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Non-Abelian Momentum-Winding Exchange

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Abstract

A non-Abelian analogue of the Abelian T-duality momentum-winding exchange is described. The non-Abelian T-duality relates σ-models living on the cosets of a Drinfeld double with respect to its isotropic subgroups. The role of the Abelian momentum-winding lattice is in general played by the fundamental group of the Drinfeld double.

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1. The Poisson-Lie (PL) T-duality [1] is a generalization of the traditional non-Abelian T-duality [2]–[5] and it proved to enjoy [1], [7]–[11] most of the structural features of the traditional Abelian T-duality [12]–[13].

The purpose of this note is to settle the global issues of the PL T-duality for closed strings. From the space-time point of view, we shall identify the targets of the mutually dual σ-models with the cosets D/G and $D/\tilde{G}$, where D denotes the Drinfeld double and G and $\tilde{G}$ two its isotropic subgroups. In the special case when the decomposition $D = G\tilde{G} = \tilde{G}G$ holds globally the corresponding cosets turn out to be the group manifolds $\tilde{G}$ and G respectively [1]. Then we shall describe the momentum and the winding states from the point of view of both targets D/G and $D/\tilde{G}$ and show how the PL T-duality interchange them.

2. For the description of the Poisson-Lie T-duality we need the crucial concept of the Drinfeld double, which is simply a Lie group D such that its Lie algebra $\mathcal{D}$ (viewed as a vector space) can be decomposed as the direct sum of two subalgebras $\mathcal{G}$ and $\tilde{\mathcal{G}}$ maximally isotropic with respect to a non-degenerate invariant bilinear form on $\mathcal{D}$ [14]. It is often convenient to identify the dual linear space to $\mathcal{G}$ ($\tilde{\mathcal{G}}$) with $\tilde{\mathcal{G}}(\mathcal{G})$ via this bilinear form.

Consider the right coset LD/D where LD denotes the loop group of the Drinfeld double. There is a natural symplectic two-form Ω on LD/D [10] given as the exterior derivative of a polarization one-form α. The latter is most naturally defined in terms of its integral along an arbitrary curve γ in the phase space, parametrized by a parameter τ. From the point of view of the Drinfeld double, this curve is a surface $l(\tau,\sigma) \in D$ with the topology of a cylinder, embedded in the double. We define

$$\int_\gamma \alpha = \frac{1}{8\pi}\int \langle \partial_\sigma l\ l^{-1}, \partial_\tau l\ l^{-1}\rangle + \frac{1}{48\pi}\int d^{-1}\langle dl\ l^{-1}, [dl\ l^{-1}, dl\ l^{-1}]\rangle. \qquad (1)$$

Here $\langle .,. \rangle$ denotes the non-degenerate invariant bilinear form on the Lie algebra $\mathcal{D}$ of the double; in the second term on the r.h.s., we recognize the two-form potential of the WZW three-form on the double. Note that this definition of α is ambiguous since the choice of the inverse exterior derivative d^{-1} is too. However, this ambiguity disappears when the exterior derivative of the one-form α is taken. In other words, the symplectic form Ω is well defined.

In this note, we study a dynamical system on the phase space $P \equiv LD/D$

given by the action[1]

$$S[l(\tau,\sigma)] = \int \alpha - \int H \; d\tau$$

$$= \frac{1}{8\pi}\int d\sigma d\tau \Big\{ \langle \partial_\sigma l\, l^{-1}, \partial_\tau l\, l^{-1}\rangle + \frac{1}{6} d^{-1}\langle dl\, l^{-1}, [dl\, l^{-1}, dl\, l^{-1}]\rangle - \langle \partial_\sigma l l^{-1}, R\partial_\sigma l l^{-1}\rangle \Big\}. \tag{2}$$

Here R is a linear idempotent selfadjoint map from the Lie Algebra $\mathcal{D}$ of the double into itself. It has two equally degenerated eigenvalues $+1$ and -1 and we choose in $\mathcal{D}$ some orthonormal basis R_+^a and R_-^a of the corresponding eigenvectors[2]:

$$\langle R_\pm^a, R_\pm^b\rangle = \pm\delta^{ab}, \qquad \langle R_+^a, R_-^b\rangle = 0. \tag{3}$$

Note that

$$R = |R_+^a\rangle\langle R_+^a| + |R_-^a\rangle\langle R_-^a|, \tag{4}$$

$$Identity = |R_+^a\rangle\langle R_+^a| - |R_-^a\rangle\langle R_-^a|. \tag{5}$$

We may also remark that the first two terms of the first-order action S give together the standard WZNW action on the double if we interpret τ and σ as the 'light-cone' variables. This means that we can conveniently use the Polyakov-Wiegmann formula [15].

Let G be an n-dimensional subgroup of $2n$-dimensional Drinfeld double D such that the Lie algebra $\mathcal{G}$ od G is isotropic (i.e. $\langle\mathcal{G},\mathcal{G}\rangle = 0$). Consider then the right coset D/G and parametrize it by the elements f of D [3]. With this parametrization of D/G we may parametrize the surface $l(\tau,\sigma)$ in the double as follows

$$l(\tau,\sigma) = f(\tau,\sigma)g(\tau,\sigma), \quad g \in G. \tag{6}$$

The action S then becomes

$$S(f, \Lambda \equiv \partial_\sigma g g^{-1}) = \frac{1}{2}I(f) - \frac{1}{2\pi}\int d\xi^+ d\xi^- \Big\{ \langle \Lambda - \frac{1}{2} f^{-1}\partial_- f, \Lambda - \frac{1}{2} f^{-1}\partial_- f\rangle$$

$$+ \langle f\Lambda f^{-1} + \partial_\sigma f f^{-1}, R_-^a\rangle\langle R_-^a, f\Lambda f^{-1} + \partial_\sigma f f^{-1}\rangle \Big\}, \tag{7}$$

[1] This action S has also a little gauge invariance corresponding to the right multiplication of $l(\sigma,\tau)$ by an arbitrary function $l(\tau) \in D$. This small gauge symmetry corresponds to the factorization LD/D.

[2] The condition of the positive or negative definiteness of the form $\langle .,.\rangle$ on the subspaces $Span(R_\pm^a)$ can be easily released; however, the form should be still non-degenerate there!

[3] If there exists no global section of this fibration, we can choose several local sections covering the whole base space D/G.

Here

$$I(f) \equiv \frac{1}{4\pi}\int d\xi^+ d\xi^- \langle \partial_+ f\, f^{-1}, \partial_- f\, f^{-1}\rangle + \frac{1}{24\pi}\int d^{-1}\langle df\, f^{-1}, [df\, f^{-1}, df\, f^{-1}]\rangle \quad (8)$$

is the standard WZNW action in the proper light-cone variables, defined by

$$\xi^{\pm} \equiv \frac{1}{2}(\tau \pm \sigma). \quad (9)$$

It is now easy to solve Λ in terms of f:

$$\Lambda_b T^b = \frac{1}{2}\langle \partial_- f\, f^{-1}, R_+^a\rangle (M_+^{-1})_{ab} T^b - \frac{1}{2}\langle \partial_+ f\, f^{-1}, R_-^a\rangle (M_-^{-1})_{ab} T^b, \quad (10)$$

where T^b is some basis of $\mathcal{G}$ and

$$M_{\pm}^{ab} \equiv \langle f T^a f^{-1}, R_{\pm}^b\rangle. \quad (11)$$

Inserting this expression back into (7), we obtain the following second-order action of a σ-model living on the coset D/G

$$S = \frac{1}{2}I(f) - \frac{1}{4\pi}\int d\xi^+ d\xi^- \langle \partial_+ f\, f^{-1}, R_-^a\rangle (M_-^{-1})_{ab}\langle f^{-1}\partial_- f, T^b\rangle. \quad (12)$$

The action of the dual σ-model on the coset $D/\tilde{G}$ has the same form; just the generators T^a of $\mathcal{G}$ are replaced by the generators $\tilde{T}_a$ of $\tilde{\mathcal{G}}$ and f will parametrize $D/\tilde{G}$ instead of D/G.

In the special case, when the decomposition $D = G\tilde{G} = \tilde{G}G$ holds globally (typically $SL(n, C)$ doubles), the corresponding cosets D/G and $D/\tilde{G}$ turn out to be the group manifolds $\tilde{G}$ and G respectively [1] and the action (12) gives the standard pair of the mutually dual lagrangians [1]

$$\tilde{L} = \frac{1}{4\pi}(\mathcal{R}^{-1} + \tilde{\Pi}(\tilde{h}))^{-1}(\partial_+ \tilde{h}\tilde{h}^{-1}, \partial_- \tilde{h}\tilde{h}^{-1}) \quad (13)$$

and

$$L = \frac{1}{4\pi}(\mathcal{R} + \Pi(h))^{-1}(\partial_+ h h^{-1}, \partial_- h h^{-1}). \quad (14)$$

Here $\tilde{h}$ and h respectively parametrize the $\tilde{G}$ and G group manifolds, $\tilde{\Pi}(\tilde{h})$ and $\Pi(h)$ are the bivector fields on the group manifold $\tilde{G}$ and G which respectively give the standard Poisson-Lie brackets on $\tilde{G}$ and G [14, 1, 10] and $\mathcal{R}$

is a bilinear form on $\tilde{\mathcal{G}}$ whose graph[4] $Span\{t+\mathcal{R}(t,.), t\in\tilde{\mathcal{G}}\}$ in $\mathcal{D}$ coincides with the $+1$-eigenspace of R.

It may seem that we have proved the canonical equivalence of the σ-models (12) on the cosets D/G and $D/\tilde{G}$. It is indeed true modulo one extremely important detail: the quantity Λ given by (10) has to fulfil a unit monodromy constraint

$$P\exp\int_\gamma \Lambda = e \tag{15}$$

where e is the unit element of the group G, P stands for the path ordered exponential and γ is a closed path around the string world-sheet with a constant τ (the completely analoguos statement is true also in the dual case $D/\tilde{G}$). This constraint follows from the obvious periodicity in σ of the field $g(\tau,\sigma)$. Thus we have established the duality between the classical σ-models with the additional non-local constraints imposed on their dynamics. What is the meaning of these constraints?

First of all we realize, that the equations of motions following from the action (12) have the zero-curvature form (this property was referred to as the 'Poisson-Lie symmetry" in [1]):

$$d\lambda - \lambda^2 = 0. \tag{16}$$

Here

$$\lambda = \lambda_+ d\xi^+ + \lambda_- d\xi^- \tag{17}$$

and

$$\lambda_\pm = -\langle \partial_\pm f\ f^{-1}, R^a_\mp\rangle (M^{-1}_\mp)_{ab} T^b. \tag{18}$$

Again, the completely analogous representation holds in the dual case. It follows that the conjugacy class of the G-monodromy

$$P\exp\int_\gamma \lambda \tag{19}$$

does not depend on the path γ and it is therefore conserved in time τ. In particular, if the momonodromy is the unit element of G, it is exactly conserved. Needless to say, the constraint (10),(15) is just the unit monodromy constraint of λ for the particular path of constant τ.

[4]Note that $\mathcal{R}(t,.)$ is an element of $\mathcal{G}$.

It may seem somewhat peculiar that we have established the classical duality of two local σ-models only when certain non-local constraints are imposed on the dynamics. What it means classically that we do not consider all possible motions of string which are allowed by the geometry of the space-time? Would not it be too difficult to quantize such a constrained theory? We believe that the answer to this question is surprisingly simple: In many relevant cases it should be enough just to quantize the unconstrained theory and the quantization itself would take care for imposing the monodromy constraint! In order to clarify this somewhat vague statement consider the well-known example of the standard Abelian $\rho \to 1/\rho$ duality.

3. Consider the Abelian Drinfeld double D_a which is just the group $U(1) \times U(1) \equiv G \times \tilde{G}$. The group manifold is topologically the ordinary torus and we choose its explicit parametrization as

$$l = e^{iaT} e^{i\tilde{a}\tilde{T}}, \tag{20}$$

where T and $\tilde{T}$ are the algebra generators of $\mathcal{G}$ and $\tilde{\mathcal{G}}$ respectively and $a, \tilde{a} \in [0, 2\pi)$. The bilinear form in $\mathcal{D}$ we define as

$$\langle T, T \rangle = \langle \tilde{T}, \tilde{T} \rangle = 0, \quad \langle T, \tilde{T} \rangle = 1. \tag{21}$$

We choose the normalized vectors $R_\pm \in \mathcal{D}$ as

$$R_\pm = \sqrt{\frac{\rho}{2}} T \pm \frac{1}{\sqrt{2\rho}} \tilde{T}, \tag{22}$$

the parametrization of $D/\tilde{G}$ clearly to be $f = e^{iaT}, a \in [0, 2\pi)$ (and analogously for D/G) and work out directly the mutually dual models (12):

$$S = -\frac{1}{4\pi\rho} \int d\xi^+ d\xi^- \partial_+ a \partial_- a, \tag{23}$$

$$\tilde{S} = -\frac{\rho}{4\pi} \int d\xi^+ d\xi^- \partial_+ \tilde{a} \partial_- \tilde{a}. \tag{24}$$

The corresponding connections $\lambda_\pm$ and $\tilde{\lambda}_\pm$ read

$$\tilde{\lambda}_\pm = \pm \frac{i}{\rho} \partial_\pm a \tilde{T}, \tag{25}$$

$$\lambda_\pm = \pm i\rho \partial_\pm \tilde{a} T. \tag{26}$$

It is now easy to quantize the free field theory (23) (the case (24) differs just by the change $\rho \to 1/\rho$). Consider the mode expansion of the field a

$$a = a_0 + p_L \xi^- + p_R \xi^+ + osc_L + osc_R; \tag{27}$$

the exact form of the oscillator terms is irrelevant for our purposes. The single-valuedness of the field $a(\sigma, \tau)$ requires that the winding number $\frac{1}{2}(p_L - p_R)$ be integer; however, classically there is no constraint on the momentum $\frac{1}{2}(p_L + p_R)$. In the quantum case, however, the spectrum of the momentum read:

$$\frac{1}{2}(p_L + p_R) = m\rho, \quad m \in \mathbf{Z} \tag{28}$$

But then from (25) and (27) we get for the monodromy

$$P \exp \oint \tilde{\lambda} = \tilde{e},$$

where $\tilde{e}$ is the unit element of the group $\tilde{G}$ (or of G in the dual picture). We witness that there is no necessity of imposing the constraint of the unit monodromy at the quantum level, because the process of the quantization itself takes care of it.

4. In our previous works [1, 10], we have referred to the monodromy $P \exp \oint \lambda$ as to the non-commutative G-valued momentum of the string. It turned out that the geometry of the targets of the dualizable σ-models allows to write the equations of motions as the zero-curvature condition for the connection λ, which in turn means that the conjugacy class of the monodromy of λ is the conserved quantity - the non-commutative momentum. In the just-described Abelian case, the zero-curvature condition coincides with the $U(1)$-current conservation equation which implies the conservation of the total momentum[5] p and hence of the monodromy $\exp 2\pi i p T$. In the Abelian quantum theory the monodromy is always e but we also observe that this does not mean that there is a single momentum state! In fact, it appears to be more natural to evaluate the monodromy in the universal covering group of G and to understand the non-commutative momentum as an element of this cover. The constraint of the unit G-valued monodromy then means that

[5]Note a terminological difficulty: the 'non-commutative momentum' becomes in the $U(1)$ case just the exponent of the $2\pi i$ times the standard momentum.

only those cover-valued monodromies which get projected to the unit element of G are allowed.

It is easy to interpret the different classical (Abelian) momentum states, which give the same unit G-monodromy. The state with the Abelian momentum n/ρ (cf. model (24)) correspond to a loop which wraps n-times along the homology cycle in the double generated by T (record that the winding states of the model (24) wrap along the cycle $\tilde{T}$). Thus we observe the perfect duality in the classical phase space: from the point of view of the model (24) the momentum and the winding states correspond to the homology cycles T and $\tilde{T}$ respectively, whileas for the dual model (23) the role of the homology cycles gets interchanged.

Consider now the fundamental group $\Gamma(D)$ of any Drinfeld double (in the Abelian case it is just $\mathbf{Z} \oplus \mathbf{Z}$). The phase space of the model (2) or (12) decomposes into disconnected sectors labelled by the elements of $\Gamma(D)$. Upon taking the coset D/G, some of the loops from $\Gamma(D)$ will remain incontractible in D/G and we naturally interpret them as the winding states in D/G target. On the other hand, those loops from $\Gamma(D)$ which become contractible after projection to the coset D/G we interpret as the momentum states. Clearly, this interpretation is not duality invariant. For instance, if we consider the dual coset $D/\tilde{G}$ in the Abelian case then the role of the momentum and the winding states gets precisely interchanged. This is the famous phenomenon of the Abelian momentum-winding exchange [12].

In the non-Abelian case the situation is very similar though few other scenarios may appear than just the complete interchange of the momentum and winding states. In what follows we shall consider only the case when all involved groups D,G and $\tilde{G}$ are connected but not necessarily simply connected. The momentum-winding interpretation is then governed by the following long exact homotopy sequence [16]:

$$\pi_2(D) = 0 \to \pi_2(D/G) \to \pi_1(G) \to \pi_1(D) \to \pi_1(D/G) \to 0 = \pi_0(G). \quad (29)$$

Note that π_2 of any Lie group vanishes. We can rewrite this sequence as follows

$$0 \to \pi_1(G)/\pi_2(D/G) \to \pi_1(D) \to \pi_1(D/G) \to 0 \quad (30)$$

and observe that the (Abelian) group $\pi_1(D)$ is an extension of the (Abelian) group $\pi_1(D/G)$ by the (Abelian) group $\pi_1(G)/\pi_2(D/G)$. Note the role of $\pi_2(D/G)$: Its possible non-vanishing means that some non-contractible cycles in G can be contracted in D upon embedding of G in D. Thus we can conclude that the winding modes of the σ-model on D/G are the elements of $\pi_1(D/G)$ and the momentum modes are the elements of $\pi_1(G)/\pi_2(D/G)$. In the dual case the winding (momentum) modes are the elements of $\pi_1(D/\tilde{G})$ $(\pi_1(\tilde{G})/\pi_2(D/\tilde{G}))$. Thus we observe that the partition of $\pi_1(D)$ into the momentum and winding piece depends on the target. There may be an element of $\pi_1(D)$ which is a momentum mode from the point of view of D/G but a winding mode for $D/\tilde{G}$.

In the case of the traditional non-Abelian duality [2, 5] there was a little room to discover the momentum-winding exchange, because the Drinfeld double is then the cotangent bundle of some compact group G and the role of $\tilde{G}$ is played by its coalgebra $\mathcal{G}^*$ viewed as the commutative group. The point is that $\pi_1(D)$ is usually quite poor in this case. For instance, for a simply connected G (and hence D) there is no trace of the non-trivial momentum or winding states whatsoever.

5. Example $D = SL(2,R) \times SL(2,R)$. Consider the Lie algebra $sl(2,R)$ defined by

$$[H, E_\pm] = \pm 2E_\pm, \qquad [E_+, E_-] = H, \tag{31}$$

and equipped with the standard Killing-Cartan non-degenerate symmetric invariant bilinear form

$$\langle E_+, E_- \rangle = 1, \qquad \langle H, H \rangle = 2. \tag{32}$$

The direct sum of the two copies of $sl(2,R)$

$$\mathcal{D} = sl(2,R) \oplus sl(2,R) \tag{33}$$

with the bilinear form (also denoted by $\langle .,. \rangle$)

$$\langle (x_1, x_2), (y_1, y_2) \rangle = \langle x_1, y_1 \rangle - \langle x_2, y_2 \rangle \tag{34}$$

is the Lie algebra of the Drinfeld double D. The notation $(x_1, x_2) \in \mathcal{D}$ obviously means that x_1 (x_2) is from the first (second) copy of $sl(2,R)$ in (33). The decomposition of the double into the pair of the maximally isotropic subalgebras is given by

$$\mathcal{D} = sl(2,R)_{diag} + b_2 \tag{35}$$

where $sl(2,R)_{diag}$ is generated by

$$\tilde{T}_0 = \frac{1}{2}(H, H), \quad \tilde{T}_+ = (E_+, E_+), \quad \tilde{T}_- = (E_-, E_-) \tag{36}$$

and b_2 (which is the Lie algebra of the Borel group B_2 consisting of upper-triangular 2×2 complex matrices with real positive diagonal elements and unit determinant) by

$$T^0 = \frac{1}{2}(H, -H), \quad T^+ = (0, -E_-), \quad T^- = (E_+, 0). \tag{37}$$

The homotopy groups of D, $SL(2,R)$, B_2 and of the relevant cosets D/B_2, $D/SL(2,R)_{diag} = SL(2,R)$ read

$$\pi_1(D) = \pi_1(D/B_2) = \mathbf{Z} \oplus \mathbf{Z}, \quad \pi_1(SL(2,R)) = \pi_1(D/SL(2,R)_{diag}) = \mathbf{Z},$$

$$\pi_1(B_2) = \pi_2(D/B_2) = \pi_2(D/SL(2,R)_{diag}) = 0. \tag{38}$$

We conclude that the σ-model (12) on D/B_2 has two types of winding states and on $D/SL(2,R)_{diag}$ [6] just one type. Under duality, the winding states of D/B_2 of one type correspond to the non-commutative momentum states of $D/SL(2,R)_{diag}$ and the winding states of the other type remain the winding states on $D/SL(2,R)_{diag}$.

6. Concluding remarks:

i) In the previous discussion we have been always talking about the Drinfeld double. However, the careful reader might have remarked that the described construction of the dual σ-models can be repeated in a more general setting. Essentially, we just require that D is a $2n$-dimensional Lie group with a non-degenerate symmetric ad-invariant bilinear form on its algebra which in turn admits at least two different n-dimensional isotropic subalgebras. Clearly, the Drinfeld double is always an example of such a situation, however, there may be more examples of this type. For instance, consider a compact connected simple Lie group G and put $D = G \times G$ with the bilinear form on $\mathcal{D} = \mathcal{G} \oplus \mathcal{G}$ given just by the difference of the standard Killing-Cartan forms on the first and second copy of $\mathcal{G}$ respectively (like in the $SL(2,R) \times SL(2,R)$-case considered previously). We may consider two different embeddings of the group G in D. First one is the standard diagonal embedding and in the second, G is identically embedded into the first copy of G and up to twist by an outer automorphism into the second copy of G. It is obvious that both embeddings are isotropic at the level of Lie algebra $\mathcal{D} = \mathcal{G} \oplus \mathcal{G}$. This

[6] For a particular choice of $R_\pm$ the model on $D/Sl(2,R)_{diag}$ is just the standard WZNW model on $SL(2,R)$ [8].

construction will most probably be connected with the Kiritsis-Obers duality [6].
ii) We hope to supply a detailed quantum picture of the presented structures in a near future. The most obvious open problem is the quantum status of the unit monodromy constraint. After previous experience with the naturalness of the structure of PL T-duality, we expect this issue will be settled and the PL T-duality will find interesting applications in both quantum field and string theories.

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