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D-Term Inflation

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Abstract

We show that inflation which is dominated by the D-term density avoids the 'slow-roll' problem of inflation in supergravity. Such an inflationary scenario can naturally emerge in theories with non-anomalous or anomalous $U(1)$ gauge symmetry. In the latter case the scale of inflation is fixed by the Green–Schwarz mechanism of anomaly cancellation. The crucial point is that the (super)gravity-mediated curvature of all the scalar fields (and, in particular, of the inflaton), which in the standard F-dominated case is of the order of the Hubble parameter, is absent in the D-term inflation case. The curvature of moduli and of all other flat directions during such an inflation crucially depends on their gauge charges.

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Most models of inflation [1] assume a period in the early history during which the Universe was dominated by a potential energy of a slowly rolling scalar field – the inflaton. For this to happen, certain slow-roll conditions must be satisfied. These conditions ensure that the curvature of the inflaton potential is smaller than the instant value of the Hubble constant (H). Inflation stops whenever this is not the case.

According to the modern view, particle physics is described below Planck scale (M_P) energies by an effective $N = 1$ supergravity theory. As was pointed by many authors [2], [3] [4], it is very difficult to naturally implement a 'slow-roll' inflation in the context of supergravity. The problem has to do with the fact that inflation, by definition, breaks global supersymmetry since it requires a non-zero cosmological constant V (false vacuum energy of the inflaton). However, in supergravity theories, the supersymmetry breaking gets transmitted to all the fields by the universal messenger which is gravity, and the resulting soft masses of the scalar fields are typically [3], [4] [5]

$$m_{soft}^2 \sim \frac{V}{M_p^2} \sim H^2. \tag{1}$$

An unnatural fine-tuning is required in order to avoid a soft mass of the same magnitude for the inflaton itself. This is usually considered as a generic problem for inflation in the context of supergravity.

The aim of the present letter is to show that the above difficulty only exists as far as inflation proceeds in a D-flat direction, with the vacuum energy being dominated by the F-term(s) of some of the superfield(s); it is not true in the case when the inflation is dominated by non-zero D-terms (as we will see below, this does not necessarily require the inflaton to be a gauge-charged field). The crucial difference is that the gravity-mediated soft scalar masses for the latter case are much smaller than H. This happens because in the universe with positive cosmological constant supersymmetry is broken, but the soft scalar masses depend whether non-zero vacuum energy comes from the F-term or D-term. In the F-type breaking the scalars are getting soft masses given by (1), whereas for the D-type breaking, these masses depend on their gauge charges: scalars charged under the corresponding gauge symmetries gain masses $>> H$, whereas others can only get gauge-mediated masses from the loops [6]. In particular, for the inflaton (which is assumed to be a singlet under gauge symmetries in question) the curvature is automatically small.

This observation can have a crucial impact also for the cosmology of the flat directions and, in particular, for the cosmological moduli problem [7]. We consider a class of inflationary scenarios where inflation is dominated by non-vanishing D-terms and we show that they naturally emerge in theories with non-anomalous or anomalous gauge $U(1)$ symmetry which include a Fayet-Iliopoulos D-term. Such scenarios can be considered as natural supersymmetric realizations of the 'hybrid' inflation invented by Linde [8]. Below we will refer to the inflation dominated by D-terms as D-inflation.

Before constructing an explicit example, let us first discuss the problems of inflation in supergravity and show why they can be avoided in the case of D-inflation. The two slow rolling conditions for the potential V of the inflaton field X can be defined in the following form [1]

$$\left|\frac{MV'}{V}\right| << 1 \tag{2}$$

and

$$|V''| << H^2 \tag{3}$$

where prime denotes the derivative with respect to X and $M = M_p/\sqrt{8\pi}$. Breakdown of one of these conditions signals the end of inflation [1]. Now, in supergravity, the scalar potential has the following form [10]

$$V = \mathrm{e}^{\frac{K}{M^2}}\left[(K^{-1})_i^j F_i F^j - 3\frac{|W|^2}{M^2}\right] + \frac{g^2}{2}\mathrm{Re}f_{AB}^{-1}D^A D^B \tag{4}$$

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[1]Needless to say, these conditions are not applicable if the inflationary state is a local minimum. This may happen if, for instance, the thermal effects force the Higgs field (which at zero temperature has a large VEV and an almost flat potential) to be temporarily trapped at the origin, but the vacuum energy dominates the thermal energy [9]. Such an inflation, unfortunately, has too few e-foldings (and a very small H) in order to solve the flatness and horizon problems.

where

$$F^i = W^i + K^i \frac{W}{M^2} \tag{5}$$

and

$$D^A = K^i (T^A)_i^j \phi_j + \xi^A . \tag{6}$$

Here K, W, and f are respectively the Kähler potential, superpotential and gauge-kinetic function. Upper (lower) indexes (i, j) denote derivative with respect to field ϕ_i (ϕ^{*i}) and T^A are generators of the gauge group in the appropriate representation. The ξ^A are Fayet-Iliopoulos D-terms, which can only exist for the $U(1)$ gauge groups. In what follows the crucial role is played by the exponential factor $e^{\frac{K}{M^2}}$ in front of the F-terms. The necessary condition for the 'slow-roll' inflation is the existence of a positive (false) vacuum energy, meaning that at least some $\langle F_i \rangle$ or some $\left\langle D^A \right\rangle$ are nonzero. Thus, inflation necessarily breaks supersymmetry. Let us assume for a moment that the inflation is dominated by some of the F_i terms and that the D^A-terms are vanishing or negligible (most of the existing scenarios in the literature assume this condition). Then the slow-roll conditions (2) and (3) can be written as

$$\frac{MV'}{V} = \frac{K_X}{M} + \text{other terms} << 1 \tag{7}$$

and

$$V'' = 3K_X^X H^2 + \text{other terms} << H^2 \tag{8}$$

respectively. Here the subscript X denotes a derivative with respect to the inflaton and we have used the relation $H^2 = V/3M^2$ to obtain the second equation. The 'other terms' in these expressions are typically of the same order as the ones written explicitly. Their precise value is model dependent, and they only cancel in some special cases (e.g. if X is a Goldstone field [11] or the dilaton or one of the moduli in some string models). Generically, however, this cancellation requires a fine tuning, which we will ignore here. In any case, neglecting these 'other terms' gives the correct order of magnitude.

Now, during inflation, unless a very special form is chosen, K_X is typically of the order of X. Thus, in principle, one can satisfy equations (1) and (7) if inflation occurs for values well below the Planck scale. This is difficult to achieve for the chaotic inflationary scenario [12]. The second condition is even more severe [3], [4]. The quantity K_X^X stands in front of the kinetic term and therefore in the true vacuum it should be normalized to one. Then it is very unlikely to expect it to be much smaller during inflation. These arguments indicate that it is not easy to implement F-type inflation in supergravity theories. All the solutions proposed that we know of involve specific non-minimal forms of the Kähler potential. While it is not excluded that such forms indeed may emerge from the underlying fundamental theory, in the present paper we will study an alternative solution, which is largely independent on the possible forms of the Kähler potential and works equally well for its minimal form ($K_X^X = 1$).

What is interesting about D-inflation is that the problems discussed above can be automatically avoided. Indeed for inflation dominated by some of the D^A-terms the slow-roll conditions can be easily satisfied: quantities $(D^A)_X$ and $(D^A)_X^X$ can be automatically zero and what one needs, just, is that X is annihilated by the corresponding gauge generator. Some other important differences emerge: first, the slope of the inflaton potential is induced from the one-loop radiative corrections and is practically independent of the details of supersymmetry breaking in the present vacuum; secondly, the inflation-induced curvature of the moduli and other flat directions crucially depends on their gauge-charges.

Let us show that such a scenario can naturally emerge in a theory with a $U(1)$ gauge symmetry. First we consider an example with a non-anomalous $U(1)$ symmetry. We introduce three chiral superfields X, ϕ_+ and ϕ_- with charges equal to 0, +1 and −1 respectively. The superpotential has the form

$$W = \lambda X \phi_+ \phi_- \tag{9}$$

which can be justified by several choices of discrete or continuous symmetries and in particular by R-symmetry. The scalar potential in the global supersymmetry limit reads:

$$V = \lambda^2 |X|^2 \left(|\phi_-|^2 + |\phi_+|^2\right) + \lambda^2 |\phi_+\phi_-|^2 + \frac{g^2}{2}\left(|\phi_+|^2 - |\phi_-|^2 + \xi\right)^2 \tag{10}$$

where g is the gauge coupling and ξ is a Fayet-Iliopoulos D-term (which we choose to be positive). This system has a unique supersymmetric vacuum with broken gauge symmetry

$$X = \phi_+ = 0, \quad |\phi_-| = \sqrt{\xi}. \tag{11}$$

Minimizing the potential, for fixed values of X, with respect to other fields, we find that for $|X| > X_c = \frac{g}{\lambda}\sqrt{\xi}$, the minimum is at $\phi_+ = \phi_- = 0$. Thus, for $|X| > X_c$ and $\phi_+ = \phi_- = 0$ the tree level potential has a vanishing curvature in the X direction and large positive curvature in the remaining two directions:

$$m_\pm^2 = \lambda^2 |X|^2 \pm g^2 \xi. \tag{12}$$

For arbitrarily large X the tree level value of the potential remains constant: $V = \frac{g^2}{2}\xi^2$. Thus X is a natural inflaton. Indeed, under the assumption of chaotic initial conditions $|X| >> X_C$ this system naturally leads to inflation. Along the inflationary trajectory all the F-terms vanish and Universe is dominated by the D-term which splits the masses of the Fermi-Bose components in the ϕ_+ and ϕ_- superfields. Such splitting results in a one-loop effective potential for the inflaton field [13], In the present case this potential can be easily evaluated and for large X it behaves as

$$V_{eff} = \frac{g^2}{2}\xi^2 \left(1 + \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 |X|^2}{\Lambda^2}\right) \tag{13}$$

Slow-roll conditions break down and inflation stops when $X \sim \sqrt{C}M$, where C is a one loop factor of order $g^2/(16\pi^2)$. In the globally supersymmetric limit, the dynamics of the above scenario is somewhat similar to the 'hybrid scenario" of [13], but it exhibits a crucial difference in the locally supersymmetric case: due to its D-type nature it escapes the problems of the standard F-type inflation. To demonstrate this explicitly let us consider the supergravity extension of our model. For definiteness we will assume the minimal structure for f and the Kähler potential

$$K = |\phi_-|^2 + |\phi_+|^2 + |X|^2. \tag{14}$$

Note that this form maximizes the problems for the F-type inflation since it gives $K_X^X = 1$. The scalar potential reads

$$\begin{aligned} V = {} & \mathrm{e}^{\frac{|\phi_-|^2 + |\phi_+|^2 + |X|^2}{M^2}} \lambda^2 \left[|\phi_+\phi_-|^2 \left(1 + \frac{|X|^4}{M^4}\right) \right. \\ & \left. + |\phi_+ X|^2 \left(1 + \frac{|\phi_-|^4}{M^4}\right) + |\phi_- X|^2 \left(1 + \frac{|\phi_+|^4}{M^4}\right) + 3\frac{|\phi_+\phi_- X|^2}{M^2} \right] \\ & + \frac{g^2}{2}\left(|\phi_+|^2 - |\phi_-|^2 + \xi\right)^2 \end{aligned} \tag{15}$$

Again for values of $|X| > X_C$, other fields than X vanish and the behaviour is much similar to the global supersymmetry case. The zero tree level curvature of the inflaton potential is not affected by the exponential factor in front of the first term in (15), since this term is vanishing during inflation. This solves the slow-rolling problems of the F-type inflation.

Let us now consider the case of an anomalous $U(1)$ symmetry. Such symmetries usually appear in the context of string theories [14] and the anomaly cancellation is due to the Green–Schwarz (GS) mechanism [15], which determines the value of the Fayet-Iliopoulos D-term as

$$\xi_{GS} = \frac{\mathrm{Tr}Q}{192\pi^2} g^2 M^2 \tag{16}$$

where the trace is taken over all charges. To see how D-inflation is realized in this case, let us consider the simple example of such a $U(1)$ symmetry under which n_+ chiral superfields ϕ_+^i and n_- superfields ϕ_-^A carry one unit of positive and negative charges respectively. For definiteness let us assume that $n_+ > n_-$, so that the symmetry is anomalous and $TrQ \neq 0$. We assume that some of the fields transform under other gauge symmetries, since the GS mechanism requires non-zero mixed anomalies. Let us introduce a single gauge-singlet superfield X. Then the most general trilinear coupling of X with the charged superfields can be put in the form:

$$W = \lambda_A X \phi_+^A \phi_-^A \tag{17}$$

(for simplicity we assume additional symmetries that forbid direct mass terms). Thus there are $n_+ - n_-$ superfields with positive charge that are left out of the superpotential. The potential has the form

$$V = \lambda_A^2 |X|^2 \left(|\phi_-^A|^2 + |\phi_+^A|^2 \right) + \lambda_A^2 |\phi_+^A \phi_-^A|^2 + \frac{g^2}{2} \left(|\phi_+^i|^2 + |\phi_+^A|^2 - |\phi_-^A|^2 + \xi_{GS} \right)^2 \tag{18}$$

where summation over $A = 1, 2..., n_-$ and $i = 1, 2..., (n_+ - n_-)$ is assumed. Again, minimizing this potential for fixed values of X we find that for $|X| > X_C = max\left(\frac{g}{\lambda_A}\right)\sqrt{\xi_{GS}}$, the minimum for all ϕ_+ and ϕ_- fields is at zero. Thus, the tree level curvature in the X direction is zero and inflation can occur. During inflation masses of $2n_-$ scalars are

$$m_\pm^2 = \lambda_A^2 |X|^2 \pm g^2 \xi_{GS}. \tag{19}$$

and the remaining $n_+ - n_-$ positively charged scalars have masses squared equal to $g^2\xi_{GS}$. We see that inflation proceeds much in the same way as for the non-anomalous $U(1)$ example discussed above. The interesting difference is that in the latter case the scale of inflation is an arbitrary input parameter (although in concrete cases it can be determined by the GUT scale), whereas in the anomalous case it is predicted by the GS mechanism.

Generalization of the above example to the case of fields with different charges and with several singlet field is straightforward. The inflaton field in such a case may not be a single gauge-singlet, but rather a combination of several fields. Such trajectories are in general model-dependent and must be studied case by case. A generic rule which leads to our scenario is that there must exist a $U(1)$-neutral direction in field space which does not appear alone in the F-terms and along which all states having charges opposite to TrQ gain positive masses.

We think that the proposed mechanism may provide a somewhat generic inflation scenario in the context of superstring models. In such models, the presence of an anomalous $U(1)$ symmetry is not unusual; in general, several fields are neutral under the $U(1)$ symmetry and thus they (or a combination of them) may play the role of X. The presence of several other non-anomalous $U(1)$ symmetries may prove to be useful in order to prevent X from contributing alone to the F-terms (say a term λX^3 in our simple model presented above). The requirement of cancellation of mixed anomalies through the Green-Schwarz mechanism may provide stringent constraints on the charges of such $U(1)$, constraints which are a priori automatically accounted for in string models.

Finally let us note some other crucial differences between the D-type and F-type inflations. Flat directions behave very differently during inflation. In the ordinary F-inflation case all flat directions, including moduli, are getting curvature at least of order H from the gravity mediated supersymmetry breaking (note that for the gauge-charged flat directions curvature can be much larger due to gauge-mediated breaking [6]). This mass is exactly of the same origin as the problematic mass for the inflaton and appears due to the exponential factor e^{K/M^2} in front of the F-terms. Obviously, no such contribution exists in the D-inflation scenario outlined above. So in this case the curvature of the flat directions during inflation crucially depends on their gauge charge. In this respect we can divide all flat directions in three categories: 1) The flat directions which carry no common charge with any of the fields charged under $U(1)$. Such flat directions will not receive any contribution to the curvature, at least up to a three loop order. 2) Flat directions that are neutral under $U(1)$, but carry some gauge charges also carried by the $U(1)$-charged fields. Such direction will receive a two-loop gauge-mediated soft masses [6]; for example, if some ϕ_+^A and ϕ_-^A superfields are transforming as fundamental and anti-fundamental representations of some $SU(N)$ gauge group, then all $SU(N)$ nonsinglet fields during inflation will get the universal (up to charges) two-loop gauge-mediated soft masses

$$m_{soft}^2 \sim \left(\frac{\alpha}{4\pi}\right)^2 \frac{g^4}{\lambda_A^2} \frac{\xi^2}{|X|^2} \sim \left(\frac{\alpha}{4\pi}\right)^2 \frac{g^2}{\lambda_A^2} \frac{M^2}{|X|^2} H^2 \tag{20}$$

where α is a gauge coupling of $SU(N)$. 3) Directions charged under $U(1)$ receive large tree level masses.

We expect that all this will have important consequences for the cosmological moduli problem and the Affleck-Dine scenario [16] of baryogenesis.

REFERENCES

[1] A.D.Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Switzerland, 1990); E.W.Kolb and M.S.Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990);

[2] See, e.g., H.Murayama, H.Suzuki, T.Yanagida and J.Yokoyama, *Phys.Rev.* **D50** (1994) R2356, and references therein.

[3] E.J.Copeland, A.R.Liddle, D.H.Lyth, E.D.Stewart and D.Wands,*Phys.Rev.* **D49** (1994) 6410.

[4] E.D.Stewart, *Phys.Rev.* **D51** (1994) 6847.

[5] In the context of the cosmological moduli problem, see, M.Dine, W.Fischler and D.Nemeschansky, *Phys.Lett.* **B136** (1984) 169; G.D.Coughlan, R.Holman, P.Ramond and G.G.Ross, *Phys.Lett.* **B140** (1984) 44; A.S.Goncharov, A.D.Linde, and M.I.Vysotsky, *Phys.Lett.* **B147** (1984) 279; O.Bertolami and G.G.Ross, *Phys.Lett.* **B183** (1987) 163; G.Dvali, preprint IFUP-TH 09/95, hep-ph/9503259; M.Dine, L.Randall and S.Thomas, *Phys.Rev.Lett.* **75** (1995) 398.

[6] G.Dvali, *Phys.Lett.***B355** (1995) 78; CERN-TH/96-129, hep-ph/9605445.

[7] G.D.Coughlan, W.Fischler, E.W.Kolb, S.Raby and G.G.Ross, *Phys.Lett.***B131** (1983) 59; J.Ellis, D.V.Nanopoulos and M.Quirós, *Phys.Lett.***B147** (1986) 176; B. de Carlos, J.A.Casas, F.Quevedo and E.Roulet, *Phys.Lett.***B318** (1993) 447.

[8] A.D.Linde, *Phys.Lett* **B259** (1991) 38; *Phys.Rev.* **D49** (1994) 748.

[9] P.Binétruy and M.K.Gaillard, *Phys.Rev.***D34** (1986) 3069; D.H.Lyth and E.D.Stewart *Phys.Rev.Lett.***75** (1995) 201.

[10] E. Cremmer *et al.*, *Phys. Lett.***79B** (1978) 231, *Nucl. Phys.***B147** (1979) 105; E. Witten and J. Bagger, *Phys. Lett.***115B** (1982) 202; E. Cremmer,S. Ferrara, L. Girardello and A. van Proeyen, *Phys. Lett.***116B** (1982) 231, *Nucl. Phys.***B212** (1983) 413; J. Bagger, *Nucl. Phys.***B211** (1983) 302.

[11] K.Freese, J.A.Frieman and A.V.Olinto, *Phys.Rev.Lett.* **65** (1990) 3233; F.C.Adams, J.R.Bond, K.Freese, J.A.Frieman and A.V.Olinto, *Phys.Rev.* **D47** (1993) 426.

[12] A.D.Linde, *Phys.Lett.***B129** (1983) 177.

[13] G.Dvali, Q.Shafi and R.Schaefer, *Phys.Rev.Lett.* **73** (1994) 1886.

[14] M.Dine, N.Seiberg and E.Witten, *Nucl. Phys.***B289** (1987) 585; J.J.Atick, L.J.Dixon and A.Sen, *Nucl. Phys.***B292** (1987) 109; M. Dine, I.Ichinose and N.Seiberg, *Nucl. Phys.***B293** (1987) 253.

[15] M.B.Green and J.H.Schwarz, *Phys. Lett.***B149** (1984) 117.

[16] I. Affleck and M. Dine, *Nucl. Phys.***B249** (1985) 361.