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HADRON SPECTRA FROM NUCLEAR COLLISIONS

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Abstract:

We describe high energy nuclear collisions by a superposition of isotropically decaying thermal sources ("fireballs") of freeze-out temperature $T = 0.15$ GeV. The longitudinal fireball superposition is taken as boost-invariant, in a rapidity range determined by the average energy loss of nucleons in $p-p$ collisions. The transverse fireball motion is assumed to be due to random walk initial state collisions; it is determined by $p-A$ data and then extrapolated to central $A-B$ interactions. We thus obtain parameter-free predictions for the rapidity and transverse momentum spectra of hadrons produced in high energy nucleus-nucleus collisions. The results account fully for the observed broadening of transverse momentum distributions, so that single-particle spectra require neither collective flow nor temperature increase.

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A very simple schematic description of high energy nucleus-nucleus collisions is obtained by a superposition of isotropically decaying thermal systems ("fireballs") [1,2]. The momentum distribution of hadrons emitted by such a thermal fireball in its rest-frame is given by

$$\frac{d^3N_0}{d^3\mathbf{p}} = \frac{V_0}{(2\pi)^3}\exp\{-p_0/T\}, \tag{1}$$

where $p_0 = \sqrt{\mathbf{p}^2+m^2}$ is the energy of the secondary, T the freeze-out temperature and V_0 the spatial volume of the fireball at freeze-out. Integrating over the momentum, we obtain

$$N_0 = \frac{V_0Tm^2}{2\pi^2}K_2(m/T) \tag{2}$$

for the number of emitted secondaries; for $m = 0$ (or in the limit $T \to \infty$) this leads to the Stefan-Boltzmann form

$$\rho_0(T) \equiv \frac{N_0}{V_0} = \frac{T^3}{\pi^2} \tag{3}$$

for the density of secondaries in the fireball at freeze-out. In terms of the longitudinal rapidity y, with $p_L = m_T \sinh y$, the transverse mass $m_T = \sqrt{\mathbf{p}_T^2+m^2}$ and the azimuthal angle ϕ, Eq. (1) becomes

$$\frac{d^3N_0}{dydp_T^2d\phi} = \frac{V_0}{2(2\pi)^3}\ m_T\cosh y\ \exp\{-(m_T\cosh y)/T\}. \tag{4}$$

To describe the production of secondaries from a nucleus-nucleus collision, we superimpose such fireballs to form a "vapour trail" of energy deposited by the collision. In the formulation of this superposition, we use cylindrical coordinates for the motion of a fireball in the collision center of mass and relative to the initial collision axis: Y specifies its longitudinal rapidity, ρ its transverse rapidity, and Φ its azimuthal orientation. The Boltzmann factor for a secondary emitted from a moving fireball is then given by [3]

$$\exp\{-p_\mu u^\mu/T\} = \exp\{-[m_T\cosh\rho\cosh(Y-y) - p_T\sinh\rho\cos(\Phi-\phi)]/T\}. \tag{5}$$

Using this, we have for the momentum distribution of secondaries emitted from the superposition of fireballs

$$\frac{d^3N}{dydp_T^2d\phi} = \frac{V_0m_T}{2(2\pi)^3}\int_{-Y_L}^{Y_L}dY\int d\rho\int d\Phi\ f(Y,\rho)\cosh(Y-y)\exp\{-p_\mu u^\mu/T\} \tag{6}$$

where we assume azimuthal symmetry for the fireball distribution. The integral over Φ can be carried out to give

$$\begin{aligned}\frac{d^2N}{dydp_T^2} &= \frac{V_0m_T}{2(2\pi)^3}\int d\rho\ I_0((p_T/T)\sinh\rho)\times \\ &\int_{-Y_L}^{Y_L}dY\ f(Y,\rho)\ \cosh(Y-y)\ \exp\{-(m_T/T)\cosh(Y-y)\cosh\rho\}.\end{aligned} \tag{7}$$

The function $f(Y,\rho)$ determines the weights of different kinematic regions in the superposition of fireballs. The ends of the trail in longitudinal rapidity, $\pm Y_L$, are determined by the energy loss of nucleons in proton-proton collisions. In the energy range of present interest, the process $p+p \to p+X$ satisfies in good approximation $d\sigma/dx_F \simeq$ const. (Feynman scaling) [2,4]. This implies

$$\frac{d\sigma}{dy} \simeq \text{const.} \cosh y, \tag{8}$$

which leads to

$$\langle y \rangle \simeq y_{in} - \coth y_{in} + \frac{1}{\sinh y_{in}}, \tag{9}$$

where $y_{in} \simeq \ln(\sqrt{s}/M)$, with M for the nucleon mass. Eq. (9) implies that at high collision energies, a proton looses on the average about one unit of rapidity in the course of a $p-p$ collision. In nuclear collisions, most of the nucleons will be stopped more; but since even a single collision leads to that much rapidity loss, we take

$$Y_L \simeq y_{in} - \coth y_{in} + \frac{1}{\sinh y_{in}} \tag{10}$$

to be the end of the trail. At high energies, we thus have $Y_L \simeq y_{in} - 1$; at lower energies, $y_{in} - Y_L$ becomes less than one.

We shall further assume a boost-invariant rapidity superposition [5], which makes the weight function $f(Y,\rho)$ independent of Y. Such a step-function cut-off at $\pm Y_L$ is probably an oversimplification, but it will give us a first idea of the resulting rapidity distributions. The remaining transverse momentum distribution of the fireballs is assumed to be normalised

$$\int d\rho f(\rho) = 1; \tag{11}$$

we will determine it from a study of hadron production in $p-A$ interactions.

It is well known that all types of secondaries, whether Drell-Yan dileptons, quarkonium resonances consisting of heavy c or b quarks, or conventional hadrons, show in $p-A$ collisions a broadening of their respective transverse momentum distribution, compared to that observed in proton-proton collisions. The virtual photons in Drell-Yan production do not interact at all with the nuclear medium after their formation, and quarkonia leave the nuclear environment before it can develop any collective effects. Hence the general broadening pattern must be at least in part an initial state effect. The primary collision of two nucleons takes place along the incident nuclear collision axis. However, when a projectile nucleon which has already undergone one scattering with a target nucleon subsequently hits a second target nucleon, the collision axis for this interaction will in general be rotated relative to the axis of the incident nuclei, and so the transverse momentum of a secondary from this collision relative to the incident beam will appear larger (Cronin effect [6]). More scattering will lead to more rotation, so that we will in general have *initial state collision broadening* of transverse momentum spectra, dependent on the mass number A of the nuclear target. It is clear that such a broadening must also arise in nucleus-nucleus collisions. If we assume the broadening to be boost-invariant, the momentum distribution of secondaries from a nuclear collision will be given by Eq. (7), with $f(\rho)$ specifying the normalised transverse rapidity distribution of the fireballs.

If the path of an incident nucleon in a $p-A$ collision follows a random walk pattern through the target, we expect the resulting transverse rapidity distribution of the sources to be Gaussian,

$$f_{pA}(\rho) = \left[\frac{4}{\pi\delta_{pA}^2}\right]^{1/2} \exp\{-\rho^2/\delta_{pA}^2\}, \tag{12}$$

with

$$\delta_{pA}^2 \equiv \{N_A - 1\}\,\delta^2 \tag{13}$$

to be determined by the broadening of the transverse momentum distributions of secondaries from $p-A$ relative to $p-p$ collisions. Here δ is the average transverse rapidity shift per collision, and N_A is the number of nucleons which the incident proton encounters on its path through the (large) target nucleus. It is given by

$$N_A \simeq (3/4)(2\pi r_0^2 R_A)\; n_0, \tag{14}$$

where $r_0 \simeq 0.8$ fm is the nucleon radius, $R_A = 1.12\; A^{1/3}$ fm that of the nucleus, and $n_0 = 0.17\;\mathrm{fm}^3$ standard nuclear density. The factor (3/4) takes into account the average of the impact parameter.

In nucleus-nucleus collisions, the initial state collision broadening will come from both target and projectile nucleon scattering, so that we now have the same Gaussian form as Eq. (12), but with a larger dispersion,

$$\delta_{AB}^2 \equiv \{N_A + N_B - 2\}\,\delta^2. \tag{15}$$

Evidently this reduces to Eq. (13) if target or projectile becomes just a single nucleon. Note that for small A the impact parameter average must be made more carefully.

For a complete description, we thus need the average transverse rapidity shift per collision, $\delta\rho$. We determine it empirically from hadron production in $p - A$ collisions; once it is fixed, we can predict the initial state transverse momentum broadening in nucleus-nucleus collisions. The transverse momentum distribution of secondaries in $p - A$ collisions, measured in the rapidity region of the target nucleus, is given by

$$\left(\frac{dN}{dy dp_T^2}\right)^{pA}_{y=y_A} = \frac{V_0 m_T}{(2\pi)^3 \pi^{1/2}\delta_{pA}} \int d\rho\; \exp\{-\rho^2/\delta_{pA}^2\}\; I_0((p_T/T)\sinh\rho)\; \times$$
$$\int_{-Y_L}^{Y_L} dY \cosh(Y-y)\;\; \exp\{-(m_T/T)\cosh(Y-y)\cosh\rho\}. \tag{16}$$

In the limit $\delta \to 0$, i.e., for collinear fireball motion, Eq. (16) reduces to

$$\left(\frac{d^2N}{dy dp_T^2}\right)^{pp} = \frac{V_0 m_T}{2(2\pi)^3}\int_{-Y_L}^{Y_L} dY \cosh(Y-y)\exp\{-(m_T/T)\cosh(Y-y)\}, \tag{17}$$

which at high energy becomes

$$\left(\frac{d^2N}{dydp_T^2}\right)^{pp} \simeq \text{ const. } m_T K_1(m_T/T). \tag{18}$$

With $T = 0.15$ GeV, Eq. (17) gives a good parametrisation for the transverse momentum distribution of secondaries emitted at central rapidity in high energy $p-p$ collisions; it essentially reproduces that obtained from ISR data [7,8]. The last integral in Eq. (16) is just the $p-p$ distribution (17) boosted in transverse rapidity.

We now keep $T = 0.15$ GeV and try to find a value of δ_{pA} that accommodates the observed $p-A$ data. In Fig. 1 we see that for $\delta_{pW} = 0.30$, the transverse momentum spectrum of charged secondaries produced by 200 GeV/c protons incident on a W target [9,10] is described very well, provided we restrict ourselves to $p_T \geq 0.4$ GeV. We have therefore normalised Eq. (16) to the data in the range $0.4 \leq p_T \leq 1.2$ GeV. The "soft pion enhancement" seen for $p_T < 0.4$ GeV is presumably due to enhanced Δ production [11].

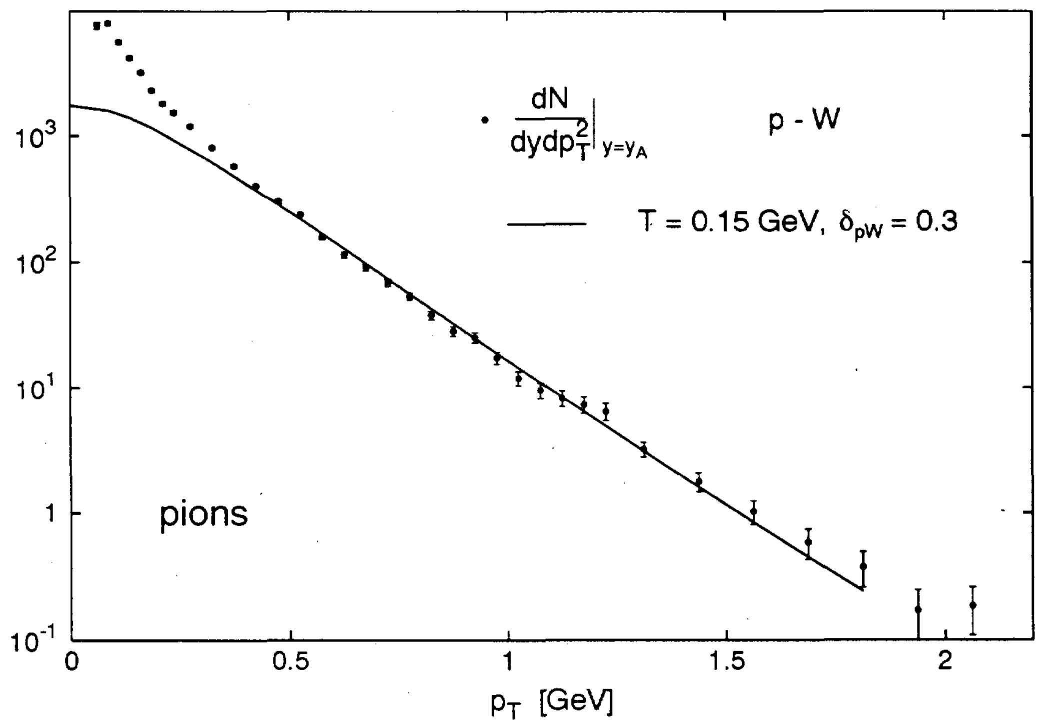

Figure 1: Transverse momentum distribution of pions in $p-W$ collisions [9]

To consider the p_T-broadening in more detail, we show in Fig. 2 the ratio of the transverse momentum spectra from $p-W$ and $p-p$ collisions, as obtained from Eqs. (16) and (17), here arbitrarily normalised to unity at $p_T = 0.375$ GeV. We compare this to the data of [9], divided by the experimental $p-p$ spectrum [7] and also normalised to unity at $p_T = 0.375$ GeV. From Figs. (1) and (2) we conclude that our superposition, with

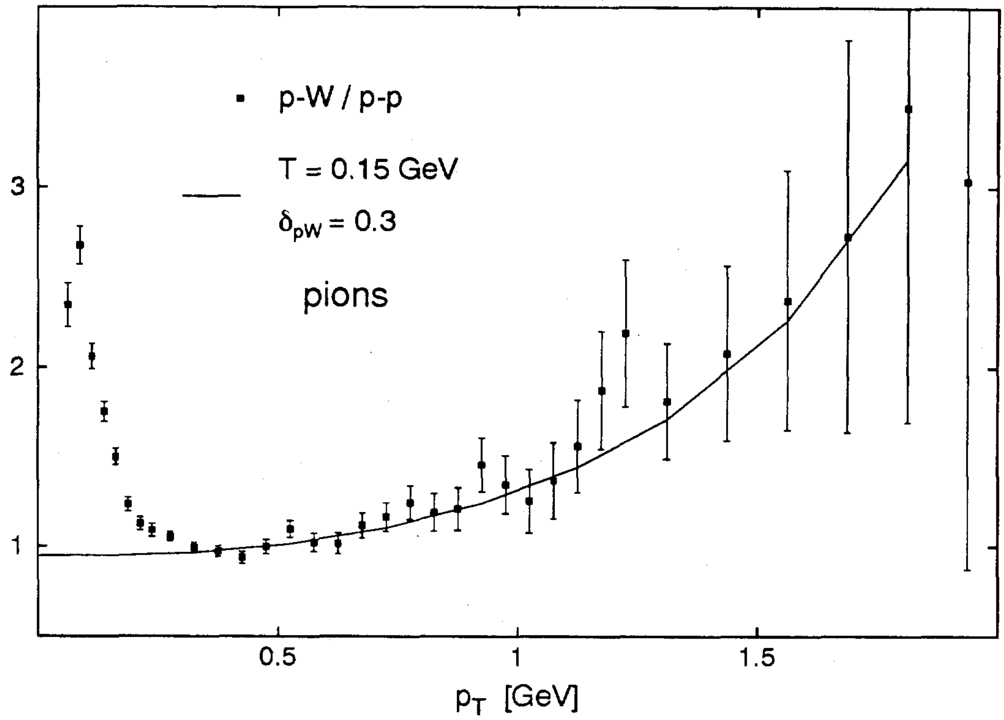

Figure 2: Ratio of $p-W$ to $p-p$ transverse momentum spectra of pions [9,7]

the noted values of T and δ_{pA}, provides an excellent description of the p_T broadening observed in $p-A$ collisions.

Before turning to $A-A$ collisions, we note that if we fit our $p-W$ p_T-distribution in the form $\exp\{-m_T/T_{\rm eff}\}$, as is often done, we obtain for pionic secondaries an effective temperature $T_{\rm eff} \simeq 0.19$ GeV at low p_T and approximately 0.2 GeV around $p_T \simeq 1.8$ GeV. This increase above the input temperature 0.15 GeV of the thermal source is, however, due to the transverse fluctuations of the sources and does not correspond to a physical temperature.

We now want to extend our considerations to $A-B$ collisions. Using Eqs. (13) and (14) for a W target (A=184), the fit value $\delta_{pW} = 0.30$ gives us $\delta \simeq 0.20$. Eq. (15) then provides the δ_{AB} needed to determine the transverse fireball distribution in nucleus-nucleus collisions. The p_T-distribution of secondaries produced in central $A-B$ collisions at rapidity $y = 0$ is given by

$$\left(\frac{dN}{dydp_T^2}\right)^{(AB)}_{y=0} = \frac{V_0 m_T}{(2\pi)^3 \pi^{1/2} \delta_{AB}} \int d\rho \, \exp\{-\rho^2/\delta_{AB}^2\} \, I_0((p_T/T)\sinh\rho) \times$$
$$\int_{-Y_L}^{Y_L} dY \cosh Y \exp\{-(m_T/T)\cosh Y \cosh\rho\}. \tag{19}$$

Let us first consider $S-W$ collisions, for which we get $\delta_{SW} \simeq 0.39$. In Fig. 3, the resulting p_T-broadening is compared to that in $p-W$ interactions, both at CERN-SPS energy. The predicted further enhancement is seen to agree very well with the measured ratio [12]. Now turning to $Pb-Pb$, we get $\delta_{PbPb} \simeq 0.44$. The resulting predictions for

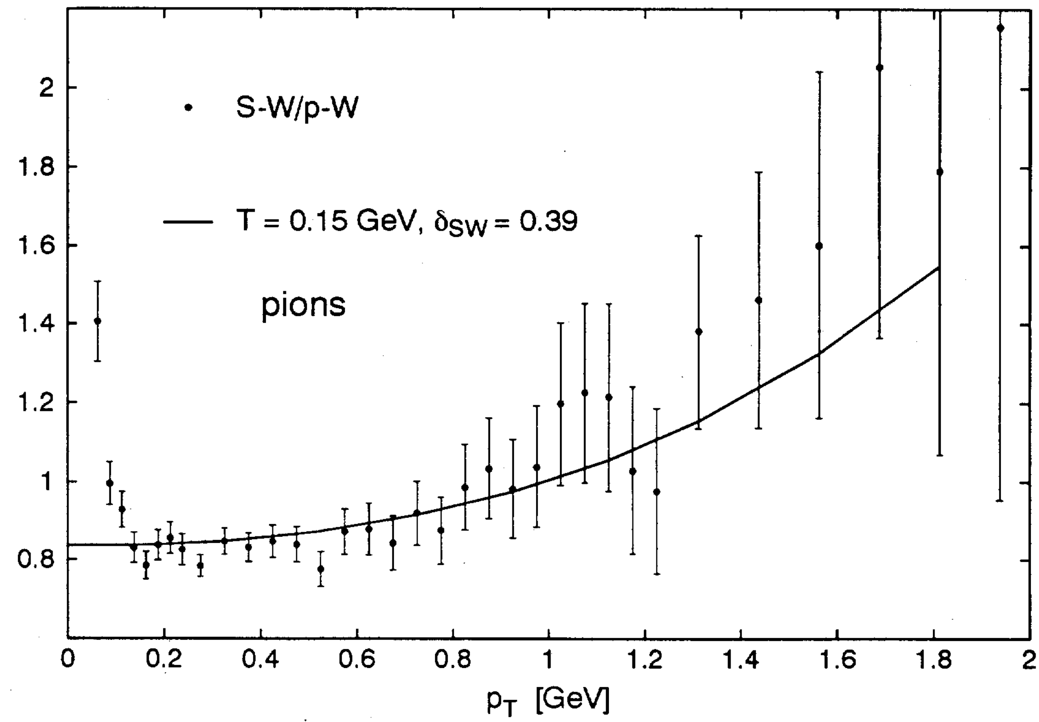

Figure 3: Ratio of transverse momentum distributions in central $S - W$ to $p - W$ collisions [12]

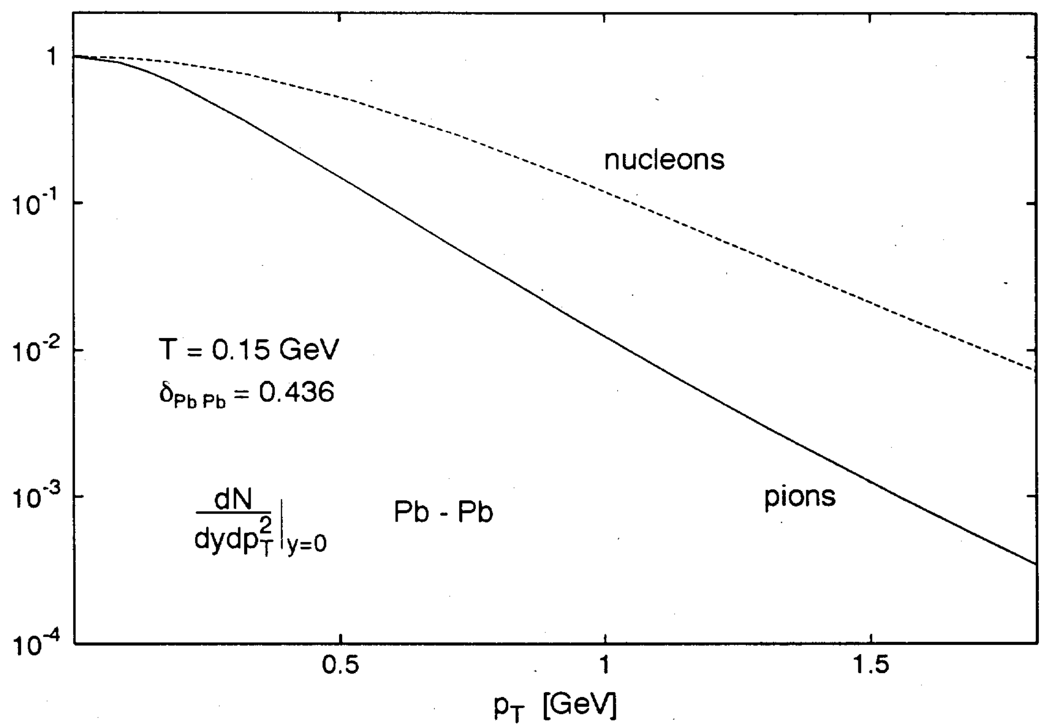

Figure 4: Predicted transverse momentum spectra for $Pb - Pb$ collisions

the normalised transverse momentum spectrum of pions and protons in this case are shown in Fig. 4. Note that the stronger broadening of the proton p_T-distribution in comparison to that for pions is to a considerable extent the consequence of m_T-scaling; we shall come back to this point shortly.

We note also here that if we insist on deriving an effective temperature from the p_T distributions provided by Eq. (19), we obtain for pions from $Pb-Pb$ collisions $T_{\text{eff}} \simeq 0.20$ GeV at low p_T, increasing to around 0.24 GeV at $p_T \simeq 1.8$ GeV. For protons, the broader spectra give 0.21 GeV at low and 0.26 GeV at high p_T. The slightly stronger proton broadening illustrates once more that this effect is not related to any enhanced overall temperature. Let us look in somewhat more detail at the relation between transverse momentum broadening and the mass of the secondary. Since a given shift in the transverse rapidity of the fireball increases the transverse momentum of secondaries in proportional to their mass,

$$p_T^i = m_i \sinh \rho \qquad (20),$$

we expect a stronger broadening for nucleons than for pions. To test the extent of this, we first have to take into account the inherent mass dependence of the broadening which follows already from m_T scaling as given by Eq. (18). We therefore form the ratio

$$R_\pi^{pA}(m_T) \equiv \frac{(d\sigma/dydm_T^2)_{pA}}{(d\sigma/dydm_T^2)_{pp}} \qquad (21)$$

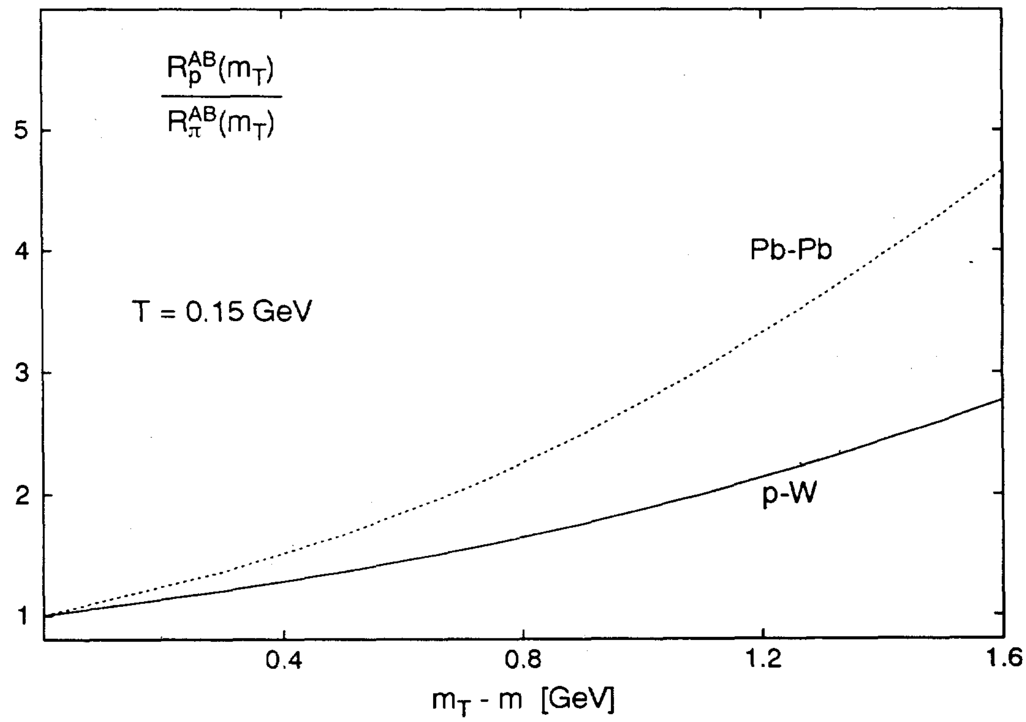

Figure 5: Ratio of normalised nucleon to pion m_T distributions

of the transverse mass distribution of pions from $p-A$ to that of pions from $p-p$ collisions. We normalize both distributions to unity at $m_T - m = 0$, so that $R_\pi^{pA}(m_T)$ shows at any given p_T the additional broadening due to the transverse motion of the fireballs. We then form the corresponding ratio for protons, $R_p^{pA}(m_T)$, and show in Fig. 5 the ratio of $R_p^{pA}(m_T)$ to $R_\pi^{pA}(m_T)$, as function of m_T; it illustrates the stronger broadening for protons due to transverse fireball motion. In Fig. 5 we also plot the same ratio for $Pb-Pb$ collisions showing a still stronger broadening for protons than for $p-W$ ones.

Very recently, first and still preliminary data have been presented for transverse momentum distributions from $Pb-Pb$ collisions at the CERN-SPS [13]. We include these in Figs. (6) and (7), with negative hadron spectra taken as "pions" and positive minus negative as "protons". The data are seen to agree quite well with our predicted forms. With some surprise we note that here the agreement between our prediction for pions and the measured pion spectrum extends down to very low p_T value, in contrast to the "low p_T enhancement" in $p-W$ collisions (Figs. 1 and 2). The weakening of this enhancement in the interaction of heavy nuclei might be due to a flattening of the transverse momentum distribution of Δ's as well as to a stronger contribution of kaons to negative secondaries.

Finally we turn to the p_T-integrated rapidity distributions resulting from our superposition (7), with Eqs. (12) and (15) for the transverse fluctuations. If we neglect these transverse fluctutations, the rapidity distribution becomes

$$\frac{dN}{dy} = \frac{V_0 T^3}{2\pi^2}\frac{N_F}{2Y_L}\int_{-Y_L}^{Y_L}\frac{dY}{\cosh^2(Y-y)}\exp\{-(m\cosh(Y-y))/T\}\times \left[1+\left(\frac{m}{T}\right)\cosh(Y-y)+\frac{1}{2}\left(\frac{m}{T}\right)^2\cosh^2(Y-y)\right], \quad (22)$$

with the limits of the integration determined by Eq. (10). The actual distribution is obtained by integration of Eq. (7) over all p_T. We have performed this integration numerically and find that the result is essentially identical (some 1 - 2 % difference) to Eq. (22), so that the longitudinal rapidity distribution remains practically unaffected by the transverse fireball fluctuations. The actual behaviour of dN/dy as function of y is shown in Figs. (8) and (9) for pions and nucleons, at AGS and SPS energies (in Figs. (8) - (10) the distributions are normalised to unity at $y=0$). For the former, Eq. (10) gives $Y_L \simeq 0.9$, with $y_{in} \simeq 1.6$; for the latter we get $Y_L \simeq 2.0$, with $y_{in} \simeq 2.9$; shown for reference are also the corresponding distributions from a single fireball. Here as well we now have preliminary data from $Pb-Pb$ collisions [13]; they are included in Fig. 9. In view of the crude model for the ends of the trail used here, the agreement is not bad. It could certainly be improved by using a more gradual y cut-off and the inclusion of some nuclear stopping effects. We further note that the rapidity distributions of the superposition are quite insensitive to temperature changes , even when the single fireball does show a temperature dependence. In Fig. 10 we illustrate this for $T = 0.1$ and 0.2 GeV, for both pions and nucleons.

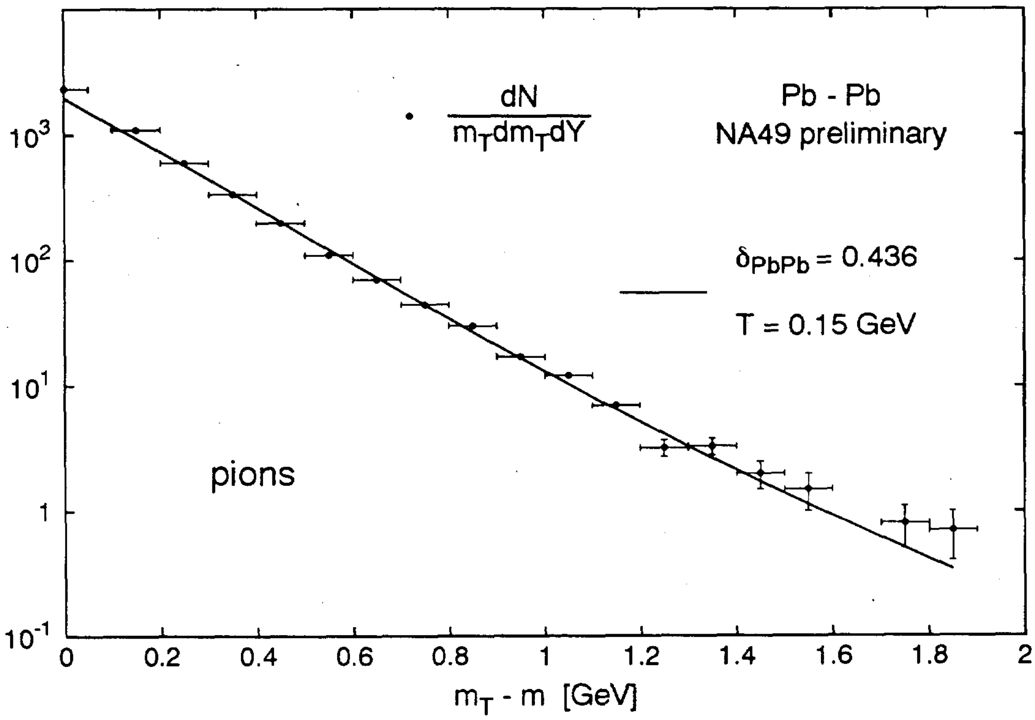

Figure 6: Pion m_T-distribution in $Pb - Pb$ collisions, with preliminary data [13]

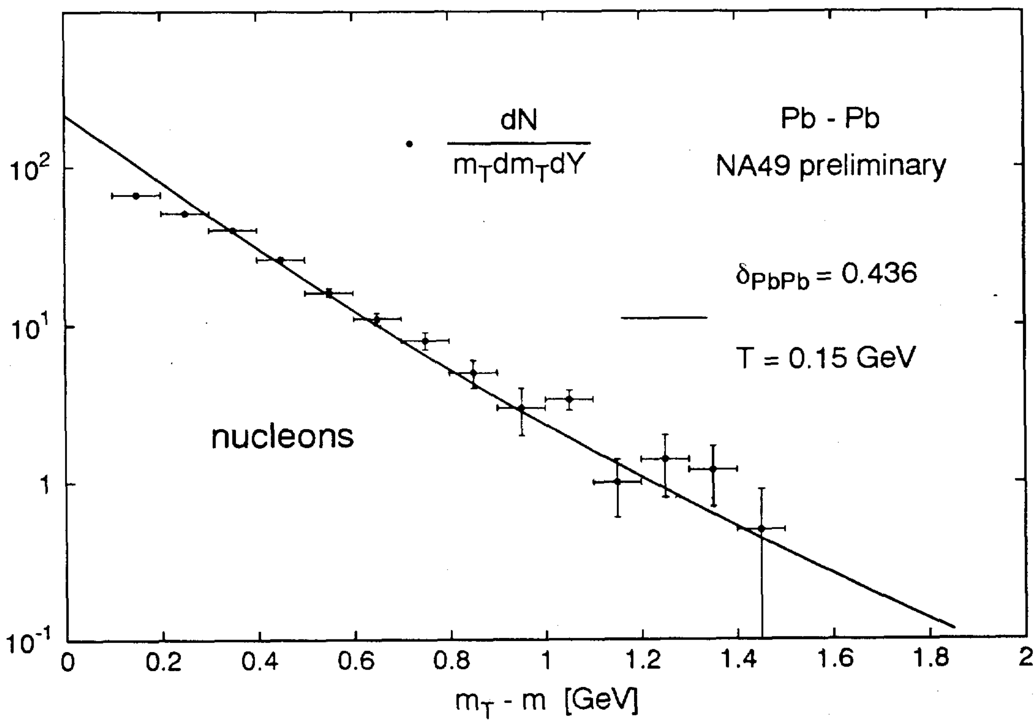

Figure 7: Nucleon m_T-distribution in $Pb - Pb$ collisions, with preliminary data [13]

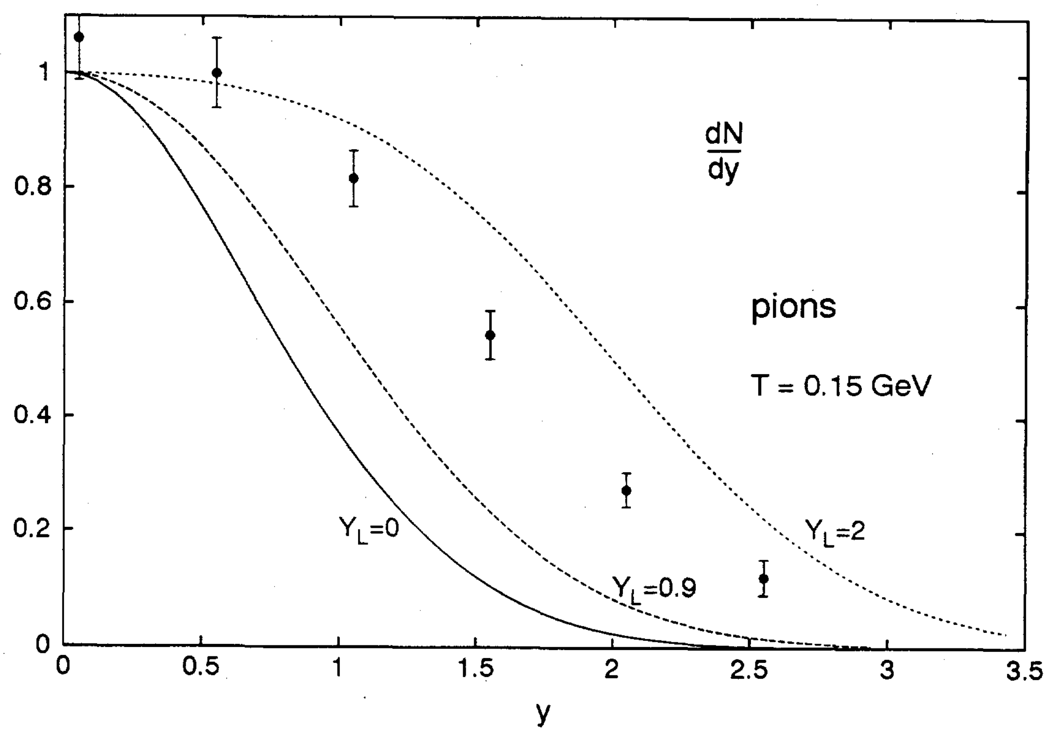

Figure 8: Predicted pion rapidity distribution in $Pb-Pb$ collisions, with preliminary data [13]

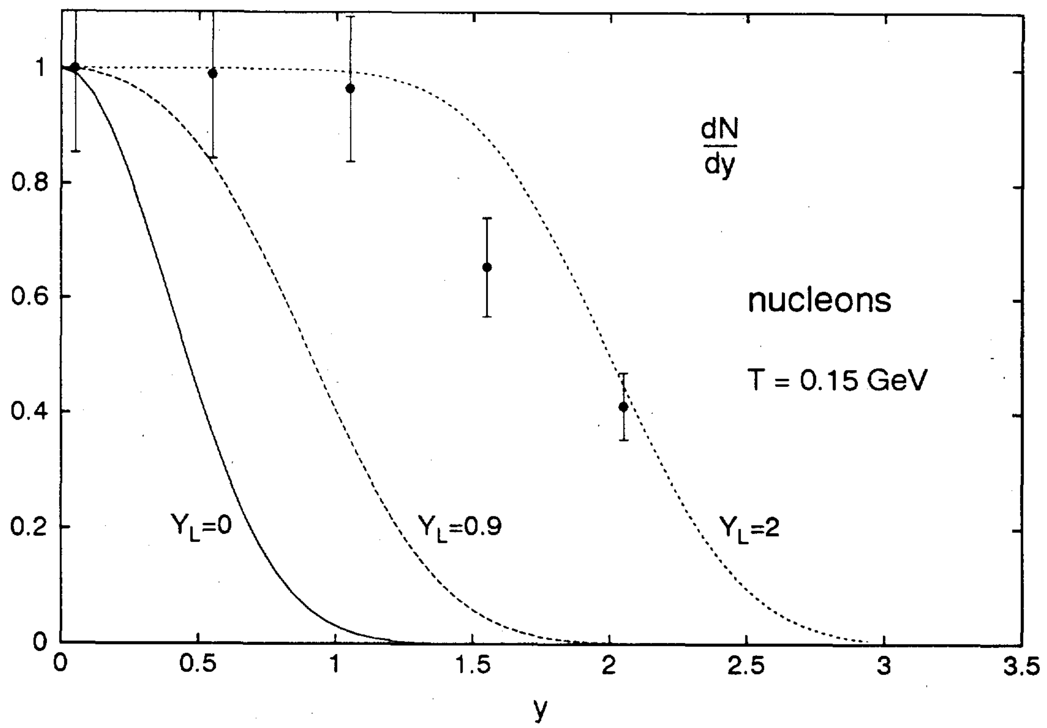

Figure 9: Predicted nucleon rapidity distribution in $Pb - Pb$ collisions, with preliminary data [13]

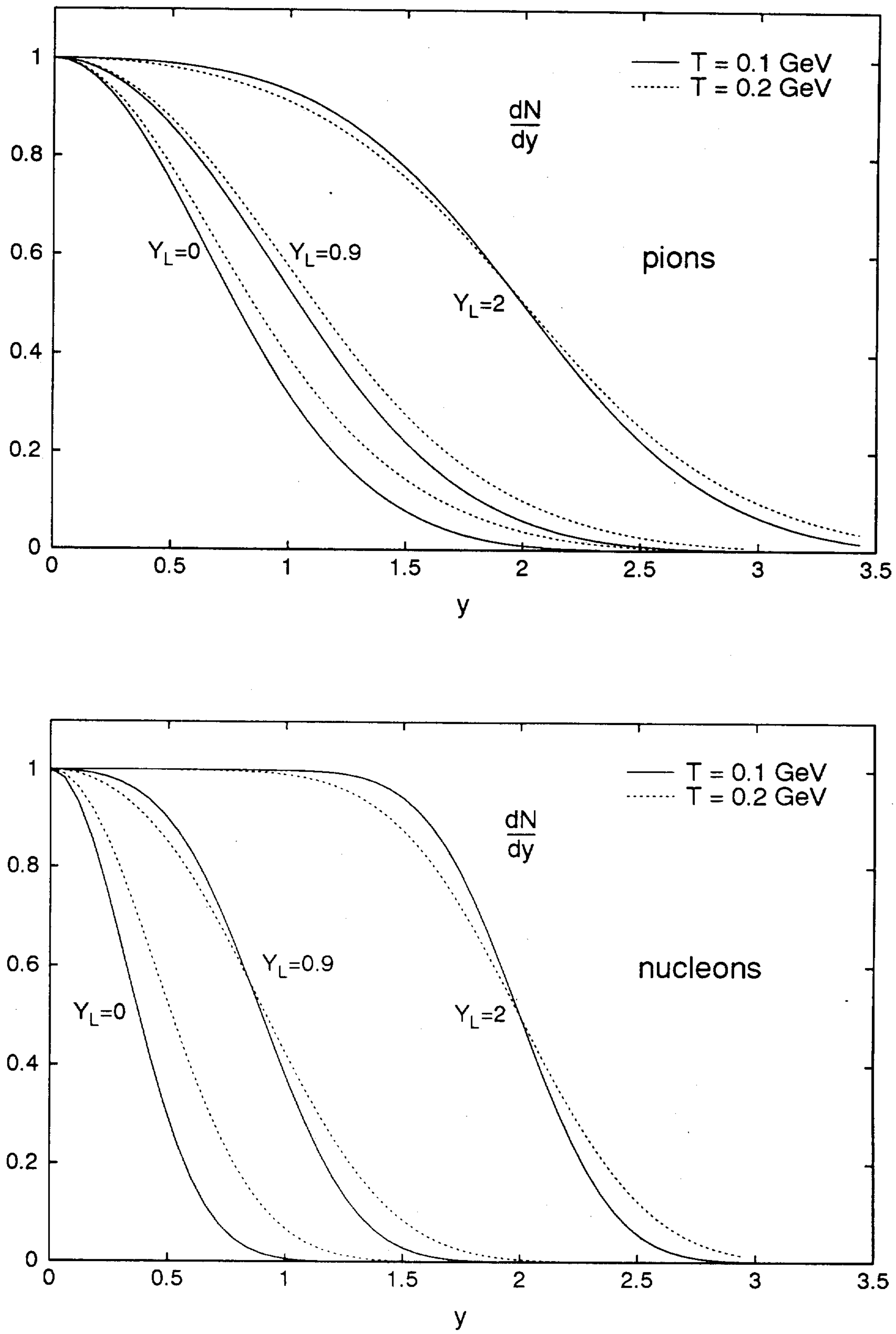

Figure 10: Temperature dependence of pion and nucleon rapidity distributions

In closing, we comment briefly on the nature of the fireballs, although a more detailed discussion of this question is beyond the scope of this paper. It has been known for many years [1] that the emission of secondaries in proton-proton collisions leads to spectra which can be interpreted as arising from a boost-invariant superposition of thermally decaying fireballs of hadronic size. These fireballs are not, however, in chemical equilibrium: they produce e.g. fewer kaons than a thermal source of $T \simeq 0.15$ GeV would lead to. The fireballs in $p - A$ collisions are formed by multiple scattering of the incident proton in the nuclear target; each fireball contains secondaries from a number of different "wounded" target nucleons. The superposition of the remnants from different wounded nucleons leads to fireballs which are bigger and have some transverse motion, due to the rotation of the collision axis in successive collisions. Multiple scattering of products from different wounded nucleons can moreover drive the system closer to chemical equilibrium as well. In nucleus-nucleus collisions, finally, the fireballs are formed by a superposition of the secondaries from still more wounded nucleons, increasing both fireball size and transverse motion further, as well as the possibility of chemical equilibration. If the production ratios of the more abundant species are correctly described by an ideal resonance gas at fixed temperature and chemical potential [14], such considerations would seem to indicate that nuclear collisions at large A indeed produce hadronic systems in thermal and chemical equilibrium.

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