^{(3,1)} + H^{(1,3)} + H^{(2,2)} \quad , \tag{4.1}$$

with Hodge numbers $h^{(4,0)} = h^{(0,4)} = 1$, $h^{(3,1)} = h^{(1,3)} = 16$, $h^{(2,2)} = 36$. According to [9], the 70 scalar fields should be assigned to the $35_s + 35_+$ of $O(8)$, where 35_s are metric deformations at fixed volume. Using $O(8)$ triality [9] one can trade the 35_s for the 35_-. All the 70 scalars ϕ_{ijkl} can then be associated to the compactified four-form A_4, and split into ϕ^+_{ijkl} and ϕ^-_{ijkl} (35_+ and 35_- of O(8)). The 12D spinors decompose as $(4, 8)$ of $O(3, 1) \times O(8)$, where $O(8)$ is naturally interpreted as the rotation group of the eight compactified dimensions. This would require 32-component Weyl spinors in $D = 12$ with signature $(11, 1)$, and in principle would allow one to obtain both type IIA and type IIB in $D = 10$. On the other hand, a single Majorana-Weyl spinor in $D = 12$ with signature $(10, 2)$ would give at most two Majorana-Weyl spinors of opposite chirality in $D = 10$, suitable for type IIA but not for type IIB [25]. Dirichlet 3-brane considerations also seem to indicate an $(11, 1)$ signature [26] for the putative 12D F-theory, although at this stage an alternative formulation with $(10, 2)$ signature may well be possible. The vector fields $V_{\mu ij}$ are naturally associated to $A^{(3)}_{\mu ij}$. For the sake of comparison, in M-theory compactified on T_7, 28 scalar fields arise from g_{ab}, 7 from $A^{(3)}_{\mu\nu a}$ and 35 from $A^{(3)}_{abc}$, while 7 vectors arise from $g_{\mu a}$ and 21 from $A^{(3)}_{\mu ab}$. The geometrical coupling of [6]

$$\int A_4 \wedge F_4 \wedge F_4 \tag{4.2}$$

induces the $D = 4$ couplings

$$\int \epsilon^{ijklmnpq} \, \phi^+_{ijkl} \, F_{mn} \wedge F_{pq} \quad , \tag{4.3}$$

where $F_{mn} = dV_{mn}$, indeed present in $N = 8$ supergravity [9, 10].

For M-theory compactified on 8-manifolds, a similar analysis was performed in [27]. A further reduction of F-theory on S_1 allows a comparison with M-theory compactified on

T_8. After suitable duality transformations, the latter contains 128 scalars that belong to the left spinor representation of $O(16)$ and parametrize the coset $E_{8(+8)}/O(16)$ [28]. The left spinor of $O(16)$ decomposes under $O(8)$ according to $128 \rightarrow 2(1) + 2(28) + 35^+ + 35^-$. This is consistent with the 4D F-theory reduction where, as we have seen, the degrees of freedom can be assigned to the 4D graviton, to 28 vectors arising from the three form A_3 with two indices tangent to T_8, and to a completely compactified four-form A_4.

The F-theory interpretation of some 6D type I vacua and their relation to non-perturbative heterotic vacua, proposed in [2], has received further support in [29]. More general classes of 6D type I models with no tensor multiplets at all [30] correspond to non-trivial K3 fibrations [2].

We can only give some hints on the relation of the open descendants of the type IIB theory on the Z orbifold [12] to the F-theory reduction on a CY fourfold. A generic CY fourfold has $SU(4)$ holonomy and leads naturally to models with $N = 1$ supersymmetry [14]. This leaves a single 4D gravitino, with a breaking pattern identical to the one induced by the Klein-bottle projection in the type I model. We have already seen that, in the unoriented closed spectrum of [12], the 37 hypermultiplets turn into 37 chiral multiplets. These allow an interpretation in terms of a fourfold with $h^{(1,1)} \neq 0$ and $h^{(1,2)} = 0$. The topological couplings

$$\int_{CY} A_{i\bar{j}l\bar{m}}\ F_{p\bar{q}}\ F_{s\bar{t}}\ \epsilon^{ilps} \epsilon^{\bar{j}\bar{m}\bar{q}\bar{t}} \tag{4.4}$$

between states associated to the $H^{(1,1)}$ and $H^{(2,2)}$ cohomology groups of the fourfold[2] give rise to $N = 1$ axion couplings. An explicit analysis of these couplings may clarify the role of the $H^{(2,2)}$ cohomology.

5 Remarks on Mirror Symmetry and the c-Map

In the type I model of [12], the Peccei-Quinn symmetries of the ten real R-R scalars have a natural geometrical interpretation. We have seen that the untwisted moduli space $Sp(8,R)/U(4)$ is the generalized Siegel upper half plane, $Im\Omega > 0$. The elements of

[2] These couplings were also discussed with R. Minasian.

$Re\Omega$ are R-R scalars, while the elements of $Im\Omega$ parametrize a real slice of the complex Kähler cone of the CY threefold. This actually suggests that open descendants of type IIB compactifications on generic CY threefolds involve a new complexification of the classical real moduli space:

$$J \rightarrow J_C^o = iIm(J) + B_R \quad , \tag{5.1}$$

where, as in eq. (2.2), ImJ is the imaginary part of the complexified Kähler class that survives the world-sheet orbifold projection, and B_R is a mixture of R-R fields. Quite differently, in the heterotic string

$$J \rightarrow J_C^h = ig + B_{NS} \quad . \tag{5.2}$$

Since the world-sheet projection of type IIB theories on CY threefolds relates $N = 1$ to $N = 2$ models, this complexification relates chiral multiplets to hypermultiplets, giving a new kind of c-map (or, more precisely, an s_n^{-1}-map). The s_n-map defined in [16] relates a special Kähler manifold [31] of complex dimension n to a quaternionic manifold of quaternionic dimension $(n + 1)$. In more physical terms, the Kähler manifold $K_n \times SU(1,1)/U(1)$ of $2(n + 1)$ NS-NS scalars is "doubled" into Q_{n+1} after the addition of $2(n + 1)$ R-R scalars. The new map for open descendants, that we shall term o-map, associates Q_{n+1} to a new Kähler manifold K_I for $(n+1)$ NS-NS and $(n+1)$ R-R real scalars. It would be interesting to analyze the intrinsic properties of the real Kähler geometry determined by the o-map, since it should be closely related to properties of the moduli spaces of (CY) fourfolds. Composing the o-map with the s-map, one would establish a direct correspondence between heterotic and type I vacua with $N = 1$ supersymmetry.

The complexification of the Kähler class of classical CY threefolds is at the heart of mirror symmetry [32], that should also have a new realization in this setting, probably related to a 12D interpretation. In this respect, it is worth recalling that in closed string theories the complexification of the Kähler cone $J \rightarrow ig + B_{NS}$ naturally implies that the classical special geometry of the $N = 2$ moduli space receives quantum corrections from string (one-brane) world-sheet instantons. The form of eq. (5.1) suggests that in this case the $N = 1$ classical Kähler manifold should receive quantum corrections from D-brane [33] world-volume instantons [34].

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