NUCLEON SPIN STRUCTURE, TOPOLOGICAL SUSCEPTIBILITY AND THE η' SINGLET AXIAL VECTOR COUPLING *)

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ABSTRACT

The observed small value of the first moment of the polarized nucleon spin structure function g_1 may be interpreted, in the Veneziano–Shore approach, as a suppression of the first moment $\chi'(0)$ of the QCD topological susceptibility. I give an extension of the Witten–Veneziano argument for the $U(1)$ problem, which yields the $O(1/N)$ correction to the $N=\infty$ relation $\chi'(0)/F_0^2 = 1$ (where F_0 is the η' axial vector coupling). The correction, although negative, seems too small to account for the data. I further argue that the $(\eta, \eta') \to \gamma\gamma$ and $J/\psi \to (\eta, \eta')\gamma$ decays indicate an enhancement rather than a suppression of F_0. A substantial gluon-like contribution in $\langle 0|\partial^\mu j^{(0)}_{\mu_5}|\gamma\gamma\rangle|_{q^2=0}$, which could parallel a similar one in the corresponding nucleon matrix element, is suggested.

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1 Introduction

The experimental discovery of a substantial suppression of the first moment $\Gamma_1^{(p,n)}$ of the polarized nucleon structure function with respect to the "naïve" (Ellis–Jaffe) OZI limit prediction [1] has spurred a lot of theoretical interest (for reviews, see Refs. [2], [3]) in recent years. Specifically, consider the full QCD expression [4] for $\Gamma_1^{(p,n)}$ (in the $\overline{\rm MS}$ scheme with $N_f = 3$):

$$\Gamma_1^{(p,n)}(Q^2) = \int_0^1 dx\ g_1^{(p,n)}(x,Q^2) = \frac{1}{6}\left(\pm G_A^{(3)} + \frac{1}{\sqrt{3}} G_A^{(8)}\right) \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.22\left(\frac{\alpha_s}{\pi}\right)^3 + \ldots\right) + \frac{1}{9} G_{A,inv}^{(0)}\left(1 - \frac{1}{3}\frac{\alpha_s}{\pi} - 0.550\left(\frac{\alpha_s}{\pi}\right)^2 + \ldots\right) \tag{1}$$

where $\alpha_s = \alpha_s(Q)$ and $G_A^{(i)}$ are zero-momentum transfer form factors in the proton matrix element of the axial vector currents: $\langle p|j_{\mu 5}^{(i)}|p\rangle = G_A^{(i)}\bar{p}\gamma_\mu\gamma_5 p + \ldots$. I stress that the radiative corrections in Eq. (1) imply that the scale independent, renormalization group–invariant singlet axial vector current $j_{\mu 5,inv}^{(0)}$ has been used to define $G_{A,inv}^{(0)}$. It is obtained from the current $j_{\mu 5}^{(0)}(\mu)$ renormalized in the standard way at scale μ in the $\overline{\rm MS}$ scheme by factorizing out the anomalous dimension factor generated by the $U(1)$ anomaly:

$$j_{\mu 5}^{(0)}(\mu) = j_{\mu 5,inv}^{(0)}\ \Big[1 + 0(\alpha_s(\mu))\Big]\ . \tag{2}$$

The crucial feature of Eq. (2) is that $j_{\mu 5}^{(0)}(\mu)$ has a "parton model" ($\mu \to \infty, \alpha_s(\mu) \to 0$) limit, owing to the special feature that the anomalous dimension starts only at $O(\alpha_s^2)$. It follows that $G_{A,inv}^{(0)}$ (also denoted as $\Delta\Sigma_{inv}$ [4] or $\Delta\Sigma_\infty$ [2]) is a physical, $\mu -$ independent constant, which stands on the same footing as $G_A^{(3)}$ and $G_A^{(8)}$, and that the whole physical Q^2 dependence is entirely contained in the (renormalization group–invariant) series in $\alpha_s(Q)$ in Eq. (1). (That $G_{A,inv}^{(0)}$ is a physical parameter should be clear from the observation that, at $Q^2 = \infty$, Eq. (1) gives the parton-model-like sum rule: $\Gamma_1^{(p,n)}(Q^2 = \infty) = \frac{1}{6}(\pm G_A^{(3)} + \frac{1}{\sqrt{3}}G_A^{(8)}) + \frac{1}{9}G_{A,inv}^{(0)}$). Experimentally, one finds [3] $G_{A,inv}^{(0)} \simeq 0.25$ (where I have taken into account the radiative corrections), to be compared with the Ellis–Jaffe value $G_{A|OZI}^{(0)} \simeq 0.58$, i.e. $G_{A,inv}^{(0)}/G_{A|OZI}^{(0)} \simeq 0.43$, roughly a factor of 2.

An interesting proposal to understand this suppression has been put forward in Ref. [5], where it has been suggested that it may be a (target-independent) effect related to the first moment of the QCD topological susceptibility $\chi(q^2)$, namely (for three flavours):

$$\frac{G_{A,inv}^{(0)}}{G_{A|OZI}^{(0)}} \simeq \frac{\sqrt{\chi'(0)}}{\frac{F_\pi}{\sqrt{6}}} \tag{3}$$

where $F_\pi = 93$ MeV,

$$\chi(q^2) \equiv \int d^4x\ e^{iq.x}\langle 0|T^*\Big(Q_{inv}(x)Q_{inv}(0)\Big)|0\rangle \tag{4}$$

and $Q_{inv}(x)$ is the anomalous divergence of the singlet axial vector current:

$$\partial^\mu j^{(0)}_{\mu 5,inv} = 3\frac{\alpha_s}{4\pi} F\tilde{F} \equiv 6Q_{inv} \ . \tag{5}$$

The basic physical assumption behind Eq. (3) is that large Zweig rule violations in $G^{(0)}_{A,inv}$ are to be found mainly in the $\sqrt{\chi'(0)}$ factor, which embodies the typical $q\bar{q} \to$ 2-gluons annihilation diagrams, which are supposed to most strongly violate the Zweig rule. In this note, I examine new ways to test this assumption. In the next section, I first derive an extension of the Witten–Veneziano argument ([6],[7]) for the solution of the $U(1)$ problem, which determines the $O(1/N)$ correction to the relation $\sqrt{\chi'(0)}/F_0|_{N=\infty} = 1$, where F_0 is the $\underline{\mathrm{physical,\ RG-invariant}}$ η' singlet axial vector coupling to $j^{(0)}_{\mu 5,inv}$ (in the chiral limit). Although the resulting correction tends indeed to suppress $\sqrt{\chi'(0)}$ with respect to F_0, it still appears to be a small perturbation on the $N=\infty$ result; it is thus likely to be insufficient to account for the observed suppression, at least as long as the nonet symmetry relation $(F_0/F_8)|_{N=\infty} = 1$ remains approximately valid at $N=3$ (the normalization is such that $F_8 \simeq F_\pi/\sqrt{6}$). Therefore, assuming $\sqrt{\chi'(0)}/F_0 \simeq 1$ the remaining possibility is that there is a large suppression of F_0/F_8 itself at finite N. I examine whether this assumption is phenomenologically viable in Section 3, where I point out that even a moderate suppression of F_0 would lead to severe difficulties with the current standard model [8] for $J/\psi \to (\eta,\eta')\gamma$ decays, given the large $\eta-\eta'$ mixing angle, which follows from an analysis of the octet electromagnetic (e.m.) sum rule for the $(\eta,\eta') \to \gamma\gamma$ decays (too strong a suppression of F_0 is not favoured either by the singlet e.m. sum rule). In Section 4, I note that the observed smallness of $G^{(0)}_{A,inv}$ might indicate a substantial glueball-like contribution to $G^{(0)}_{A,inv}$, which should then cancel against that of the η', assuming the latter to be of typical $G^{(0)}_{A|OZI}$ size if F_0 is not suppressed (and could thus be identified to the quark spin piece ([9]–[11]) of the nucleon in the chiral limit). I then draw a parallel with the occurrence of a sizeable violation of the $\eta-\eta'$ saturation hypothesis in the $\langle 0|\partial^\mu j^{(0)}_{\mu 5,inv}|\gamma\gamma\rangle|_{q^2=0}$ matrix element.

2 $\chi'(0)$ at large N

Consider the dispersion relation:

$$\chi(q^2) = \chi(0) + \chi'(0)q^2 + \frac{q^4}{\pi}\int_{q_0^2}^{\infty} \frac{dq'^2}{q'^2}\ \frac{Im\chi(q'^2)}{q'^2-q^2} \tag{6}$$

where two subtractions are needed, since $\chi(q^2)$, which is of dimension 4, is $O(q^4)$ at large q^2. In the quarkless Yang-Mills theory, one can thus write (symbolically):

$$\chi_{YM}(q^2) = A_{YM} + B_{YM}q^2 + \frac{F_G^2 M_G^4}{M_G^2 - q^2} + \dots \tag{7}$$

where F_G and M_G are the coupling and mass of the lowest-lying glueball state, and the dots stand for more massive glueballs as well as continuum contributions, whereas in the presence of quarks, splitting out the η' contribution:

$$\chi(q^2) = \frac{F_0^2 m_0^4}{m_0^2 - q^2} + \left[A + Bq^2 + \frac{F_G^2 M_G^4}{M_G^2 - q^2} + \ldots \right] \tag{8}$$

where m_0 is the η' mass in the chiral limit, and the subtraction constants $(A_{YM}, B_{YM}), (A, B)$ (which are not reducible to the glueballs contribution) have been introduced. Taking the $N \to \infty$ limit in Eq. (8) at fixed $q^2 \neq 0$, one then expects (since quark loops are subleading and decouple) $\chi(q^2) \to \chi_{YM}(q^2)$. Indeed, the η' contribution drops out, given that $F_0^2 = O(N)$, if one assumes [6] $m_0^2 = O(1/N)$, whereas the quantity within brackets in Eq. (8) approaches $\chi_{YM}(q^2)|_{N=\infty}$, i.e., $A \to A_{YM\ |N=\infty}$, $B \to B_{YM\ |N=\infty}$ (and glueballs $\to$ glueballs$|_{N=\infty}$). The implication for $\chi'(q^2)$ is obtained by expanding Eq. (8) around $q^2 = 0$:

$$\begin{aligned} \chi(q^2) &= \left[F_0^2 m_0^2 + (A + F_G^2 M_G^2 + \ldots)\right] + q^2 \left[F_0^2 + (B + F_G^2 + \ldots)\right] + 0(q^4) \\ &\equiv \chi(0) + q^2 \chi'(0) + O(q^4) \ . \end{aligned} \tag{9}$$

The basic QCD constraint (for massless quarks) $\chi(0) = 0$ then gives:

$$\chi(0) = F_0^2 m_0^2 + (A + F_G^2 M_G^2 + \ldots) = 0 \ . \tag{10}$$

Letting $N \to \infty$ in Eq. (10), one first recovers the relation ([6],[7]):

$$F_0^2 m_0^2|_{N=\infty} = -(A_{YM} + F_G^2 M_G^2 + \ldots)|_{N=\infty} \equiv -\chi_{YM}(0)|_{N=\infty} \ , \tag{11}$$

whereas for $\chi'(0)$ one obtains from Eq. (9) the additional relation[1]:

$$(\chi'(0) - F_0^2)|_{N=\infty} = (B_{YM} + F_G^2 + \ldots)|_{N=\infty} \equiv \chi'_{YM}(0)|_{N=\infty} \ . \tag{12}$$

Since $\chi'_{YM}(0)$ is $O(1)$ and F_0^2 is $O(N)$, Eq. (12) requires $\chi'(0)$ to be $O(N)$ and positive, in order that a cancellation takes place with F_0^2. On the other hand, a lattice calculation [13], in agreement with a QCD sum rule analysis [12], yields $\chi'_{YM}(0) < 0$. Writing Eq. (12) as:

$$\frac{\chi'(0)}{F_0^2} \underset{N \to \infty}{\simeq} 1 + \frac{\chi'_{YM}(0)}{F_0^2} \tag{13}$$

one thus finds the second term on the right-hand side gives the $O(1/N)$ correction to the OZI limit relation $\chi'(0)/F_0^2|_{N=\infty} = 1$, and indeed tends to suppress $\chi'(0)$ with respect to F_0^2, since $\chi'_{YM}(0) < 0$. However, the correction appears numerically small (from Refs. [12] and [13] one gets $-\chi'_{YM}(0)/F_0^2 \simeq 0.1$), which suggests that the OZI violations in $\chi'(0)/F_0^2$ are probably small and that the large-N expansion is reliable for this ratio. In the next section, I investigate whether the assumption that there are instead large OZI violations that strongly suppress the ratio F_0/F_8 at $N = 3$ is phenomenologically viable.

The results of this section suggest a simple model for the structure of $\chi(q^2)$ at finite N [2] in the presence of massless quarks, where it is written as the sum of the η' pole contribution and

[1] This relation was first discovered in Ref. [12], where it was (interestingly) suggested by a QCD sum rule analysis of $\chi_{YM}(0)$.

[2] I am indebted to G. Veneziano for stressing this point.

the Yang–Mills topological susceptibility: $\chi(q^2) = \chi_{YM}(q^2) + F_0^2 m_0^4/(m_0^2 - q^2)$, and $\chi_{YM}(q^2)$ is further approximated by dropping the glueballs contribution, and keeping only the subtraction terms, namely taking:

$$\chi_{YM}(q^2) \equiv A_{YM} + B_{YM} q^2 \equiv \chi_{YM}(0) + \chi'_{YM}(0) q^2 \; .$$

We thus get:

$$\chi(q^2) = \chi_{YM}(0) + \chi'_{YM}(0) q^2 + \frac{F_0^2 m_0^4}{m_0^2 - q^2} \; . \tag{14}$$

The constraint $\chi(0) = 0$ yields $\chi_{YM}(0) + F_0^2 m_0^2 = 0$ [cf. Eq. (11)]. Since $\chi'(0) = F_0^2 + \chi'_{YM}(0)$ [cf. Eq. (12)], Eq. (14) then becomes, after eliminating m_0^2:

$$\chi(q^2) = q^2 \left(\frac{\chi_{YM}(0)}{q^2 + \frac{\chi_{YM}(0)}{F_0^2}} + \chi'_{YM}(0) \right) \; . \tag{15}$$

If $\chi'_{YM}(0)$ is dropped, i.e. if one assumes $\chi_{YM}(q^2) \equiv \chi_{YM}(0)$, one recovers an ansatz given in Ref. [14], which yields $\chi'(0) = F_0^2$; the additional term $\chi'_{YM}(0)$ accounts for the OZI violation in this model.

3 Implication of a small F_0 for $J/\psi \to (\eta, \eta')\gamma$ and $(\eta, \eta') \to \gamma\gamma$ decays

An analysis [15] of $\eta - \eta'$ mixing using the anomalous Ward identities does indicate that a large suppression of F_0/F_8 is indeed possible at large mixing angles, and at least appears to favour a moderate suppression in this region (these results may, however, be changed by taking into account [16] the recently calculated [17] $O(m_q^2)$ quark mass corrections at $N = \infty$). A large mixing angle is itself supported [15] by the data on $(\eta, \eta') \to \gamma\gamma$. However, even a modest suppression of F_0 is in strong disagreement with the current standard model [8] for $J/\psi \to (\eta, \eta')\gamma$. The argument ([15], [16]) can be summarized as follows. From the octet e.m. anomaly sum rule (assuming $\eta - \eta'$ saturation):

$$F_{8\eta} A(\eta \to \gamma\gamma) + F_{8\eta'} A(\eta' \to \gamma\gamma) = \frac{1}{\sqrt{3}} \tag{16}$$

[where $F_{8p} (p = \eta, \eta')$ are the couplings to $j^{(8)}_{\mu 5}$], one can extract $F_{8\eta'}$, using as input the experimentally determined amplitudes [18]: $A(\eta \to \gamma\gamma) = (0.993 \pm 0.030) F_\pi^{-1}$ and $A(\eta' \to \gamma\gamma) = (1.280 \pm 0.085) F_\pi^{-1}$, as well as the crucial perturbation theory estimate [19]: $F_{8\eta}/F_\pi = 1.3 \pm 0.05$. As an indication, using $F_{8\eta}/F_\pi = 1.25$, one gets $\sin\theta \equiv -F'_{8\eta}/F_\pi = 0.52$, a rather large value.

Furthermore, the singlet couplings F_{0p} can be constrained with the $J/\psi \to (\eta, \eta')\gamma$ decays. Indeed the current standard model [8] for the ratio $\Gamma(J/\psi \to \eta'\gamma)/\Gamma(J/\psi \to \eta\gamma)$ relates it to $\tilde{f}_{\eta'}/\tilde{f}_\eta$ (where the $\tilde{f}_p$'s are the anomalous divergence couplings: $\langle 0|Q|p\rangle = \tilde{f}_p m_p^2$):

$$R \equiv \frac{\tilde{f}_{\eta'}}{\tilde{f}_\eta} \simeq \left[\frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma)} \; \frac{(M^2_{J/\psi} - m_\eta^2)^3}{(M^2_{J/\psi} - m_{\eta'}^2)^3} \right]^{1/2} \; \frac{m_\eta^2}{m_{\eta'}^2} \; ; \tag{17}$$

This gives, using [18] $\Gamma(J/\psi \to \eta'\gamma)/\Gamma(J/\psi \to \eta\gamma) = 5.0 \pm 0.6$: $R_{\rm exp} = 0.81 \pm 0.05$. But $\tilde{f}_p$ can be expressed [15] in terms of the corresponding axial vector couplings F_{ip} and the quark mass ratios β/γ and γ/α:

$$\begin{aligned}
\frac{\tilde{f}_{\eta'}}{F_\pi} &\equiv \frac{F_{0\eta'}}{F_\pi} - \frac{f_{0\eta'}}{F_\pi} \simeq \frac{F_{0\eta'}}{F_\pi} - \frac{\beta}{\gamma}\ \frac{F_{8\eta'}}{F_\pi} \\
\frac{\tilde{f}_{\eta}}{F_\pi} &\equiv \frac{F_{0\eta}}{F_\pi} - \frac{f_{0\eta}}{F_\pi} \simeq \frac{F_{0\eta}}{F_\pi} - \frac{\gamma}{\alpha}\ \frac{F_{8\eta}}{F_\pi}
\end{aligned} \tag{18}$$

where the f_{0p}'s are the "naïve divergence" couplings, $\alpha \equiv (2/3)(m_u+m_d+4m_s)$, $\beta \equiv (4/3)(m_u+m_d+m_s)$, $\gamma \equiv -\sqrt{2}(\alpha-\beta)$, and we have the estimates [19] $\beta/\gamma \simeq 0.79$ and $\gamma/\alpha \simeq -0.67$. I shall simply use the value (obtained by putting $m_u = m_d = 0$): $\beta/\gamma = \gamma/\alpha = -1/\sqrt{2}$. Then one gets:

$$\frac{\tilde{f}_{\eta'}}{\tilde{f}_\eta} \simeq \frac{\frac{F_{0\eta'}}{F_\pi} - \frac{\beta}{\gamma}\ \frac{F_{8\eta'}}{F_\pi}}{\frac{F_{0\eta}}{F_\pi} - \frac{\gamma}{\alpha}\ \frac{F_{8\eta}}{F_\pi}} = \frac{\frac{F_{0\eta'}}{F_\pi} - \frac{1}{\sqrt{2}}\ \sin\theta}{\frac{F_{0\eta}}{F_\pi} + \frac{1}{\sqrt{2}}\ \frac{F_{8\eta}}{F_\pi}}\ . \tag{19}$$

Assuming again $F_{8\eta}/F_\pi = 1.25$ and $\sin\theta \equiv -F'_{8\eta}/F_\pi = 0.52$ from the octet sum rule, and taking $\tilde{f}_{\eta'}/\tilde{f}_\eta = R_{\rm exp} \simeq 0.81$, Eq. (19) then fixes $F_{0\eta}$ as a function of $F_{0\eta'}$, and one finds that unrealistically small values of $F_{0\eta}$ are required to fit $R_{\rm exp}$. For instance, assuming the moderately suppressed value $F_{0\eta'}/F_\pi = 0.90$, Eq. (19) gives $F_{0\eta}/F_\pi = -0.23$, which violates, even in sign, the large-N expectation [15]: $F_{0\eta} \simeq -F_{8\eta'}$! Also, one still gets $F_{0\eta} = 0$ even for $F_{0\eta'}/F_\pi$ as large as 1.1. Clearly, the model of Eq. (17) is <u>incompatible with any kind of suppression whatsoever of the singlet coupling</u> $F_{0\eta'}$, given the large input values of the octet couplings $F_{8\eta}$ and $-F_{8\eta'}$. Since the quark mass corrections that relate $F_{0\eta'}$ to its chiral limit F_0 are small (they have been estimated [15] to be $F_{0\eta'} \simeq 1.16F_0$), this observation probably rules out the possibility that $F_0/F_8 \simeq F_0/F_\pi$ be substantially suppressed. In fact, for $F_{0\eta}/F_\pi = 0.50 (\simeq -F_{8\eta'}/F_\pi)$, Eq. (19) gives $F_{0\eta'}/F_\pi = 1.49$, hence $F_0/F_\pi = 1.28$, an <u>enhancement</u>! On the other hand, the singlet e.m. sum rule (<u>assuming again</u> η, η' <u>saturation</u>):

$$F_{0\eta}A(\eta \to \gamma\gamma) + F_{0\eta'}A(\eta' \to \gamma\gamma) = 2\sqrt{\frac{2}{3}} \tag{20}$$

does favour a (<u>moderate</u>)[3] suppression of $F_{0\eta'}$, e.g., if one again assumes $F_{0\eta}/F_\pi = 0.50$, one deduces from Eq. (20) $F_{0\eta'}/F_\pi \simeq 0.89$ (this suppression would not be sufficient anyway to explain the magnitude of $G^{(0)}_{A,inv}$). However, Eq. (19) then gives (taking the same values as above for $F_{8\eta}$ and $F_{8\eta'}$) $R \simeq 0.38$, still a factor of 2 below $R_{\rm exp}$, in accordance with the previous remarks (this potential conflict between the singlet e.m. sum rule and $R_{\rm exp}$ is further commented upon in the next section).

[3] That is, too strong a suppression, such as the one needed to explain $G^{(0)}_{A,inv}$, is not favoured either by Eq. (20), which would then lead to values of $F_{0\eta}$ too large, typically $F_{0\eta}/F_\pi \simeq F_{0\eta'}/F_\pi \simeq 0.7$!

4 On gluonic contributions to $\langle 0|\partial^\mu j^{(0)}_{\mu_5}|N\bar{N}\rangle$ and $\langle 0|\partial^\mu j^{(0)}_{\mu_5}|\gamma\gamma\rangle$

The smallness of $G^{(0)}_{A,inv}$ may alternatively be seen as the result of a cancellation [20] between the η' and the (glueball + continuum) contributions (I consider for simplicity the chiral limit, where the η decouples from $G^{(0)}_A$). This picture can be given a precise content by using the invariant definition of the singlet current (which removes [see also below] the inconsistencies with renormalization group invariance discussed in Refs. [5] and [21]). One can define, splitting out the η' contribution ($g_{\eta' NN}$ is the η'-nucleon coupling):

$$\langle 0|\partial^\mu j^{(0)}_{\mu_5,inv}|N\bar{N}\rangle \propto \Delta\Sigma_{inv} \equiv G^{(0)}_{A,inv} \equiv F_0\; g_{\eta' NN} + \Delta\Gamma_{inv}\ , \tag{21}$$

where $\Delta\Gamma_{inv}$ represents the (glueball + continuum) contribution, and all quantities in Eq. (21) are renormalization group-invariant. It is then attractive to identify the "quark contribution" $\Delta\Sigma'_{inv}$ to the nucleon spin with the η' contribution[4] $F_0\; g_{\eta' NN}$, while $\Delta\Gamma_{inv}$ would represent the "gluon contribution" ([9]–[11]). If F_0 is indeed not suppressed, one might further assume, in the line of the latter references, that

$$\Delta\Sigma'_{inv} = F_0\; g_{\eta' NN} \sim (F_0\; g_{\eta' NN})|_{OZI} = G^{(0)}_A|_{OZI} \tag{22}$$

and attribute the small value of $G^{(0)}_{A,inv}$ to the effect of a substantial (negative) $\Delta\Gamma_{inv}$, i.e. $G^{(0)}_{A,inv} \simeq G^{(0)}_{A|_{OZI}} + \Delta\Gamma_{inv}$ (it could also be that both $F_0\; g_{\eta' NN}$ and $\Delta\Gamma_{inv}$ are suppressed, in which case $F_0\; g_{\eta' NN}$ would still differ from $\Delta\Sigma'_{inv}$ by some additional, non-perturbative ([22], [2]) contributions, e.g. $F_0\; g_{\eta' NN} = \Delta\Sigma'_{inv} - N_f\Omega$) [5]. Such a proposal, which identifies $\Delta\Gamma_{inv}$ to $G^{(0)}_{A,inv} - F_0\; g_{\eta' NN}$, is complementary to the QCD-improved parton model approach of Refs. ([9]-[11]), which starts from the implicit assumption that it is possible to define a (perturbatively wise [24]) unique physical gluon distribution, which could independently be measured in suitable hard processes.

There is an interesting parallel with the situation for the $\langle 0|\partial^\mu j^{(0)}_{\mu_5,inv}|\gamma\gamma\rangle|_{q^2=0}$ matrix element: the above-mentioned conflict between the singlet e.m. sum rule (which favours a moderately suppressed F_0) and $R_{\rm exp}$ (which favours an unsuppressed, or even enhanced, F_0) may be resolved by assuming that F_0 is indeed not suppressed. The resulting discrepancy in the singlet sum rule (where the η and η' contribution now by itself exceeds the right-hand side) can then be attributed to a substantial (negative) gluon-like contribution, as in the nucleon channel. Actually, considering the standard μ-dependent renormalization of the singlet current, Eq. (2), one may suspect that an independent subtraction constant $\Delta_0(\mu)$ enters the sum rule, in addition to the glueball contribution, i.e. Eq. (20) should be replaced by:

$$\langle 0|\partial^\mu j^{(0)}_{\mu_5}(\mu)|\gamma\gamma\rangle|_{q^2=0} \qquad \propto F_{0\eta}(\mu)A(\eta\to\gamma\gamma)$$

[4] In the presence of $SU(3)$ breaking, one should also add the η contribution.

[5] Alternatively one could have $\Delta\Sigma'_{inv} = F_0\; g_{\eta' NN}$ small, with $g_{\eta' NN}$ suppressed, as suggested by the Skyrme model [23].

$$+F_{0\eta'}(\mu)A(\eta'\to\gamma\gamma)+[F_G(\mu)A(G\to\gamma\gamma)+\ldots]$$
$$+\Delta_0(\mu)\equiv 2\sqrt{\frac{2}{3}}\,. \qquad (23)$$

The introduction of $\Delta_0(\mu)$ appears necessary[6] to resolve the conflict [21] between the multiplicative renormalizability of all the singlet axial vector couplings in the left-hand side of Eq. (23), and the μ-independence of the right-hand side (then, letting $\mu\to\infty$ one recovers the equation written in terms of the invariant couplings, with $\Delta_0(\mu)\to\Delta_{0,inv}$). This Δ_0 is also welcome to explain the conjectured existence of a sizeable discrepancy in the singlet sum rule with respect to the η,η' saturation hypothesis, since the glueballs by themselves are expected to couple too weakly to the photons to be responsible for the entire discrepancy (Δ_0 also recalls the necessary subtraction constant ([6],[15]) needed to cancel [see Eq. (10)] the η' contribution and implement the constraint $\chi(q^2=0)=0$ in the chiral limit of QCD; a subtraction constant has, however, no reason to be present in $\langle 0|\partial^\mu j^{(0)}_{\mu_5}|N\bar{N}\rangle$).

To conclude, the present analysis offers only scarce evidence for the suppression of $\chi'(0)$ as implied by Eq. (3). Although $\chi'(0)/F_0^2$ is indeed suppressed at next-to-leading order in $1/N$, the correction appears small, and of typical perturbative (in $1/N$) size. Furthermore, F_0 itself does not seem to be suppressed on phenomenological grounds, compared with F_8 (and may even turn out to be predicted enhanced at large $\sin\theta$ once the $O(m_q^2)$ corrections are taken into account in the Ward identity analysis of $\eta-\eta'$ mixing ([15],[16]). Assuming that F_0 is not suppressed, an intriguing picture of large (negative) gluon-like contributions to $\langle 0|\partial^\mu j^{(0)}_{\mu_5}|N\bar{N}\rangle|_{q^2=0}$, $\langle 0|\partial^\mu j^{(0)}_{\mu_5}|\gamma\gamma\rangle|_{q^2=0}$ and $\chi(q^2=0)$ tentatively emerges.

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[6] The origin of $\Delta_0(\mu)$ can probably be traced back to the additional ultraviolet divergence in $\langle 0|\partial^\mu j^{(0)}_{\mu_5}|\gamma\gamma\rangle$ arising first at the $O(\alpha_s^2\alpha)$ level from the (lowest-order in electromagnetism) $O(\alpha)$ coupling to $\gamma\gamma$, which is distinct from the standard one responsible for the QCD anomalous dimension of $j^{(0)}_{\mu_5}$ due to the strong anomaly.

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