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MORE ON SOFTLY BROKEN $N = 2$ QCD

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Abstract

We extend previous work on the soft breaking of $N = 2$ supersymmetric QCD. We present the formalism for the breaking due to a dilaton spurion for a general gauge group and obtain the exact effective potential. We obtain some general features of the vacuum structure in the pure $SU(N)$ Yang-Mills theory and we also derive a general mass formula for this class of theories, in particular we present explicit results for the mass spectrum in the $SU(2)$ case. Finally we analyze the vacuum structure of the $SU(2)$ theory with one massless hypermultiplet. This theory presents dyon condensation and a first order phase transition in the supersymmetry breaking parameter driven by non-mutually local BPS states. This could be a hint of Argyres-Douglas-like phases in non-supersymmetric gauge theories.

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1 Introduction and conclusions

Recently there has been great progress in understanding the dynamics of supersymmetric gauge theories in four dimensions. For $N = 1$ theories exact results have been obtained [1] using the holomorphy properties of the superpotential and the gauge kinetic function, culminating in Seiberg's non-abelian duality conjecture [2]. In two remarkable papers [3, 4], Seiberg and Witten obtained exact information on $N = 2$ Yang-Mills theories with gauge group $SU(2)$ and $N_f \leq 4$ flavour multiplets. Their work was extended to other groups in [5, 6, 7, 8]. One of the crucial advantages of using $N = 2$ supersymmetry is that the low-energy effective action in the Coulomb phase up to two derivatives (*i.e.* the Kähler potential, the superpotential and the gauge kinetic function in $N = 1$ superspace language) are determined in terms of a single holomorphic function called the prepotential [9], which was exactly determined in [3, 4] using some plausible assumptions and many consistency conditions. For $SU(2)$ the solution is neatly presented by associating to each case an elliptic curve together with a meromorphic differential of the second kind whose periods completely determine the prepotential. For other gauge groups [5, 8] the solution is again presented in terms of the period integrals of a meromorphic differential on a Riemann surface whose genus is the rank of the group considered. It was also shown in [3, 4] that by soft breaking $N = 2$ down to $N = 1$ (by adding a mass term for the adjoint $N = 1$ chiral multiplet in the $N = 2$ vector multiplet) confinement follows due to monopole condensation [10].

With all this new information it is also tempting to analyze the dynamics of non-supersymmetric gauge theories in order to determine to what extent these results depend on the supersymmetry structure, and perhaps to obtain exact information about ordinary QCD. A useful avenue to explore is soft supersymmetry breaking. The structure of soft supersymmetry breaking in $N = 1$ theories has been known for some time [11]. In [12, 13] soft breaking terms are used to explore $N = 1$ supersymmetric QCD (SQCD) with gauge group $SU(N_c)$ and N_f flavours of quarks, and to extrapolate the exact results in [1] concerning the superpotential and the phase structure of these theories in the absence of supersymmetry. This leads to expected and unexpected predictions for non-supersymmetric theories which may eventually be accessible to lattice computations. Since the methods of [3, 4] provide us with the effective action up to two derivatives, the possibility was explored in [14] of breaking $N = 2$ SQCD directly to $N = 0$ preserving the analyticity properties of the Seiberg-Witten solution. It has been shown in [14] that a natural way to accomplish this task is, essentially, to make the dynamical scale Λ of the $N = 2$ theory a function of an $N = 2$ vector multiplet. This multiplet is then frozen to become a spurion whose F and D-components break softly $N = 2$ down to $N = 0$, in a way compatible with the Seiberg-Witten monodromies. The spurion can be interpreted in terms of the string derivation of the Seiberg-Witten solution in [15, 16], based on type II-heterotic duality. In this spirit, the soft breaking terms were rederived in [14] starting from the theory coupled to $N = 2$ supergravity with a simple superpotential which breaks spontaneously supersymmetry. The resulting theory at $N = 0$ has a more restricted structure than those used in [12, 13], and it is possible to compute the exact effective potential. The $SU(2)$ case with $N_f = 0$ and $N_f = 2$ massless quark hypermultiplets were analyzed in detail, and it turned out that these softly broken theories have a unique ground state (near the massless monopole points of [3, 4]) and confinement occurs through monopole condensation.

In this paper we extend the results of [14] in three different directions. First of all, we present the general formalism for the breaking of supersymmetry due to a dilaton spurion for a general gauge group with massless hypermultiplets, and we study the symplectic transformations of the various quantities involved. The results agree with the general structure derived in [17] concerning the modification of the symplectic transformations of special geometry in the presence of background $N = 2$ vector superfields. We also obtain the exact effective potential for the Coulomb phase in this general setting, and we derive some general features of the vacuum structure in the $SU(N)$ case studying the theory around the $N = 1$ points described in [7]. Another important issue addressed in this paper is the mass spectrum of the softly broken theories. We prove a general mass formula for this class of theories, stating that the graded trace of the squared mass matrix is zero, as it happens in supersymmetric theories and also in some restricted models of $N = 1$ soft supersymmetry breaking [18]. We also obtain explicit results for the mass spectrum in the case of $SU(2)$ theories. Finally, we consider the softly broken $SU(2)$ theory with one massless hypermultiplet. The vacuum structure of this model is very interesting, as it presents two degenerate ground states with dyon condensation. The corresponding BPS states are not mutually local and as the supersymmetry breaking parameter is turned on the regions where these states get a VEV begin to overlap. The possibility of this kind of behaviour was pointed out in [14], and here we find an explicit realization. We argue that the effective potential description of this situation is still reliable as long as the condensates do not attain the singularities associated to the other states, and this allows us to obtain some hints of the dynamics involving non-local objects. Within this range of validity we find a first order phase transition to a new ground state located in the region where the two condensates overlap. Although the interpretation of this new vacuum is not clear to us, it may correspond to a new kind of phase similar to the Argyres-Douglas phase [19]. The dynamics of this phase transition looks very much like the phase transitions in the theta angle leading to an oblique confinemente phase. Other possible interpretation would be the formation of a bound state condensing in the new vacuum.

We see that the study of $N = 2$ soft supersymmetry breaking gives a rich variety of dynamical structures. We believe that the introduction of bare masses for the quark hypermultiplets will also lead to interesting field theoretic phenomena that may shed some light on the exact dependence on the quark masses of the effective Goldstone boson lagrangian describing chiral symmetry breaking. This situation is presently under study.

The organization of this paper is as follows: In section two we extend the formalism of [14] to arbitrary gauge groups with massless hypermultiplets. In section three we derive the exact effective potential in the Coulomb phase and the basic tools to analyze the vacuum structure. In section four we analyze in some detail the case of the $SU(N)$ Yang-Mills theory without hypermultiplets. In section five we present a general mass sum rule, and also obtain explicit results for the masses in the $SU(2)$ case. In section six, we consider the $SU(2)$ theory with one massless hypermultiplet.

2 Breaking $N = 2$ with a dilaton spurion: general gauge group

In this section we present the generalization of the procedure introduced in [14] to $N = 2$ Yang-Mills theories with a general gauge group G of rank r and massless matter hypermultiplets.

The low energy theory description of the Coulomb phase [3] involves r

abelian $N=2$ vector superfields A^i, $i=1,\cdots,r$ corresponding to the unbroken gauge group $U(1)^r$. The holomorphic prepotential $\mathcal{F}(A^i,\Lambda)$ depends on the r superfields A^i and the dynamically generated scale of the theory, Λ. The low energy effective lagrangian takes the form (in $N=1$ notation) [3]:

$$\mathcal{L}=\frac{1}{4\pi}\mathrm{Im}\Big[\int d^4\theta\frac{\partial\mathcal{F}}{\partial A^i}\overline{A}^i+\frac{1}{2}\int d^2\theta\frac{\partial^2\mathcal{F}}{\partial A^i\partial A^j}W_\alpha^i W^{\alpha j}\Big], \tag{2.1}$$

We define the dual variables, as in the $SU(2)$ case, by

$$a_{D,i}\equiv\frac{\partial\mathcal{F}}{\partial a^i}. \tag{2.2}$$

The Kähler potential and effective couplings associated to (2.1) are:

$$\begin{aligned} K(a,\bar a) &= \frac{1}{4\pi}\mathrm{Im}a_{D,i}\bar a^i, \\ \tau_{ij} &= \frac{\partial^2\mathcal{F}}{\partial a^i\partial a^j}, \end{aligned} \tag{2.3}$$

and the metric of the moduli space is given accordingly by:

$$(ds)^2=\mathrm{Im}\frac{\partial^2\mathcal{F}}{\partial a^i\partial a^j}da^i d\overline{a}^j. \tag{2.4}$$

We introduce now a complex space $\mathbf{C}^{2r}$ with elements of the form

$$v=\left(\begin{array}{c} a_{D,i} \\ a^i \end{array}\right). \tag{2.5}$$

The metric (2.4) can then be written as

$$\begin{aligned} (ds)^2 &= -\frac{i}{2}\sum_i(da_{D,i}d\overline{a}^i-d\overline{a}_{D,i}da^i) \\ &= -\frac{i}{2}\left(\begin{array}{cc} da_{D,i} & da^i \end{array}\right)\begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0\end{pmatrix}\left(\begin{array}{c} d\overline{a}_{D,i} \\ d\overline{a}^i \end{array}\right), \end{aligned} \tag{2.6}$$

which shows that the transformations of v preserving the form of the metric are matrices $\Gamma\in Sp(2r,\mathbf{Z})$. They verify $\Gamma^{\mathrm{T}}\Omega\Gamma=\Omega$, where Ω is the $2r\times 2r$ matrix appearing in (2.6), and can be written as:

$$\begin{pmatrix} A & B \\ C & D\end{pmatrix} \tag{2.7}$$

where the $r \times r$ matrices A, B, C, D satisfy:

$$A^{\mathrm{T}}D - C^{\mathrm{T}}B = \mathbf{1}_r, \quad A^{\mathrm{T}}C = C^{\mathrm{T}}A, \quad B^{\mathrm{T}}D = D^{\mathrm{T}}B. \tag{2.8}$$

The vector v transforms then as:

$$\begin{pmatrix} a_D \\ a \end{pmatrix} \rightarrow \Gamma \begin{pmatrix} a_D \\ a \end{pmatrix} = \begin{pmatrix} Aa_D + Ba \\ Ca_D + Da \end{pmatrix}. \tag{2.9}$$

From this we can obtain the modular transformation properties of the prepotential $\mathcal{F}(a^i)$ (see [21]). Since

$$\begin{aligned} \frac{\partial \mathcal{F}_\Gamma}{\partial a^k} &= \frac{\partial a^i_\Gamma}{\partial a^k}\frac{\partial \mathcal{F}_\Gamma}{\partial a^i_\Gamma} = \Big(C^{ip}\tau_{pk} + D^i_k\Big)\Big(A^j_i a_{D,j} + B_{ij}a^j\Big) \\ &= (D^{\mathrm{T}}B)_{kj}a^j + (D^{\mathrm{T}}A)^j_k \frac{\partial \mathcal{F}}{\partial a^j} + (C^{\mathrm{T}}B)^p_j \frac{\partial a_{D,p}}{\partial a^k}a^j \\ &+ (C^{\mathrm{T}}A)^{pj}\frac{\partial a_{D,p}}{\partial a^k}a_{D,j}, \end{aligned} \tag{2.10}$$

using the properties (2.8) of the symplectic matrices we can integrate (2.10) to obtain:

$$\begin{aligned} \mathcal{F}_\Gamma &= \mathcal{F} + \frac{1}{2}a^k(D^{\mathrm{T}}B)_{kj}a^j + \frac{1}{2}a_{D,k}(C^{\mathrm{T}}A)^{pj}a_{D,j} \\ &+ a^k(B^{\mathrm{T}}C)^j_k a_{D,j}. \end{aligned} \tag{2.11}$$

Starting with (2.11) we can prove that the quantity $\mathcal{F} - 1/2\sum_i a^i a_{D,i}$ is a monodromy invariant, and evaluating it asymptotically, one obtains the relation [20, 21, 22]:

$$\mathcal{F} - \frac{1}{2}\sum_i a^i a_{Di} = -4\pi i b_1 u, \tag{2.12}$$

where b_1 is the coefficient of the one-loop β-function (for $SU(N_c)$ with N_f hypermultiplets in the fundamental representation, $b_1 = (2N_c - N_f)/16\pi^2$) and $u = \langle \mathrm{Tr}\phi^2 \rangle$. With the normalization for the electric charge used in [4] and [7], the r.h.s. of (2.12) is $-2\pi i b_1 u$.

As in the $SU(2)$ case, presented in [14], we break $N = 2$ supersymmetry down to $N = 0$ by making the dynamical scale Λ a function of a background vector superfield S, $\Lambda = \mathrm{e}^{iS}$. This must be done in such a way that s, $s_D = \partial \mathcal{F}/\partial s$ be monodromy invariant. To see this, we will derive a series of relations analogous to the ones in the $SU(2)$ case, starting with the following expression for the prepotential in terms of local coordinates:

$$\mathcal{F} = \sum_{ij} a^i a^j f_{ij}(a^l/\Lambda), \tag{2.13}$$

where we take $f_{ij} = f_{ji}$. We define now a $(r+1)\times(r+1)$ matrix of couplings including the dilaton spurion $a^0 = s$:

$$\tau_{\alpha\beta} = \frac{\partial^2 \mathcal{F}}{\partial a^\alpha a^\beta}. \tag{2.14}$$

Greek indices α, β go from 0 to r, and latin indices i, j from 1 to r. We obtain:

$$\begin{aligned} a_{D,k} &= 2\sum_i a^i f_{ik} + \frac{1}{\Lambda}\sum_{ij} a^i a^j f_{ij,k}, \\ \tau_{ij} &= 2f_{ij} + \frac{2}{\Lambda}\sum_k a^k (f_{ik,j} + f_{jk,i}) + \frac{1}{\Lambda^2} a^k a^l f_{kl,ij}, \\ \tau_{0i} &= -\frac{i}{\Lambda}\sum_{jk} a^j a^k (2f_{ij,k} + f_{jk,i}) - \frac{i}{\Lambda^2}\sum_{jkl} a^j a^k a^l f_{jk,li}, \\ \tau_{00} &= -\frac{1}{\Lambda}\sum_{ijk} a^i a^j a^k f_{ij,k} - \frac{1}{\Lambda^2}\sum_{ijkl} a^i a^j a^k a^l f_{ij,kl}, \end{aligned} \tag{2.15}$$

and the dual spurion field is given by:

$$s_D = \frac{\partial \mathcal{F}}{\partial s} = -\frac{i}{\Lambda}\sum_{ijk} a^i a^j f_{ij,k} \tag{2.16}$$

The equations (2.15) and (2.16) give the useful relations:

$$\begin{aligned} \tau_{0i} = i(a_{D,i} - \sum_j a^j \tau_{ji}), \qquad \frac{\partial \tau_{0i}}{\partial a^k} &= -i\sum_j a^j \frac{\partial \tau_{ij}}{\partial a^k}, \\ \frac{\partial \tau_{00}}{\partial a^k} = i\tau_{0k} - \sum_{ij} a^i a^j \frac{\partial \tau_{ij}}{\partial a^k}&. \end{aligned} \tag{2.17}$$

Using now (2.12) one can prove that s_D is a monodromy invariant,

$$\frac{\partial \mathcal{F}}{\partial s} = i\Big(2\mathcal{F} - \sum_i a^i a_{D,i}\Big) = 8\pi b_1 u \tag{2.18}$$

and from (2.17) and (2.18) we get

$$\tau_{0i} = 8\pi b_1 \frac{\partial u}{\partial a^i},$$

$$\tau_{00} = 8\pi i b_1 \Big(2u - \sum_i a^i \frac{\partial u}{\partial a^i}\Big) \tag{2.19}$$

Now we will present the transformation rules of the gauge couplings τ_{ij} under a monodromyy matrix Γ in $Sp(2r, \mathbf{Z})$. In terms of the local coordinates $a^i_\Gamma = C^{ip} a_{D,p}(a^j, s) + D^i_q a^q$ we have the couplings

$$\tau^\Gamma_{\alpha\beta} = \frac{\partial^2 \mathcal{F}}{\partial a^\alpha_\Gamma \partial a^\beta_\Gamma}. \tag{2.20}$$

The change of coordinates is given by the matrix:

$$\begin{pmatrix} \dfrac{\partial a^i_\Gamma}{\partial a^j} & \dfrac{\partial a^i_\Gamma}{\partial s} \\[2ex] \dfrac{\partial s}{\partial a^j} & \dfrac{\partial s}{\partial s} \end{pmatrix} = \begin{pmatrix} C^{ip}\tau_{pj} + D^i_j & C^{ip}\tau_{0p} \\ 0 & 1 \end{pmatrix}, \tag{2.21}$$

with inverse

$$\begin{pmatrix} \dfrac{\partial a^i}{\partial a^j_\Gamma} & \dfrac{\partial a^i}{\partial s} \\[2ex] \dfrac{\partial s}{\partial a^j_\Gamma} & \dfrac{\partial s}{\partial s} \end{pmatrix} = \begin{pmatrix} \Big((C\tau + D)^{-1}\Big)^i_j & -\Big((C\tau + D)^{-1}\Big)^i_k C^{kp}\tau_{p0} \\ 0 & 1 \end{pmatrix}. \tag{2.22}$$

Therefore we have:

$$\Big(\frac{\partial}{\partial a^j_\Gamma}\Big)_{\Gamma-\mathrm{basis}} = \Big((C\tau + D)^{-1}\Big)^i_j \frac{\partial}{\partial a^i},$$

$$\Big(\frac{\partial}{\partial s}\Big)_{\Gamma-\text{basis}} = \frac{\partial}{\partial s} - \Big[(C\tau + D)^{-1} C\tau\Big]_0^i \frac{\partial}{\partial a^i}; \tag{2.23}$$

which lead to the transformation rules for the couplings:

$$\begin{aligned} \tau^\Gamma_{ij} &= \Big(A\tau + B\Big)\Big(C\tau + D\Big)^{-1}_{ij}, \qquad \tau^\Gamma_{0i} = \tau_{0j}\Big((C\tau + D)^{-1}\Big)^j_i, \\ \tau^\Gamma_{00} &= \tau_{00} - \tau_{0i}\Big[(C\tau + D)^{-1} C\tau\Big]^i_0. \end{aligned} \tag{2.24}$$

3 Effective potential and vacuum structure

In this section we will obtain, starting from the formalism developed in the previous section, the effective potential in the Coulomb phase of the softly broken $N = 2$ theory, for a general group of rank r.

To break $N = 2$ down to $N = 0$ we freeze the spurion superfield to a constant. The lowest component is fixed by the scale Λ, and we only turn on the auxiliary F^0 (*i.e.* we take $D^0 = 0$). We must include in the effective lagrangian $r + 1$ vector multiplets, where r is the rank of the gauge group:

$$A^\alpha = (A^0, A^I), \qquad I = 1, \cdots, r. \tag{3.1}$$

There are submanifolds in the moduli space where extra states become massless and we must include them in the effective lagrangian. They are BPS states corresponding to monopoles or dyons, so we introduce n_H hypermultiplets near these submanifolds in the low energy description:

$$(M_i, \widetilde{M}_i), \qquad i = 1, \cdots, n_H \tag{3.2}$$

We suppose that these BPS states are mutually local, hence we can find a symplectic transformation such that they have $U(1)^r$ charges $(q_i^I, -q_i^I)$ with respect to the I-th $U(1)$ (we follow the $N = 1$ notation). The full $N = 2$ effective lagrangian contains two terms:

$$\mathcal{L} = \mathcal{L}_{\rm VM} + \mathcal{L}_{\rm HM}, \tag{3.3}$$

where $\mathcal{L}_{\rm VM}$ is given in (2.1), and

$$\begin{aligned} \mathcal{L}_{\rm HM} &= \sum_i \int d^4\theta (M_i^* \mathrm{e}^{2q_i^I V^{(I)}} M_i + \widetilde{M}_i^* \mathrm{e}^{-2q_i^I V^{(I)}} \widetilde{M}_i) \\ &+ \sum_{I,i} \Big(\int d^2\theta \sqrt{2} A^I q_i^I M_i \widetilde{M}_i + \text{h.c.} \Big) \end{aligned} \tag{3.4}$$

The terms in (3.3) contributing to the effective potential are

$$\begin{aligned} V &= b_{IJ}F^I\overline{F}^J + b_{0I}(F^0\overline{F}^I + \overline{F}^0 F^I) + b_{00}|F^0|^2 \\ &+ \frac{1}{2}b_{IJ}D^I D^J + D^I q_i^I(|m_i|^2 - |\widetilde{m}_i|^2) + |F_{m_i}|^2 + |F_{\widetilde{m}_i}|^2 \\ &+ \sqrt{2}\Big(F^I q_i^I m_i\widetilde{m}_i + a^I q_i^I m_i F_{\widetilde{m}_i} + a^I q_i^I \widetilde{m}_i F_{m_i} + \text{h.c.}\Big), \end{aligned} \tag{3.5}$$

where all repeated indices are summed and $b_{\alpha\beta} = \mathrm{Im}\tau_{\alpha\beta}/4\pi$. We eliminate the auxiliary fields and obtain:

$$D^I = -(b^{-1})^{IJ} q_i^J(|m_i|^2 - |\widetilde{m}_i|^2),$$

$$\begin{aligned} F^I &= -(b^{-1})^{IJ} b_{0J} F^0 - \sqrt{2}(b^{-1})^{IJ} q_i^J \overline{m}_i\overline{\widetilde{m}}_i, \\ F_{m_i} &= -\sqrt{2}\overline{a}^I q_i^I \overline{\widetilde{m}}_i, \qquad F_{\widetilde{m}_i} = -\sqrt{2}\overline{a}^I q_i^I \overline{m}_i. \end{aligned} \tag{3.6}$$

We denote $(q_i, q_j) = \sum_{IJ} q_i^I (b^{-1})^{IJ} q_j^I$, $(q_i, b_0) = \sum_{IJ} q_i^I (b^{-1})^{IJ} b_{0J}$, $a\cdot q_i = \sum_I a^I q_i^I$. Substituting in (3.5) we obtain:

$$\begin{aligned} V &= \frac{1}{2}\sum_{ij}(q_i,q_j)(|m_i|^2 - |\widetilde{m}_i|^2)(|m_j|^2 - |\widetilde{m}_j|^2) + 2\sum_{ij}(q_i,q_j) m_i\widetilde{m}_i\overline{m}_j\overline{\widetilde{m}}_j \\ &+ 2\sum_i |a\cdot q_i|^2(|m_i|^2 + |\widetilde{m}_i|^2) + \sqrt{2}\sum_i (q_i, b_0)\Big(F^0 m_i\widetilde{m}_i + \overline{F}^0\overline{m}_i\overline{\widetilde{m}}_i\Big) \\ &- |F^0|^2\frac{\det b_{\alpha\beta}}{\det b_{IJ}}, \end{aligned} \tag{3.7}$$

where $\det b_{\alpha\beta}/\det b_{IJ} = b_{00} - b_{0I}(b^{-1})^{IJ} b_{0J}$ is the cosmological term. This term in the potential is a monodromy invariant. To prove this it is sufficient to prove invariance under the generators of the symplectic group $Sp(2r,\mathbf{Z})$:

$$\begin{pmatrix} A & 0 \\ 0 & (A^{\mathrm{T}})^{-1} \end{pmatrix}, \qquad A \in Gl(r,\mathbf{Z}),$$

$$T_\theta = \begin{pmatrix} \mathbf{1} & \theta \\ 0 & \mathbf{1} \end{pmatrix}, \qquad \theta_{ij}\in\mathbf{Z}, \qquad \Omega = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}. \tag{3.8}$$

Invariance under T_θ and the matrix involving only A is obvious, and for Ω one can check it easily.

The vacuum structure is determined by the minima of (3.7). As in [14], we first minimize with respect to m_i, $\overline{m}_i$:

$$\begin{aligned} \frac{\partial V}{\partial \overline{m}_i} &= \sum_j (q_i,q_j)(|m_j|^2-|\widetilde{m}_j|^2)m_i + 2|a\cdot q_i|^2 m_i \\ &+ 2\sum_j (q_i,q_j) m_j \widetilde{m}_j \overline{\widetilde{m}}_i + \sqrt{2}\overline{F}^0 (q_i,b_0)\overline{\widetilde{m}}_i = 0, \end{aligned} \tag{3.9}$$

$$\begin{aligned} \frac{\partial V}{\partial \overline{\widetilde{m}}_i} &= \sum_j (q_i,q_j)(-|m_j|^2+|\widetilde{m}_j|^2)\widetilde{m}_i + 2|a\cdot q_i|^2 \widetilde{m}_i \\ &+ 2\sum_j (q_i,q_j) m_j \widetilde{m}_j \overline{m}_i + \sqrt{2}\overline{F}^0 (q_i,b_0)\overline{m}_i = 0. \end{aligned} \tag{3.10}$$

Multiplying (3.9) by $\overline{m}_i$, (3.10) by $\overline{\widetilde{m}}_i$ and substracting, we get

$$\sum_j (q_i,q_j)(|m_j|^2-|\widetilde{m}_j|^2)(|m_i|^2+|\widetilde{m}_i|^2) + 2|a\cdot q_i|^2(|m_i|^2-|\widetilde{m}_i|^2) = 0. \tag{3.11}$$

Suppose now that, for some indices $i \in I$, $|m_i|^2+|\widetilde{m}_i|^2>0$. Multiplying (3.11) by $|m_i|^2-|\widetilde{m}_i|^2$ and summing over i we obtain

$$\sum_{ij}(q_i,q_j)(|m_i|^2-|\widetilde{m}_i|^2)(|m_j|^2-|\widetilde{m}_j|^2) = -\sum_{i\in I}\frac{2|a\cdot q_i|^2}{|m_i|^2+|\widetilde{m}_i|^2}(|m_i|^2-|\widetilde{m}_i|^2)^2. \tag{3.12}$$

The matrix $(b^{-1})^{IJ}$ is positive definite, and if the charge vectors q_i^I are linearly independent it follows that the matrix (q_i,q_j) is positive definite too. Then the l.h.s. of (3.12) is ≥ 0 while the r.h.s. is ≤ 0. The only way for this equation to be consistent is if

$$|m_i| = |\widetilde{m}_i|, \qquad i=1,\cdots,n_H. \tag{3.13}$$

In this case we can write the equation (3.9), after absorbing the phase of $F^0 = f_0 e^{i\gamma}$ in $\widetilde{m}_i$, as:

$$2|a\cdot q_i|^2 m_i + 2\sum_j (q_i,q_j) m_j \widetilde{m}_j \overline{\widetilde{m}}_i + \sqrt{2} f_0 (q_i,b_0)\overline{\widetilde{m}}_i = 0. \tag{3.14}$$

Multiplying by $\overline{m}_i$ and summing over i, we obtain

$$2\sum_i |a\cdot q_i|^2|m_i|^2 + \sqrt{2} f_0 \sum_i (q_i,b_0)\overline{m}_i\overline{\widetilde{m}}_i = -2\sum_{ij}(q_i,q_j) m_j \overline{m}_i \widetilde{m}_j \overline{\widetilde{m}}_i, \tag{3.15}$$

hence $\sqrt{2}f_0\sum_i(q_i,b_0)\overline{m}_i\overline{\widetilde{m}}_i$ is real. We can insert in (3.7) and get the following expression for the effective potential:

$$V=-f_0^2\frac{\det b_{\alpha\beta}}{\det b_{IJ}}-2\sum_{ij}(q_i,q_j)m_j\overline{m}_i\widetilde{m}_j\overline{\widetilde{m}}_i. \tag{3.16}$$

If (3.13) holds, we can fix the gauge in the $U(1)^r$ factors and write

$$m_i=\rho_i,\qquad \widetilde{m}_i=\rho_i\mathrm{e}^{i\phi_i} \tag{3.17}$$

and (3.14) reads:

$$\rho_i^2\Big(|a\cdot q_i|^2+\sum_j(q_i,q_j)\rho_j^2\mathrm{e}^{i(\phi_j-\phi_i)}+\frac{f_0(q_i,b_0)}{\sqrt{2}}\mathrm{e}^{-i\phi_i}\Big)=0. \tag{3.18}$$

Apart form the trivial solution $\rho_i=0$, we have:

$$|a\cdot q_i|^2+\sum_j(q_i,q_j)\rho_j^2\mathrm{e}^{i(\phi_j-\phi_i)}+\frac{f_0(q_i,b_0)}{\sqrt{2}}\mathrm{e}^{-i\phi_i}=0 \tag{3.19}$$

and we can have a monopole (or dyon) VEV in some regions of the moduli space. Notice that for groups of rank $r>1$ there is a coupling between the different $U(1)$ factors and one needs a numerical study of the equation above once the values of the charges q_i^I are known. In addition, the moduli space is in that case very complicated and explicit solutions for the prepotential and gauge couplings of the $N=2$ theory are difficult to find. However we still can have some qualitative information in many cases under some mild assumptions, as we will see.

4 Vacuum structure of the $SU(N)$ Yang-Mills theory

The moduli space of vacua of the $N=2$ $SU(N)$ Yang-Mills can be parametrized in a gauge-invariant way by the elementary symmetric polynomials s_l, $l=2,\cdots,N$ in the eigenvalues of $\langle\phi\rangle$, ϕ_i. The vacuum structure of the theory is associated to the hyperelliptic curve [5]:

$$y^2=P(x)^2-\Lambda^{2N},$$

$$P(x) = \frac{1}{2}\det(x - \langle\phi\rangle) = \frac{1}{2}\prod_i (x - \phi_i), \tag{4.1}$$

where Λ is the dynamical scale of the $SU(N)$ theory and $P(x)$ can be written in terms of the variables s_l as $P(x) = 1/2\sum_l(-1)^l s_l x^{N-l}$. Once the hyperelliptic curve is known, one can compute in principle the metric on the moduli space and the exact quantum prepotential, but explicit solutions are difficult to find (they have been obtained in [6] for the $SU(3)$ case). However, as in the $SU(2)$ case [14], one expects that the minima of the effective potential for the $SU(N)$ theory are near the $N=1$ points (at least for a small supersymmetry breaking parameter). The physics of the $N=1$ points in $SU(N)$ theories has a much simpler description because it involves only small regions of the moduli space, and has been studied in [7]. The $N=1$ points correspond to points in the moduli space where $N-1$ monopoles or dyons coupling to each $U(1)$ become massless simultaneously. From the point of view of the hyperelliptic curve, the $N=1$ point where $N-1$ monopoles become massless ($a_{D,I}=0$) corresponds to a simultaneous degeneration of the $N-1$ α-cycles, associated to magnetic monopoles. This means in turn that the polynomial $P(x)^2 - \Lambda^{2N}$ must have $N-1$ double zeros and two single zeros. If we set $\Lambda = 1$, this can be achieved with the Chebyshev polynomials

$$P(x) = \cos\Big(N \arccos\frac{x}{2}\Big), \tag{4.2}$$

and the corresponding eigenvalues are $\phi_i = 2\cos\pi(i - \frac{1}{2})/N$. The other $N-1$ points, corresponding to the simultaneous condensation of $N-1$ mutually local dyons, are obtained with the action of the anomaly-free discrete subgroup $\mathbf{Z}_{4N} \subset U(1)_R$. One can perturb slightly the curve (4.2) to obtain the effective lagrangian (or equivalently, the prepotential) at lowest order. What is found is that, in terms of the dual monopole variables $a_{D,I}$, the $U(1)$ factors are decoupled and $\tau^D_{IJ} \sim \delta_{IJ}\tau_I$. Near the $N=1$ point where $N-1$ monopoles become massless one can then simplify the equation (3.19) for the monopole VEVs, because $q^I_i = \delta^I_i$, $(b^{-1})^{IJ} = \delta^{IJ} b_I^{-1}$. The equation reduces then to $r = N-1$ $SU(2)$-like equations, and in particular the phase factors $\mathrm{e}^{-i\phi_I}$ must be real. We then set $\mathrm{e}^{-i\phi_I} = \epsilon_I$, $\epsilon_I = \pm 1$. The VEVs are determined by:

$$\rho_I^2 = -b_I|a_{D,I}|^2 - \frac{f_0 b_{0I}\epsilon_I}{\sqrt{2}}, \qquad I = 1, \cdots, r. \tag{4.3}$$

The effective potential (3.16) reads:

$$V = -f_0^2\Big(b_{00} - \sum_I \frac{b_{0I}^2}{b_I}\Big) - 2\sum_I \frac{1}{b_I}\rho_I^4. \tag{4.4}$$

The quantities that control, at least qualitatively, the vacuum structure of the theory, are b_{0I} and b_{00}. If $b_{0I} \neq 0$ at the $N=1$ points, we have a monopole VEV for ρ_I around this point. If $b_{0I}=0$, we still can have a VEV, as it happens in the $SU(2)$ case in the dyon region, but we expect that it will be too tiny to produce a local minimum [14]. When one has monopole or dyon condensation at one of these $N=1$ points in all the $U(1)$ factors, the value of the potential at this point is given by

$$V = -f_0^2 b_{00}, \tag{4.5}$$

and if the local minimum is very near to the $N=1$ point, we can compare the energy of the different $N=1$ points according to (4.5) and determine the true vacuum of the theory. Hence, to have a qualitative picture of the vacuum structure, and if we suppose that the minima of the effective potential will be located near the $N=1$ points, we only need to evaluate b_{0I}, b_{00} at these points. This can be done using the explicit solution in [7] and the expressions (2.19).

To obtain the correct normalization of the constant appearing in (2.18) we can evaluate $\sum_I (a_{D,I} da/du - a da_{D,I}/du)$ in the $N=1$ points, obtaining the constant value $4\pi i b_1$. The value of the quadratic Casimir at the $N=1$ point described by (4.2) is

$$u = \langle \mathrm{Tr}\phi^2\rangle = 4\sum_{i=1}^{N}\cos^2\frac{\pi(i-1/2)}{N} = 2N, \tag{4.6}$$

and the values at the other $N=1$ points are given by the action of $\mathbf{Z}_N$ (u has charge 4 under $U(1)_R$): $u^{(k)} = 2\omega^{4k}N$, $\omega = \mathrm{e}^{\pi i/2N}$ with $k = 0,\cdots,N-1$. To compute τ_{0I} we must also compute $\partial u/\partial a_{D,I}$. Using the results of [7], we have:

$$\frac{\partial u}{\partial a_{D,I}} = -4i\sin\frac{\pi I}{N}, \tag{4.7}$$

and using $b_1 = 2N/16\pi^2$, we obtain

$$\tau_{0I} = 4\pi b_1 \frac{\partial u}{\partial a_{DI}} = -\frac{2Ni}{\pi}\sin\frac{\pi I}{N}. \tag{4.8}$$

At the $N=1$ point where $N-1$ monopoles condense, $a_{D,I}=0$, therefore

$$\tau_{00} = 8\pi i u = \frac{2i}{\pi}N^2. \tag{4.9}$$

(4.8) indicates that monopoles condense at this point in all the $U(1)$ factors, but with different VEVs. This is a consequence of the spontaneous breaking of the S_N symmetry permuting the $U(1)$ factors [7].

To study the other $N=1$ points we must implement the $\mathbf{Z}_N$ symmetry in the u-plane. The local coordinates $a_I^{(k)}$ vanishing at these points are given by a $Sp(2r,\mathbf{Z})$ transformation acting on the coordinates a_I, $a_{D,I}$ around the monopole point. The $\mathbf{Z}_N$ symmetry implies that

$$\frac{\partial u}{\partial a_I^{(k)}}(u^{(k)}) = \omega^{2k}\frac{\partial u}{\partial a_{D,I}}(u^{(0)}), \tag{4.10}$$

and then we get

$$\begin{aligned} b_{0I}^{(k)} &= \frac{1}{4\pi}\mathrm{Im}\tau_{0I}^{(k)} = -\frac{N}{2\pi^2}\cos\frac{\pi k}{N}\sin\frac{\pi I}{N}, \\ b_{00}^{(k)} &= \frac{1}{4\pi}\mathrm{Im}\tau_{00}^{(k)} = \frac{1}{2}\Big(\frac{N}{\pi}\Big)^2\cos\frac{2\pi k}{N}. \end{aligned} \tag{4.11}$$

The first equation tells us that generically we will have dyon condensation at all the $N=1$ points, and the second equation together with (4.5) implies that the condensate of $N-1$ monopoles at $u=2N$ is energetically favoured, and then it will be the true vacuum of the theory. Notice that the $\mathbf{Z}_N$ symmetry works in such a way that the size of the condensate, given by $|\cos\frac{\pi k}{N}|$, corresponds to an energy given by $-\cos\frac{2\pi k}{N}$: as one should expect, the bigger the condensate the smaller its energy. In fact, for N even the $N=1$ point corresponding to $k=N/2$ has no condensation. In this case the energy is still given by (4.5), as the effective potential equals the cosmological term with $b_{0I}=0$, and is the biggest one.

5 Mass formula in softly broken $N=2$ theories

5.1 A general mass formula

In some cases the mass spectrum of a softly broken supersymmetric theory is such that the graded trace of the square of the mass matrix is zero as it happens in supersymmetric theories [18]. We will see in this section that this is also the case when we softly break $N = 2$ supersymmetry with a dilaton spurion.

We will then compute the trace of the squared mass matrix which arises from the effective lagrangian (3.3), once the supersymmetry breaking parameter is turned on. The fermionic content of the theory is as follows: we have fermions ψ^I, λ^I coming from the $N = 2$ vector multiplet A^I (in $N = 1$ language, ψ^I comes from the $N = 1$ chiral multiplet and λ^I from the $N = 1$ vector multiplet). We also have "monopolinos" ψ_{m_i}, $\psi_{\widetilde{m}_i}$ from the n_H matter hypermultiplets. To obtain the fermion mass matrix, we just look for fermion bilinears in (3.3). From the gauge kinetic part and the Kähler potential in $\mathcal{L}_{\rm VM}$ we obtain:

$$\frac{i}{16\pi} F^\alpha \partial_\alpha \tau_{IJ} \lambda^I \lambda^J + \frac{i}{16\pi} \overline{F}^\alpha \partial_\alpha \tau_{IJ} \psi^I \psi^J. \tag{5.12}$$

where $F^0 = f_0$ and the auxiliary fields F^I are given in (3.6). From the kinetic term and the superpotential in $\mathcal{L}_{\rm HM}$ we get:

$$\begin{aligned} & i\sqrt{2} \sum_i q_i \cdot \lambda (\overline{m}_i \psi_{m_i} - \overline{\widetilde{m}}_i \psi_{\widetilde{m}_i}) \\ - \quad & \sqrt{2} \sum_i \Big(a \cdot q_i \psi_{m_i} \psi_{\widetilde{m}_i} + q_i \cdot \psi \psi_{\widetilde{m}_i} m_i + q_i \cdot \psi \psi_{m_i} \widetilde{m}_i \Big) \end{aligned} \tag{5.13}$$

If we order the fermions as $(\lambda, \psi, \psi_{m_i}, \psi_{\widetilde{m}_i})$ and denote $\mu^{IJ} = iF^\alpha \partial_\alpha \tau_{IJ}/4\pi$, $\hat{\mu}^{IJ} = i\overline{F}^\alpha \partial_\alpha \tau_{IJ}/4\pi$, the "bare" fermionic mass matrix reads:

$$M_{1/2} = \begin{pmatrix} \mu/2 & 0 & i\sqrt{2} q_i^I \overline{m}_i & -i\sqrt{2} q_i^I \overline{\widetilde{m}}_i \\ 0 & \hat{\mu}/2 & -\sqrt{2} q_i^I \widetilde{m}_i & -\sqrt{2} q_i^I m_i \\ i\sqrt{2} q_i^I \overline{m}_i & -\sqrt{2} q_i^I \widetilde{m}_i & 0 & -\sqrt{2} a \cdot q_i \\ -i\sqrt{2} q_i^I \overline{\widetilde{m}}_i & -\sqrt{2} q_i^I m_i & -\sqrt{2} a \cdot q_i & 0 \end{pmatrix}, \tag{5.14}$$

but we must take into account the wave function renormalization for the fermions λ^I, ψ^I and consider

$$\mathcal{M}_{1/2} = Z M_{1/2} Z, \qquad Z = \begin{pmatrix} b^{-1/2} & 0 & 0 & 0 \\ 0 & b^{-1/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5.15}$$

The trace of the squared fermionic matrix can be easily computed:

$$\begin{aligned} \mathrm{Tr}\mathcal{M}_{1/2}\mathcal{M}^\dagger_{1/2} &= \frac{1}{4}\mathrm{Tr}[\mu b^{-1}\overline{\mu} b^{-1} + \hat{\mu} b^{-1}\overline{\hat{\mu}} b^{-1}] \\ &+ 4\sum_i |a \cdot q_i|^2 + 8\sum_i (q_i, q_i)(|m_i|^2 + |\widetilde{m}_i|^2). \end{aligned} \tag{5.16}$$

The scalars in the model are the monopole fields m_i, $\widetilde{m}_i$ and the lowest components of the $N = 1$ chiral superfields in the A^I, a^I. To compute the trace of the scalar mass matrix we need

$$\frac{\partial^2 V}{\partial m_i \partial \overline{m}_i} = \sum_l (q_i, q_l)(|m_l|^2 - |\widetilde{m}_l|^2) + (q_i, q_i)(|m_i|^2 + 2|\widetilde{m}_i|^2) + 2|a \cdot q_i|^2,$$

$$\frac{\partial^2 V}{\partial \widetilde{m}_i \partial \overline{\widetilde{m}}_i} = -\sum_l (q_i, q_l)(|m_l|^2 - |\widetilde{m}_l|^2) + (q_i, q_i)(2|m_i|^2 + |\widetilde{m}_i|^2) + 2|a \cdot q_i|^2,$$

$$\begin{aligned} \frac{\partial^2 V}{\partial a^I \partial \overline{a}^J} &= f_0^2 \frac{\partial^2 (b_0, b_0)}{\partial a^I \partial \overline{a}^J} + 2\sum_{k,l} \frac{\partial^2 (q_k, q_l)}{\partial a^I \partial \overline{a}^J} m_k \widetilde{m}_k \overline{m}_l \overline{\widetilde{m}}_l \\ &+ 2\sum_k q_k^I q_k^J (|m_k|^2 + |\widetilde{m}_k|^2) \\ &+ \sqrt{2}\sum_k \frac{\partial^2 (q_k, b_0)}{\partial a^I \partial \overline{a}^J} f_0 (m_k \widetilde{m}_k + \overline{m}_k \overline{\widetilde{m}}_k). \end{aligned} \tag{5.17}$$

In the last expression we used that, due to the holomorphy of the couplings $\tau_{\alpha\beta}$, $\partial^2_{I\overline{J}} b_{\alpha\beta} = 0$. If we assume that we are in the conditions of section 2, at the minimum we have $|m_i| = |\widetilde{m}_i|$, and the trace of the squared scalar matrix is

$$\mathrm{Tr}\mathcal{M}_0^2 = 6\sum_i (q_i, q_i)(|m_i|^2 + |\widetilde{m}_i|^2) + 8\sum_i |a \cdot q_i|^2 + 2(b^{-1})^{IJ} \frac{\partial^2 V}{\partial a^I \partial \overline{a}^J}, \tag{5.18}$$

where we have included the wave function renormalization for the scalars a^I. The mass of the dual photon is given by the monopole VEV through the magnetic Higgs mechanism:

$$\mathrm{Tr}\mathcal{M}_1^2 = 2\sum_i (q_i, q_i)(|m_i|^2 + |\widetilde{m}_i|^2). \tag{5.19}$$

Taking into account all these contributions, the graded trace of the squared matrix is:

$$\begin{aligned}\sum_j (-1)^{2j}(2j+1)\mathrm{Tr}\mathcal{M}_j^2 = &-\frac{1}{2}\mathrm{Tr}[\mu b^{-1}\overline{\mu} b^{-1} + \hat{\mu} b^{-1}\overline{\hat{\mu}} b^{-1}] \\ &+2f_0^2 \mathrm{Tr} b^{-1}\partial\overline{\partial}(b_0, b_0) + 4\sum_{k,l}\mathrm{Tr} b^{-1}\partial\overline{\partial}(q_k, q_l) m_k \widetilde{m}_k \overline{m}_l \overline{\widetilde{m}}_l \\ &+2\sqrt{2}\sum_k \mathrm{Tr} b^{-1}\partial\overline{\partial}(q_k, b_0) f_0 (m_k \widetilde{m}_k + \overline{m}_k \overline{\widetilde{m}}_k).\end{aligned} \tag{5.20}$$

To see that this is zero, we write the bilinears in the monopole fields in terms of the auxiliary fields F^I, $\overline{F}^I$, using (3.6):

$$\sum_i q_i^I \overline{m}_i \overline{\widetilde{m}}_i = -\frac{1}{\sqrt{2}}(b_{IJ}F^J + b_{0I} f_0). \tag{5.21}$$

Then we can group the terms in (5.20) depending on the number of F^I, $\overline{F}^I$, and check that they cancel separately. For instance, for the terms with two auxiliaries, we have from the first term in (5.20):

$$-2(F^I\overline{F}^J + \overline{F}^I F^J)\partial_I b_{MN}(b^{-1})^{NP}\partial_{\overline{J}} b_{PQ}(b^{-1})^{QM} \tag{5.22}$$

and from the third term

$$\begin{aligned} &2F^I\overline{F}^J \partial_M b_{JN}(b^{-1})^{NP}\partial_{\overline{Q}} b_{PI}(b^{-1})^{QM} \\ + \quad &2F^I\overline{F}^J \partial_M b_{PI}(b^{-1})^{NP}\partial_{\overline{Q}} b_{JN}(b^{-1})^{QM}.\end{aligned} \tag{5.23}$$

Taking into account the holomorphy of the couplings and the Kähler geometry, we have $\partial_M b_{PI} = \partial_I b_{PM}$, $\partial_{\overline{Q}} b_{JN} = \partial_{\overline{J}} b_{QN}$, so (5.22) and (5.23) add up to zero. With a little more algebra one can verify that the terms with one F^I (and their conjugates with $\overline{F}^I$) and without any auxiliaries add up to zero too. The result is then:

$$\sum_j (-1)^{2j}(2j+1)\mathrm{Tr}\mathcal{M}_j^2 = 0. \tag{5.24}$$

5.2 Mass spectrum in the $SU(2)$ case

In the $SU(2)$ case we can obtain much more information about the mass matrix and also determine its eigenvalues. First we consider the fermion mass matrix. Taking into account that at the minimum of the effective potential $m = \overline{m} = \rho$, $\tilde{m} = \epsilon m$, we can introduce the linear combination:

$$\eta_{\pm} = \frac{1}{\sqrt{2}}(\psi_m \pm \epsilon \psi_{\tilde{m}}). \tag{5.25}$$

With respect to the new fermion fields $(\lambda, \eta_+, \psi, \eta_-)$, the bare fermion mass matrix reads:

$$M_{1/2} = \begin{pmatrix} \frac{1}{2}\mu & -2\epsilon\rho & 0 & 0 \\ -2\epsilon\rho & -\sqrt{2}\epsilon a & 0 & 0 \\ 0 & 0 & \frac{1}{2}\mu & 2i\rho \\ 0 & 0 & 2i\rho & -\sqrt{2}\epsilon a \end{pmatrix}, \tag{5.26}$$

Notice that, in the $SU(2)$ case, the auxiliary field F is real and $\mu = \hat{\mu}$. $\mathcal{M}_{1/2}\mathcal{M}^{\dagger}_{1/2}$ can be easily diagonalized. From (5.26) it is easy to see that the squared fermion mass matrix is block-diagonal with the same 2×2 matrix in both entries:

$$\begin{pmatrix} b_{11}^{-2}\mu\overline{\mu}/4 + 4b_{11}^{-1}\rho^2 & -\epsilon b_{11}^{-3/2}\mu\rho + 2\sqrt{2}\overline{a}\rho \\ -\epsilon b_{11}^{-3/2}\overline{\mu}\rho + 2\sqrt{2}a\rho & 4b_{11}^{-1}\rho^2 + 2|a|^2 \end{pmatrix}. \tag{5.27}$$

Hence there are two different doubly degenerate eigenvalues. In terms of the determinant and trace of (5.27),

$$\begin{aligned} \alpha &= (m_1^F)^2 + (m_2^F)^2 = \frac{1}{4b_{11}^2}\mu\overline{\mu} + 2|a|^2 + \frac{8}{b_{11}}\rho^2 \\ \beta &= (m_1^F)^2(m_2^F)^2 = \frac{1}{b_{11}^2}|4\rho^2 + \frac{\epsilon}{\sqrt{2}}a\mu|^2, \end{aligned} \tag{5.28}$$

the eigenvalues are:

$$(m_{1,2}^F)^2 = \frac{\alpha}{2} \pm \frac{1}{2}\sqrt{\alpha^2 - 4\beta}. \tag{5.29}$$

The computation of the scalar mass matrix is more lengthy. First we must compute the second derivatives of the effective potential, evaluated at the minimum. To obtain more simple expressions, we can use the identities

(2.19) to express all the derivatives of the couplings in terms only of $\partial b_{11}/\partial a$, $\partial^2 b_{11}/\partial a^2$. The results are:

$$
\begin{aligned}
\frac{\partial^2 V}{\partial m \partial \overline{m}} &= \frac{3}{b_{11}}\rho^2 + 2|a|^2, \quad \partial^2 V/\partial m^2 = \frac{\partial^2 V}{\partial \widetilde{m}^2} = \frac{1}{b_{11}}\rho^2, \\
\frac{\partial^2 V}{\partial m \partial \widetilde{m}} &= \frac{\epsilon}{b_{11}}\rho^2 + \frac{\sqrt{2} b_{01}}{b_{11}} f_0, \quad \frac{\partial^2 V}{\partial m \partial \overline{\widetilde{m}}} = \frac{\epsilon}{b_{11}}\rho^2 \\
\frac{\partial^2 V}{\partial m \partial a} &= 2\rho \Big[\overline{a} - (b_{11} \frac{\partial}{\partial a} \frac{1}{b_{11}})(|a|^2 - \frac{i\epsilon}{\sqrt{2}} a f_0) \Big] \\
\frac{\partial^2 V}{\partial a^2} &= -b_{11}^2 (\frac{\partial}{\partial a} \frac{1}{b_{11}}) f_0 (a f_0 + 2\sqrt{2} i\epsilon |a|^2) \\
& \quad -b_{11}^2 (\frac{\partial^2}{\partial a^2} \frac{1}{b_{11}})(a f_0 + \sqrt{2} i\epsilon |a|^2)^2, \\
\frac{\partial^2 V}{\partial \widetilde{m} \partial a} &= \epsilon \frac{\partial^2 V}{\partial m \partial a}, \quad \frac{\partial^2 V}{\partial \overline{m} \partial a} = \frac{\partial^2 V}{\partial m \partial a}, \quad \frac{\partial^2 V}{\partial \overline{\widetilde{m}} \partial a} = \frac{\partial^2 V}{\partial \widetilde{m} \partial a}, \\
\frac{\partial^2 V}{\partial \overline{a} \partial a} &= 4\rho^2 + \frac{1}{2 b_{11}} \mu \overline{\mu},
\end{aligned}
\tag{5.30}
$$

and the rest of the derivatives are obtained through complex conjugation. In the last line we used the result of the previous section. To obtain the bosonic mass matrix we must take into account the wave-function renormalization of the a, $\overline{a}$ variables, as in (5.18). Its eigenvalues are as follows: we have a zero eigenvalue corresponding to the Goldstone boson of the spontaneously broken $U(1)$ symmetry. There is also an eigenvalue with degeneracy two given by:

$$
2(\frac{\partial^2 V}{\partial m \partial \overline{m}} - \frac{\partial^2 V}{\partial \widetilde{m}^2}) = -\frac{2\sqrt{2}\epsilon}{b_{11}} f_0 b_{01}. \tag{5.31}
$$

Notice that this is always positive if we have a non-zero VEV for ρ. The other three eigenvalues are best obtained numerically, as they are the solutions to a third-degree algebraic equation.

As an application of these general results, we can plot the mass spectrum as a function of the supersymmetry breaking parameter f_0 in the $SU(2)$ Yang-Mills case, where the minimum corresponds to the monopole region and $\epsilon = -1$. We have only to compute the derivatives of the magnetic coupling. Using the elliptic function representation of the Seiberg-Witten

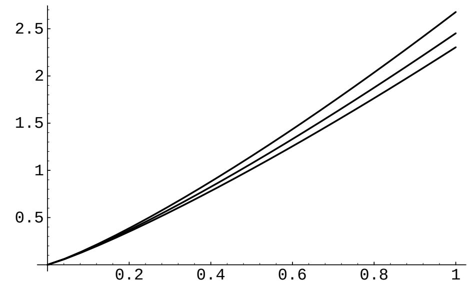

Figure 1: Fermion masses (5.29) (top and bottom) and photon mass (5.19) (middle) in softly broken $SU(2)$ Yang-Mills, as a function of $0 \leq f_0 \leq 1$.

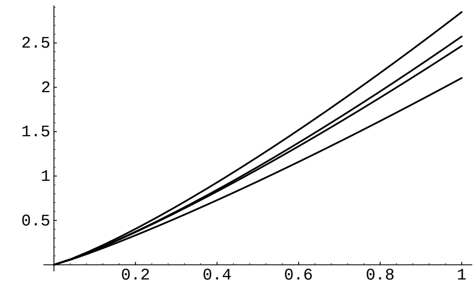

Figure 2: Masses of the scalars in softly broken $SU(2)$ Yang-Mills, as a function of $0 \leq f_0 \leq 1$.

solution [14] we obtain:

$$\frac{\partial \tau_{11}^{(m)}}{\partial a^{(m)}} = \frac{\pi^2}{8} \frac{k}{k'^2 K'^3}, \qquad \frac{\partial^2 \tau_{11}^{(m)}}{\partial a^{(m)2}} = -\frac{\pi i}{32} \frac{k^2}{k'^4 K'^4} \Big(k'^2 - k^2 + \frac{3E'}{K'} \Big), \tag{5.32}$$

where we set $\Lambda = 1$.

These derivatives diverge at the monopole singularity $u = 1$, and we may think that this can give some kind of singular behaviour for the masses near this point. In fact this is not so. The position of the minimum, u_0, behaves almost linearly with respect to f_0, $u_0 - 1 \sim f_0$, and this guarantees that the behaviour very near to $u = 1$ (corresponding to a very small f_0) is perfectly smooth, as one can see in the figures. In fig. 1 we plot the fermion masses (5.29) (top and bottom) and the photon mass given in (5.19) (middle). In fig. 2 we plot the masses of the scalars, where the second one from the top corresponds to the doubly degenerate eigenvalue (5.31).

6 $SU(2)$ theory with one massless hypermultiplet

The low energy effective action of $N = 2$ supersymmetric QCD with one massless hypermultiplet is described by the elliptic curve [4]:

$$y^2 = x^3 - ux^2 - \frac{1}{64}\Lambda_1^6 \tag{6.1}$$

with discriminant

$$\Delta = -\frac{\Lambda_1^6}{16}[u^3 + \frac{27}{256}\Lambda_1^6] \tag{6.2}$$

and there are three singularities in the u-plane corresponding to the roots of the equation $u^3 = -27\Lambda_1^6/256$. These singularities are related by a $\mathbf{Z}_3$ symmetry coming from the anomaly-free discrete subgroup $\mathbf{Z}_{12} \subset U(1)_R$. In what follows we will normalize the dynamical scale of the theory as $\Lambda_1^6 = 256/27$, so the three singularities of the u-plane are located at the three roots $u_1 = \mathrm{e}^{-i\pi/3}$, $u_2 = \mathrm{e}^{i\pi/3}$ and $u_3 = -1$. These singularities correspond to massless BPS states with quantum numbers $(n_e, n_m) = (0,1)$, $(1,1)$ and $(2,1)$. The periods of the curve (6.1) $a(u)$, $a_D(u)$ satisfy the Picard-Fuchs equation [24]:

$$\frac{d^2\omega}{du^2} + \frac{1}{4}\frac{u}{u^3+1}\omega = 0, \tag{6.3}$$

and as in the $N_f = 0$ case the wronskian $a da_D/du - a_D da/du$ is a constant (this is in fact the case for all the $SU(2)$ theories with $N_f \leq 3$). This constant appears in the r.h.s. of (2.18) and depends on the normalization chosen for the periods. The normalization of [4] is such that the electric charges are integers, and then the periods behave near infinity as

$$\begin{aligned} a(u) &\sim \frac{1}{2}\sqrt{2u} \\ a_D(u) &\sim i\frac{4-N_f}{2\pi}a(u)\ln\frac{u}{\Lambda_f^2}. \end{aligned} \tag{6.4}$$

This gives for the wronskian the constant value $i(4-N_f)/4\pi$, hence in the r.h.s. of (2.18) and with this normalization we must use $4\pi b_1$, where $b_1 = (4-N_f)/16\pi^2$ is the coefficient of the one-loop β-function.

The solution to (6.3) has been obtained in [24] and further ellaborated in [25], where the global issues are carefully analyzed. It is expressed in terms of hypergeometric functions and the branch cuts must be taken into account. For $a(u)$ we have:

$$a(u) = \frac{1}{2}\sqrt{2u}F(-\frac{1}{6}, \frac{1}{6}, 2; -\frac{1}{u^3}), \tag{6.5}$$

with one branch cut along the negative real axis corresponding to the square root, and three branch cuts going from the three singularities to the origin and corresponding to the branch cut of the hypergeometric function. To obtain the solution for $a_D(u)$ we must choose at what singularity the monopole $(0,1)$ becomes massless, and then the asymptotic behaviour fixes the solution. Following [25], we choose $u_1 = \mathrm{e}^{-i\pi/3}$ as the monopole singularity. If we now define the function

$$f_D(u) = \frac{\sqrt{12}}{12}(u^3+1)F(\frac{5}{6}, \frac{5}{6}, 2; 1+u^3), \tag{6.6}$$

the other period $a_D(u)$ will be given by the analytic continuation of $f_D(u)$ through the branch cuts of the hypergeometric function in (6.6). Explicitly:

$$\begin{aligned}
-\frac{2\pi}{3} < \mathrm{Arg}u < 0 \quad &: \quad a_D(u) = \mathrm{e}^{-2\pi i/3} f_D(u), \\
0 < \mathrm{Arg}u < \frac{2\pi}{3} \quad &: \quad a_D(u) = \mathrm{e}^{-2\pi i/3} f_D(u) - a(u), \\
\frac{2\pi}{3} < \mathrm{Arg}u < \pi \quad &: \quad a_D(u) = f_D(u) - 2a(u), \\
-\pi < \mathrm{Arg}u < -\frac{2\pi}{3} \quad &: \quad a_D(u) = -f_D(u) + a(u).
\end{aligned} \tag{6.7}$$

As $f_D(u) = 0$ vanishes at the three singularities, we see that the good variable around u_2 is $a_D(u) + a(u)$, and thus corresponds to the dyon $(1,1)$. For the dyon becoming massless at $u_3 = -1$ there is a branch cut and two different descriptions, one with quantum numbers $(2,1)$ in the upper half of the complex u-plane (and the corresponding coordinate is $a_D(u)+2a(u)$), and another one with quantum numbers $(-1,1)$ in the lower half, corresponding to the coordinate $a_D(u)-a(u)$. We then see that $a_D(u)$ has two branch cuts, one along the negative real axis, and another one going from u_2 to the origin. If we recall that $a_D(u)$ is the good coordinate to describe the $(0,1)$ monopole, we can see that these cuts are due to the singularities associated with BPS states which are non-local with respect to the monopole. Another interesting feature of (6.7) is the explicit realization of the $\mathbf{Z}_3$ symmetry acting on the u-plane.

In the softly broken theory $N_f = 1$ with a dilaton spurion we must compute the couplings (2.14) using (2.19) with the right normalization. All we need are the explicit expressions for da/du, df_D/du. Using the properties of hypergeometric functions, one obtains:

$$\begin{aligned}
\frac{da}{du} &= \frac{1}{2\sqrt{2u}} F(\frac{5}{6}, \frac{1}{6}, 1; -\frac{1}{u^3}) \\
\frac{df_D}{du} &= \frac{\sqrt{2}}{4} u^2 \{ \frac{1}{6} F(\frac{5}{6}, \frac{5}{6}, 2; 1 + u^3) + \frac{5}{6} F(\frac{11}{6}, \frac{5}{6}, 2; 1 + u^3) \}.
\end{aligned} \tag{6.8}$$

With (6.5)-(6.8) we can compute the gauge couplings for the Higgs variables $a^{(h)} = a$, $a_D^{(h)} = a_D$ and then use the monodromy transformations (2.24) to obtain the couplings for the variables associated to the BPS states becoming massless. We then have:

i) $(0, 1)$ monopole:

$$\begin{gathered}
\begin{pmatrix} a_D^{(m)} \\ a^{(m)} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} a_D^{(h)} \\ a^{(h)} \end{pmatrix}, \\
\tau_{11}^{(m)} = -\frac{1}{\tau_{11}^{(h)}}, \qquad \tau_{01}^{(m)} = -\frac{\tau_{01}^{(h)}}{\tau_{11}^{(h)}}, \\
\tau_{00}^{(m)} = \tau_{00}^{(h)} - \frac{(\tau_{01}^{(h)})^2}{\tau_{11}^{(h)}}.
\end{gathered} \tag{6.9}$$

ii) $(1, 1)$ dyon:

$$\begin{gathered}
\begin{pmatrix} a_D^{(d)} \\ a^{(d)} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_D^{(h)} \\ a^{(h)} \end{pmatrix}, \\
\tau_{11}^{(d)} = -\frac{1}{\tau_{11}^{(h)} + 1}, \qquad \tau_{01}^{(d)} = \frac{\tau_{01}^{(h)}}{\tau_{11}^{(h)} + 1}, \\
\tau_{00}^{(d)} = \tau_{00}^{(h)} - \frac{(\tau_{01}^{(h)})^2}{\tau_{11}^{(h)} + 1}.
\end{gathered} \tag{6.10}$$

iii) $(2,1)$ dyon:

$$\begin{pmatrix} a_D^{(d_1)} \\ a^{(d_1)} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} a_D^{(h)} \\ a^{(h)} \end{pmatrix},$$

$$\tau_{11}^{(d_1)} = -\frac{1}{\tau_{11}^{(h)}+2}, \qquad \tau_{01}^{(d_1)} = \frac{\tau_{01}^{(h)}}{\tau_{11}^{(h)}+2},$$

$$\tau_{00}^{(d_1)} = \tau_{00}^{(h)} - \frac{(\tau_{01}^{(h)})^2}{\tau_{11}^{(h)}+2}. \tag{6.11}$$

For the other description of the $(2,1)$ dyon we have similar expressions.

Before studying numerically the effective potential, we can extract some qualitative information about the vacuum structure and the condensates as in sect. 4. As in the $N_f = 0$, $SU(2)$ case [14], we expect that the minima of the effective potential will be located near the singularities (the $N = 1$ points). The monopole condensate at u_1 is given essentially by $\mathrm{Im}\tau_{01}^{(m)}(u_1)$, and we can evaluate the $(1,1)$ and $(2,1)$ condensates at the other singularities using the $\mathbf{Z}_3$ symmetry in the moduli space, as in (4.10). From (6.7) we obtain

$$\tau_{01}^{(m)}(u_1) \sim \frac{du}{da^{(m)}} = -2\sqrt{2}\mathrm{e}^{-2\pi i/3}, \tag{6.12}$$

with a non-zero imaginary part, hence we have monopole condensation at $u_1 = \mathrm{e}^{-\pi i/3}$. The $\mathbf{Z}_3$ symmetry gives

$$\tau_{01}^{(d)}(u_2) = -\mathrm{e}^{\pi i/3}\tau_{01}^{(m)}(u_1), \qquad \tau_{01}^{(d_1)}(u_3) = -\mathrm{e}^{2\pi i/3}\tau_{01}^{(m)}(u_1), \tag{6.13}$$

and we get a condensate at $u_2 = \mathrm{e}^{\pi i/3}$ with the same size than the one at u_1, and no condensate at $u_3 = -1$. Therefore we expect two minima for the effective potential, located near u_1 and u_2. This is consistent with the value of the effective potential at the singularities, given by (4.5). As $b_{00} \sim -\mathrm{Re}u$, the singularity at u_3 is energetically less favourable than the singularities at u_1, u_2, which have moreover the same energy. Again we find a precise correlation between the size of the condensate and the corresponding value of the effective potential.

Now we proceed to the precise numerical analysis of the vacuum structure. In the Higgs region the effective potential is given by the cosmological

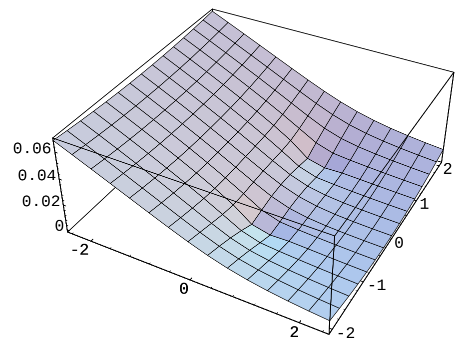

Figure 3: Effective potential $V^{(h)}$, (6.14).

constant,

$$V^{(h)} = -\frac{\det b^{(h)}}{b_{11}^{(h)}} f_0^2, \tag{6.14}$$

which is a monodromy invariant. and we plot it in fig. 3. Its shape is very similar to the $N_f = 0$ case studied in [14]. It has two cusp singularities located at u_1, u_2, where we expect an important contribution from the condensates. In fact, one should include the monopole and dyon contribution in order to smooth out the behavior of the effective potential near these singularities, and consider

$$V_{\rm eff} = -\frac{\det b}{b_{11}} f_0^2 - \frac{2}{b_{11}^{(m)}} \rho_{(m)}^2 - \frac{2}{b_{11}^{(d)}} \rho_{(d)}^2. \tag{6.15}$$

This is precisely the expected behaviour from the Wilsonian point of view: near the points where extra states become massless one should include the relevant degrees of freedom to have a non-singular effective potential.

Therefore, we must analyze the condensates around the three singularities. For the $(2,1)$ (or $(-1,1)$) dyon state we find, according to our previous estimates in (6.13), a very tiny condensate, with the same pattern obtained for the $(-1,1)$ dyon in the pure Yang-Mills case analyzed in [14]. In particular it does not produce a minimum in the effective potential once its contribution is included. For the $(0,1)$ monopole and $(1,1)$ dyon associated to the singularities at u_1, u_2, respectively, we find condensates in symmetric regions with respect to the Reu axis, as one can see in fig. 4. As long as $f_0 < 0.25$ the two regions don't overlap. Once these contributions are in-

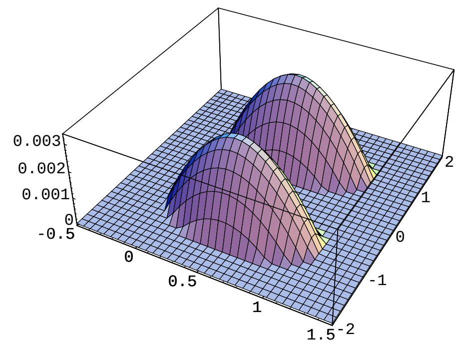

Figure 4: Monopole and dyon VEVs for $f_0 = 0.1$ on the u-plane.

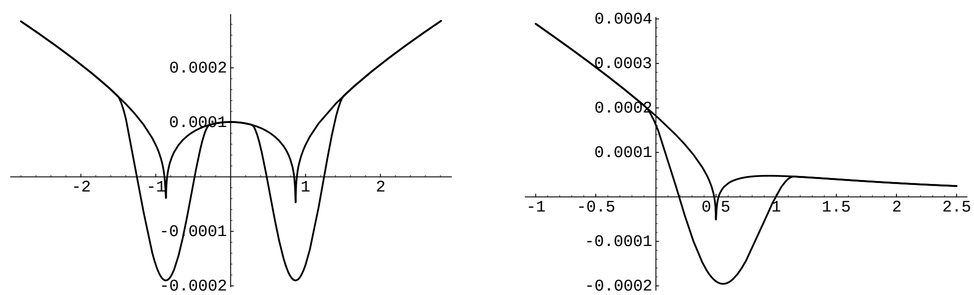

Figure 5: Effective potential (6.14) (top), and (6.15) (bottom) for $u = 1/2 + iy$ (left) and for $u = x - i\sqrt{3}/2$ (right). Both are plotted for $f_0 = 0.1$.

cluded in the effective potential, for this range of f_0, we get two degenerate minima at complex conjugate points u_0, u_0^*. The minimum associated to the monopole condensate has $\mathrm{Im} u_0 < 0$, and the one associated to the dyon condensate has $\mathrm{Im} u_0^* > 0$.

Of course, once the contribution of the condensates is taken into account, the singularities are smoothed out, as one can see in fig. 5. The position of the minimum moves away from the singularities as the supersymmentry breaking parameter f_0 is turned on.

There are two features which are worth noticing. The symmetry between the monopole and dyon condensates, as well as the degeneracy of the potential, are due to the following symmetry properties of the monopole and dyon variables and couplings:

$$
\begin{aligned}
a^{(m)}(u) &= \overline{a}^{(d)}(u^*), \qquad \tau_{11}^{(m)}(u) = -\overline{\tau}_{11}^{(d)}(u^*), \\
\tau_{01}^{(m)}(u) &= \overline{\tau}_{01}^{(d)}(u^*), \qquad \tau_{00}^{(m)}(u) = -\overline{\tau}_{00}^{(d)}(u^*).
\end{aligned} \tag{6.16}
$$

With our choice of the monopole and dyon coordinates, $\epsilon^{(m)} = -1$, $\epsilon^{(d)} = 1$. From (6.16), (5.28) and (5.30) one can check that the mass spectrum is the same for the monopole and dyon minima.

The other interesting property of these minima is the following. As it was shown in [23], magnetic monopoles labeled by the quantum numbers (n_e, n_m) have anomalous electric charge given by $q = n_e + (\theta_{\rm eff}/\pi) n_m$ (in the normalization of [4]). The theta parameter we must take into account in this case is the low-energy one, and as we have been labeling the BPS states with quantum numbers referred to the Higgs variables a, a_D, it will be given by [3]

$$
\theta_{\rm eff} = \pi {\rm Re} \frac{da_D}{da}. \tag{6.17}
$$

In the $N_f = 0$ case analyzed in [14], the minimum of the effective potential was associated to the $(0,1)$ state and occurred along the real u-axis with $u > 1$. In this region $\theta_{\rm eff} = 0$, $q = 0$, and we have stricitly speaking monopole condensation and confinement of colour electric charge, as it happens when the theory is broken down to $N = 1$ theory and the minima are locked at the singularities [26]. In the softly broken $N_f = 1$ theory the situation is different: for the minimum associated to the $(0,1)$ BPS state one has $\theta_{\rm eff}(u_0) < 0$ as soon as the supersymmetry breaking parameter is different from zero (at the singularity $\theta_{\rm eff}(u_1) = 0$), and for the minimum located at u_0^*, associated to the $(1,1)$ BPS state, one has $\theta_{\rm eff}(u_0^*) = -1 - \theta_{\rm eff}(u_0)$ (which in fact is a consequence of the symmetry (6.16)). In this way the monopole and dyon have anomalous electric charges given by

$$
q^{(m)} = \frac{\theta_{\rm eff}(u_0)}{\pi}, \qquad q^{(d)} = -\frac{\theta_{\rm eff}(u_0)}{\pi}. \tag{6.18}
$$

As the supersymmetry breaking parameter is turned on, the minimum moves in such a way that $|\theta_{\rm eff}(u_0)|$ increases starting from zero : the condensing states have greater electric charges with opposite sign. Hence we have dyon condensation properly speaking in both minima. This must produce some

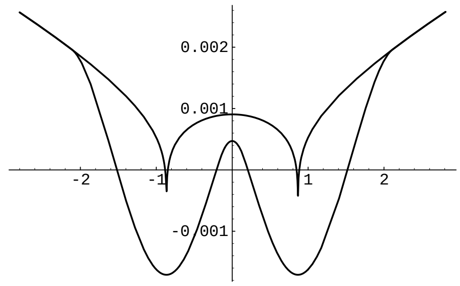

Figure 6: Effective potential (6.15) for $f_0 = 0.3$ along $u = 1/2 + iy$.

screening of the electric sources and correspondingly a smaller string tension, although on general grounds we still expect confinement of colour electric charge.

As $f_0 \sim 0.25$, the regions where monopole and dyon condensation occur begin to overlap on the real u-axis. This kind of behaviour for the softly broken $N = 2$ theories was already noticed in [14]. At this point it is clear that in the overlapping region there are non-mutually local degrees of freedom which must be taken into account simultaneously, and as it is well known it is difficult to find an effective lagrangian description of this situation. Nevertheless it is interesting to notice that we are studying the vacuum structure with the effective potential (6.15), and in fact we still have a smooth description of the low-energy physics when we add the cosmological constant and the condensates contribution, even in the overlapping region. In fig. 6 $V_{\rm eff}$ is plotted along the Imu direction, for Re$u = 1/2$, and we can see that the overlapping of monopole and dyon condensates lowers the energy on the real u-axis. A reliable description in terms of (6.15) is possible because of the monodromy invariance of the cosmological constant, which takes into account the gauge-field degrees of freedom independently of the description we choose. In the contributions coming from the BPS states we are not considering the monopole and dyon variables as independent variables, as the only independent parameter in (6.15) is the gauge-invariant order parameter u. Hence we can try to extrapolate our analysis to a wider range of the supersymmetry breaking parameter using the effective potential description. This could give a hint of the dynamics induced by the interaction of non-mutually local objects in field theory. The situation we find here is similar to the one considered by Argyres and Douglas in [7], concerning certain points in the moduli space of the $N = 2$, $SU(3)$ Yang-Mills theory where non-local BPS states become simultaneously massless. They also showed that, even if it is not possible to write a well-defined lagrangian describing these objects, one can make sense of other quantities such as the β-function. We think that (6.15) should be considered on the same footing.

However, the monopole and dyon condensates appearing in (6.15) are defined in terms of functions which have branch cuts connecting the origin of the moduli space with the other singularities. In other words, the terms associated to the BPS states in (6.15) are not monodromy invariant. The monopole (dyon) condensate attains these branch cuts precisely when the overlapping occurs, and one could think that this invalidates the effective potential description as soon as $f_0 \sim 0.25$. But actually one can solve this problem performing an analytic continuation through the cuts. This is closely related to the fact that inside the curve of marginal stability the BPS states are described by different quantum numbers, depending on the region under consideration [25]. Finally, this also indicates what is the breakdown of our approximation: as soon as the monopole (dyon) condensate attains the other singularities, the analytic continuation cannot be done in a consistent way. Moreover, the condensates have a non-smooth behaviour at the singularities associated to the other, non-mutually local BPS states. This is an indication that we are not taking into account the relevant degrees of freedom in the description provided by (6.15). This breakdown occurs for $f_0 \sim 0.8$, therefore we can study the dynamics induced by the supersymmetry breaking parameter before it reaches this critical value.

For $0.25 \leq f_0 \leq 0.6$ the vacuum structure remains qualitatively the same, with the only difference that the minima of the effective potential move away from the singularities and correspondingly $|\theta_{\rm eff}(u_0)|$ increases its value. For $f_0 \sim 0.6$, a new minimum develops on the real u-axis, due to the overlapping of the monopole and dyon VEVs. If we increae f_0 we find a first order phase transition for $f_0 \sim 0.68$: the new minimum becomes the absolute minimum and therefore the true vacuum of the theory, and the vacuum degeneracy is lifted. In fig. 7 $V_{\rm eff}$ is plotted for $f_0 = 0.68$ along the Imu direction with Re$u = 0.95$, and for $f_0 = 0.7$ on the u-plane. The possibility of this kind of behaviour due to the presence in the effective potential of non-mutually local states was pointed out in [14]. This minimum persists until the region where the monopole (dyon) develops a VEV attains the other singularities. At this point our description breaks down and we can not longer trust the

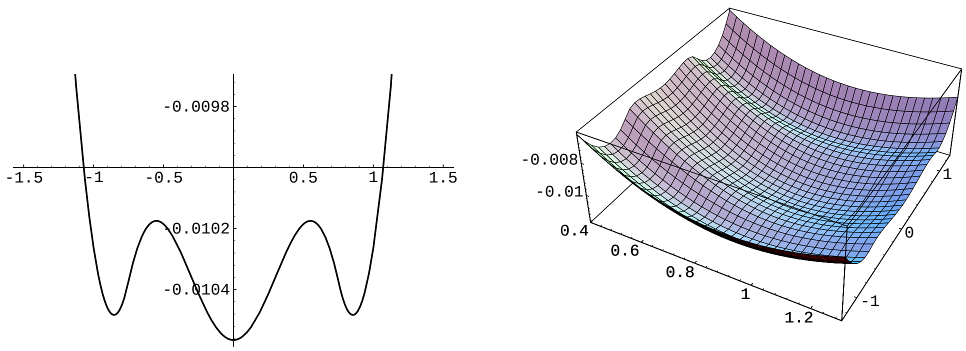

Figure 7: Effective potential (6.15) for $u = 0.95 + iy$, $f_0 = 0.68$ (left), and for $f_0 = 0.7$ on the u-plane (right).

effective potential.

What can be the interpretation of this phase transition? A possible explanation may be the fact that the BPS states which are condensing have an anomalous electric charge due to the effective theta parameter. One should expect that the additional electric charge makes the condensation less and less favourable energetically, as it happens in many models. The dynamics looks very much like the phase transition in the theta angle leading to an oblique confinement phase. In these cases the anomalous electric charge makes more favourable enegetically the condensation of a bound state of dyons with opposite electric charges [27]. In our model, the theta angle transition is induced by the supersymmetry breaking parameter f_0, and the new minimum seems to correspond to a simultaneous condensation of dyons which also have opposite electric charges: the theta parameter along the real u-axis is $\theta_{\text{eff}} = \pi/2$, and the monopole and dyon have anomalous electric charge $q^{(m)} = -q^{(d)} = -1/2$.

It is clear that a thorough understanding of this kind of vacua, with non-mutually local states, is still lacking. But we think that the analysis presented here can give some hints about the possibility of these new phases and the rich dynamics associated to them. Perhaps the transition to this new vacuum can be understood in more traditional terms as the condenstion of a bound state with zero electric charge, but the ubiquity of these phenomena in supersymmetric theories [19, 28] raises the possibility of new phases of gauge theories that may be relevant to the description of the QCD vacuum.

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