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INTERFACES IN HOT GAUGE THEORY

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Consider Hamiltonian $Z(2)$ gauge theory in a heat bath with temperature T . The stringtension at low T and the free energy of domainwalls at high T can be computed from one and the same observable. We show by explicit calculation that domain walls in hot $Z(2)$ gauge theory have good thermodynamical behaviour. This is due to roughening of the wall, which expresses the restoration of translational symmetry.

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1 Introduction

Traditionally, spontaneously broken symmetries are restored at high enough temperature. High enough means the scale of the energy gap in the theory. Nature provides us with such systems abundantly, such as superconductivity, ferromagnetism and the electroweak sector of the standard model of particle physics.

Much less known is the fact that not only nature, but also the laboratory of Lagrangian field theory, provides us with systems where the opposite happens. Weinberg¹ quotes the example of Seignette salt, a ferroelectric compound; below the critical temperature it orthorombic and above only isoclinic. Also, during the last two decades examples in field theory have been given mainly in connection with particle physics and cosmology.

Independently of these developments, the study of $SU(N)$ gauge theories has led to the notion of spontaneously broken centergroup ($Z(N)$) symmetry⁴ taking place above the deconfinement transition. This notion has been useful in understanding certain universal properties of the transition, like critical exponents⁵.

Other consequences of the spontaneous breaking are domain walls between coexisting phases. These questions have been addressed at very high temperatures where semi-classical techniques are possible⁹. It was soon discovered that these walls showed thermodynamic anomalies¹⁰; and other criticism sprang up, going as far as to deny the concept of broken $Z(N)$ symmetry.

In this paper we want to restore the balance and point out that, in simple gauge systems, a fully controlled calculation gives perfectly sensible answers to thermodynamical behaviour of the walls. The model in question is $Z(2)$ gauge theory, formulated with a lattice Hamiltonian in two space dimensions. The reason one has control is that the model can be reformulated as a two-dimensional Ising model. Its three-dimensional version is not analytically solvable, but similar properties hold.

Although the reader may think $Z(2)$ theory a little outlandish, we believe the results will carry over to an $SU(2)$ theory, because the main actors are electric fluxes and are present in both theories.

In the next section we will formulate the problem in a general way. The subsequent section deals with the $Z(2)$ case. Most of Sections 2 and 3 has been known for 15 years. Only from Sec. 4 on are there developments which to the best of our knowledge have not appeared in the literature: a more precise formulation of boundary conditions, and the roughening of the surface.

2 Formulation of the Problem

Let us start with some simple observations on a domain wall in between two ordered phases.

If we have a system that breaks the symmetry at low temperature, the wall reflects the properties of the high temperature phase. This implies that entropy and internal energy of the wall are higher than those in the surroundings. Needless to say, the free energy of the wall is *higher* because otherwise the system would be unstable against wall formation.

Now the case under study: the symmetry breaks at high temperature. Then the wall has the properties of the low T phase- hence a lower entropy and internal energy! Of course stability requires that the resulting free energy be higher, just as in the previous case.

Local free energy and entropy are not unambiguously defined inside the wall. Only the internal energy is locally defined as the expectation value of the local Hamiltonian.

The free energy F of a gauge theory is related to the Gibbs sum Z over physical states:

$$\exp -\frac{F}{T} \equiv Z = \sum_{phys} \langle phys | \exp \frac{-H}{T} | phys \rangle. \quad (1)$$

A physical state in a gauge theory can be written as the average over all gauge transformations $\Omega(\vec{x})$:

$$|phys\rangle = \int D\Omega |\vec{A}^\Omega\rangle. \quad (2)$$

We suppose that boundary conditions on Ω have been imposed, such that any product of two of them again obeys the boundary conditions.

With this definition a physical state is gauge-invariant. A gauge transformation in Hilbert space is generated by the Gauss operator $G(\vec{x}) \equiv \vec{D} \cdot \vec{E}(\vec{x})$ in the following way:

$$|\vec{A}^\Omega\rangle = \exp i\omega \cdot G|\vec{A}\rangle \quad (3)$$

The dot means summation over all indices, such as space and colour and $\Omega \equiv \exp i\omega$. The Gauss operator commutes with the Hamiltonian, and the gauge invariance of a physical state leads to:

$$\langle \vec{A}^\Omega | \exp \frac{-H}{T} | phys \rangle = \langle \vec{A} | \exp \frac{-H}{T} | phys \rangle. \quad (4)$$

In other words, the presence of the physical state renders the gauge transformation on other $|\vec{A}\rangle$ states redundant. This is an important property and

corresponds, in the Euclidean path integral version of the free energy, to the freedom in dropping integrations over A_0 variables in the Euclidean time direction without changing the free energy.

Here we want to avoid Euclidean path integrals altogether. Let it suffice to mention that in the path integral one can define the Polyakov loop as the ordered product of all the A_0 potentials along the line in the Euclidean time direction from 0 to $\frac{1}{T}$. This formulation is useful numerically, but is obscuring the issues at hand, i.e. the formation of surfaces in space. Due to the above invariance the Wilson line can be reduced to the sole gauge transform Ω . Nowhere will we introduce Euclidean time.

Instead, we will combine Eqs. (1), (2) and (4) and do the integrations over \vec{A} to get an effective action in the *same* dimensionality as the original Hamiltonian:

$$\int D\vec{A} \langle \vec{A} | \exp \frac{-H}{T} | \vec{A}^\Omega \rangle \equiv \exp -S_{eff} \quad (5)$$

so that

$$\exp \frac{-F}{T} = \int D\Omega \exp -S_{eff}(\Omega). \quad (6)$$

The gauge transformation lives in the same space as that of the quantum states and the Hamiltonian. The effective action describes the interactions between the gauge group variables. There is no Euclidean time in Eq. (6).

What purpose does the effective action serve?

It defines averages such as $\langle \Omega(\vec{0}) \rangle$ (the average of the Wilson line) and more generally correlations. These correlations are, in the language of the quantum statistical Gibbs sum (8), simply the correlations of heavy charges³. These correlations teach us about the forces present in the system for various temperatures. The import of the effective action is now clear, and in Sec. 3 these correlations are worked out for the simple Z(2) model.

3 The Z(2) Gauge Model

3.1 The Model

We study a system with gauge group Z(2) on a spatial lattice with continuous time at equilibrium with a thermostat at temperature T. We will partly follow Kiskis⁷ The Hamiltonian reads

$$H = \sum_l K(1 - \sigma_x(l)), \quad (7)$$

K is the coupling and $\sigma_x(l)$ is the first Pauli matrix defined on the link l . It acts as such on the two-component wave function in the link l and acts as unity on wave functions on the other links. The system is represented by the product of those two-component ψ 's:

$$\Psi = \prod_l \psi(l) .$$

The Hamiltonian contains only a kinetic term analogous to an electric term in an $SU(N)$ gauge theory. We omit the magnetic plaquette term for simplicity. It is not essential to the argument of this paper.

The ensuing theory is not trivial! The partition function Z in Eq. (1) has physical states satisfying Gauss's law

$$G(\vec{x})|phys\rangle \equiv \prod_{\{l \leftarrow l_{\vec{x}}\}} \sigma_x(l)|phys\rangle = |phys\rangle \quad (8)$$

We can diagonalize all the σ_x and obtain on any link an eigenvalue $2K$ or 0 . We then have

$$Z(T/K) = \sum'_{\{\phi(l)\}} \prod_l \left\{ \exp -\frac{2K}{T} \phi(l) \right\}, \quad (9)$$

where the prime denotes the Gauss's constraint and $\phi(l)$ takes the values 0 or 1 . If the latter we say there is "flux" on link l .

It is easy to see that a physical state can be represented by a configuration of fluxes such that an even number enters into every vertex of the lattice. One can define the charge

$$Q(\vec{x}) = \sum_{\{l \leftarrow l_{\vec{x}}\}} \phi(l) \equiv \Phi(\vec{x}), \text{ mod } 2 \quad (10)$$

so Gauss's law in Eq. (8) does not allow charge.

3.2 Order Parameter

We will now establish the relation between correlations of Wilson lines and the correlations between heavy test charges. First we need states with a test charge at \vec{y} . To this end we introduce the two projectors :

$$P_e(\vec{x}) \equiv \frac{1}{2} \sum_{\omega(\vec{x})=0,1} G(\vec{x})^{\omega(\vec{x})} \quad (11a)$$

and

$$P_o(\vec{x}) \equiv \frac{1}{2} \sum_{\omega(\vec{x})=0,1} (\exp i\pi\omega(\vec{x})) G(\vec{x})^{\omega(\vec{x})} \quad (11b)$$

We introduced here the notation

$$\Omega(\vec{x}) \equiv \exp i\pi\omega(\vec{x}) = \pm 1, \quad (12)$$

using $G(\vec{x})$ as defined in Eq. (8).

In the basis where the Hamiltonian and hence the Gauss operator are diagonal, the projectors read:

$$P_e(\vec{x}) = \frac{1}{2} \sum_{\Omega(\vec{x})=\pm 1} \Omega(\vec{x})^{\Phi(\vec{x})} \quad (13a)$$

and

$$P_o(\vec{x}) = \frac{1}{2} \sum_{\Omega(\vec{x})=\pm 1} \Omega(\vec{x})^{\Phi(\vec{x})+1}. \quad (13b)$$

The first projector renders the number of fluxes ending in \vec{x} even, the second one admits only an odd number, so creates a charge mod 2. The projectors obviously commute with the Hamiltonian.

Consider a quantum state of the gauge field, which is physical everywhere except at \vec{y} , where an odd number of fluxes ends (so where the charge sits). It can be written with the help of the projectors as:

$$P_o(\vec{y})\Pi_{\vec{x}\neq\vec{y}}P_e(\vec{x})|\vec{A}\rangle \equiv |Q(\vec{y}), \vec{A}\rangle. \quad (14)$$

The free energy F_Q of the system with a charge is defined by:

$$\exp -\frac{F_Q}{T} = \sum_{\{\vec{A}\}} \langle \vec{A}, Q(\vec{y}) | \exp -\frac{H}{T} | \vec{A}, Q(\vec{y}) \rangle. \quad (15)$$

The projectors on the bra can be commuted through the Boltzmann factor and give, with Eqs. (11):

$$\exp -\frac{F_Q}{T} = \sum_{\{\Omega\}} \langle \vec{A} | \exp -\frac{H}{T} \Omega(\vec{y}) | \vec{A}^{\Omega} \rangle$$

Together with the definition of S_{eff} in Eq. (5) we find:

$$\exp -\frac{F_Q}{T} = \sum_{\{\Omega\}} \Omega(\vec{y}) \exp -S_{eff} \quad (16)$$

The reader will have noticed that an important point has been glossed over. It is the fact that a state with only one charge in some point \vec{y} is hard

to realise, with Gauss' law satisfied everywhere else. In terms of the fluxes: imagine one flux leaving \vec{y} (never to return there anymore) it has necessarily an end point (or rather on odd number of them). These end points must be on the boundary, if we want Gauss's law everywhere inside our volume. Thus we need at least one charge on the boundary, on which the flux can land. More generally we need an odd number of charges on the boundary. So Gauss's law forces us in computing *correlations* of an even number of Wilson lines, and the free energy F_Q is obtained exponentially fast with distance.

4 Effective Action and the Ising Model

We start from Eq. (9). We can undo the Gauss constraint on the sum by introducing the projectors of Eqs. (13) and find for the partition function:

$$Z(T/K) = 2^{-V} \sum_{\{\Omega\}} \prod_l \exp -2 \frac{\phi(l)}{T} \prod_{\vec{x}} (\Omega(\vec{x}))^{\Phi(\vec{x})}. \quad (17)$$

V and L stand in this section for number of points and number of links on the lattice.

The summation over all gauge field configurations $\{\phi(l)\}$ is now easily done and leaves us with:

$$\exp -S_{eff} = \prod_l \left(1 + (\Omega\Omega')_l \exp -2 \frac{K}{T} \right). \quad (18)$$

In this equation only products of nearest-neighbour gauge transforms appear because a given flux $\phi(l)$ in Eq. (10) is present as power in both, in Eqn. (17).

Clearly the Ising model has emerged up to some factors. We find with the relation

$$\exp -2 \frac{K}{T} = \tanh J \quad (19)$$

that the r.h.s. of Eq. (17) can be rewritten as

$$\exp JL \frac{1}{\cosh J^L} \prod_l \exp -J(1 - (\Omega\Omega')_l). \quad (20)$$

The relation between the parameters can be extended to a local one: Eq. (19) is true linkwise. Note that the dual coupling J^* in the Ising model equals $\frac{K}{T}$.

This is the main result of this section. As expected from the previous section we have at high T (large J) spontaneous breakdown of the Z(2) symmetry.

We note in Eq. (17) that the Boltzmann weight of the gauge fields is accompanied by a factor depending on $\Omega(\vec{x})$ which is negative when there is a

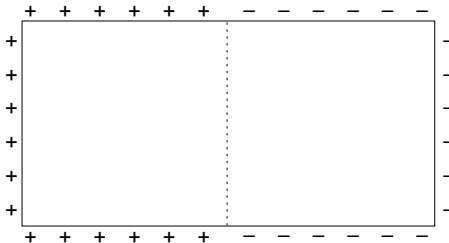


Figure 1: The system with boundary spins frozen, such that a domainwall develops in the middle. Multiplying all spins on one side of the wall with -1 will generate a dislocation along the centerline, and render the boundary conditions homogeneous.

charge in \vec{x} (only after integrating over the gauge transform do we recover the projectors, which give of course non negative factors). The thermodynamical behaviour of the gauge fields in terms of S_{eff} can therefore be quite unexpected. On the other hand the density matrix in terms of the variables Ω , Eq. (20) is positive.

Note that the behaviour is dual; it is obvious that at high T many quantum states contribute to the partition sum, whereas at large J few classical spin configurations contribute, So the quantum entropy of the gauge system is high whilst the classical entropy of the Ising model is low. We know that the Ising model allows coexistence of different phases, separated by domain walls. One can therefore ask how this translates into the gauge model at high temperature.

5 Coexisting Phases

There is a well-known device to create a domain wall in spin models: consider the boundary spins. Fix the spins on one side to be up, on the other side to be down (see Fig. 1).

In the Ising model we have to introduce an external field h_b at the boundary points b , which freezes the spins in question. Its counterpart in the gauge model is a chemical potential μ_b that couples to the charge $Q(b)$ of Eq. (10). The relation between the two is like that between the link couplings in Eq. (19):

$$\exp -\frac{\mu_b}{T} = \tanh h_b \quad (21).$$

Freezing spins with a positive (negative) field means $\mu_b = 0^+(0^+ + i\pi)$. It is clear that at large J (high T in the gauge model) the spins on both sides of this wall will have opposite sign. We have created a domain wall. To detect

the wall one can put a charge on one side, and measure the correlation with another charge. When going through the wall the correlation will go through zero and change sign when emerging on the other side.

Of course, the configuration of chemical potentials implies that any flux line passing through the dislocation an *odd* number of times will contribute with a minus sign to the partition function Z_{+-} ! This is so because such a flux will necessarily end with an odd number of end points on the boundary on both sides of the wall, picking up an odd number of minus signs. This is perhaps easier to see by transforming all the spin variables on one side of the wall with a minus sign. Then the border spins are all frozen the same way, and there is no minus sign from the border. But now it comes from the antiferromagnetic dislocation ($J \rightarrow -J$) we created by the transformation on the spins and by using the relation (19) between J and T ! Below the critical T_c there is no wall. Correspondingly, the correlation drops exponentially fast with distance and is controlled by the string tension $\rho(J)$.

The ratio of the partition functions with and without mixed boundary conditions signals in what phase we are:

$$\begin{aligned} \frac{Z_{+-}}{Z_{++}} &= 1 - \exp -\rho(J)L \quad \text{if } J < J_c \\ &= \exp -L_{tr}J\alpha(J) \quad \text{if } J > J_c \end{aligned}$$

The string tension has good thermodynamical properties⁶. We now analyse those of the domain wall.

6 Domain Wall and Roughening

In the Ising model for $d = 2$ the total free energy $\alpha(J)$ of the wall is known and equals $\alpha(J) = 2(1 - \frac{J^*}{J})$ for $J \geq J^*$. When the two are equal, we have the Curie point, where the surface stops to exist.

The location of the wall is subject to large fluctuations, of the order of $\sqrt{L_{tr}}$ for $d = 2$. This is known as roughening and its consequence is that the scale over which the profile is varying becomes $\sqrt{L_{tr}}$ in lattice units. The same is true for the energy density profile of the wall in the Ising model. The result is known analytically⁸ and reads:

$$\epsilon_I(l) = \frac{a}{\sqrt{L_{tr}}} \exp \left\{ -\sinh(2(J - J^*)) \frac{z^2}{L_{tr}} \right\}. \quad (22)$$

We will write $\frac{z^2}{L_{tr}} = \zeta^2$. The link l is supposed to run from z to $z+1$. Important is to note that the distance over which the interface is varying is of the order of

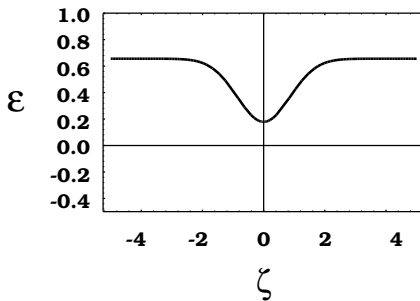
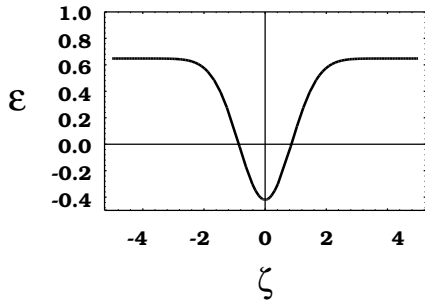


Figure 2: $\epsilon(\zeta) = \langle 1 - \sigma_x(l) \rangle(\zeta)$ is the mean value of the local energy operator at physical temperature T with $K/T = 0.3$, Eqn. (23). ζ is the rescaled variable z/\sqrt{L} . The length of the wall is $L_{tr} = 5$ in lattice units.

Figure 3: As in Fig. 2 but now for $L_{tr} = 25$.

$\sqrt{L_{tr}}$. As the integrated profile gives us the same total energy, the amplitude of the energy profile *decreases* like $\sqrt{L_{tr}}$.

This formula reflects the random walk our one dimensional surface is making. It is universal up to the typical Ising factor in front of the exponent. The constant a is such that upon integration we get the total energy $2(1 - \frac{\partial J^*}{\partial J})$. To translate this result into the average $\epsilon(\zeta)$ of the energy density $1 - \sigma_x(l)$ in the gauge model is straightforward. The result is:

$$\epsilon(\zeta) = \epsilon_0(T) - \left(-\frac{\partial J}{\partial J^*}\right)\epsilon_I(T). \quad (23)$$

The first term is the link energy density of the ground state of the gauge model, and is simply the Ising ground state energy per link, which is analytically known⁸.

The Jacobian- reflecting the change from the variable J to the variable $KT^{-1} = J^*$ - is always negative so the energy density inside the wall is smaller than outside. This is one of the hallmarks of a broken symmetry at high T , as we argued in the introduction. In Figs. 2 and 3 we show the dependence on L_{tr} .

7 Conclusions

Domain walls and string tension in gauge theory can be described by the asymptotic properties of the same partition function, for different temperature

regimes. When the system is large enough both string tension and wall have reasonable thermodynamical properties, the latter thanks to roughening. Its origin is the restoration of the translation invariance broken by the interface. In $d = 3$ the $\sqrt{L_{tr}}$ behaviour becomes logarithmic, and is only present for a range of temperatures not too far above T_c .

If one considers bubbles of radius R , the width of the wall will grow as \sqrt{R} or as $\log R$. The total energy of the wall profile depends only on temperature.

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